MTH100 Definations

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VUWAYS Team

Welcome To All New Members

important subjective notes

1) **Write the following expression in the form of** $a + ib$.

$\frac{3+i}{2-i}$, $\frac{4+10i}{-9+7i}$

Answer:

$$\frac{3+i}{2-i} = \frac{3+i}{2-i} * \frac{2+i}{2+i} = \frac{6+3i+2i+i^2}{4+2i-2i-i^2} = \frac{6+5i+(-1)}{4-(-1)} = \frac{5+5i}{5}$$

$$\frac{4+10i}{-9+7i} * \frac{-9-7i}{-9-7i} = \frac{-36-28i-90i-70i^2}{81+63i-63i-49i^2} = \frac{-36-118i+70}{81+49} = \frac{34-118i}{130}$$

2) **Write in set builder notation**
   A= Set of first 100 odd numbers
   A= Set of Natural numbers starting from 2 to 40.
   A= Set of all even numbers greater than 50.

Answer:

A = {x| x is set of first 100 odd numbers}

A = {x|2≤x≤40 and x is set of natural numbers}

A = {x|x>50 and x is set of even numbers}

3) **List the elements of the set (roster method)**

$A = \{x \mid x \in E^+ \wedge x \le 10\}$

$A = \{x \mid x^2 = 9,\ x\ is\ even\}$

$A = \{x \mid x \in N\ \wedge 2 \le x \le 10\}$

A = {x | **x is a person taller than 15** feet}
B = {x | x^2= 16, x is odd}
Set of first five multiples of 3.

Answer:

A = {0,2,4,6,8,10}

A = ∅

A = {2,3,4,5,6,7,8,9,10}

A= ∅ A=∅

A = {3,6,9,12,15}

4) Let A be the set of all even numbers. Let B be the set of all Number less than 100. Is A a subset of B? or Is B a subset of A?

Answer:

A⊄B

B⊄A

List all elements in these sets and verify the result.

5) Let A={2,4,6,8,10,12,14,16,18,20}
B={1,3,5,7,9,11,13,15,17,19,21}
Find A ∩ B and A Ụ B.

Answer:

A ∩ B = { ∅ } and A Ụ B = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21}

6) Let A be all multiples of 6 and B be all multiples of 3. Is A a subset of B? And is B a subset of A?

Answer:

A= {6,12,18,24,30,36, ... }
B={3,6,9,12,15,18,21,24,..... }
A is subset of B but **B is not subset of A**.

7) The complex numbers Z **is givenby** $Z = \frac{i-1}{i}$ **Find modulus of** Z.

Answer:

$$Z = \frac{i-1}{i}$$

$$\frac{i-1}{i} * \frac{i}{i} = \frac{i^2 - i}{i^2} = \frac{-(1-i)}{-1} = \frac{1+i}{1}$$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

8) Identify rational numbers.

(a) 5.3333333333……. Rational

(b) 4.131 Rational

(c) **8.69854763214598456324896631**……… Irrational

9) Evaluate

$(2+i)(3+i)$

$(4+5i)(-2-2i)$

$(-1-i)(-2-2i)$

i^3

Solution:

$$(2+i)(3+i) = 6 + 2i + 3i + i^2 = 6 + 5i(-1) = 5 + 5i$$

$$(4+5i)(-2-2i) = -8 - 8i - 10i - 10i^2 = -8 - 18i - 10(-1) = -8 - 18i + 10 = 2 - 18i$$

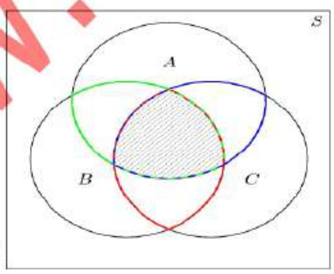
$$(-1-i)(-2-2i) = 2 + 2i + 2i + 2i^2 = 2 + 4i + 2(-1) = 4i$$

$$i^3 = i * i * i = (i)^2 (i) = (-1)(\sqrt{-1}) = -i$$

10) The Venn diagram given below shows the sets A, B, and C. Shade the following sets

i. $A \cup (B \cap C)$

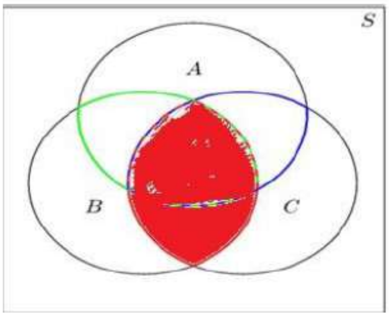
ii. $A \cup (B \cap C)^c$



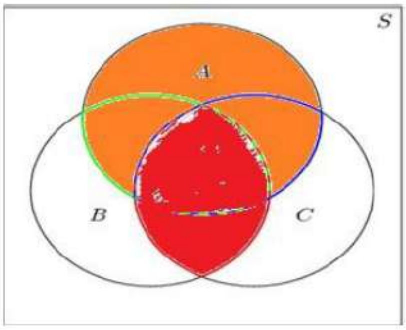
iii. $A \cap (B \cup C)$

Solution:

i. $A \cup (B \cap C)$



B intersection C is shown in red. If we take the union of this red portion with the A then it will be like



So the total colored region (red + orange) is the required region.

Solve all other on the same lines.

11) What is the cardinality of these sets?

A = Set of even integers ∞

B = Set of prime numbers less than 30 ∞

C={11,12,13,14,15,16,17,18}

12) State the Associative Law for union and intersection.

A∪ (B∪C) = (A∪B) ∪C

A∩ (B∩C) = (A∩B) ∩C

13) $z=-5+8i$ **then find** $z-z^*$

Solution:

$z = -5 + 8i$

$z^* = -5 - 8i$

$z - z^* = -5 + 8i - (-5 - 8i)$

$= -5 + 8i + 5 + 8i = 16i$

14) If $z = 3 + 6i$ then find $z.z^*$

Solution:

$z = 3 + 6i$

$z^* = 3 - 6i$

$z.z^* = (3 + 6i)(3 - 6i)$

$= 9 - 18i + 18i + 36 = 45$

15) Evaluate

$$\left|\frac{4 + 7i}{-5 + 6i}\right|$$

Solution:

$$\frac{4 + 7i}{-5 + 6i} * \frac{-5 - 6i}{-5 - 6i} = \frac{(4 + 7i)(-5 - 6\text{ i})}{(-5 + 6i)(-5 - 6i)}$$

$$= \frac{-20 - 24i - 35i - 42i^2}{(-5)^2 - (6i)^2}$$

$$= \frac{22 - 59i}{25 + 36} = \frac{22 - 59i}{61} = \frac{22}{61} - \frac{59}{61}i$$

$$\left|\frac{4 + 7i}{-5 + 6i}\right| = \sqrt{\left(\frac{22}{61}\right)^2 + \left(\frac{59}{61}\right)^2} = \sqrt{\frac{484 + 3481}{3721}} = \sqrt{\frac{3965}{3721}}$$

16) Write the domain and range of $f(x) = 6x + 9$

Solution:

Domain of f(x)= All real no's

Range of f(x)= All real no's

17) Write the domain and range of $g(x) = \dfrac{5}{x + 6}$

Solution:

Domain of g(x)= $R - \{-6\}$

Range of f(x)= All real no's except zero

18) Prove that $f(\text{x}) = x^4 + 5x^2 + 3$ **is an even function**

Solution:

$$f(\text{x}) = x^4 + 5x^2 + 3$$
$$f(-x) = (-x)^4 + 5(-x)^2 + 3$$
$$f(-x) = x^4 + 5x^2 + 3 = f(x)$$

19) Prove that $f(\text{x}) = x^3 - 8x$ **is an odd function**

Solution:

$$f(\mathrm{x}) = x^3 - 8x$$
$$f(-x) = (-x)^3 - 8(-x)$$
$$f(-x) = -x^3 + 8x = -(x^3 - 8x) = -f(x)$$

20) Express the quadratic polynomial $x^2 + 4x + 6$ in complete square form.

Solution:

$x^2 + 4x + 6 = x^2 + 4x + 4 - 4 + 6 = (x+2)^2 + 2$

21) If the functions *f* is defined by

$$f(x) = \frac{1}{x},\ 0 < x \le 5,$$

then find the range of *f.*

Solution:

Since $x \le 5$, $1/x \ge 1/5$.. i.e., $f(x) \ge 1/5$.

22) If the functions *f* and *g* are defined by

$$f(x) = \frac{3}{x}, \qquad 0 < x \le 3,$$
$$g(x) = 5x - 1,\ x \in R,$$

then Calculate *gf(2).*

Solution:

gf(x) = g(f(x)) = g(3/x) = 5(3/x) – 1 = (15/x) – 1.

So, *gf(2) = (15/2) – 1 =13/2*

23) If the functions *g* is defined by $g(x) = 8x - 5,\ \ x \in R,$ then find an expression in terms of *x* for $g^{-1}(x)$

Solution:

To find $g^{-1}(x)$, we write *y = 8x – 5*

→ *y + 5 = 8x* → *x = (y + 5)/8* Therefore, $g^{-1}(x) = (x + 5)/8$

24) Let $f(x) = 6x + 1$ and $g(x) = -9x + 5$. Find the product of f & g *i.e* $(f.g)(x)$.

Solution:

$(f.g)(x) = f(x).g(x) = (6x+1).(-9x+5)$

$= -54x^2 + 21x + 5 \ (Ans.)$

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25) Let f and g be the functions defined by $f(x) = 3x+3$ & $g(x) = 3x+5$ then find

$f \square g(x)$ and $g \square f(x)$.

Solution:

$f \square g(x) = f(g(x))$

$= f(3x+5)$
$= 3(3x+5)+3$
$= 9x+18$
$g \square f(x) = g(f(x))$
$= g(3x+3)$
$= 3(3x+3)+5$
$= 9x+14$

26) Determine whether the equation has one, two or zero real solution.

$5x^2 + 4x - 6 = 0$

solution :
$a = 5, b = 4, c = -6$
$b^2 - 4ac = 16 + 120 = 136 > 0$

So there will be two distinct real solutions.

27) Determine whether the equation has one, two or zero real solution.

$x^2 + 3x + 6 = 0$

solution :
$a = 1, b = 3, c = 6$
$b^2 - 4ac = 9 - 24 = -15 < 0$ So there will be no real solutions.

28) The equation $tx^2 + 5x - 6 = 0$ has two real **solutions. What can be deduced about the value of** t?

Solution:

$$tx^2 + 5x - 6 = 0$$
$$a = t, b = 5, c = -6$$

For two real solutions the discriminant should be >0 i.e

$$b^2 - 4ac > 0$$
$$25 + 24t > 0$$
$$24t > -25$$
$$t > \frac{-25}{24}$$

29) The equation $kx^2 - 8x + 9 = 0$ has exactly one solution. What can be deduced **about the value of k?**

Solution:

$$kx^2 - 8x + 9 = 0$$
$$a = k, b = -8, c = 9$$

For exactly one solution the discriminant should be equal to 0 i.e

$$b^2 - 4ac = 0$$
$$64 - 36k = 0$$
$$64 = 36k$$
$$\frac{64}{36} = k$$
$$\frac{32}{18} = k$$

30) Determine whether the function is even, odd, neither even nor odd.

1. $f(x) = x^2 + 3x$
2. $f(x) = \sqrt{x^2 + 9}$
3. $f(x) = x^2 + 4$
4. $f(x) = \dfrac{x}{x^2 - 1}$

5. $f(x) = 6x^5 - x^3$
6. $g(x) = x^4 + x^6$

31) Express the quadratic polynomial in complete square form $2x^2 + 8x + 6$

32) If $f(x) = 4x - 3 \quad \& \; g(x) = x^3 + 3$ Then find $gof(x) \; and \; fog(x)$.

33) Determine whether the following equations have one, two or zero real solution.

1. $7x^2 + 9x - 3 = 0$
2. $4x^2 + 2x + 1 = 0$

34) If the following equation has one solution then find the value of a $ax^2 + 5x + 2 = 0$

35) Given matrices A and B such that $A = \begin{bmatrix} 1 & 2 \\ -3 & -6 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix}$ Find $2A - 3B$

36) Check whether the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 1 \end{bmatrix} \; and \; B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ can be multiplied? Justify your answer.

37) Find the inverse $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$

38) Multiply the matrix: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; and \; B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

39) Find the transpose of matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -2 & 9 \\ -5 & 7 & -3 \end{bmatrix}$$

40) How many 3-permutation are there of {A, B, C,D}?

41) The letters of the word PEN taken all at a time can be written in _____

42) $^{15}c_9 = \ldots$

43) What is the relation between permutations and combinations?

44) There are total 12 players in a cricket team. The captain needs to choose 8 players to play in a match. How many possible teams can be chosen?

45) Given the sequence, 150, 300, 450, 600, …, find the 15th term.
Answer: 2250

46) What is the general term of the sequence 1,8,27,64,125….
Answer: an=n3

47) The 3rd and the 4th terms of a geometric progression are 18 and 6 respectively. Find the sum to infinity of the progression.
Answer: 243

48) If first term , common difference , then what is the value of 10th term in an arithmetic sequence?
Answer: 22

49) Suppose , is an arithmetic sequence . Write its corresponding arithmetic series.
Hint: Just sum (add) all the terms to find the corresponding series.

50) If then find the first four terms of the sequence .
Answer: 9,16,25,36

51) Determine whether the sequence 9, 11, 13, 15, 17, ... is arithmetic or geometric. Give reason for your answer.
Answer: The sequence is arithmetic.

52) Find the 5th term of the geometric sequence with a1 = 20 and a2 = 40 .
Answer: 320

53) If , then compute the summation:
Answer: 10

54) **There are three juniors and three seniors in the Service Club. The club is to send four representatives to the State Conference. How many different ways are there to select a group of four students to attend the conference?**
Solution: Choose 4 students from the total number of students. Order is not important. There are 6 total students. So number of choices is $^6C_4 = 15$

55) **There are four juniors and three seniors in the Service Club.The club is to send three representatives to the State Conference. If the members of the club decide to send one junior and two seniors, how many different groupings are possible?**

Solution: Choose 1 junior and 2 seniors: $4C_1 . 3C_2 = 12$

56) **Suppose that X = {*a*, *b*,*c*}. Then list all 3-combinations of the set *X*, and hence find the value of *C*(3,3).**
Solution: *X* = {*a*, *b*,*c*}. Then 3-combinations of the 3 elements of the set *X* is: {*a*, *b*,*c*}. Hence *C*(3,3) = 1.

57) **If *W* = {*x*, *y*, *z*, *t*}. Then list all 3-combinations of the set *W*, and hence find the value of *C*(4,3).**

Solution:

W = {*x*, *y*, *z*,*t*}.

Then 3-combinations of the 4 elements of the set *X* are:

{*x*,*y*,*z*}, {*x*, *y*,*t*}, {x,z,t} and {*y*, *z*,*t*}. Hence *C*(4,3) = 4.

58) **Compute: *C*(9, 6).**

Solution:

$$\rightarrow \quad {}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

$$\Rightarrow \quad {}^{9}C_{6} = \frac{9!}{(9-6)!\,6!}$$

$$= \frac{9!}{3!\,6!}$$

$$= \frac{362880}{6.(720)}$$

$$= \frac{362880}{4320}$$

$$= 84$$

59) There are total 14 players in a cricket team. The captain needs to choose 10 players to play in a match. How many possible teams can be chosen?

Solution:

This is a combination problem as the order in which the teams are chosen is not important. The number of combinations is

$$\rightarrow \quad {}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

$$\Rightarrow \quad {}^{14}C_{10} = \frac{14!}{(14-10)!\,10!}$$

$$= \frac{14!}{4!\,10!}$$

$$= \frac{87178291200}{24.2628800}$$

$$= 1001$$

60) In how many ways can a team of 5 candidate can be formed out of a total of 12 candidates such that two particular candidates should be included in each department?

Solution:

Two particular candidates should be included in each team.

i.e., we have to select 5-2 = 3 persons from 12-2 = 10persons Hence, the required number of ways are:

$$\rightarrow \quad {}^{n}C_{r} = \frac{n!}{(n-r)!\, r!}$$

$$\Rightarrow \quad {}^{10}C_{3} = \frac{10!}{(10-3)!\, 3!}$$

$$= \frac{10!}{7!\, 3!} = 120$$

61) Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

62) In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

63) From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there in the committee. In how many ways can it be done?

64) In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

65) In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

66) In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?

67) In how many different ways can the letters of the word 'MATHEMATICS' be arranged such that the vowels must always come together?

68) There are 8 men and 10 women and you need to form a committee of 5 men and 6 women. In how many ways can the committee be formed?

69) How many 3-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

70) In how many different ways can the letters of the word 'LEADING' be arranged such that the vowels should always come together?

71) In how many different ways can the letters of the word 'DETAIL' be arranged such that the vowels must occupy only the odd positions?

72) A bag contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the bag, if at least one black ball is to be included in the draw?

73) In how many different ways can the letters of the word 'JUDGE' be arranged such that the vowels always come together?

74) In how many ways can the letters of the word 'LEADER' be arranged?

75) How many arrangements can be made out of the letters of the word 'ENGINEERING'?

76) How many 3 digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9 which are divisible by 5 and none of the digits is repeated?

77) There are 6 periods in each working day of a school. In how many ways can one organize 5 subjects such that each subject is allowed at least one period?

78) How many 6 digit telephone numbers can be formed if each number starts with 35 and no digit appears more than once?

79) An event manager has ten patterns of chairs and eight patterns of tables. In how many ways can he make a pair of table and chair?

80) 25 buses are running between two places P and Q. In how many ways can a person go from P to Q and return by a different bus?

81) A box contains 4 red, 3 white and 2 blue balls. Three balls are drawn at random. Find out the number of ways of selecting the balls of different colors?

82) A question paper has two parts P and Q, each containing 10 questions. If a student needs to choose 8 from part P and 4 from part Q, in how many ways can he do that?

83) In how many different ways can 5 girls and 5 boys form a circle such that the boys and the girls alternate?

84) Find out the number of ways in which 6 rings of different types can be worn in 3 fingers?

85) In how many ways can 5 man draw water from 5 taps if no tap can be used more than once?

86) There are 6 persons in an office. A group consisting of 3 persons has to be formed. In how many ways can the group be formed?

87) There are 5 yellow, 4 green and 3 black balls in a bag. All the 12 balls are drawn one by one and arranged in a row. Find out the number of different arrangements possible.

88) In how many ways can a team of 5 persons be formed out of a total of 10 persons such that two particular persons should not be included in any team?

89) A company has 11 software engineers and 7 civil engineers. In how many ways can they be seated in a row so that no two of the civil engineers will sit together?

90) How many signals can be made using 6 different coloured flags when any number of them can be hoisted at a time?

91) In how many ways can 5 distinguishable balls be put into 8 distinguishable boxes if no box can contain more than one ball?

92) Find the first two terms in the expansion of (4 – x)6 , in ascending powers of x.

Solution: =4096-6144x+3840x2+…… Check first two terms only which are 4096-6144x

93) Find the value of the constant a for which the coefficient of x in the expansion of (1 + ax)(2 – x)6 is 128.

Solution: (1 + ax)(2 – x)6 = (1 + ax)(64 – 192 x + 240x2).

Collecting the x terms only, we get, -192x +64ax = (64a – 192)x. Since the coefficient of x is given to be 64, we get 64a – 192 = 128 → a = 5

94) Calculate the coefficient of x8y3 and xy10 in the expansion of (x+y)11

Solution: 11C8 = 165

11C1 = 11

Final term important and practice exercises questions

1) What is the domain and range of y = arcsin x and *arccosx?*

Solution:

The domain of y = arcsin x is [-1,1]

Range of y = arcsin x = [-pi/2, pi/2]

Domain of y = arccos x = [-1,1]

Range of y = arccos x = [0, pi]

2) Prove the Pythagorean Identities.

Solution:

$$\tan^2\theta + 1 = \sec^2\theta$$

$$(\frac{\sin\theta}{\cos\theta})^2 + 1 = \frac{\sin^2\theta}{\cos^2\theta} + 1$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

3) $1 + \cot^2\theta = \csc^2\theta$

Solution:

Use $\cot\theta = \frac{\cos\theta}{\sin\theta}$ and solve as above.

4) Expand the sum and difference formula Sin (A+B) and Sin (A-B)

Solution: Given in handouts.

5) Use trigonometric identities to prove that.

$$\frac{\tan y}{\sin y} = \sec y$$

Solution:

$$\frac{\tan y}{\sin y} = \frac{\sin y}{\cos y} \cdot \frac{1}{\sin y} = \frac{1}{\cos y} = \sec y$$

6) Without using the calculator, find $arc\sin\frac{1}{\sqrt{2}}$ in radians.
Solution:

$arc\sin\frac{1}{\sqrt{2}} = \frac{\pi}{4}$ since $\frac{\pi}{4}$ is the angle whose sine is $\frac{1}{\sqrt{2}}$.

7) **Without using the** calculator, find the exact value of Cos 210°.

Solution:

$$\begin{aligned}\cos(210°) &= \cos(180° + 30°)\\ &= \cos 180° \cos 30° - \sin 180° \sin 30°\\ &= (-1)\left(\frac{\sqrt{3}}{2}\right) - (0)\left(\frac{1}{2}\right)\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

8) Given that's $\sin\theta = \frac{\sqrt{3}}{2}$, find all possible values of θ in the range *-180 ≤ ❑ ≤ 180.*

Solution: $\theta = \sin^{-1}(\frac{\sqrt{3}}{2}) = 60$❑

Also by using symmetry property:

→$\sin(180-\theta) = \sin(\theta) \Rightarrow \sin(180-(-60)) = \sin(240)$ to get 240 as a solution.

Note that 240 must be a solution but it does not lies in our range *-180 ≤ ❑ ≤ 180.*

Now use the periodic property: *sin(240) = sin(240 - 360).* So another solution is *-120*degrees.

All the solutions are ± 60 and ± 120 degrees.

9) Verify that the following: $\frac{1-\tan\theta}{1-\cot\theta} = -\tan\theta$.

Solution: