LIMITS A FORMAL DEFINITION

- Formal definition of limit
- Examples

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We say that $f(x)$ *approaches the limit* L *as* x *approaches* x_0, and write

$$\lim_{x \to x_0} f(x) = L,$$

if f is defined on some deleted neighborhood of x_0 and, for every $\epsilon > 0$, there is a $\delta > 0$ such that

$$|f(x) - L| < \epsilon, \qquad \text{whenever} \qquad 0 < |x - x_0| < \delta.$$

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If $c \neq 0$, this yields

$$|f(x) - cx_0| < \epsilon$$

if

$$|x - x_0| < \delta,$$

where δ is any number such that $0 < \delta \leq \epsilon/|c|$.

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If $c = 0$, then $f(x) - cx_0 = 0$ for all x, so $|f(x) - cx_0| < \epsilon$ holds for all x.