

STA301

Lec # 23

How to find probability.

x	$P(x)$
3	6
4	1
5	1
6	1
$\Sigma x =$ 13	

$$\therefore P(x) = \frac{x}{\Sigma x} \quad \therefore \frac{3}{18} = \frac{1}{6}$$

$$\mu = \Sigma x p(x) \quad \therefore \mu = E(x) = \text{Expected value}$$

$$\text{variance} = \sigma^2 = \Sigma x^2 p(x) + (\Sigma x p(x))^2$$

$$\text{Standard deviation} = \sigma = \sqrt{\Sigma x^2 p(x) + (\Sigma x p(x))^2}$$

$$E(ax + b) = aE(x) + b$$

Chebyshev's Theorem

$$(k-1) \leq P(X \leq \mu + k\sigma) \leq 1 - \frac{1}{k^2}$$

$$\int_{-\infty}^{\infty} f(x) dx$$

Leibniz #25

Moments $\mu'_1 = E(X)$

$$\mu'_2 = E(X^2)$$

$$\mu'_3 = E(X^3)$$

$$\mu'_4 = E(X^4)$$

$$\int_{-\infty}^{\infty} x f(x) dx$$

To find mean

$$\mu_2 = 0$$

$$\mu_1 = \mu'_1 - (\mu'_1)^2$$

Moment Ratio = $\frac{\mu'_2}{\mu_1^2}$

$$\frac{\mu'_2}{\mu_1^2} > 1$$

Joint probability distributions

$$g(x_i, y_j) = \sum_{j=1}^m f(x_i, y_j)$$

$$h(y_j) = \sum_{i=1}^m f(x_i, y_j)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{f(x_i, y_j)}{g(x_i)}$$

$$f(x_i, y_j) = f(x_i, y_j) \cdot h(y_j)$$

$$\sum f(x_i, y_j) = 1$$

$$f(x_i, y_j) > 0$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$f(x, y) = g(x) \cdot h(y) \text{ Independent}$$

$$f(x, y) \neq g(x) \cdot h(y) \text{ non-Independent}$$

3

$$E(X+Y) = E(X) + E(Y)$$

$$E(X-Y) = E(X) - E(Y)$$

$$E(XY) = E(X) \cdot E(Y) \Rightarrow$$

for
independence

$$E(X) = \sum x g(x)$$

$$E(Y) = \sum y h(y)$$

$$E(X^2) = \sum x^2 g(x)$$

$$E(Y^2) = \sum y^2 h(y)$$

$$\text{variance}(X) = \sigma^2 = E(X^2) - [E(X)]^2$$

$$\text{variance}(Y) = \sigma^2 = E(Y^2) - [E(Y)]^2$$

$$E[XY] = \sum \sum x \cdot y f(x, y)$$

$$g(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

$$h(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

$$\mu = \mu', E(X) = \int_{-\infty}^{\infty} x \cdot g(x) dx$$

$$\mu = \mu'', E(X^2) = \int_{-\infty}^{\infty} x^2 g(x) dx$$

$$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x, y) f(x, y) dy dx$$

$$\text{cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$$

Binomial Distribution

$$P(X=x) = {}^n C_x p^x q^{n-x} \quad \because x=0, 1, 2, \dots, n$$

It has two parameters n, p

$$\mu = np \quad \sigma^2 = npq$$

$$\mu = E(X) \quad \sigma = \sqrt{npq}$$

* If value of p, q is ~~not~~ equal it is called symmetrical B.D.

* If $p < q$ it is called positively skewed

* If $p > q$ it is negatively skewed

Hypergeometric Experiment:-

$$P(X=x) = \frac{{}^k C_x \cdot {}^{N-k} C_{n-x}}{{}^N C_n}$$

It has 3 params N, n, k

* $\mu = n \cdot \frac{k}{N}$

* $\sigma^2 = n \cdot \frac{k}{N} \cdot \frac{N-k}{N} \cdot \frac{N-n}{N-1}$

* $C.V = \frac{\sigma}{\mu} \times 100$

→ in with replacement → trials become independent
 ↓ Binomial distribution (ghata hidi)

	Population	Finite	Infinite
* Sampling			
with replacement	BD	BD	
without replacement	H.G.D	BD	

(WOR)
 * if $n < \frac{5}{100} N$ then Binomial formula
 * if $n > \frac{5}{100} N$ then Hypergeometric (WOR)

* Poisson Distribution

$$P(X=x) = \frac{e^{-\mu} \mu^x}{x!} \quad \text{--- } x=0, 1, 2, 3, \dots$$

∴ $\mu = np$

∴ $e = 2.71828$

* Poisson Process:

$$P(X=x) = \frac{e^{-\lambda t} (\lambda t)^x}{x!} \quad \text{here } \mu = \lambda t$$

- * $E(X) = \mu = \text{Var}(X)$
- * poisson can never be negatively skewed

* Distribution

$\mu = \frac{a+b}{2}$ and variance $= \frac{\sigma^2(b-a)^2}{12}$

- * Normal Distribution = $f(x) = \frac{1}{\sigma(\sqrt{2\pi})} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$ $-\infty < x < \infty$

$\pi \approx 3.1416$

$e \approx 2.71828$

- * Properties from handout

$\mu_4 = 3\mu_2^2$ $\mu_3 = 0$

- * $B_2 > 3$ Lepto kurtic
- * $B_2 < 3$ platy kurtic
- * $B_2 = 3$ meso kurtic

* $P(\mu - \sigma \leq x \leq \mu + \sigma) = 0.6826$

$\mu + 2\sigma = 95.44\%$

$\mu + 3\sigma = 99.73\%$

Process of Standardization:

$$Z = \frac{X - \mu}{\sigma}$$

$$\therefore \bar{Z} = 0, \text{ S.D.}(Z) = 1$$

(X)

Population = N

Sample = n

Numbers of samples with replacement = N^n

Numbers of samples without replacement = C_n^N

Verification with Replacement:-

$$\bullet \mu_x = \mu$$

$$\bullet \sigma_x^2 = \frac{\sigma^2}{n}$$

$$\bullet \sigma_x^2 = \frac{\sigma^2}{n}$$

Verification without Replacement:-

$$\bullet \mu_x = \mu$$

$$\bullet \sigma_x^2 = \frac{\sigma^2}{n} \cdot \left(\frac{N-n}{N-1} \right)$$

$$\bullet \sigma_x = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

$$\mu_{\bar{x}} = E(\bar{x}) = \sum \bar{x} f(\bar{x})$$

$$\sigma_{\bar{x}}^2 = \text{var}(\bar{x}) = E(\bar{x}^2) - [E(\bar{x})]^2$$

$$\sigma_{\bar{x}} = \sqrt{\sigma_{\bar{x}}^2}$$

(4)

* Pop. Size = $N = 310$

Pop. mean = Pop. Average = $\mu = 24,500$

Pop. SD = $\sigma = 5000$

Sample size = $n = 100$

Sample mean = Sample Average = $\bar{x} = 24,500$

$P(\bar{x} > 24,500) = ?$

$$\mu_{\bar{x}} = \mu = 24,500$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5000}{\sqrt{100}} = 500$$

$$Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{24,500 - 24,500}{500} = 0$$

$$P(\bar{x} > 24,500) = P(Z > 0) = 0.5$$

$$= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \geq \frac{245000 - 240000}{412.20}\right)$$

$$= P(Z \geq 1.21)$$

$$= 0.50 - P(0 < Z < 1.21)$$

$$= 0.50 - 0.3869 = 0.1131 = 11.31\%$$

* Population proportion = $\frac{p \times X}{N}$

* Sample Proportion = $\frac{\sum x}{n}$

Example

Pop > 1, 3, 6, 8, 9, 12

N > 6, n = 3 .WOR

$$m \Rightarrow {}^N C_n = {}^6 C_3 = 20$$

(2) $\hat{p} = \frac{\sum x}{n}$

Sample \hat{p} samples

1, 3, 6 $\hat{p} = \frac{1}{3}$ 3, 6, 8 $\hat{p} = \frac{2}{3}$

1, 3, 8 $\hat{p} = \frac{1}{3}$ 3, 6, 9 $\hat{p} = \frac{1}{3}$

1, 3, 9 $\hat{p} = \frac{2}{3}$ 3, 6, 12 $\hat{p} = \frac{2}{3}$

1, 3, 12 $\hat{p} = \frac{1}{3}$ 8, 8, 9 $\hat{p} = \frac{1}{3}$

1, 6, 8 $\hat{p} = \frac{2}{3}$ 3, 8, 12 $\hat{p} = \frac{2}{3}$

1, 6, 9 $\hat{p} = \frac{1}{3}$ 3, 9, 12 $\hat{p} = \frac{1}{3}$

1, 6, 12 $\hat{p} = \frac{2}{3}$ 6, 8, 9 $\hat{p} = \frac{2}{3}$

1, 8, 9 $\hat{p} = \frac{1}{3}$ 6, 8, 12 $\hat{p} = \frac{2}{3}$

1, 8, 12 $\hat{p} = \frac{2}{3}$ 6, 9, 12 $\hat{p} = \frac{2}{3}$

1, 9, 12 $\hat{p} = \frac{1}{3}$ 8, 9, 12 $\hat{p} = \frac{2}{3}$

Sampling of \hat{p}

\hat{p}	f	f(P)	Pf(P)	$\hat{p}^2 f(P)$
$\frac{1}{3}$	1	$\frac{1}{20}$	$\frac{1}{60}$	$\frac{1}{180}$
$\frac{2}{3}$	9	$\frac{9}{20}$	$\frac{9}{60}$	$\frac{36}{180}$
$\frac{1}{3}$	9	$\frac{9}{20}$	$\frac{9}{60}$	$\frac{9}{180}$
$\frac{2}{3}$	1	$\frac{1}{20}$	$\frac{3}{60}$	$\frac{4}{180}$
$\frac{3}{3}$	1	$\frac{1}{20}$	$\frac{3}{60}$	$\frac{4}{180}$
			<u>$\frac{45}{180}$</u>	<u>$\frac{45}{180}$</u>

Ex#

MTWTFSS

Lec# 33

DATE: _____

A	B
$\mu_1 = 4.3$	$\mu_2 = 4.0$
$\sigma_1 = 0.6$	$\sigma_2 = 0.4$
$n_1 = 49$	$n_2 = 36$
\bar{X}_1	\bar{X}_2

$P(\bar{X}_1 - \bar{X}_2 \geq 0.5) = ?$

First

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 = 4.3 - 4.0 = 0.3$$

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = 0.1086$$

$$P(\bar{X}_1 - \bar{X}_2 \geq 0.5)$$

$$= P\left(\frac{\bar{X}_1 - \bar{X}_2 - \mu_{\bar{X}_1 - \bar{X}_2}}{\sigma_{\bar{X}_1 - \bar{X}_2}} \geq \frac{0.5 - 0.3}{0.1086}\right)$$

①

MTWTFSS

DATE: _____

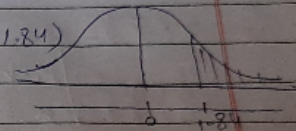
$$= P(Z \geq 1.84)$$

$$= 0.5 - P(0 < Z < 1.84)$$

$$= 0.5 - 0.4671$$

$$= 0.0329$$

$$= 3.29\%$$



$$= \bar{X}_1 - \bar{X}_2 = d$$

Example:-

A	B
$P_1 = 30\% = 0.30$	$P_2 = 20\% = 0.20$
$n_1 = 100$	$n_2 = 100$
$\hat{p}_1 = 0.34$	$\hat{p}_2 = 0.13$
$\hat{p}_1 - \hat{p}_2 = 0.34 - 0.13$	
$\quad \quad \quad = 0.21$	

$P(\hat{p}_1 - \hat{p}_2 \geq 0.21) = ?$

$$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2 = 0.30 - 0.20 = 0.10$$

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{p_1 q_1 + p_2 q_2}$$

$$= \sqrt{\frac{0.30(0.70)}{150} + \frac{0.20(0.80)}{150}} = 0.037$$

Now!

$$P(\hat{p}_1 - \hat{p}_2 \geq 0.21)$$

$$P\left(\frac{(\hat{p}_1 - \hat{p}_2) - \mu_{\hat{p}_1 - \hat{p}_2}}{\sigma_{\hat{p}_1 - \hat{p}_2}} \geq \frac{0.21 - 0.10}{0.037}\right)$$

$$P(Z \geq 1.83)$$

$$= 0.50 - P(0 < Z \leq 1.83) \\ = 0.50 - 0.4664 \\ = 0.0336 \\ = 3.36\%$$

Point Estimation: =

pop. parameter (constant val)

$\theta = \mu, \sigma^2, \sigma, \rho, \text{etc}$

sample statistic / random variable

$\hat{\theta} = \bar{X}, s^2/s^2, S/S, \hat{p} \text{ etc}$

$$\bar{X} = \frac{\sum X}{n}$$

$$\bar{X} \sim T = \hat{\theta} = f(X)$$

Efficiency:

$$\rightarrow E(\bar{X}) = \mu \quad E(\tilde{X}) = \mu$$

$$\text{var}(\bar{X}) = ? \quad \text{var}(\tilde{X}) = ?$$

$$\rightarrow \text{var}(\bar{X}) = E(\bar{X}^2) - [E(\bar{X})]^2$$

$$\rightarrow \text{var}(\tilde{X}) = E(\tilde{X}^2) - [E(\tilde{X})]^2$$

$$\rightarrow \text{var}(\bar{X}) < \text{var}(\tilde{X})$$

\bar{X} is more efficient than \tilde{X} for population mean-estimator

OR

$$\rightarrow E_f = \frac{\text{var}(\tilde{X})}{\text{var}(\bar{X})} = \frac{3}{2} = 1.5 > 1$$

$$\rightarrow E_f = \frac{\text{var}(\tilde{X})}{\text{var}(\bar{X})} = \frac{2}{3} = 0.67 < 1$$

Point estimation: Ex#

$$N = 1000, \quad \mu = \text{pop. mean weight} \Rightarrow$$

$$n = 100 \rightarrow \bar{X} = \frac{\sum x}{n} = \frac{6500}{100} = 65 \text{ kg}$$

on the average students have weight 65 kg.

Methods of Estimation: =

* Methods of Moments: =

$$m'_1 = \sum x' = \bar{X} = \text{sample moment about origin}$$

$$\text{Population moment about origin} \mu'_1 = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

$$= \int_0^{\infty} x \cdot \frac{1}{\theta} dx$$

$$= \frac{1}{\sigma} \left[\frac{x^2}{2} \right]_0^{\sigma}$$

$$= \frac{1}{\sigma} \left[\frac{\sigma^2}{2} - \frac{0^2}{2} \right]$$

$$\mu_1 = \frac{1}{\sigma} \cdot \frac{\sigma^2}{2} = \frac{\sigma}{2}$$

Equating $m_1' = \mu_1$

$$\frac{\sum x}{n} = \frac{\sigma}{2}$$

$$\bar{x} = \frac{\sigma}{2}$$

$$2\bar{x} = \sigma$$

put hab

$$\sigma = 2\bar{x}$$

$$m_1' = \frac{\sum x^1}{n}$$

$$m_1' = \frac{\sum x^1}{n} = \bar{x}$$

$$m_2' = \frac{\sum x^2}{n}$$

$$\mu_1' = \int_{-\infty}^{\infty} x^1 f(x) dx$$

$$\mu_1', \mu_2'$$

As

$$\mu_1' = \mu$$

$$\mu_2' = \mu_2' - \mu^2$$

$$\sigma^2 = \mu_2' + \mu^2$$

$$\sigma^2 + \mu^2 = \mu_2'$$

Equating

$$\mu_1' = m_1'$$

$$\mu = \bar{x}$$

$$\mu = \bar{x}$$

also

$$\mu_2' = m_2'$$

$$\sigma^2 + \mu^2 = \frac{\sum x^2}{n} \Rightarrow \sigma^2 = s^2$$

$$\sigma^2 + (\bar{x})^2 = \frac{\sum x^2}{n}$$

$$\sigma^2 = \frac{\sum x^2}{n} - (\bar{x})^2$$

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2$$

$$\sigma^2 = s^2$$

method of least square

$$X = a + bx$$

method of maximum likelihood

poisson distribution MLE = \bar{X}

$$\text{G.D} = \frac{1}{X}$$

$$\text{B.D} = \bar{X}^X$$

Interval Estimation:-

$$* X \sim N(\mu, \sigma^2) = N(\mu, \sigma^2)$$

$$* \bar{X} \sim N(\mu_{\bar{X}}, \sigma_{\bar{X}}^2) = N(\mu, \frac{\sigma^2}{n})$$

$$* \hat{p} \sim N(\mu_{\hat{p}}, \sigma_{\hat{p}}^2) = (p, \frac{pq}{n})$$

$$* (\bar{X}_1 - \bar{X}_2) \sim N(\mu_{\bar{X}_1 - \bar{X}_2}, \sigma_{\bar{X}_1 - \bar{X}_2}^2)$$

$$* (\hat{p}_1 - \hat{p}_2) \sim N(\mu_{\hat{p}_1 - \hat{p}_2}, \sigma_{\hat{p}_1 - \hat{p}_2}^2)$$

$$(\bar{X}_1 - \bar{X}_2) \sim N(\mu_{\bar{X}_1 - \bar{X}_2}, \sigma_{\bar{X}_1 - \bar{X}_2}^2) = \left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} \right)$$

$$* Z = \frac{X - \mu}{\sigma}$$

$$* Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$* Z \sim N(0, 1)$$

$$\mu_Z = \bar{Z} = 0, \quad \sigma_Z^2 = 1$$

$$* P(-1.96 < Z < 1.96) = 0.95$$

$$* P\left(-1.96 < \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} < 1.96\right) = 0.95$$

* C.I for mean (pop) (μ)

$$\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$\therefore n \geq \frac{z^2 \sigma^2}{E^2}$

* Confidence interval difference
Between the means of Two
populations: -

$$(\bar{X}_1 - \bar{X}_2) \pm z_{\alpha/2} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

when $n_1 \geq 30, n_2 \geq 30$

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= \hat{p} = \frac{x}{n}$$

confidence interval for $P_1 - P_2$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$\hat{q}_1 = 1 - \hat{p}_1$$

sample size for estimating population mean

$$* n = \left(\frac{z_{\alpha/2} \sigma}{e} \right)^2 \quad e = |\bar{x} - \mu|$$

$\rightarrow z_{\alpha/2} = 1.96$

sample size for estimating population proportions

$$n = \frac{(z_{\alpha/2})^2 \hat{p} \hat{q}}{e^2}$$

Hypothesis test

Null Hypothesis

Alternative Hypothesis

Also local
Sankhyat

Type-I and Type II ERROR

$$\alpha = P(\text{TYPE I Error})$$

$$= P(\text{Reject } H_0 | H_0 \text{ is True})$$

$$\beta = P(\text{TYPE II Error})$$

$$= P(\text{Accept } H_0 | H_0 \text{ is False})$$

Pop. size	Sample size
parameter $N > 10000 \Rightarrow$ Popul.	$n = 100$ Statistics
$\mu = \frac{\sum X}{N}$ Pop. mean	$\bar{X} = \frac{\sum x}{n}$ Statistics
	$n \cdot \Downarrow$ Random Variable

next from Handout
and also: click apic in whtp-

s^2 is unbiased estimator of σ^2

~~σ^2 is unbiased~~

S^2 is biased estimator

$$\text{mean } E(S^2) \neq \sigma^2$$

$$E(s^2) = \sigma^2$$

also

- * $n < 30$ use s^2 & t-distribution
- * $n \geq 30$ use s^2 or S^2 , z-distribution

- * $\mu_1 - \mu_2 = 0$ Insignificant difference
- * $\mu_1 - \mu_2 \neq 0$ Significant difference

$$Z = \frac{X \pm \frac{1}{2} - nP_0}{\sqrt{nP_0(1-P_0)}}$$

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - \mu_{\hat{P}_1 - \hat{P}_2}}{\sigma_{\hat{P}_1 - \hat{P}_2}}$$

$$= \frac{(\hat{P}_1 - \hat{P}_2) - (\hat{P}_1 - \hat{P}_2)}{\sqrt{P_1 q_1 + P_2 q_2}}$$

Example $\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$

$$= \frac{(\hat{P}_1 - \hat{P}_2) - 0}{\sqrt{\hat{P}_1 \hat{q}_1 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$\mu_{P_c} = \frac{X_1 + X_2}{n_1 + n_2}$$

$$\hat{P}_c = \frac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2}$$

T-Distribution

formula:-

$$y = f(x) = \frac{1}{\sqrt{v} B\left(\frac{1}{2}, \frac{v}{2}\right)} \left(\frac{1+x^2}{v} \right)^{-\frac{v+1}{2}}$$

It has only one parameter v , which is known as degree of freedom $v = n - 1$

variance of t-distribution:

$$\sigma^2 = \frac{v}{v-2} \text{ for } v > 2$$

Confidence Interval for μ (population mean)

σ^2 known

σ^2 unknown

$n \geq 30$ $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$

$n < 30$ $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$

Confidence Interval for $(\mu_1 - \mu_2)$ (Diff b/w pop mean)

σ_1^2, σ_2^2 known

σ_1^2, σ_2^2 unknown

$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2, n-1} \cdot s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

C.I for Proportion (p)

$\hat{q} = 1 - \hat{p}$

$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

C.I for Diff B/w Pop. proportion

$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

* σ unknown & $n < 30$

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \quad \therefore \nu = n - 1$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{(\alpha/2, \nu)} \cdot s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

u)

Example:-

$$\textcircled{1} \quad \begin{array}{ll} \mu_d = \mu_1 - \mu_2 = 0 & \mu_1 = \mu_2 \\ \mu_d = \mu_1 - \mu_2 \neq 0 & \mu_1 \neq \mu_2 \end{array}$$

$$\textcircled{1} \quad \begin{array}{l} H_0: \mu_d = 0 \\ H_1: \mu_d \neq 0 \end{array}$$

$$\textcircled{2} \quad \alpha = 0.05$$

$$\textcircled{3} \quad t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}} \quad t = 2.09$$

$$\textcircled{5} \quad t_{\alpha/2, \nu} = t_{(0.025, 0)}$$

$$\textcircled{6} \quad \begin{array}{l} H_0 \text{ Accepted} \\ H_0: \mu_d = 0 \end{array}$$