PRACTICE QUESTIONS

By Pin2

MTH632

Lecture No. 2

Modules (5 – 7)

Q1: Verify commutative law with respect to multiplication for complex numbers $z_1 = 2 + i$, $z_2 = -3 + 9i$.

Solution: The commutative law is $z_1 + z_2 = z_2 + z_1$

Consider $z_1 + z_2 = (2 + i) + (-3 + 9i)$

$= 2 - 3 + i + 9i$

$= -1 + 10i$

Now $z_2 + z_1 = (-3 + 9i) + (2 + i)$

$= -3 + 2 + 9i + i$

$= -1 + 10i$

Hence $z_1 + z_2 = z_2 + z_1$

Q2: Verify the associative law with respect to multiplication for complex numbers $z_1 = 2 + i$, $z_2 = -3 + 9i$ and $z_3 = -8 + i$.

Solution: The associative law is $(z_1\ z_2)\ z_3 = z_1(z_2\ z_3)$

Consider $z_1\ z_2 = (2 + i)(-3 + 9i)$

$= 2(-3) + 2(9i) + i(-3) + (i)(9i)$

$= -6 + 18i - 3i + 9i^2$

$= -6 + 18i - 3i + 9(-1)$

$= -6 - 9 + 18i - 3i$

$= -12 + 15i$

Now $(z_1\ z_2)\ z_3 = (-12 + 15i)(-8 + i)$

$= -12(-8) + (-12(i)) + 15i(-8) + (15i)(i)$

$= 96 - 12i - 120i + 15i^2$

$= 96 - 12i - 120i + 15(-1)$

$= 81 - 132i$

Similarly, you can find $z_1(z_2\ z_3)$.

Q3: Find the multiplicative inverse z^{-1} of the complex numbers $z = -5 + i$.

Solution:

$$z^{-1} = \frac{1}{z} = \frac{1}{-5+1} = \frac{-5}{(-5)^2+1^2} - i\frac{1}{(-5)^2+1^2} \quad \text{by formula}$$

$$= \frac{-5}{25+1} - i\frac{1}{25+1}$$

$$= \frac{-5}{26} - i\frac{1}{26}$$

Q4: Write down the Distributive law of multiplication over addition.

Solution: The Distributive law is $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$

Q5: Let $z_1 = -8 + i$, $z_2 = -3 + 8i$, then calculate $\frac{z_1}{z_2}$.

Solution:

$$\frac{z_1}{z_2} = (x_1 + iy_1)\left(\frac{x_2}{x_2^2+y_2^2} - i\frac{y_2}{x_2^2+y_2^2}\right)$$

$$= \left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}, \frac{y_1x_2-x_1y_2}{x_2^2+y_2^2}\right)$$

$$= \left(\frac{(-8)(-3)+(1)(8)}{(-3)^2+(8)^2}, \frac{(1)(-3)-(-8)(8)}{(-3)^2+(8)^2}\right)$$

$$= \left(\frac{32}{(-3)^2+(8)^2}, \frac{61}{(-3)^2+(8)^2}\right)$$

Q6: Find the modulus $|z|$ of the complex numbers $z = -7 + 5i$.

Solution: $|z| = |-7 + 5i| = \sqrt{7^2+5^2} = \sqrt{49+25} = \sqrt{74}$

Q7: Find the distance of the complex number $z_1 = 2 + i$ from $z_2 = -3 + 9i$.

Solution: Distance $= \sqrt{(2-(-3))^2+(1-9)^2} = \sqrt{(5)^2+(-8)^2}$

$= \sqrt{25+64} = \sqrt{89}$

Q8: Show that $|z_1 + z_2| \geq \big||z_1| - |z_2|\big|$

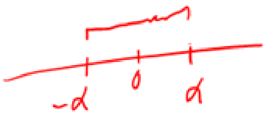
Proof

$|z_1| = |z_1 + z_2 - z_2| \leq |z_1 + z_2| + |-z_2| \quad \because |z| = |-z|$

$\Rightarrow |z_1| - |z_2| \leq |z_1 + z_2| \quad — I$

Interchanging z_1 & z_2

$|z_2| - |z_1| \leq |z_1 + z_2|$



$-\alpha < x < \alpha$

$|x| < \alpha$

$\Rightarrow -(|z_1| - |z_2|) \leq |z_1 + z_2|$

$|z_1| - |z_2| \geq -|z_1 + z_2| \quad — II$

$-|z_1 + z_2| \leq |z_1| - |z_2| \leq |z_1 + z_2| \quad — III$