Q.1

Let $W = \text{span}(x_1, x_2)$, where $x_1 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix}$, constract an orthogonal basis (V_1, V_2) for W? M.M

Solution:-

Let $V_1 = x_1$ and compute $V_2 = x_2 - P$

$$\therefore P = \frac{x_2 \times V_1}{V_1 \times V_1} \times V_1$$

then,

$$V_2 = \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} - \frac{\begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}}{\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}} \times \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} - \frac{30}{10} \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} - 3 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 8-9 \\ 5-0 \\ -6+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix}$$

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Thus, the orthogonal basis are,

$$(V_1, V_2) = \left(\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix} \right)$$ Ans.

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Q.2

Find A^2, given that $A = PDP^{-1}$ where $A = \begin{pmatrix} 2 & 6 \\ -4 & 12 \end{pmatrix}$, $P = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$, $P^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$

and $D = \begin{pmatrix} 6 & 0 \\ 0 & 8 \end{pmatrix}$?

Solution:-

M.M

since,

$A = PDP^{-1}$

$A^2 = PDP^{-1} \times PDP^{-1} = PDP^{-1}PDP^{-1}$ $\quad \because PP^{-1} = I = 1$

$A^2 = PDDP^{-1} = PD^2P^{-1}$

therefore,

$$A^2 = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 6^2 & 0 \\ 0 & 8^2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 36 & 0 \\ 0 & 64 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 108+0 & 0+64 \\ 72+0 & 0+64 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 108 & 64 \\ 72 & 64 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 108-128 & -108+192 \\ 72-128 & -72+192 \end{pmatrix}$$

$$= \begin{pmatrix} -20 & 84 \\ -56 & 120 \end{pmatrix}$$ Ans

Q.3

Find the dominant eigen pair (i-e the Eigenvalue and Eigenvector) by using the power method for the following matrix (Perform only 1st iteration)

$$A=\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}, \quad x_0=\begin{bmatrix}1\\0\end{bmatrix}$$

Solution:-

since 1st iteration is

$$u_1 = Ax_0$$

$$u_1=\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}5+0\\1+0\end{bmatrix}=\begin{bmatrix}5\\1\end{bmatrix}$$

Now we find the eigenvalue and eigenvector

$$u_1 = 5\begin{bmatrix}1\\\frac{1}{5}\end{bmatrix}=q_1x_1$$

here "q_1" is eigenvalue and eigenvector is "x_1"

thus

$$q_1 = \text{eigenvalue} = 5$$

$$x_1 = \text{eigenvector} = \begin{bmatrix}1\\\frac{1}{5}\end{bmatrix}$$

Q.4

Determine whether the set of vectors are orthogonal or not, $\begin{bmatrix}5\\-4\\0\\3\end{bmatrix}, \begin{bmatrix}-4\\1\\-3\\8\end{bmatrix}, \begin{bmatrix}3\\3\\5\\1\end{bmatrix}$

Solution:-

Let

$a = \begin{bmatrix}5\\-4\\0\\3\end{bmatrix}, b = \begin{bmatrix}-4\\1\\-3\\8\end{bmatrix}, C = \begin{bmatrix}3\\3\\5\\1\end{bmatrix}$

a, b, c will be orthogonal if

$a^t b = 0 \longrightarrow i)$

$a^t C = 0 \longrightarrow ii)$

$b^t C = 0 \longrightarrow iii)$

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by (i)

$a^t b = \begin{bmatrix}5 & -4 & 0 & 3\end{bmatrix}\begin{bmatrix}-4\\1\\-3\\8\end{bmatrix}$

$= -20 - 4 + 0 + 24 = 0$

thus, the vector "a" and "b" are orthogonal.

Same as we can find "$a^t C = 0$" and "$b^t C = 0$" which are not orthogonal.

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Q.5

If $A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix}$ and $\det(A^t) = 6$

Find the determinent of the matrix

Solution:-

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 4 & 7 \\ 4 & 5 & 8 \\ 5 & 6 & 8 \end{bmatrix}$$

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$$\det(A^t) = \begin{vmatrix} 1 & 4 & 7 \\ 4 & 5 & 8 \\ 5 & 6 & 8 \end{vmatrix}$$

$$\det(A^t) = 1(40-48) - 4(32-40) + 7(24-25)$$

$$= -8 + 32 - 7$$

$$= 32 - 15$$

$$= 17 \quad Ans$$

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Q.6

Is the following set of vectors is orthogonal with respect to the Euclidean inner product on $\mathbb{R}^3$?

$$\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right), \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$$

Let

$$V_1 = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right), \quad V_2 = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$$

V_1 and V_2 will be orthogonal w.r.t the euclidean inner product if.

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$$\langle V_1 \cdot V_2 \rangle = 0$$

$$\left\langle \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right) \cdot \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right) \right\rangle = 0$$

$$\left\langle \frac{1}{\sqrt{6} \times \sqrt{2}} - \frac{1}{\sqrt{6} \times \sqrt{2}} + 0 \right\rangle = 0$$

$$\langle 1 - 1 + 0 \rangle = 0$$

$$\langle 0 \rangle = 0$$

$$0 = 0$$

Thus V_1 and V_2 are orthogonal.

Q.7

Determine whether the vector $y = \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix}$ and $z = \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$ are orthogonal?

Solution:-

The vectors "y" and "z" will be orthogonal if $y^t z = 0$ or $z^t y = 0$

Proof:-

$$y^t z = \begin{bmatrix} -3 & 7 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$$

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$$y^t z = -3 - 56 + 60 + 0$$

$$y^t z = 60 - 59$$

$$y^t z = 1$$

here

$$y^t z \neq 0$$

therefore,

"y" and "z" vectors are not orthogonal.

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Q.8

$A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$ and $B = \{b_1, b_2\}$ where

$b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, Find B matrix for the transformation $x \rightarrow Ax$?

since,

$$M = \left[[T(b_1)]_c \ [T(b_2)]_c \cdots [T(b_n)]_c \right]$$

here

$$[T(b_1)]_c = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$[T(b_2)]_c = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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therefore

$$B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$ Ans.

Q.9

Find a distance between $x = \begin{bmatrix} 10 \\ -3 \end{bmatrix}$ and $y = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$?

Solution:-

since,

$dist(x, y) = ||x - y|| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$

therefore

$dist(x, y) = \sqrt{(10+1)^2 + (-3+5)^2}$

$= \sqrt{121 + 4}$

$= \sqrt{125}$ Ans.

M.M