MTH632 MID TERM SOLVED MCQS

(VISIT VURANK FOR MORE)

	Module # 001
1.	Complex Analysis related to complex numbers and their functions. We have different types of numb pIntegers, Rational Number i.e. $\mathbb{Q} = \{\frac{a}{b} : a, b \epsilon \mathbb{Z}, b \neq 0\}$, Irrational Numbers that cannot be written as
2.	The decimal expression is non-terminating and non-periodic.
3.	Real Numbers: Union of rational and irrational numbers are called real numbers i.e. $\mathbb{R} = \mathbb{Q} U \mathbb{Q}'$
4.	Real numbers are not enough to solve our all problems if we consider an equation $x^2+2x+2=0$ then w

5.	By including i we get numbers of the form $a + ib$: $a, b \epsilon \mathbb{R}$ and such numbers are called complex num
6.	"Cardano" was first to introduce complex numbers $a + \sqrt{-1}\, b$ into algebra but had misgivings abou
7.	"Rafael Bombelli authored I'Algebra (1572 and 1579) a set of three books "At first , the things seem searched until I found the proof".
8.	L.Euler (1701-1783) introduced the notation $i = -1$

	Module # 002
9.	A complex number z can be defined as an ordered pair z=(x,y) where x,y are real numbers.
10.	Complex numbers can be interpreted as points in the complex plane.
11.	Real numbers is a subset of complex numbers.
12.	When real numbers x are displayed as points (x,0) on the real axis the set of complex numbers inclu
13.	Complex numbers of the form (0,y) corresponds to y-axis and are pure imaginary numbers when y
14.	For complex number z=(x,y) the real numbers x,y are known as the real and imaginary parts of z re
15.	Two complex numbers z1 and z2 are equal whenever they have the same real parts and the same in
16.	The statement z1=z2 means that z1 and z2 corresponds to same point in the complex plane or z-pla
17.	Let z1=(x1,y1), z2=(x2,y2) then z1=z2 iff x1=x2 and y1=y2.

Module # 003

18.	Two ways to represent complex numbers are Addition of Complex numbers and Product of Comple
19.	Commutative Law w.r.t Addition and Associative Law w.r.t Addition holds in Complex Numbers.
20.	**Addition of two complex numbers:** For z1=(x1,y1), z2=(x2,y2) the addition is defined as z1+z2=(x
21.	**Product of two complex numbers:** For z1=(x1,y1), z2=(x2,y2) the product is defined as z1.z2=(x1,
22.	Any complex number z=(x,y) can be written as z=(x,y)=(x,0)+(0,y)

23.	Any complex number z=(x,y) can be written as z=(x,y)=(x,0)+(0,y), Also we have (0,1)(y,0)=(0,y) so The numbers of the form (x,0) are real numbers so can simply denoted by x, numbers of the form (y number denoted by "i" then z=(x,0)+(0,1)(y,0)=x+iy
24.	(0,1) is such a complex number when it is multiplied with itself then it gives -1. i.e (0,1)(0,1)=(0-1,0
25.	Z=x+iy is used to represent the complex numbers and this representation is due to the mathematici

	Module # 004
26.	Properties of Addition of Complex Numbers: i. The addition of complex numbers follows all the properties of real numbers. ii. **Additive Identity**: There is a unique complex number 'w' such that z+w=w+z=z such numbe complex number z=x+iy there is a unique ξ such that z+$\xi = 0$, obviously $\xi = -x - iy$ and is know
27.	Complex numbers can be represented by two ways. i. Ordered pair representation ii. z=x+iy form. O foundational definition on which complex number system is built while z=x+iy is used or preferred

	Module # 005
28.	Properties of Multiplication of Complex Numbers: i. The product of complex numbers follows all the properties of real numbers. ii. The commutative law z1z2=z2z1. The commutative implies iy=yi so z=x+iy and z=x+yi are sa The associative law z1(z2z3)=(z1z2)z3. We can write z1z2z3 without parenthesis.

	iv. **Multiplicative Identity**: There is a unique complex number 'w' such that zw=wz=z such nun iv. **Multiplicative Inverse**: For every complex number z=x+iy other than (0,0) there is a unique
29.	Division of Complex numbers can be calculated using $\frac{z1}{z2} = \left(\frac{x1x2+y1y2}{x2^2+y2^2}, \frac{y1x2-x1y2}{x2^2+y2^2}\right)$
30.	**Distributive Law**: The Operation of multiplication is distributive over addition that is z1(z2+z3)=z
31.	**Binomial Formula**: For any two complex numbers z1, z2 we have $(z1+z2)^n=\sum_{k=0}^{n}\binom{n}{k} z_1^k z_2^{n-k}\ for\ n$

	Module # 006
32.	A complex number 'z' can be associated with the directed line segment or vector from origin to the p
33.	Multiplication of two complex number is neither dot product nor vector product.
34.	The vector interpretation helps us defining modulus or absolute value of complex number.
35.	The modulus or absolute value of a complex number z=(x,y)=x+iy is defined and denoted as $\|Z\| = \sqrt{}$

	Module # 007-008
36.	Properties of modulus of a complex number: An important consequence of triangular inequality is |
37.	The complex conjugate or simple conjugate of a complex number z=x+iy is denoted by$z$ and is giver

	Module # 008
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	Module # 009
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	Module # 011
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Module # 012	
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Module # 29	
48.	Continuity means without any break or sudden jump or holes at any point.
49.	If the small change in the input of a function brings to a small change in the output of that function th
50.	A function f(x) is continuous at a point X_0 if □ $\lim_{x\to x0} f(x)$ Exists. • $f(x_0)$ Exists. • $\lim_{x\to x_0} f(x) = f(x_0)$

51.	A function is said to be continuous on a set A if it is continuous at each point of A.
52.	In order to calculate limit at a point x_0 function f(x) must be defined in neighborhood of x_0.
53.	The function $f(x) = \{^{x^2+1}_{x} \ ^{x\leq 0}_{x>0}$ is continuous at x=1 but not continuous at x=0
54.	Let U(x,y) be a real valued function of the two real variables x and y then U is continuous at a point (• $\lim_{(x,y)\to(x_0,y_0)} U(x,y)$ Exists. • $U(x_0, y_0)$ Exists. • $\lim_{(x,y)\to(x_0,y_0)} U(x,y) = U(x_0,y_0)$
55.	The function $U(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} = 0 & (x,y) \neq (0,0) \\ 0 & (x,y) \neq (0,0) \end{cases}$ is continuous at (0,0)
56.	A complex function f(z) of complex variable z for all values of z in some neighborhood of z_0 is contin $\lim_{z\to z_0} f(z)$ Exists. • $f(z_0)$ Exists. • $\lim_{z\to z_0} f(z) = f(z_0)$
57.	For every $> 0\ t\ \ ere\ exist\ a\ \delta > 0$ such that $\lvert f(z) - L\rvert <$ whenever $0 < z - z_0 < \delta$ Where $\lim_{z\to z_0} f(z) = f(z_0) \implies \lim_{z\to z_0} f(z) = L$
58.	A complex function f(z) of complex variable z for all values of z in some neighborhood of z_0 is conti $\lvert f(z) - f(z_0)\rvert <$ whenever $0 < z - z_0 < \delta$

	Module # 30
59.	Let a function $f(z) = u(x,y) + i\ v(x,y)$ be defined in some neighborhood of z_0 then f is continuous a
60.	If f and g are continuous at the point z_0 then the following functions are continuous at z_0. • The sum f+g, where (f+g)(z)=f(z)+g(z) • The difference f-g, where (f-g)(z)=f(z)-g(z) The Product fg, where (fg)(z)=f(z)g(z) • The quotient $\frac{f}{g}$, where $\left(\frac{f}{g}\right)(z) = \frac{f(z)}{g(z)}$ provided $g(z) \neq 0$
61.	Any polynomial $p(z) = a_0 + a_1 z + a_2 z^2 + \cdots \ldots \ldots .. + a_n z^n (a_n \neq 0)$ is continuous at each point Z_0 in th

62.	$\lim_{z\to i} \frac{z^2+2}{z^2-2z+1} = \frac{1}{-2i}$

	Module # 31
63.	Consider two complex valued functions f(z) and g(z) then its composition is denoted by '**g of (z)**' a
64.	If f and g are continuous at the point z_0 then 'gof' is continuous at z_0.

	Module # 32
65.	If a function f(z) is continuous and non zero at a point z_0 then f(z) $\neq$ 0 throughout some neighborho
66.	If a function f is continuous at throughout a region R that is both closed and bounded then there exis where equality holds for at least one such Z.

	Module # 33
67.	The mapping $\frac{1}{Z}$ is called the reciprocal function. In exponential notation it can be expressed as $z = r$ Case 1: If z lies on a unit circle. Case 2: If z lies outside the unit circle. Case 3: If z lies inside the unit circle.
68.	Image of the half plane {z:Re(z)≥ $\frac{1}{2}$ under the mapping $w = \frac{1}{Z}$ is the closed disk i.e. {w:|w-1|≤ 1
69.	For transformation $\frac{1}{Z}$ image of portion of the half plane Re(z) ≥ $\frac{1}{2}$ is inside the closed disk i.e. {z:|z

	Module # 34
70.	The derivative of 'f' at x_0 is written as $f'(x_0)$and is defined as $f'(x_0) = \lim_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0}$ provided th
71.	The derivative of f at z_0 is written as$f'(z_0) = \lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ provide the limit exists. The function f
72.	If we write $\Delta_z = z - z_0$, *then the funciton* $f'(z_0) = \lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ can be expressed as $f'(z_0) = \text{li}$

73.	If we let w=f(z) and $\Delta w = f(z) - f(z_0)$ then we can use the Leibniz notation $\frac{dw}{dz}$ for the derivative Hence $f'(z_0) = \lim_{\Delta_z \to 0} \frac{\Delta w}{\Delta z} = \lim_{\Delta_z \to 0} \frac{f(z_0+\Delta_z)-f(z_0)}{\Delta_z}$ and often we drop the subscript z_0 and write the
74.	If $f(z) = z^2$ then $f'(z) = 2z$. **(PROVE IT-PAGE 66)**
75.	The function f(z)=Re(z) is differentiable nowhere. **(PROVE IT-PAGE 67)**

Module # 35	
76.	Consider the real valued function f(z)=|z|^2. Find the points where f(z) is differentiable. **(PROVE IT-**
77.	If f(z) is differentiable at z_0 then f(z) is continuous at z_0. **(PROVE IT-PAGE 73)**

Module # 36	
78.	Chain rule is useful when we want to differentiate composition of two functions.
79.	Given two functions f(z) and g(z) composition of f and g is given by; fog(z)=f(g(z)) or $\frac{d}{dz}(f(g(z)) = f'(g(z)).g'(z))$

Module # 37 – The Cauchy – Riemann Eq	
80.	Augustin-Lonis Cauchy (French Mathematician, Engineer and Physicist – 21 March 1789 to 23 May
81.	Cauchy – Riemann equations are a pair of equations that first order partial derivative of the compon satisfy at a point (x_0,y_0)when derivative of f exist there.
82.	If Cauchy – Riemann equations are not satisfied at z_0 then function is not differentiable at z_0.
83.	A Cauchy – Riemann equations are satisfied at a point z_0 then function may or may not be differenti

Module # 38	
84.	If f(z)=f(x+iy)=u(x,y)+iv(x,y) differentiable at a point $z_0=x_0+iy_0$ then u and v satisfy Cauchy-Riemann $U_x(x_0,y_0)=V_y(x_0,y_0)$ and $U_y(x_0,y_0)=-V_x(x_0,y_0)$
85.	

Module # 39	

86.	Let (x_0,y_0) be an arbitrary point defined in some neighborhood of the point $Z_0 = x_0+iy_0$ and suppose t $U(x,y)=e^{-y}Cosx$ ------- $Ux(x,y)=-e^{-y}Sinx$ -------$Uy(x,y)= -e^{-y}Cosx$ and $V(x,y)=e^{-y}Sinx$ ------- $Vx(x,y)=-e^{-y}$(
87.	The above partial derivatives are continuous at (X0,Y0) and satisfy Cauchy – Riemann equations Ux $Ux=)=-e^{-y}Sinx$ =Vy and $Uy=-e^{-y}Cosx$ =-Vx

	Module # 40
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	Module # 41
90.	A complex function f is analytic at the point Z_0 provided there is some > 0 such that $f'(z)$ exist in
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	Module # 42
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	Module # 43
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	Module # 44
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	Module # 45
98.	Harmonic Functions are real valued functions but they are very strongly related to complex valued
99.	**Harmonic Function Definition**: A real valued function H of two variables x and y is said to be harm conditions are satisfied by function; 1. Throughout the domain it has continuous partial derivatives of the first order and second or 2. It satisfied following partial differential equation $Hxx(x,y)+Hyy(x,y)=0$ or $\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$

100.	Notations: The sum $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}$ of two partial derivatives is denoted by $\nabla^2\varphi$ and is called the Laplacia
101.	Laplacian equation or sometimes known as Potential equation is $\nabla^2\varphi = 0$ **(PREPARE EXAMPLE**

Module # 46

102.	If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then its component functions u and v a
103.	**<u>Harmonic Conjugate</u>**: Let $u(x, y)$ and $v(x, y)$ are two functions such that $u(x, y)$ and $v(x, y)$ are harm Riemann equations throughout D then V is said to be harmonic conjugate of U in D.
104.	A function f(z)=U(x,y)+iv(x,y) is analytic in a domain D if and only iff V is harmonic conjugate of U.
105.	The function $f(z)=z^2$ i.e $f(x+iy)=(x+iy)^2=(x^2-y^2)+i(2xy)$ is analytic everywhere. It means U(x,y)= x^2-y
106.	The function 2xy and x^2-y^2 are harmonic functions but $f(x+iy)=(x+iy)^2=(2xy)+i(x^2-y^2)$ is not harmon

	Module # 47
107.	Find Harmonic Conjugate of $U(x,y)=x^3-3xy^2-5y$. **(PREPARE EXAMPLE - PAGE 32)**

	Module # 48
108.	Harmonic Functions are very important in applied mathematics for solving real life problems e.g. fo
109.	Incompressible means density w.r.t times remains constant or density does not change with time ar walls is zero.
110.	An analytic function has analytic anti-derivative.
111.	The curves given by $\{(x,y): \varphi(x, y) = constant\}$ are called equi-potentials or stream lines.
112.	For $F(x,y)=\varphi(x, y) + i\Psi(x, y)$ the component $\varphi(x, y)$ is harmonic also $\varphi_y = -\Psi_x$

	Module # 49
113.	**Sequence:** A sequence is any collection of objects, events or numbers that has some pattern that we
114.	A sequence is a function whose domain is a positive integer and whose range is a subset of complex
115.	$f: z^+ \to \mathbb{C}$ or $f: \mathbb{N} \to \mathbb{C}$ is a sequence
116.	Examples of complex valued sequence: $f_1(n) = n + i2n, \quad n \in \mathbb{N}$
117.	A sequence is usually denoted as $\{Z_n\}_{n=1}^{\infty}$ OR $\{Z_n\}_1^{\infty}$

	Module # 50
118.	A sequence is function from natural number to complex number.
119.	**Limit of a sequence**: A sequence {Xn} has P as its limit as n approcehes to infinity provided the te enough.
120.	A sequence of real numbers {Xn} converges of P if for every > 0 there exist N such that |Xn-P|< v
121.	A sequence of complex numbers {Zn} converges to if for each > 0 there exist N scuh that |Zn- |< w

	Module # 51
122.	A complex sequence {Zn} is bounded provided that there exist a positive real number R and an inte
123.	If {Zn} is a convergent sequence then {Zn} is bounded.
124.	**Cauchy Sequence**: The sequence {Zn} is a Cauchy Sequence if for every > 0 there is a positive
125.	{Zn} is a Cauchy Seqence if and only if {Zn} converges.

	Module # 52
126.	
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	Module # 53
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	Module # 54
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