MTH100 - Mathematics

Practice Questions Final Term

1 - 75

Solved by Alif

Practice Questions
for Lecture no. 23 to 45

(1) $2x+3y=8$, $3y=-2x+8$, $y=-\frac{2}{3}x+\frac{8}{3}$

$m_1=-\frac{2}{3}$, so, $m_2=3/2$ and with $(6,7)$, eq. is:-

$y-7=\frac{3}{2}(x-6) \Rightarrow y-7=\frac{3}{2}x-9 \Rightarrow \boxed{y=\frac{3}{2}x-2}$

(2) $(-5,-2), (-6,7)$ $m=\frac{7+2}{-6+5}=\frac{9}{-1}=\boxed{-9}$

(3) $6x-8y=26 \Rightarrow -8y=-6x+26 \Rightarrow y=\frac{3}{4}x-13/4$

So, slope is $m=3/4$

(4) $2x+3y=7$, $3x+5y=3$

$6x+9y=21$

$\pm 6x \pm 10y = \pm 6$

$-y=27$

$-y=15$

$y=-15$

$2x+3(-15)=7$

$2x-45=7$

$2x=7+45$

$x=52/2$

$\boxed{x=26}$

Date: Day:

(5) $A(2,8), B(6,2) \rightarrow M = \left(\frac{2+6}{2}, \frac{8+2}{2}\right) = (4,5)$

$m = \frac{8-2}{2-6} = \frac{6}{-4} = -3/2 \Rightarrow m_2 = 2/3$

Equation:- $y - 5 = \frac{2}{3}(x-4)$

(6) $(-8,2), (6,-1) \Rightarrow m = \frac{-1-2}{6+8} = \boxed{-3/14}$

(7) $(x+5)^2 + (y-3)^2 = 36$, $r = 6$, center $= (-5,3)$

(8) $4x+5y=6$, $5x-4y=2$ $\quad m_1 \cdot m_2 = -1$

$m_1 = -4/5$ $\quad m_2 = 5/4$ $\quad \therefore$ Perpendicular

$-\frac{4}{5} \times \frac{5}{4} = -1$

(9) $3x+y=-4$, $3x+3y=1 \rightarrow 3x+y=-4$

$m_1 = -3$ $\quad m_2 = -1$ $\quad \pm 3x \pm 3y = \pm 1$

$3x + \frac{5}{2} = -4 \leftarrow -2y = -5 \Rightarrow \boxed{y = +5/2}$

$3x = -4 - \frac{5}{2} \Rightarrow 3x = \frac{-8-5}{2} \Rightarrow 3x = \frac{-13/2}{3} = \boxed{-13/6}$

Practice Questions

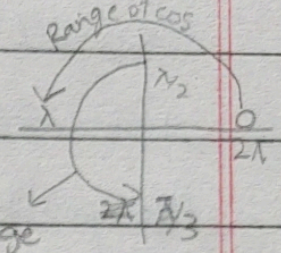
☆ Lecture # 23 to 45:-

10- Domain of $y = \arcsin x$ is $(-1, 1)$

Range of $y = \arcsin x$ is $(-\pi/2, \pi/2)$

Domain of $y = \arccos x$ is $(-1, 1)$

Range of $y = \arccos x$ is $(0, \pi)$



11- (i) $\tan^2\theta + 1 = \sec^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \sec^2\theta$

$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \sec^2\theta$

$\frac{1}{\cos^2\theta} = \sec^2\theta$

$\boxed{\sec^2\theta = \sec^2\theta}$

(ii) $1 + \cot^2\theta = \csc^2\theta$

$1 + \frac{\cos^2\theta}{\sin^2\theta} = \csc^2\theta$

$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \csc^2\theta$

$\frac{1}{\sin^2\theta} = \csc^2\theta$

$\boxed{\csc^2\theta = \csc^2\theta}$

12- $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

13- $\frac{\tan y}{\sin y} = \sec y \Rightarrow \frac{\sin y/\cos y}{\sin y} = \sec y \Rightarrow \frac{\sin y}{\sin y \cos y} = \sec y$

$\frac{1}{\cos y} = \sec y \Rightarrow \boxed{\sec y = \sec y}$

14- $\arcsin \frac{1}{\sqrt{2}} = ? \Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{(\sqrt{2})^2} = \frac{\sqrt{2}}{2} = 45° = \frac{\pi}{4}$

$\sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$ since $\frac{\pi}{4}$ is the angle whose sine is $\frac{1}{\sqrt{2}}$

15- $\cos 120° = ?$ $\cos(180-60) = \cos 180\cos 60 - \sin 180\sin 60$

$= (-1)(\frac{\sqrt{3}}{2})(\frac{1}{2}) - (0)(\frac{\sqrt{3}}{2}) = \boxed{-1/2}$

$\cos(210°) = \cos(180°+30°) = \cos 180°\cos 30° - \sin 180°\sin 30°$

$= (-1)(\sqrt{3}/2) - (0)(\frac{1}{2}) = \boxed{-\sqrt{3}/2}$

16- $\sin\theta = \sqrt{3}/2 \quad -180 \le \theta \le 180$

$\theta = \sin^{-1}\sqrt{3}/2 = 60°$, $\sin(180-\theta) = \sin\theta \Rightarrow \sin(180-60) = 120°$ ✓

~~180~~ $60+360 = 420°$ ✗, $60-360 = -300°$ ✗

Date:

$120+360=480^{\circ}$ ✗, ~~$+120-360=140$~~

$\sin(-\theta)=-\sin\theta = \sin(-60)=-\sin 60 = -60^{\circ}$ ✓

$180-(-60)=180+60=240^{\circ}$ ✗

~~$+60-360=$~~ $240-360=-120^{\circ}$ ✓

All possible solutions are $\pm 60^{\circ}, \pm 120^{\circ}$.

17- $\dfrac{1-\tan\theta}{1-\cot\theta} = -\tan\theta$ ~~⇒~~

L.H.S:- $= \dfrac{1-\tan\theta}{1-\frac{\cos\theta}{\sin\theta}} = \dfrac{1-\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} = \dfrac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} = \dfrac{\cos\theta-\sin\theta}{\cos\theta} \times \dfrac{\sin\theta}{-(\cos\theta-\sin\theta)}$

$= -\sin\theta/\cos\theta$ $\boxed{= -\tan\theta = R.H.S}$

18- Verify that $\cos 4A = \cos^2 2A - \sin^2 2A$

L.H.S:- $\cos 4A = \cos(2A+2A) = \cos 2A\cos 2A - \sin 2A \sin 2A$

$\boxed{= \cos^2 2A - \sin^2 2A = R.H.S}$

19- $\tan 2A = ?$ $\tan 2A = \dfrac{\tan A+\tan A}{1-\tan A\tan A} = \dfrac{2\tan A}{1-\tan^2 A}$

~~20-~~ 21- $\cos 15^{\circ} = ?$ $\sin 15^{\circ} = ?$ $45-30$

$\cos(45-30) = \cos 45\cos 30 + \sin 45\sin 30 = \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\times\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

$180-15=165$

$=180-165$

$=165^{\circ}$

$=180^{\circ}$

$\cos 15^{\circ} = \cos\left(\frac{30^{\circ}}{2}\right) = \cos 30^{\circ} = \sqrt{3}/2 \Rightarrow \cos\left(\frac{30}{2}\right) = \frac{\sqrt{3}/2}{2} = \boxed{\frac{\sqrt{3}}{4}}$

$\sin(45-30) = \sin 45\cos 30 - \cos 45\sin 30 = \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\times\frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\sin 15^{\circ} = \sin\left(\frac{30^{\circ}}{2}\right) = \sin 30^{\circ} = \frac{1}{2} \Rightarrow \sin\frac{30^{\circ}}{2} = \frac{1/2}{2} = \boxed{\frac{1}{4}}$

22 Prove: $\dfrac{\cot\theta-\tan\theta}{\sin\theta\cos\theta} = \operatorname{cosec}^2\theta - \sec^2\theta$

L.H.S:- $= \dfrac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{\sin\theta\cos\theta} = \dfrac{\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta-\sin^2\theta}{\sin^2\theta\cos^2\theta}$

R.H.S:-

$= \dfrac{1}{\sin^2\theta} - \dfrac{1}{\cos^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta} - \dfrac{\sin^2\theta}{\sin^2\theta\cos^2\theta} =$

$= \dfrac{1}{\sin^2\theta} - \dfrac{1}{\cos^2\theta}$ $\boxed{\csc^2\theta - \sec^2\theta = R.H.S}$

Day:

23- ? 25- ? 26-?

24- $\cos\theta = 1/2$, $0 \le \theta \le 360^\circ$

$\theta = \cos^{-1} 1/2 = 60^\circ$ ✓, ~~$\cos(-\theta) = \theta = \cos(60^\circ) = -60^\circ$~~ ×

$-60^\circ + 360^\circ = 300^\circ$ ✓, $\cos(180^\circ - 60^\circ) = -\cos 60^\circ$

~~$180 - 60 =$~~ $\theta = 60^\circ, 300^\circ$

Center of circle

27- $(4,4)$ ⇒ equation? ⇒ $y - y_1 = m(x - x_1)$

↳ $(4,0), (0,4)$ ⇒ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-4} = \frac{4}{-4} = -1$

$y - y_1 = m(x_2 - x_1) \Rightarrow y - 0 = -1(x - 4)$ $y - 4 = -1(x - 0)$

$y = -x + 4$ → verify $y = -x + 4$

28- m=? through $(3,-1), (-2,-3)$.

$m = \frac{-3+1}{-2-3} = \frac{-2}{-5} = \frac{2}{5}$

29- $2x - 4y = 10$, m=? $-4y = -2x + 10$

$y = \frac{1}{2}x - \frac{5}{2}$ ⇒ $m = \frac{1}{2}$

30- $x + 2y = 10 \Rightarrow 2y = -x + 10 \Rightarrow y = -\frac{1}{2}x + 5$

$6x - 3y = 7 \Rightarrow -3y = -6x + 7 \Rightarrow y = 2x - \frac{7}{3}$

both these lines are perpendicular.

31- $\cos 150^\circ = ? = \cos(180 - 30)^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

~~$\cos 15^\circ = \cos(180^\circ - 165)^\circ = \cos 165^\circ = \cos 180 - 15$~~

26- $(1-2x)^5 = {}^5C_0(1)^5(-2x)^0 + {}^5C_1(1)^4(-2x)^1 = (1)(1)(1) + 5(1)(-2x)$

$= 1 - 10x$

25- $a_1 = 2$, $a_2 = 3\frac{1}{2}$, $d = -0.5$, $a_{20} = 2 + (20-1)(-0.5)$

$a_{20} = 2 + (19)(-0.5) = 2 + (-9.5) = -7.5$

Practic Questions

Lecture # 23-45 :-

(32) Find Relative Frequencies :-

X	30-39	40-49	50-59	60-69	70-79	80-89	Total
F	6	11	15	18	17	21	88
R.F	0.068	0.125	0.170	0.204	0.193	0.238	

(33) Find Arithmetic Mean:

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	Total
f	5	11	18	22	14	10	7	$\Sigma f = 87$
x	15	25	35	45	55	65	75	
fx	75	275	630	990	770	650	525	$= 3915$

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{3915}{87} = \boxed{45}$$

(34) Average Growth rate of salary ?

$$GM = (1.02 \times 1.0225 \times 1.03)^{1/3} = (1.0742385)^{1/3}$$

(35) Find mean, median, mode and geometric mean of following data.

16, 9, 7, 11, 8, 9, 16, 19, 14, 13, 17, 16

$\bar{x} = \frac{\Sigma x}{n} = \frac{155}{12} = \boxed{12.9}$, A.O → 7, 8, 9, 9, 11, 13, 14, 16, 16, 16, 17, 19

Meadian $= \frac{13+14}{2} = \frac{27}{2} = \boxed{13.5}$, Mode $= \boxed{16}$

$G.M = \sqrt[12]{7\times8\times9\times9\times11\times13\times14\times16\times16\times16\times17\times19} = \sqrt[12]{(1.2014)^{13}} = \boxed{12.302}$

$= \text{Antilog}\left(\frac{\Sigma \log x}{12}\right)$

(36) Find mean, median and mode:

Values	51-55	56-60	61-65	66-70	Total
f	2	7	8	4	21
x	53	58	63	68	
fx	106	406	504	272	1288
CF	2	9	17	21	

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$M = L + \frac{h}{f}\left(\frac{n}{2} - cf\right)$

$M_o = L + \frac{\Delta_1}{\Delta_1+\Delta_2} h$

$\bar{x} = \frac{1288}{21} = \boxed{61.33}$, Median class is $\frac{21}{2} = 10.5 \rightarrow$ C.F → 17, Class 61-65

So, $M = 61 + \frac{4}{8}\left(\frac{21}{2} - 9\right) = 61 + \frac{1}{2}\left(\frac{3}{2}\right) = 61 + \frac{3}{4} = \frac{244+3}{4} = \frac{247}{4}, \boxed{61.75}$

~~$M_o = 6$~~ Modal class is the class with the highest frequency So, its same class.

$M_o = 61 - \frac{(8-7)}{(8-7)+(8-4)} \times 4 = 61 - \frac{1}{1+4} \times 4 = 61 - \frac{4}{5} = \frac{305-4}{5} = \frac{301}{5} = \boxed{60.2}$

(37) Find mean, median and mode of given grouped data.

Length (mm)	150-154	155-159	160-164	165-169	170-174	175-179	180-184	185-189	Total
Frequency (f)	5	2	6	8	9	11	6	3	50
Mid point (x)	152	157	162	167	172	177	182	187	
fx	760	314	972	1336	1548	1947	1092	561	8530
CF	5	7	13	21	30	41	47	50	

$\bar{x} = \frac{8530}{50} = \boxed{170.6}$, Median is $\frac{50}{2} = 25$th values which lies

class $\Rightarrow$ 170-174. So, $M = 170 + \frac{4}{9}\left(\frac{50}{2} - 21\right) = 170 + \frac{4}{9}(4)$

$M = \frac{1530+16}{9} = \frac{1546}{9} = \boxed{171.78}$

Modal class is the class with highest frequency, So, here is 175-179.

$M_0 = 175 + \frac{(11-9)}{(11-9)+(11-6)} \times 4 = 175 + \frac{2}{2+5} \times 4 = 175 + \frac{8}{5+0} = \frac{875+4}{5} = \frac{879}{5}$

$\boxed{M_0 = 175.8}$

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(38) 10, 19, 25, 17, 33, 29, 11, 40, 38, 15, 35

A.O → 10, 11, 15, 17, 19, 25, 29, 33, 35, 38, 40

$M_{min} = 10$, $M_{max} = 40$, Median = 25, $Q_1 = 15$, $Q_3 = 35$

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(39) Find MAD and SD :- 25, 22, 27, 29, 35, 39, 41, 37, 45, 51, 43

A.O → 22, 25, 27, 29, 35, 37, 39, 41, 43, 45, 51

$MAD = \frac{\Sigma|x_i - \mu|}{n}$ $SD = \sqrt{\frac{\Sigma(x_i-\mu)^2}{n}}$, $\mu = \frac{\Sigma x_i}{n} = \frac{394}{11} = \boxed{35.8}$

x	22	25	27	29	35	37	39	41	43	45	51	Total.
$x-\mu$	-13.8	-10.8	-8.8	-6.8	-0.8	1.2	3.2	5.2	7.2	9.2	15.2	
$\lvert x-\mu \rvert$	13.8	10.8	8.8	6.8	0.8	1.2	3.2	5.2	7.2	9.2	15.2	82.2
$(x-\mu)^2$	182.16	116.64	77.44	46.24	0.64	1.44	10.24	27.04	51.84	84.64	231.04	829.36

MAD = $\frac{82.2}{11} = \boxed{7.47}$, SD = $\sqrt{\frac{829.36}{11}} = \sqrt{75.3963} = \boxed{8.68}$

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(40) ~~This data is positively skewed. B/c in +ve skewed data, mean is of~~

This is negatively skewed data. B/c is ~~an~~ a negatively skewed data:-

mean < median < mode

(41) ✓ (44) Find CF

Output in tons	50-59	60-69	70-79	8089	90-99	100-109
f	3	11	19	28	20	15
CF	3	14	33	61	81	96

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(45) → This diagram represents bar chart.

(46) F.D = $\frac{\text{frequency}}{\text{class width}}$, 0.3 = $\frac{15}{\text{Class Width}} = \frac{15}{0.3} = \boxed{50}$

(47) F.D = $\frac{f}{C.W} = \frac{24}{6} = \boxed{4}$

(48)

Class-interval	50-59	60-69	70-79	80-89	90-99	100-109
f	4	11	18	28	22	15
FD	4/9	11/9	18/9	28/9	22/9	15/9

(49)

Output in tons	50-59	60-69	70-79	80-89	90-99	100-109	Total
f	15	12	10	28	25	10	100
~~FD~~ RF	0.15	0.12	0.1	0.28	0.25	0.1	

(50) Find A.M

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	Total
f	5	16	12	25	17	8	6	89
x	15	25	35	45	55	65	75	
fx	75	400	420	1125	935	520	450	3925

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{3925}{89} = \boxed{44.10}$$

(51) Find CF

Class Limits	50-59	60-69	70-79	80-89	90-99	100-109
f	5	18	10	27	19	12
CF	5	23	33	60	79	91

(52)

Find RF:-

Class Limits	50-59	60-69	70-79	80-89	90-99	100-109
f	5	18	10	27	19	12
RF	5/91	18/91	10/91	27/91	19/91	12/91

(53) Find angles for pie chart:

Medium of Institute	Frequency	Angle
Urdu	436	130.8
English	764	229.2
Total	1200	360

$130.8 = \frac{43600}{1200} \times 360°$

$229.2 = \frac{76400}{1200} \times 360$

(54) L_o = lower class boundary of median class, h = class width (median)
f_o = frequency of median class, CF = cumulative frequency of pre-median class

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(55) Find median:-

Profits	0-10	10-20	20-30	30-40	40-50	50-60	Total
f	5	18	22	16	7	2	70
CF	5	23	45	61	68	70	

M·class is $\frac{70}{2} = 35$ (20-30)

$$M_e = L + \frac{h}{f}\left(\frac{n}{2} - CF\right) = 20 + \frac{10}{22}\left(\frac{70}{2} - 23\right) = 20 + \frac{5}{11}(35-23)$$

$$M_e = 20 + \frac{5}{11}(12) = \frac{220+60}{11} = \frac{280}{11} = \boxed{25.45}$$

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(56) A·O → 25, 29, 31, 36, 38, 42, 49, 53, 57, 61, 67, 71, 73, 84, 87

50th percentile is $= (n+1)(y/100) = (15+1)(50/100) = \frac{16}{2} = 8$ th value

So 50th percentile is $\boxed{53}$.

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(57) A·O → 14, 15, 17, 18, 19, 25, 35, 37, 42, 45, 51

$M_{min} = 14$, $M_{max} = 51$, $M_e = 25$, $Q_1 = 17$, $Q_3 = 42$

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(58) Find Arithmetic mean:

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	Total
f	9	10	15	25	32	5	7	103
x	15	25	35	45	55	65	75	
fx	135	250	525	1125	1760	325	525	4645

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{4645}{103} = \boxed{45.09}$$

(59) Find mean, median and mode.

A·O → 45, 45, 48, 49, 49, 50, 51, 52, 52, 56

$\bar{x} = \frac{497}{10} = \boxed{49.7}$, Mode = 45, 49, 52, Median = $\frac{49+50}{2} = \boxed{49.5}$

(60) (61) Find 50th percentile.

A·O → 25, 29, 31, 38, 38, 42, 49, 53, 57

50th percentile = $(9+1)(50/100) = \frac{10}{2}$ = 5th value = $\boxed{38}$

(62) IQR = 45 − 33 = $\boxed{12}$

(63) Find SD:−

A·O → 25, 29, 38, 43, 45, 47, 52, 59, 62

$\mu = 400/9 = \boxed{44.4}$

x	25	29	38	43	45	47	52	59	62	Total
$x-\mu$	-19.4	-15.4	-6.4	-1.4	0.6	2.6	7.6	14.6	17.6	
$(x-\mu)^2$	376.36	237.16	40.96	1.96	0.36	6.76	57.76	213.16	309.76	1244.15

$SD = \sqrt{\frac{\Sigma(x-\mu)^2}{n}} = \sqrt{\frac{1244.15}{9}} = \sqrt{138.24} = \boxed{11.76}$

(64) (65) SD = 8.5, Mean = 25, $CV = \frac{8.5}{25} \times 100 = \boxed{34}$

(66) Mean = 32, median = 25, mode = ?

Mode = 3Median − 2Mean = 3(25) − 2(32) = 75 − 64 = $\boxed{11}$

(67) $T=13, B=5, R=8, P(B)=?$

$P(B) = \boxed{\frac{5}{13}}$

(68) $SD=?$ A.O $\rightarrow$ 29, 29, 36, 41, 42, 45, 47, 52, 59, 63

37

$\mu = \frac{480}{11} = \boxed{43.6}$

x	29	29	36	37	41	42	45	47	52	59	63	Total
$x-\mu$	-14.6	-14.6	-7.6	-6.6	-2.6	-1.6	1.4	3.4	8.4	15.4	19.4	
$(x-\mu)^2$	213.16	213.16	57.76	43.56	6.76	2.56	1.96	11.56	70.56	237.16	376.36	1234.47

$SD = \sqrt{\frac{\Sigma(x-\mu)^2}{n}} = \sqrt{\frac{1234.47}{11}} = \sqrt{112.22} = \boxed{10.59}$

(69) $MAD=?$ $x = 3, 4, 2$, $\bar{x} = 3$

x	3	4	2	Total
$x-\bar{x}$	0	1	-1	
$\lvert x-\bar{x}\rvert$	0	1	1	2

$MAD = \frac{\Sigma|x-\bar{x}|}{n} = \frac{2}{3} = \boxed{0.667}$

(70) (71) $\bar{x} = 18, SD = 6.5, CV=?$ $CV = \frac{SD}{\bar{x}} \times 100 = \frac{6.5}{18} \times 100 = \boxed{36.11}$

(72) Mean = 18, Meadian = 25, Mode = ?

Mode = 3Median − 2Mean = 3(25) − 2(18) = 75 − 36 = $\boxed{39}$

(73) Mean = \$1395, Median = \$1350, Mode = \$65

Mean > Median > Mode → So it is positively skewed.

(74) $T=9, B=4, R=5$ $P(R)=?$ $P(R) = \boxed{5/9}$

(75) Mean = 285, Median = 330, Mode = 340

Mean < Median < Mode → So it is negatively skewed.

End of this course