

Mid term Practice exercises and important Questions

1) Write the following expression in the form of $a + ib$.

$$\frac{3+i}{2-i}, \frac{4+10i}{-9+7i}$$

Answer:

$$\frac{3+i}{2-i} = \frac{3+i}{2-i} * \frac{2+i}{2+i} = \frac{6+3i+2i+i^2}{4+2i-2i-i^2} = \frac{6+5i+(-1)}{4-(-1)} = \frac{5+5i}{5}$$

$$\frac{4+10i}{-9+7i} * \frac{-9-7i}{-9-7i} = \frac{-36-28i-90i-70i^2}{81+63i-63i-49i^2} = \frac{-36-118i+70}{81+49} = \frac{34-118i}{130}$$

2) Write in set builder notation

A= Set of first 100 odd numbers

A= Set of Natural numbers starting from 2 to 40.

A= Set of all even numbers greater than 50.

Answer:

$A = \{x \mid x \text{ is set of first 100 odd numbers}\}$

$A = \{x \mid 2 \leq x \leq 40 \text{ and } x \text{ is set of natural numbers}\}$

$A = \{x \mid x > 50 \text{ and } x \text{ is set of even numbers}\}$

3) List the elements of the set (roster method)

$$A = \{x \mid x \in E^+ \wedge x \leq 10\}$$

$$A = \{x \mid x^2 = 9, x \text{ is even}\}$$

$$A = \{x \mid x \in N \wedge 2 \leq x \leq 10\}$$

A = {x | x is a person taller than 15 feet}

B = {x | x² = 16, x is odd}

Set of first five multiples of 3.

Answer:

$A = \{0, 2, 4, 6, 8, 10\}$

$A = \emptyset$

$A = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$A = \emptyset \quad A = \emptyset$$

$$A = \{3, 6, 9, 12, 15\}$$

- 4) Let A be the set of all even numbers. Let B be the set of all Number less than 100. Is A a subset of B? or Is B a subset of A?

Answer:

$$A \not\subset B$$

$$B \not\subset A$$

List all elements in these sets and verify the result.

- 5) Let $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$
 $B = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$
Find $A \cap B$ and $A \cup B$.

Answer:

$$A \cap B = \{ \emptyset \} \text{ and } A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21\}$$

- 6) Let A be all multiples of 6 and B be all multiples of 3. Is A a subset of B? And is B a subset of A?

Answer:

$$A = \{6, 12, 18, 24, 30, 36, \dots\}$$

$$B = \{3, 6, 9, 12, 15, 18, 21, 24, \dots\}$$

A is subset of B but B is not subset of A.

- 7) The complex numbers Z is given by $Z = \frac{i-1}{i}$ Find modulus of Z .

Answer:

$$Z = \frac{i-1}{i}$$

$$\frac{i-1}{i} * \frac{i}{i} = \frac{i^2 - i}{i^2} = \frac{-(1-i)}{-1} = \frac{1+i}{1}$$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

8) Identify rational numbers.

(a) 5.333333333..... Rational

(b) 4.131 Rational

(c) 8.6985476321459845632489631..... Irrational

9) Evaluate

$$(2+i)(3+i)$$

$$(4+5i)(-2-2i)$$

$$(-1-i)(-2-2i)$$

$$i^3$$

Solution:

$$(2+i)(3+i) = 6 + 2i + 3i + i^2 = 6 + 5i(-1) = 5 + 5i$$

$$(4+5i)(-2-2i) = -8 - 8i - 10i - 10i^2 = -8 - 18i - 10(-1) = -8 - 18i + 10 = 2 - 18i$$

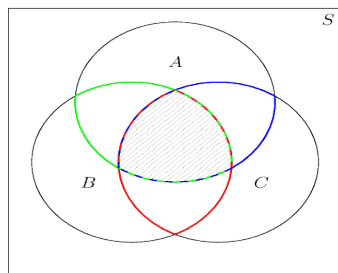
$$(-1-i)(-2-2i) = 2 + 2i + 2i + 2i^2 = 2 + 4i + 2(-1) = 4i$$

$$i^3 = i * i * i = (i)^2(i) = (-1)(\sqrt{-1}) = -i$$

10) The Venn diagram given below shows the sets A, B, and C. Shade the following sets

i. $A \cup (B \cap C)$

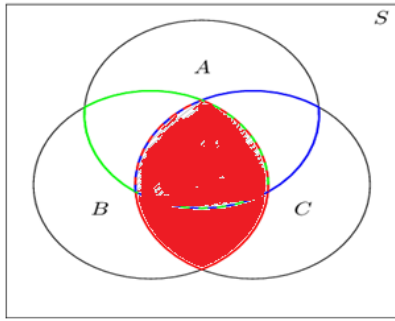
ii. $A \cup (B \cap C)^c$



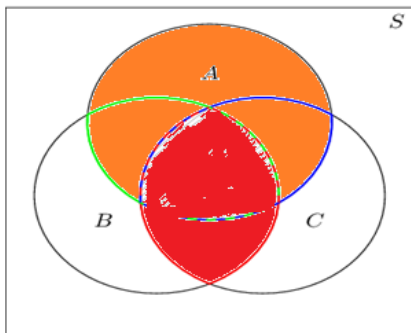
iii. $A \cap (B \cup C)$

Solution:

i. $A \cup (B \cap C)$



B intersection C is shown in red. If we take the union of this red portion with the A then it will be like



So the total colored region (red + orange) is the required region.

Solve all other on the same lines.

11) What is the cardinality of these sets?

A = Set of even integers ∞

B = Set of prime numbers less than 30 ∞

C = {11, 12, 13, 14, 15, 16, 17, 18}

12) State the Associative Law for union and intersection.

$$A \cup (B \cap C) = (A \cup B) \cap C$$

$$A \cap (B \cup C) = (A \cap B) \cup C$$

13) $z = -5 + 8i$ then find $z - z^*$

Solution:

$$z = -5 + 8i$$

$$z^* = -5 - 8i$$

$$\begin{aligned} z - z^* &= -5 + 8i - (-5 - 8i) \\ &= -5 + 8i + 5 + 8i = 16i \end{aligned}$$

14) If $z = 3 + 6i$ then find $z \cdot z^*$

Solution:

$$z = 3 + 6i$$

$$z^* = 3 - 6i$$

$$\begin{aligned} z \cdot z^* &= (3 + 6i)(3 - 6i) \\ &= 9 - 18i + 18i + 36 = 45 \end{aligned}$$

15) Evaluate

$$\left| \frac{4 + 7i}{-5 + 6i} \right|$$

Solution:

$$\begin{aligned} \frac{4 + 7i}{-5 + 6i} \cdot \frac{-5 - 6i}{-5 - 6i} &= \frac{(4 + 7i)(-5 - 6i)}{(-5 + 6i)(-5 - 6i)} \\ &= \frac{-20 - 24i - 35i - 42i^2}{(-5)^2 - (6i)^2} \\ &= \frac{22 - 59i}{25 + 36} = \frac{22 - 59i}{61} = \frac{22}{61} - \frac{59}{61}i \\ \left| \frac{4 + 7i}{-5 + 6i} \right| &= \sqrt{\left(\frac{22}{61}\right)^2 + \left(\frac{59}{61}\right)^2} = \sqrt{\frac{484 + 3481}{3721}} = \sqrt{\frac{3965}{3721}} \end{aligned}$$

16) Write the domain and range of $f(x) = 6x + 9$

Solution:

Domain of $f(x)$ = All real no's

Range of $f(x)$ = All real no's

17) Write the domain and range of $g(x) = \frac{5}{x + 6}$

Solution:

Domain of $g(x)$ = $R - \{-6\}$

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Range of $f(x)$ = All real no's except zero

18) Prove that $f(x) = x^4 + 5x^2 + 3$ is an even function

Solution:

$$f(x) = x^4 + 5x^2 + 3$$

$$f(-x) = (-x)^4 + 5(-x)^2 + 3$$

$$f(-x) = x^4 + 5x^2 + 3 = f(x)$$

19) Prove that $f(x) = x^3 - 8x$ is an odd function

Solution:

$$f(x) = x^3 - 8x$$

$$f(-x) = (-x)^3 - 8(-x)$$

$$f(-x) = -x^3 + 8x = -(x^3 - 8x) = -f(x)$$

20) Express the quadratic polynomial $x^2 + 4x + 6$ in complete square form.

Solution:

$$x^2 + 4x + 6 = x^2 + 4x + 4 - 4 + 6 = (x+2)^2 + 2$$

21) If the functions f is defined by

$$f(x) = \frac{1}{x}, \quad 0 < x \leq 5,$$

then find the range of f .

Solution:

Since $x \leq 5$, $1/x \geq 1/5$. i.e., $f(x) \geq 1/5$.

22) If the functions f and g are defined by

$$f(x) = \frac{3}{x}, \quad 0 < x \leq 3,$$

$$g(x) = 5x - 1, \quad x \in R,$$

then Calculate $gf(2)$.

Solution:

$$gf(x) = g(f(x)) = g(3/x) = 5(3/x) - 1 = (15/x) - 1.$$

$$\text{So, } gf(2) = (15/2) - 1 = 13/2$$

23) If the functions g is defined by $g(x) = 8x - 5$, $x \in R$, then find an expression in terms of x for $g^{-1}(x)$.

Solution:

To find $g^{-1}(x)$, we write $y = 8x - 5$

$$\rightarrow y + 5 = 8x \rightarrow x = (y + 5)/8 \quad \text{Therefore, } g^{-1}(x) = (x + 5)/8$$

24) Let $f(x) = 6x + 1$ and $g(x) = -9x + 5$. Find the product of f & g i.e $(f \cdot g)(x)$.

Solution:

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) = (6x+1) \cdot (-9x+5) \\ &= -54x^2 + 21x + 5 \quad (\text{Ans.})\end{aligned}$$

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25) Let f and g be the functions defined by $f(x) = 3x + 3$ & $g(x) = 3x + 5$ then find $f \circ g(x)$ and $g \circ f(x)$.

Solution:

$$\begin{aligned}f \circ g(x) &= f(g(x)) \\ &= f(3x+5) \\ &= 3(3x+5)+3 \\ &= 9x+18 \\ g \circ f(x) &= g(f(x)) \\ &= g(3x+3) \\ &= 3(3x+3)+5 \\ &= 9x+14\end{aligned}$$

26) Determine whether the equation has one, two or zero real solution.

$$5x^2 + 4x - 6 = 0$$

solution:

$$a = 5, b = 4, c = -6$$

$$b^2 - 4ac = 16 + 120 = 136 > 0$$

So there will be two distinct real solutions.

27) Determine whether the equation has one, two or zero real solution.

$$x^2 + 3x + 6 = 0$$

solution:

$$a = 1, b = 3, c = 6$$

$$b^2 - 4ac = 9 - 24 = -15 < 0 \quad \text{So there will be no real solutions.}$$

28) The equation $tx^2 + 5x - 6 = 0$ has two real solutions. What can be deduced about the value of t ?

Solution:

$$tx^2 + 5x - 6 = 0$$

$$a = t, b = 5, c = -6$$

For two real solutions the discriminant should be >0 i.e

$$b^2 - 4ac > 0$$

$$25 + 24t > 0$$

$$24t > -25$$

$$t > \frac{-25}{24}$$

29) The equation $kx^2 - 8x + 9 = 0$ has exactly one solution. What can be deduced about the value of k ?

Solution:

$$kx^2 - 8x + 9 = 0$$

$$a = k, b = -8, c = 9$$

For exactly one solution the discriminant should be equal to 0

i.e

$$b^2 - 4ac = 0$$

$$64 - 36k = 0$$

$$64 = 36k$$

$$\frac{64}{36} = k$$

$$\frac{32}{18} = k$$

30) Determine whether the function is even, odd, neither even nor odd.

1. $f(x) = x^2 + 3x$

2. $f(x) = \sqrt{x^2 + 9}$

3. $f(x) = x^2 + 4$

4. $f(x) = \frac{x}{x^2 - 1}$

5. $f(x) = 6x^5 - x^3$

6. $g(x) = x^4 + x^6$

31) Express the quadratic polynomial in complete square form $2x^2 + 8x + 6$

32) If $f(x) = 4x - 3$ & $g(x) = x^3 + 3$ Then find $gof(x)$ and $fog(x)$.

33) Determine whether the following equations have one, two or zero real solution.

1. $7x^2 + 9x - 3 = 0$

2. $4x^2 + 2x + 1 = 0$

34) If the following equation has one solution then find the value of a $ax^2 + 5x + 2 = 0$

35) Given matrices A and B such that $A = \begin{bmatrix} 1 & 2 \\ -3 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix}$ Find $2A - 3B$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

36) Check whether the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 1 \end{bmatrix}$ can be multiplied? Justify your answer.

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$$

37) Find the inverse

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

38) Multiply the matrix:

39) Find the transpose of matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -2 & 9 \\ -5 & 7 & -3 \end{bmatrix}$$

40) How many 3-permutation are there of {A, B, C, D}?

41) The letters of the word PEN taken all at a time can be written in _____

42) ${}^{15}C_9 = \dots$

43) What is the relation between permutations and combinations?

44) There are total 12 players in a cricket team. The captain needs to choose 8 players to play in a match. How many possible teams can be chosen?

45) Given the sequence, 150, 300, 450, 600, ..., find the 15th term.

Answer: 2250

46) What is the general term of the sequence 1, 8, 27, 64, 125,

Answer: $an = n^3$

47) The 3rd and the 4th terms of a geometric progression are 18 and 6 respectively.
Find the sum to infinity of the progression.

Answer: 243

48) If first term , common difference , then what is the value of 10th term in an arithmetic sequence?

Answer: 22

49) Suppose , is an arithmetic sequence . Write its corresponding arithmetic series.
Hint: Just sum (add) all the terms to find the corresponding series.

50) If then find the first four terms of the sequence .

Answer: 9,16,25,36

51) Determine whether the sequence 9, 11, 13, 15, 17, ... is arithmetic or geometric.
Give reason for your answer.

Answer: The sequence is arithmetic.

52) Find the 5th term of the geometric sequence with $a_1 = 20$ and $a_2 = 40$.

Answer: 320

53) If , then compute the summation:

Answer: 10

54) There are three juniors and three seniors in the Service Club. The club is to send four representatives to the State Conference. How many different ways are there to select a group of four students to attend the conference?

Solution: Choose 4 students from the total number of students. Order is not important. There are 6 total students. So number of choices is ${}^6C_4 = 15$

55) There are four juniors and three seniors in the Service Club. The club is to send three representatives to the State Conference. If the members of the club decide to send one junior and two seniors, how many different groupings are possible?

Solution: Choose 1 junior and 2 seniors: $4C_1 \cdot 3C_2 = 12$

56) Suppose that $X = \{a, b, c\}$. Then list all 3-combinations of the set X , and hence find the value of $C(3,3)$.

Solution: $X = \{a, b, c\}$. Then 3-combinations of the 3 elements of the set X is: $\{a, b, c\}$. Hence $C(3,3) = 1$.

57) If $W = \{x, y, z, t\}$. Then list all 3-combinations of the set W , and hence find the value of $C(4,3)$.

Solution:

$$W = \{x, y, z, t\}.$$

Then 3-combinations of the 4 elements of the set X are:

$\{x, y, z\}$, $\{x, y, t\}$, $\{x, z, t\}$ and $\{y, z, t\}$. Hence $C(4,3) = 4$.

58) Compute: $C(9, 6)$.

Solution:

$$\rightarrow {}^n C_r = \frac{n!}{(n-r)! r!}$$

$$\begin{aligned}\Rightarrow {}^9 C_6 &= \frac{9!}{(9-6)! 6!} \\ &= \frac{9!}{3! 6!} \\ &= \frac{362880}{6 \cdot (720)} \\ &= \frac{362880}{4320} \\ &= 84\end{aligned}$$

59) There are total 14 players in a cricket team. The captain needs to choose 10 players to play in a match. How many possible teams can be chosen?

Solution:

This is a combination problem as the order in which the teams are chosen is not important. The number of combinations is

$$\rightarrow {}^n C_r = \frac{n!}{(n-r)! r!}$$

$$\begin{aligned}\Rightarrow {}^{14} C_{10} &= \frac{14!}{(14-10)! 10!} \\ &= \frac{14!}{4! 10!} \\ &= \frac{87178291200}{24 \cdot 2628800} \\ &= 1001\end{aligned}$$

60) In how many ways can a team of 5 candidate can be formed out of a total of 12 candidates such that two particular candidates should be included in each department?

Solution:

Two particular candidates should be included in each team.

i.e., we have to select $5-2 = 3$ persons from $12-2 = 10$ persons. Hence, the required number of ways are:

$$\begin{aligned}\rightarrow {}^nC_r &= \frac{n!}{(n-r)! r!} \\ \Rightarrow {}^{10}C_3 &= \frac{10!}{(10-3)! 3!} \\ &= \frac{10!}{7! 3!} = 120\end{aligned}$$

- 61) Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?
- 62) In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?
- 63) From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there in the committee. In how many ways can it be done?
- 64) In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?
- 65) In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?
- 66) In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
- 67) In how many different ways can the letters of the word 'MATHEMATICS' be arranged such that the vowels must always come together?
- 68) There are 8 men and 10 women and you need to form a committee of 5 men and 6 women. In how many ways can the committee be formed?
- 69) How many 3-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

- 70) In how many different ways can the letters of the word 'LEADING' be arranged such that the vowels should always come together?
- 71) In how many different ways can the letters of the word 'DETAIL' be arranged such that the vowels must occupy only the odd positions?
- 72) A bag contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the bag, if at least one black ball is to be included in the draw?
- 73) In how many different ways can the letters of the word 'JUDGE' be arranged such that the vowels always come together?
- 74) In how many ways can the letters of the word 'LEADER' be arranged?
- 75) How many arrangements can be made out of the letters of the word 'ENGINEERING'?
- 76) How many 3 digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9 which are divisible by 5 and none of the digits is repeated?
- 77) There are 6 periods in each working day of a school. In how many ways can one organize 5 subjects such that each subject is allowed at least one period?
- 78) How many 6 digit telephone numbers can be formed if each number starts with 35 and no digit appears more than once?
- 79) An event manager has ten patterns of chairs and eight patterns of tables. In how many ways can he make a pair of table and chair?
- 80) 25 buses are running between two places P and Q. In how many ways can a person go from P to Q and return by a different bus?
- 81) A box contains 4 red, 3 white and 2 blue balls. Three balls are drawn at random. Find out the number of ways of selecting the balls of different colors?
- 82) A question paper has two parts P and Q, each containing 10 questions. If a student needs to choose 8 from part P and 4 from part Q, in how many ways can he do that?

- 83) In how many different ways can 5 girls and 5 boys form a circle such that the boys and the girls alternate?
- 84) Find out the number of ways in which 6 rings of different types can be worn in 3 fingers?
- 85) In how many ways can 5 men draw water from 5 taps if no tap can be used more than once?
- 86) There are 6 persons in an office. A group consisting of 3 persons has to be formed. In how many ways can the group be formed?
- 87) There are 5 yellow, 4 green and 3 black balls in a bag. All the 12 balls are drawn one by one and arranged in a row. Find out the number of different arrangements possible.
- 88) In how many ways can a team of 5 persons be formed out of a total of 10 persons such that two particular persons should not be included in any team?
- 89) A company has 11 software engineers and 7 civil engineers. In how many ways can they be seated in a row so that no two of the civil engineers will sit together?
- 90) How many signals can be made using 6 different coloured flags when any number of them can be hoisted at a time?
- 91) In how many ways can 5 distinguishable balls be put into 8 distinguishable boxes if no box can contain more than one ball?
- 92) Find the first two terms in the expansion of $(4 - x)^6$, in ascending powers of x .

Solution: $=4096-6144x+3840x^2+\dots$. Check first two terms only which are $4096-6144x$

- 93) Find the value of the constant a for which the coefficient of x in the expansion of $(1 + ax)(2 - x)^6$ is 128.

Solution: $(1 + ax)(2 - x)^6 = (1 + ax)(64 - 192x + 240x^2)$.

Collecting the x terms only, we get, $-192x + 64ax = (64a - 192)x$. Since the coefficient of x is given to be 128, we get $64a - 192 = 128 \rightarrow a = 5$

- 94) Calculate the coefficient of x^8y^3 and xy^{10} in the expansion of $(x+y)^{11}$

Solution: ${}^{11}C_8 = 165$

${}^{11}C_1 = 11$

Final term important and practice exercises questions

- 1) What is the domain and range of $y = \arcsin x$ and $\arccos x$?

Solution:

The domain of $y = \arcsin x$ is $[-1, 1]$

Range of $y = \arcsin x = [-\pi/2, \pi/2]$

Domain of $y = \arccos x = [-1, 1]$

Range of $y = \arccos x = [0, \pi]$

- 2) **Prove the Pythagorean Identities.**

Solution:

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\begin{aligned} \left(\frac{\sin \theta}{\cos \theta}\right)^2 + 1 &= \frac{\sin^2 \theta}{\cos^2 \theta} + 1 \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta \end{aligned}$$

3) $1 + \cot^2 \theta = \csc^2 \theta$

Solution:

Use $\cot \theta = \frac{\cos \theta}{\sin \theta}$ and solve as above.

- 4) Expand the sum and difference formula $\sin (A+B)$ and $\sin (A-B)$

Solution: Given in handouts.

- 5) Use trigonometric identities to prove that.

$$\frac{\tan y}{\sin y} = \sec y$$

Solution:

$$\frac{\tan y}{\sin y} = \frac{\sin y}{\cos y} \cdot \frac{1}{\sin y} = \frac{1}{\cos y} = \sec y$$

- 6) Without using the calculator, find $\arcsin \frac{1}{\sqrt{2}}$ in radians.
Solution:

$$\arcsin \frac{1}{\sqrt{2}} = \frac{\pi}{4} \text{ since } \frac{\pi}{4} \text{ is the angle whose sine is } \frac{1}{\sqrt{2}}.$$

- 7) **Without using the calculator**, find the exact value of $\cos 210^\circ$.

Solution:

$$\begin{aligned} \cos(210^\circ) &= \cos(180^\circ + 30^\circ) \\ &= \cos 180^\circ \cos 30^\circ - \sin 180^\circ \sin 30^\circ \\ &= (-1) \left(\frac{\sqrt{3}}{2} \right) - (0) \left(\frac{1}{2} \right) \\ &= -\frac{\sqrt{3}}{2} \end{aligned}$$

- 8) Given that's $\sin \theta = \frac{\sqrt{3}}{2}$, find all possible values of θ in the range $-180 \leq \theta \leq 180$.

$$\theta = \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = 60^\circ$$

Solution:

Also by using symmetry property:

$$\rightarrow \sin(180 - \theta) = \sin(\theta) \Rightarrow \sin(180 - (-60)) = \sin(240) \text{ to get } 240 \text{ as a solution.}$$

Note that 240 must be a solution but it does not lie in our range $-180 \leq \theta \leq 180$.

Now use the periodic property: $\sin(240) = \sin(240 - 360)$. So another solution is -120 degrees.

All the solutions are ± 60 and ± 120 degrees.

- 9) Verify that the following: $\frac{1 - \tan \theta}{1 - \cot \theta} = -\tan \theta$.

Solution:

$$\rightarrow \frac{1 - \tan \theta}{1 - \cot \theta} = -\tan \theta$$

$$LHS = \frac{1 - \tan \theta}{1 - \cot \theta}$$

$$= \frac{1 - \frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}}$$

$$= \frac{\frac{\cos \theta - \sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}}$$

$$= \frac{\cos \theta - \sin \theta}{\cos \theta} \times \frac{\sin \theta}{-(\cos \theta - \sin \theta)}$$

$$= -\frac{\sin \theta}{\cos \theta}$$

$$\frac{1 + \tan \theta}{1 + \cot \theta} = -\tan \theta = RHS \quad (\text{verified})$$

10) Verify that $\cos 4A = \cos^2 2A - \sin^2 2A$.

Solution:

$$\begin{aligned} \cos 4A &= \cos(2A + 2A) \\ &= \cos 2A \cos 2A - \sin 2A \sin 2A \\ &= \cos^2 2A - \sin^2 2A \quad (\text{verified}) \end{aligned}$$

11) Write the double angle formula for \tan function.

Solution $\tan 2A = \frac{\tan A + \tan A}{1 - \tan A \tan A} = \frac{2 \tan A}{1 - \tan^2 A}$

Question no.1

Find the equation of the line that passes through the point (6, 7) and is perpendicular to line $2x + 3y = 8$.

Solution:

The line l can be written as $y = -2x/3 + 8/3$.

So its slope is $-2/3$.

Slope of the line perpendicular to l is $3/2$.

The equation is $y - 7 = 3/2(x - 6)$

- 12) Find the slope of the line through the points $(-5, -2)$ and $(-6, 7)$
- 13) Find slope of the line $6x - 8y = 26$.
Slope is $6/8$ or $3/4$
- 14) Find the point of intersection of the lines $2x + 3y = 7$ and $3x + 5y = 3$.
Solution To find the point of intersection we need (x, y) which lie on both the lines, i.e. which satisfy the two equations simultaneously. We need to solve the equations simultaneously. Solving the equations simultaneously we get $x = 26$ and $y = -15$.
- 15) Find the equation of the perpendicular bisector of the line joining the points $A(2, 8)$ and $B(6, 2)$.

Let M be the mid-point of AB . Then $M(4,5)$

Gradient of AB is $(8 - 2)/(2-6) = 6/-4 = -3/2$.

So slope of required line is $2/3$. Equation of required line is $y - 5 = 2/3(x - 6)$

- 16) Find the slope for the following graph:
- 17) Find the center and radius of the given equation of circle:
- 18) Are the lines parallel, perpendicular, or neither?
- 19) Given the equations of the lines . Do these lines intersect? If they do, find the point of intersection.
- 20) Find the equation of the line that passes through the point $(6, -7)$ and is perpendicular to line $2x + 3y = 8$.
- 21) Find the slope of the line through the points $(2, -5)$ and $(1, 4)$
- 22) Find slope of the line $7x - 2y = 11$.
- 23) Identify the center and radius of the given circle $x^2 + (y - 7)^2 = 13$
- 24) What is the domain and range of $y = \arcsin x$ and $\arccos x$?
- 25) Prove the Pythagorean Identities.

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

26) Expand the sum and difference formula $\sin(A+B)$ and $\sin(A-B)$

$$\frac{\tan y}{\sin y} = \sec y$$

27) Use trigonometric identities to prove that.

28) Write and memorize Pythagorean identities.

29) Without using calculator find the value of $\cos 15^\circ$ and $\sin 15^\circ$

$$\frac{\cot \theta - \tan \theta}{\sin \theta \cos \theta} = \operatorname{cosec}^2 \theta - \sec^2 \theta$$

30) Prove that

31) If first and fourth term of arithmetic progression is the first two terms of a geometric progression then find the common ratio of geometric progression. Take first term of arithmetic progression as 'a' and $d = 4/3a$.

$$\cos \theta = \frac{1}{2}$$

32) Given that $\cos \theta = \frac{1}{2}$, find all possible values of θ in the range $0 \leq \theta \leq 360$.

33) If first and second term of arithmetic progression is $2s$ and $3/2s$ respectively, then find the 20th term.

34) Find the first two terms in the expansion of $(1-2x)^5$, in ascending powers of x .

35) If a circle has center $(4, 4)$ and touches the origin then find the equation of circle.

36) Find the slope of the line which is passing through the points $(3, -1)$ and $(-2, -3)$.

37) Find the slope of the given line $2x - 4y = 10$.

38) Are the lines $x + 2y = 10$ and $6x - 3y = 7$ parallel, perpendicular, or neither?

39) Without using the calculator, find the exact value of $\cos 150^\circ$.

40) Find the Relative frequencies for the frequency distribution given below:

X	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89
F	6	11	15	18	17	21

41) Following is the group frequency distribution of 87 passengers. Find the arithmetic mean.

Class Interval	No. Of Passengers
10-20	5
20-30	11
30-40	18
40-50	22
50-60	14
60-70	10
70-80	07

42) Salary of an employee increased by 2%, 2.25%, and 3% in three consecutive years. What is the average growth rate of salary?

43) Find the mean, median, mode, and geometric mean of the following data.

16, 9, 7, 11, 8, 9, 16, 19, 14, 13, 17, 16

44) Find the mean, mode and median of the given grouped data

Values	Frequency
51 - 55	2
56 - 60	7
61 - 65	8
66 - 70	4

45) Find the mean, mode and median of the given grouped data

Length (mm)	Frequency
150 - 154	5
155 - 159	2
160 - 164	6
165 - 169	8
170 - 174	9
175 - 179	11
180 - 184	6
185 - 189	3

46) Give a five number summary of the data. i.e. Minimum, First Quartile, Median, Third Quartile, and maximum value.

10, 19, 25, 17, 33, 29, 11, 40, 38, 15, 35

47) Find mean absolute deviation and standard deviation for the following data
25, 22, 27, 29, 35, 39, 41, 37, 45, 51, 43

48) If the mean is 285, the median is 330 and the mode is 340, then find that either the distribution positively skewed, negatively skewed, or symmetrical? Give reason.

49) Find standard deviation for the following data
36, 42, 47, 29, 41, 45, 52, 37, 59, 63, 29

Solution:

Mean of the data = 43.63

x	x - 43.63	(x - 43.63) ²
36	-7.63	58.2169
42	-1.63	2.6569
47	3.37	11.3569
29	-14.63	214.0369
41	-2.63	6.9169
45	1.37	1.8769
52	8.37	70.0569
37	-6.63	43.9569
59	15.37	236.2369
63	19.37	375.1969
29	-14.63	214.0369
Total		1234.5459

Now variance = $1234.5459 / (11) = 112.2314$

Standard Deviation = 10.5939

50) Find Mean absolute Deviation from Mean of 3, 4 and 2 with mean = 3.

Solution:

X	$ X - \bar{X} $
3	0
4	1
2	1
Total	2

Mean = 3

$$M.D = \frac{\sum |X - \bar{X}|}{n}$$
$$= \frac{2}{3}$$

- 51)** What is the essential information which must be given to give a five number summary of the data set?
Solution: First quartile Median Third quartile Lowest value Highest value
- 52)** Average sale and standard deviation for a store are 18 and 6.5 respectively .Find coefficient of variation.
Solution =36.11
- 53)** Mean and median of frequency distributions are 18 and 25 respectively. Find mode of the distribution.
Solution:
Mode= 3 median – 2 mean
Mode = 3 (25) –2(18) = 75-36=39
- 54)** An automobile dealership pays to its salespeople, a salary plus a commission on sales. The mean biweekly commission is \$1395, the median is \$1350 and the mode is \$65. Is the distribution of commissions positively skewed, negatively skewed, or symmetrical? Give reason.
Solution: Since, Mean > Median > Mode Hence the distribution of commission is positively skewed.
- 55)** A bag contains 9 balls. Among them 4 are blue and 5 are red. One ball is taken out at random from the bag. Find the probability the ball is red.
Solution: Total number of balls = 9 Probability that the first ball drawn is blue = 4/9.
- 56) Without using the calculator, find the exact value of Cos 210° .**

$$\begin{aligned}\cos(210^\circ) &= \cos(180^\circ + 30^\circ) \\ &= \cos 180^\circ \cos 30^\circ - \sin 180^\circ \sin 30^\circ \\ &= (-1)\left(\frac{\sqrt{3}}{2}\right) - (0)\left(\frac{1}{2}\right) \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$