
Assignment No.2 (Course BIO401) SPRING 2024

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Question 1:

Part a): Find $E(X)$ and $SD(X)$ of a binomial distribution when $n = 10$, $p = 1/3$, $q = 2/3$

Part b): In a hyper geometric distribution the $N=15$, $n=4$, and $k=3$, find $E(X)$ and $SD(X)$.

Solution:

Part a)

For a binomial distribution, the expected value $E(X)$ and standard deviation $SD(X)$ can be calculated by using the formula:

$$E(X) = np$$

$$SD(X) = \sqrt{np(1-p)}$$

Given:

◦ $n = 10$

◦ $p = \frac{1}{3}$

◦ $q = \frac{2}{3}$

Calculations:

Expected value ($E(X)$):

$$E(X) = 10 * \frac{1}{3}$$

$$E(X) = \frac{10}{3}$$

$$E(X) \approx 3.33$$

Standard deviation ($SD(X)$):

$$SD(X) = \sqrt{10 * \frac{1}{3} \left(1 - \frac{1}{3}\right)}$$

$$SD(X) = \sqrt{10 * \frac{1}{3} * \frac{2}{3}}$$

$$SD(X) = \sqrt{\frac{20}{9}}$$

$$SD(X) \approx 1.49$$

Part b)

For a hyper geometric distribution, the expected value $E(X)$ and standard deviation $SD(X)$ can be calculated by using the formula:

$$E X = n * \frac{k}{N}$$

$$SD X = \sqrt{n * \frac{k}{N} \left(1 - \frac{k}{N}\right) * \frac{N-n}{N-1}}$$

Given:

- $n = 4$ number of draws
- $N = 15$ total population
- $k = \frac{2}{3}$ number of successes in the population

Calculations:

Expected value ($E X$):

$$E X = 4 * \frac{3}{15}$$

$$E X = 4 * \frac{1}{5}$$

$$E X = 0.8$$

Standard deviation ($SD X$):

$$SD X = \sqrt{4 * \frac{3}{15} \left(1 - \frac{3}{15}\right) * \frac{15-4}{15-1}}$$

$$SD X = \sqrt{4 * \frac{1}{5} * \frac{4}{5} * \frac{15-4}{15-1}}$$

$$SD X = \sqrt{\frac{17}{35}}$$

Question 2:

A computer store's stock comprises 3 fully operational units and 2 defective units. If two units are randomly selected in succession without replacement, compute the probability that:

- i) both units selected are fully operational.
- ii) The selection results in one fully operational unit and one defective unit.

Solution:

Probability that both units selected are fully operational:

When two units are selected without replacement, the sequence matters. For the 1st unit to be fully operational, we have 2 good ones out of 5:

$$P \text{ 1st operational} = \frac{3}{5}$$

After selecting one fully operational unit, we are left 2 fully operational units and a total of 4:

$$P \text{ 2nd operational} \mid \text{1st operational} = \frac{2}{4} = \frac{1}{2}$$

So, the probability of both units being fully operational is:

$$\begin{aligned}
 P \text{ both operational} &= P \text{ 1st operational} * P \text{ 2nd operational} | \text{ 1st operational} \\
 &= \frac{3}{5} * \frac{1}{2} \\
 &= \frac{3}{10}
 \end{aligned}$$

Probability that the selection results in one fully operational unit and one defective:

This event can occur in two different sequences:

1. The 1st unit is selected is fully operational and 2nd is defective
2. The 1st unit is selected is defective and 2nd is fully operational

1st case (operational then defective)

Probability that 1st unit fully operational:

$$P \text{ 1st operational} = \frac{3}{5}$$

Probability that 2nd unit is defective:

$$P \text{ 2nd operational} | \text{ 1st operational} = \frac{2}{4} = \frac{1}{2}$$

2nd case (defective then operational)

Probability that 1st unit is defective:

$$P \text{ 1st defective} = \frac{2}{5}$$

Probability that 2nd unit operational:

$$P \text{ 2nd operational} | \text{ 1st operational} = \frac{3}{4}$$

Adding these probabilities:

$$\begin{aligned}
 P \text{ operational then defective} &= \frac{3}{5} * \frac{1}{2} \\
 &= \frac{3}{10}
 \end{aligned}$$

$$\begin{aligned}
 P \text{ defective then operational} &= \frac{2}{5} * \frac{3}{4} \\
 &= \frac{6}{20} \\
 &= \frac{3}{10}
 \end{aligned}$$

The total probability is:

$$\begin{aligned}
 P &= \frac{3}{10} + \frac{3}{10} \\
 &= \frac{6}{20} \\
 &= \frac{3}{10}
 \end{aligned}$$

