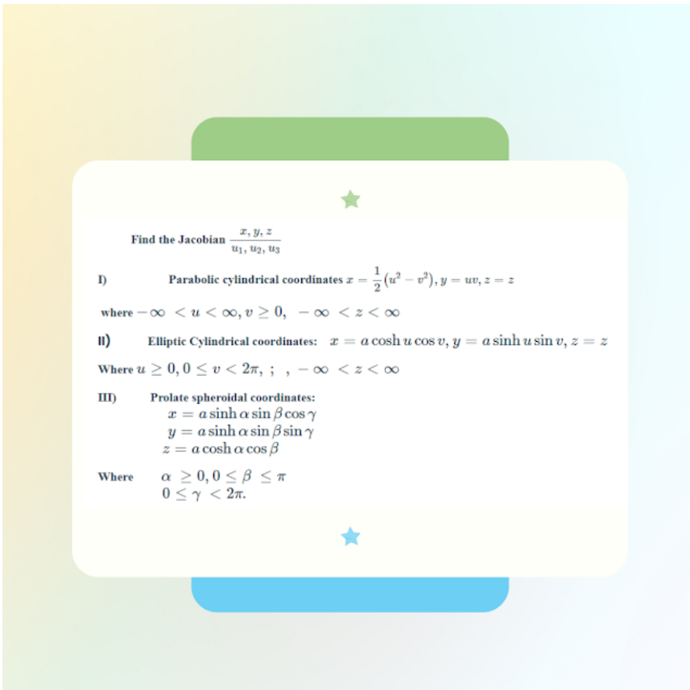
Find the Jacobian

Query Point Tuesday, November 22, 2022



Find the Jacobian $\frac{x, y, z}{u_1, u_2, u_3}$

I) Parabolic cylindrical coordinates $x = \frac{1}{2}(u^2 - v^2), y = uv, z = z$

where $-\infty < u < \infty, v \geq 0, \quad -\infty < z < \infty$

II) Elliptic Cylindrical coordinates: $x = a\cosh u \cos v, y = a \sinh u \sin v, z = z$

Where $u \geq 0, 0 \leq v < 2\pi, \; ; \; , -\infty < z < \infty$

III) Prolate spheroidal coordinates:

$x = a\sinh\alpha \sin\beta \cos\gamma$
$y = a\sinh\alpha \sin\beta \sin\gamma$
$z = a\cosh\alpha \cos\beta$

Where $\alpha \geq 0, 0 \leq \beta \leq \pi$
$0 \leq \gamma < 2\pi.$

Find the Jacobian $\frac{x, y, z}{u_1, u_2, u_3}$

I) **Parabolic cylindrical coordinates** $x = \frac{1}{2}(u^2 - v^2), y = uv, z = z$

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III) **Prolate spheroidal coordinates:**

$$x = a\sinh\alpha \sin\beta \cos\gamma$$
$$y = a\sinh\alpha \sin\beta \sin\gamma$$
$$z = a\cosh\alpha \cos\beta$$

Where $\alpha \geq 0, 0 \leq \beta \leq \pi$
$0 \leq \gamma < 2\pi.$

Solution:

I) Parabolic cylindrical coordinates $x = \frac{1}{2}(u^2 - v^2), y = uv, z = z$

where $-\infty < u < \infty, v \geq 0, \quad -\infty < z < \infty$

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$$let \quad u = u_1, \quad v = u_2 \quad z = u_3 \quad and \quad we \quad know \quad that$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = \begin{vmatrix} \frac{\partial x}{\partial u_1} & \frac{\partial x}{\partial u_2} & \frac{\partial x}{\partial u_3} \\ \frac{\partial y}{\partial u_1} & \frac{\partial y}{\partial u_2} & \frac{\partial y}{\partial u_3} \\ \frac{\partial z}{\partial u_1} & \frac{\partial z}{\partial u_2} & \frac{\partial z}{\partial u_3} \end{vmatrix} ------(1)$$

$$\frac{\partial x}{\partial u_1} = \frac{1}{2}(2u - 0) = u \quad \frac{\partial x}{\partial u_2} = \frac{1}{2}(0 - 2v) = -v \quad \frac{\partial x}{\partial u_3} = \frac{1}{2}(0 - 0) = 0$$

$$\frac{\partial y}{\partial u_1} = 1(v) \quad \frac{\partial y}{\partial u_2} = u(1) = u \quad \frac{\partial y}{\partial u_3} = 0$$

$$\frac{\partial z}{\partial u_1} = 0 \quad \frac{\partial z}{\partial u_2} = 0 \quad \frac{\partial z}{\partial u_3} = 1$$

$$by \quad putting \quad all \quad values \quad in \quad (1)$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = \begin{vmatrix} u & -v & 0 \\ v & u & 0 \\ 0 & 0 & 1 \end{vmatrix} = u(u) - v(v) + 0(u^2 - v^2) = u^2 - v^2$$

II) Elliptic Cylindrical coordinates: $x = a\cosh u \cos v, y = a \sinh u \sin v, z = z$

Where $u \geq 0, 0 \leq v < 2\pi,\ ;\ \ , -\infty\ < z < \infty$

$$let \quad \alpha = u_1, \quad \beta = u_2 \quad \gamma = u_3 \quad and \quad we \quad know \quad that$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = \begin{vmatrix} \frac{\partial x}{\partial u_1} & \frac{\partial x}{\partial u_2} & \frac{\partial x}{\partial u_3} \\ \frac{\partial y}{\partial u_1} & \frac{\partial y}{\partial u_2} & \frac{\partial y}{\partial u_3} \\ \frac{\partial z}{\partial u_1} & \frac{\partial z}{\partial u_2} & \frac{\partial z}{\partial u_3} \end{vmatrix} ------(2)$$

$$\frac{\partial x}{\partial u_1} = \alpha \sinh u(\cos v) \quad \frac{\partial x}{\partial u_2} = -(\alpha \cosh u)\sin v \quad \frac{\partial x}{\partial u_3} = 0$$

$$\frac{\partial y}{\partial u_1} = (\alpha \cosh u)\sin v \quad \frac{\partial y}{\partial u_2} = \alpha \sinh u(\cos v) \quad \frac{\partial y}{\partial u_3} = 0$$

$$\frac{\partial z}{\partial u_1} = 0 \quad \frac{\partial z}{\partial u_2} = 0 \quad \frac{\partial z}{\partial u_3} = 1$$

$$by \quad putting \quad all \quad values \quad in \quad (2)$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = \begin{vmatrix} \alpha \sinh u(\cos v) & -(\alpha \cosh u)\sin v & 0 \\ (\alpha \cosh u)\sin v & \alpha \sinh u(\cos v) & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = (\alpha \sinh u(\cos v))((\alpha \sinh u)\cos v) + ((\alpha \cosh u)\sin v)((\alpha \cosh u)\sin v)$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = (\alpha \sinh u(\cos v))^2 + ((\alpha \cosh u)\sin v)^2$$

III) Prolate spheroidal coordinates:

$x = a \sinh\alpha \sin\beta \cos\gamma$

$y = a \sinh\alpha \sin\beta \sin\gamma$

$z = a \cosh\alpha \cos\beta$ Where $\alpha\ \geq 0, 0 \leq \beta\ \leq \pi$

$0 \leq \gamma\ < 2\pi.$

$$let \quad \alpha = u_1, \quad \beta = u_2 \quad \gamma = u_3 \quad and \quad we \quad know \quad that$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = \begin{vmatrix} \frac{\partial x}{\partial u_1} & \frac{\partial x}{\partial u_2} & \frac{\partial x}{\partial u_3} \\ \frac{\partial y}{\partial u_1} & \frac{\partial y}{\partial u_2} & \frac{\partial y}{\partial u_3} \\ \frac{\partial z}{\partial u_1} & \frac{\partial z}{\partial u_2} & \frac{\partial z}{\partial u_3} \end{vmatrix} ------(3)$$

$$\frac{\partial x}{\partial u_1} = \alpha \cosh \alpha \sin \beta \cos \gamma \quad \frac{\partial x}{\partial u_2} = \alpha \sinh \alpha \cos \beta \cos \gamma \quad \frac{\partial x}{\partial u_3} = -\alpha \sinh \alpha \sin \beta \sin \gamma$$

$$\frac{\partial y}{\partial u_1} = \alpha \cosh \alpha \sin \beta \sin \gamma \quad \frac{\partial y}{\partial u_2} = \alpha \sinh \alpha \cos \beta \sin \gamma \quad \frac{\partial y}{\partial u_3} = \alpha \sinh \alpha \sin \beta \cos \gamma$$

$$\frac{\partial z}{\partial u_1} = \alpha \sinh \alpha \cos \beta \quad \frac{\partial z}{\partial u_2} = -\alpha \cosh \alpha \sin \beta \quad \frac{\partial z}{\partial u_3} = 0$$

by putting all values in (3)

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = \begin{vmatrix} \alpha \cosh \alpha \sin \beta \cos \gamma & a \sinh a \cos \beta \cos \gamma & -\alpha \sinh \alpha \sin \beta \sin \gamma \\ \alpha \cosh \alpha \sin \beta \sin \gamma & \alpha \sinh \alpha \cos \beta \sin \gamma & \alpha \sinh \alpha \sin \beta \cos \gamma \\ \alpha \sinh \alpha \cos \beta & -\alpha \cosh \alpha \sin \beta & 0 \end{vmatrix}$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = (\alpha \sinh \alpha \cos \beta) \begin{vmatrix} \alpha \sinh \alpha \cos \beta \cos \gamma & -\alpha \sinh \alpha \sin \beta \sin \gamma \\ \alpha \sinh \alpha \cos \beta \sin \gamma & \alpha \sinh \alpha \sin \beta \cos \gamma \end{vmatrix}$$

$$-(-\alpha \cosh \alpha \sin b) \begin{vmatrix} \alpha \cosh \alpha \sin \beta \cos \gamma & -\alpha \sinh \alpha \sin \beta \sin \gamma \\ \alpha \cosh \alpha \sin \beta \sin \gamma & \alpha \sinh \alpha \sin \beta \cos \gamma \end{vmatrix} + (0) \begin{vmatrix} \alpha \cosh \alpha \sin \beta \cos \gamma & \alpha \sinh \alpha \cos \beta \cos \gamma \\ \alpha \cosh \alpha \sin \beta \sin \gamma & \alpha \sinh \alpha \cos \beta \sin \gamma \end{vmatrix}$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = (\alpha \sinh \alpha \cos \beta)\left[\left(\alpha^2 \sinh^2 \alpha \sin \beta \cos \beta \cos^2 \gamma\right) + \left(\alpha^2 \sinh^2 \alpha \sin \beta \cos \beta \sin^2 \gamma\right)\right]$$

$$(\alpha \cosh \alpha \sin b)\left[\left(\alpha^2 \cosh \alpha \sinh \beta \sin^2 \beta \cos^2 \gamma\right) + \left(\alpha^2 \cosh \alpha \sinh \alpha \sin^2 \beta \sin^2 \gamma\right)\right]$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = (\alpha \sinh \alpha \cos \beta)\left[\left(\alpha^2 \sinh^2 \alpha \sin \beta \cos \beta\right)\left(\cos^2 g + \sin^2 \gamma\right)\right]$$

$$+(\alpha \cosh \alpha \sin \beta)\left[\left(\alpha^2 \cosh \alpha \sinh \beta \sin^2 \beta\right)\left(\cos^2 g + \sin^2 \gamma\right)\right]$$

$$\therefore \cos^2 \gamma + \sin^2 \gamma = 1$$

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = (\alpha \sinh \alpha \cos \beta)\left[\left(\alpha^2 \sinh^2 \alpha \sin \beta \cos \beta\right)\right] + (\alpha \cosh \alpha \sin \beta)\left[\left(\alpha^2 \cosh \alpha \sinh \beta \sin^2 \beta\right)\right]$$

So,

$$j\frac{(x,y,z)}{(u_1,u_2,u_3)} = (\alpha \sinh \alpha \cos \beta)\left[\left(\alpha^2 \sinh^2 \alpha \sin \beta \cos \beta\right)\right] + (\alpha \cosh \alpha \sin \beta)\left[\left(\alpha^2 \cosh \alpha \sinh \beta \sin^2 \beta\right)\right]$$

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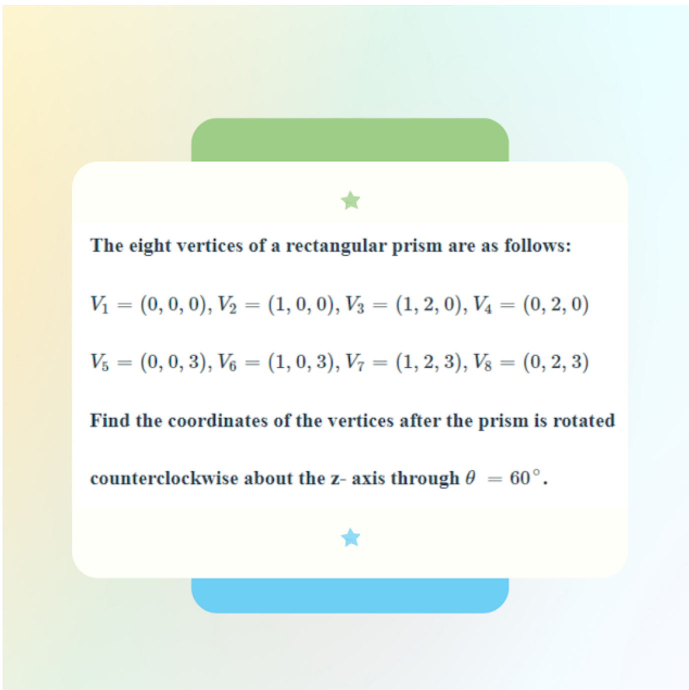
The eight vertices of a rectangular prism are as follows, Find the coordinates of the vertices after the prism is rotated counterclockwise about the z- axis.

Query Point Thursday, November 24, 2022



Question

The eight vertices of a rectangular prism are as follows:

$V_1 = (0,0,0), V_2 = (1,0,0), V_3 = (1,2,0), V_4 = (0,2,0)$

$V_5 = (0,0,3), V_6 = (1,0,3), V_7 = (1,2,3), V_8 = (0,2,3)$

Find the coordinates of the vertices after the prism is rotated counterclockwise about the z- axis through $\theta = 60^\circ$.

Solution

Here $\theta = 60$

and

The prism is rotated counterclockwise about the z -axis

then

now we first find $V_1^{-} = (x_1^{-}, y_1^{-}, z_1^{-})$

$Here \quad V_1 = (0,0,0)$

then

$V_1^{-} = (0,0,0)$

Now $V_2^{-} = = (x_2^{-}, y_2^{-}, z_2^{-})$

here $V_2 = (1,0,0)$

$$x_2 = 1, y_2 = 0, \text{ and } z_2 = 0$$

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$$x_2^- = x_2 \cos 60 - y_2 \sin 60 = (1)\left(\frac{1}{2}\right) - (0)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2}$$

$$y_2^- = x_2 \sin 60 + y_2 \cos 60 = (1)\left(\frac{\sqrt{3}}{2}\right) + (0)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

$$z_2^- = z_2 = 0$$

So,

$$V_2^- = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$$

Now

$$V_3^- = \left(x_3^-, y_3^-, z_3^-\right)$$

here

$$V_3 = (1, 2, 0)$$

$$x_3 = 1, y_3 = 2, z_3 = 0$$

$$x_3^- = x_3 \cos 60 - y_3 \sin 60 = (1)\left(\frac{1}{2}\right) - (2)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2} - \sqrt{3}$$

$$y_3^- = x_3 \sin 60 + y_3 \cos 60 = (1)\left(\frac{\sqrt{3}}{2}\right) + (2)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2} - 1$$

$$z_3^- = z_3 = 0$$

So,

$$V_3^- = \left(\frac{1}{2} - \sqrt{3}, \frac{\sqrt{3}}{2} - 1, 0\right)$$

Now

$$V_4^- = \left(x_4^-, y_4^-, z_4^-\right)$$

$$V_4 = (0, 2, 0)$$

$$x_4 = 0, y_4 = 2, z_4 = 0$$

$$x_4^- = x_4 \cos 60 - y_4 \sin 60 = (0)\left(\frac{1}{2}\right) - (2)\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$$

$$y_4^- = x_4 \sin 60 + y_4 \cos 60 = (0)\left(\frac{\sqrt{3}}{2}\right) + (2)\left(\frac{1}{2}\right) = -1$$

$$z_4^- = z_4 = 0$$

So,

$$V_4^- = \left(-\sqrt{3}, -1, 0\right)$$

Now $V_5^- = \left(x_5^-, y_5^-, z_5^-\right)$

here

$$V_5 = (0, 0, 3)$$

$$x_5 = 0, y_5 = 0, z_5 = 3$$

$$x_5^- = x_5 \cos 60 - y_5 \sin 60 = (0)\left(\frac{1}{2}\right) - (0)\left(\frac{\sqrt{3}}{2}\right) = 0$$

$$y_5^- = x_5 \sin 60 + y_5 \cos 60 = (0)\left(\frac{\sqrt{3}}{2}\right) + (0)\left(\frac{1}{2}\right) = 0$$

$$z_5^- = z_5 = 3$$

So,

$$V_5^- = (0, 0, 3)$$

Now

$$V_6^- = \left(x_6^-, y_6^-, z_6^-\right)$$

here $V_6 = (1, 0, 3)$

$$x_6 = 1, y_6 = 0, z_6 = 3$$

$x_6^- \quad = x_6 \cos 60 - y_6 \sin 60 = (1)\left(\frac{1}{2}\right) - (0)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2}$

$y_6^- \quad = x_6 \sin 60 + y_6 \cos 60 = (1)\left(\frac{\sqrt{3}}{2}\right) + (0)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

$z_6^- \quad = z_6 = 3$

So,

$$V_6^- \quad = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 3\right)$$

Now $V_7^- \quad = (x_7^-, y_7^-, z_7^-)$

here $V_7 = (1, 2, 3)$

$x_7 = 1, y_7 = 2, z_7 = 3$

$x_7^- \quad = x_7 \cos 60 - x_7 \sin 60 = (1)\left(\frac{1}{2}\right) - (2)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2} - \sqrt{3}$

$y_7^- \quad = x_7 \sin 60 + x_7 \cos 60 = (1)\left(\frac{\sqrt{3}}{2}\right) + (2)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2} - 1$

$z_7^- \quad = z_7 = 3$

So,

$V_7^- \quad = \left(\frac{1}{2} - \sqrt{3}, \frac{\sqrt{3}}{2} - , 3\right)$

Now $V_8^- \quad = (x_8^-, y_8^-, z_8^-)$

$here\ V_8 = (0, 2, 3)$

$x_8 = 0,\ y_8 = 2,\ \text{and}\ z_8 = 3$

$x_8^- \quad = x_8 \cos 60 - y_8 \sin 60 = (0)\left(\frac{1}{2}\right) - (2)\left(\frac{\sqrt{3}}{2}\right) = \ -\sqrt{3}$

$y_8^- \quad = x_8 \sin 60 + y_8 \cos 60 = (0)\left(\frac{\sqrt{3}}{2}\right) + (2)\left(\frac{1}{2}\right) = \ -1$

$z_8^- \quad = z_8 = 3$

So,

$V_8^- \quad = (-\sqrt{3},\ -1, 3)$

\]

Hence,

The coordinates of the vertices after the prism are rotated counterclockwise about the z-axis through $\theta \ = 60^\circ$ are

$V_1^- \quad = (0, 0, 0), V_2^- \quad = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right),$

$V_3^- \quad = \left(\frac{1}{2} - \sqrt{3}, \frac{\sqrt{3}}{2} - 1, 0\right), V_4^- \quad = (-\sqrt{3}, -1, 0),$

$V_5^- \quad = (0, 0, 3), V_6^- \quad = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 3\right),$

$V_7^- \quad = \left(\frac{1}{2} - \sqrt{3}, \frac{\sqrt{3}}{2} - , 3\right), V_8^- \quad = (-\sqrt{3}, -1, 3)$

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Solved the following function

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