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Mth631
Best Of luck

MTH631 Real Analysis II

Question No : 10 of 25

A function of the form $L(X) = m_1 x_1 + m_2 x_2 + \dots + m_n x_n$ for some constants m_i's is called ———

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Answer (Please select your correct option)

- linear function
- quadratic function
- constant function
- None of these

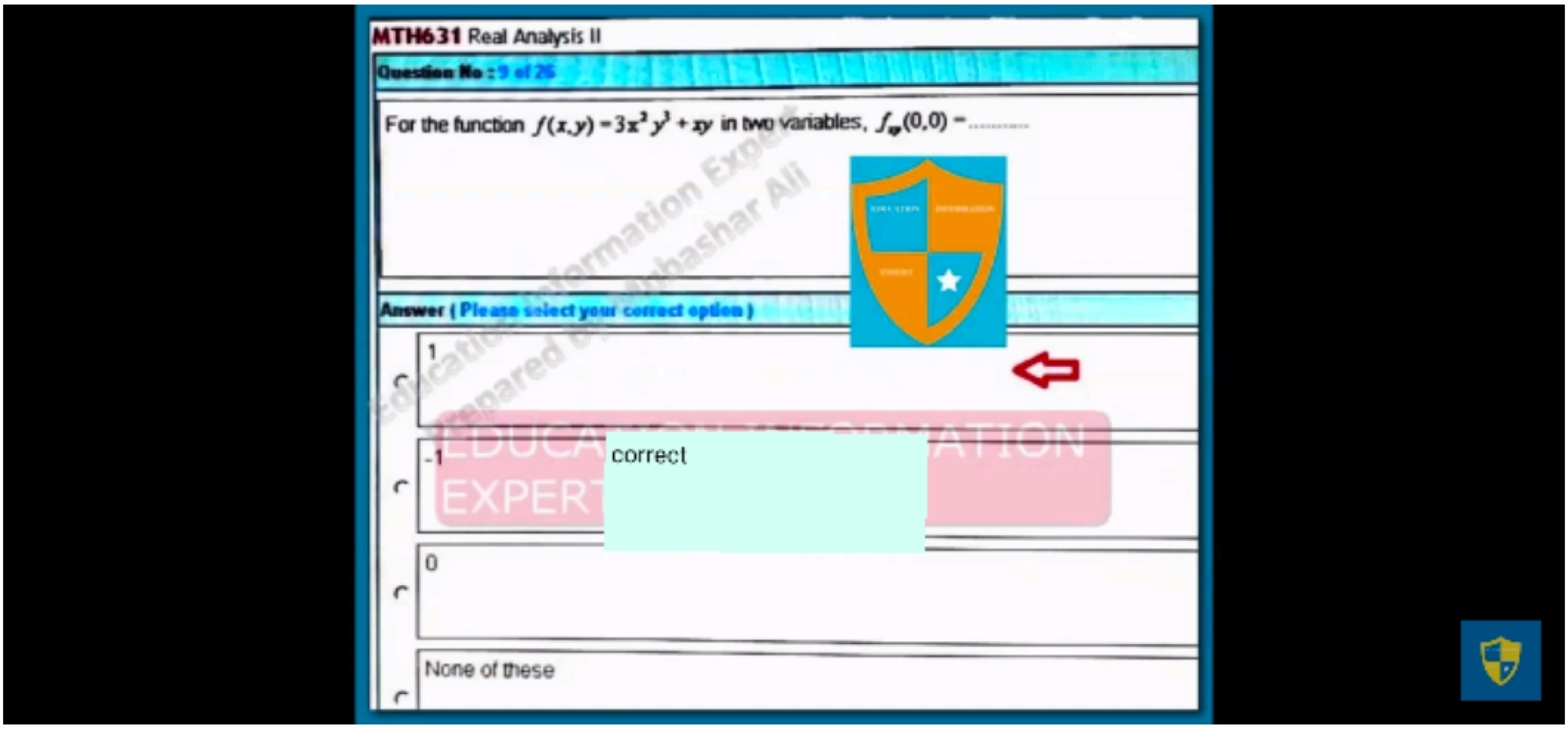
2.17.1 Linear Function

A *linear function* is a function of the form

$$L(\mathbf{X}) = m_1x_1 + m_2x_2 + \cdots + m_nx_n,$$

where m_1, m_2, …, m_n are constants. From definition of differentiable at $\mathbf{X}_0$ if and only if there is a linear function L s can be approximated so well near $\mathbf{X}_0$ by

$$L(\mathbf{X}) - L(\mathbf{X}_0) = L(\mathbf{X} - \mathbf{X}_0)$$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Question No : 9 of 25
For the function $f(x,y) = 3x^2y^3 + xy$ in two variables, $f_{xy}(0,0) =$ ....
Answer (Please select your correct option)
1
correct
-1
0
None of these

it is an *rth-order partial derivative of f*. The function

$$f(x,y) = 3x^2y^3 + xy$$

has partial derivatives everywhere. Its first-order partial derivatives are

$$f_x(x,y) = 6xy^3 + y, \quad f_y(x,y) = 9x^2y^2 + x.$$

Its second-order partial derivatives are

$$\begin{aligned} f_{xx}(x,y) &= 6y^3, & f_{yy}(x,y) &= 18x^2y, \\ f_{xy}(x,y) &= 18xy^2 + 1, & f_{yx}(x,y) &= 18xy^2 + 1. \end{aligned}$$

There are eight third-order partial derivatives. Some examples are

$$f_{xxy}(x,y) = 18y^2, \quad f_{xyx}(x,y) = 18y^2, \quad f_{yxx}(x,y) = 18y^2.$$

Compute $f_{xx}(0,0)$, $f_{yy}(0,0)$, $f_{xy}(0,0)$, and $f_{yx}(0,0)$ if

$$f(x,y) = \begin{cases} \frac{(x^2y+xy^2)\sin(x-y)}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

If $(x,y) \neq (0,0)$, the ordinary rules for differentiation, applied sepa
and y, yield

$$\begin{aligned} f_x(x,y) &= \frac{(2xy+y^2)\sin(x-y)+(x^2y+xy^2)\cos(x-y)}{x^2+y^2} \\ &\quad - \frac{2x(x^2y+xy^2)\sin(x-y)}{(x^2+y^2)^2}, \quad (x,y) \neq (0,0), \end{aligned}$$

and

$$\begin{aligned} f_y(x,y) &= \frac{(x^2+2xy)\sin(x-y)-(x^2y+xy^2)\cos(x-y)}{x^2+y^2} \\ &\quad - \frac{2y(x^2y+xy^2)\sin(x-y)}{(x^2+y^2)^2}, \quad (x,y) \neq (0,0). \end{aligned}$$

These formulas do not apply if $(x,y) = (0,0)$, so we find $f_x(0,0)$ and $f_$
their definitions as difference quotients:

$$\begin{aligned} f_x(0,0) &= \lim_{x\to 0} \frac{f(x,0)-f(0,0)}{x} = \lim_{x\to 0} \frac{0-0}{x} = 0, \\ f_y(0,0) &= \lim_{y\to 0} \frac{f(0,y)-f(0,0)}{y} = \lim_{y\to 0} \frac{0-0}{y} = 0. \end{aligned}$$

Setting $y = 0$ in (2.32) and (2.33) yields

$$f_x(x,0) = 0, \quad f_y(x,0) = \sin x, \quad x \neq 0,$$

$$f_y(0,0) = \lim_{y\to 0} \frac{f(0,y) - f(0,0)}{y} = \lim_{y\to 0} \frac{0-0}{y} = 0.$$

Setting $y = 0$ in (2.32) and (2.33) yields

$$f_x(x,0) = 0, \quad f_y(x,0) = \sin x, \quad x \neq 0,$$

so

$$f_{xx}(0,0) = \lim_{x\to 0} \frac{f_x(x,0) - f_x(0,0)}{x} = \lim_{x\to 0} \frac{0-0}{x} = 0.$$

$$f_{yx}(0,0) = \lim_{x\to 0} \frac{f_y(x,0) - f_y(0,0)}{x} = \lim_{x\to 0} \frac{\sin x - 0}{x} = 1.$$

Setting $x = 0$ in (2.32) and (2.33) yields

$$f_x(0,y) = -\sin y, \quad f_y(0,y) = 0, \quad y \neq 0,$$

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so

$$f_{xy}(0,0) = \lim_{y\to 0} \frac{f_x(0,y) - f_x(0,0)}{y} = \lim_{y\to 0} \frac{-\sin y - 0}{y} = -1,$$

$$f_{yy}(0,0) = \lim_{y\to 0} \frac{f_y(0,y) - f_y(0,0)}{y} = \lim_{y\to 0} \frac{0-0}{y} = 0.$$

2.16.2 Equality of Mixed Partial Derivatives

Theorem: Suppose that f, f_x, f_y, and f_{xy} exist on a neighborhood N o
and f_{xy} is continuous at (x_0, y_0).

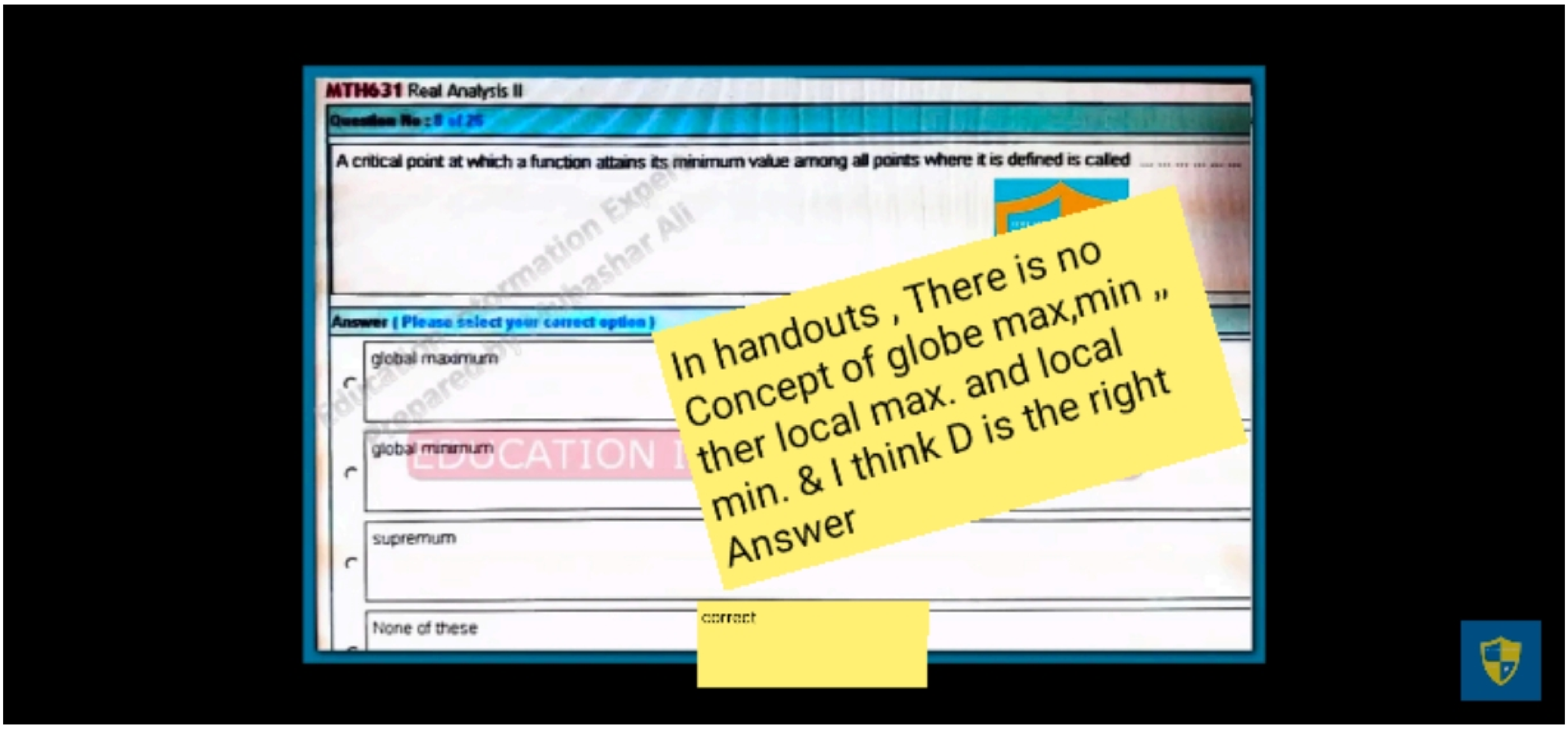
Then $f_{yx}(x_0, y_0)$ exists, and

$$f_{yx}(x_0, y_0) = f_{xy}(x_0, y_0).$$

Proof: Suppose that $\varepsilon > 0$. Choose $\delta > 0$ so that the ope

$$S_\delta = \{(x,y) : |x - x_0| < \delta, |y - y_0| < \delta\}$$

is in N.



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Question No :
A critical point at which a function attains its minimum value among all points where it is defined is called
Answer (Please select your correct option)
global maximum
global minimum
supremum
None of these
In handouts , There is no Concept of globe max,min ,, ther local max. and local min. & I think D is the right Answer
correct

2.18 Maxima and Minima

We say that $\mathbf{X}_0$ is a *local extreme point* of f if there is a $\delta > 0$ such that

$$f(\mathbf{X}) - f(\mathbf{X}_0)$$

does not change sign in $S_\delta(\mathbf{X}_0) \cap D_f$.

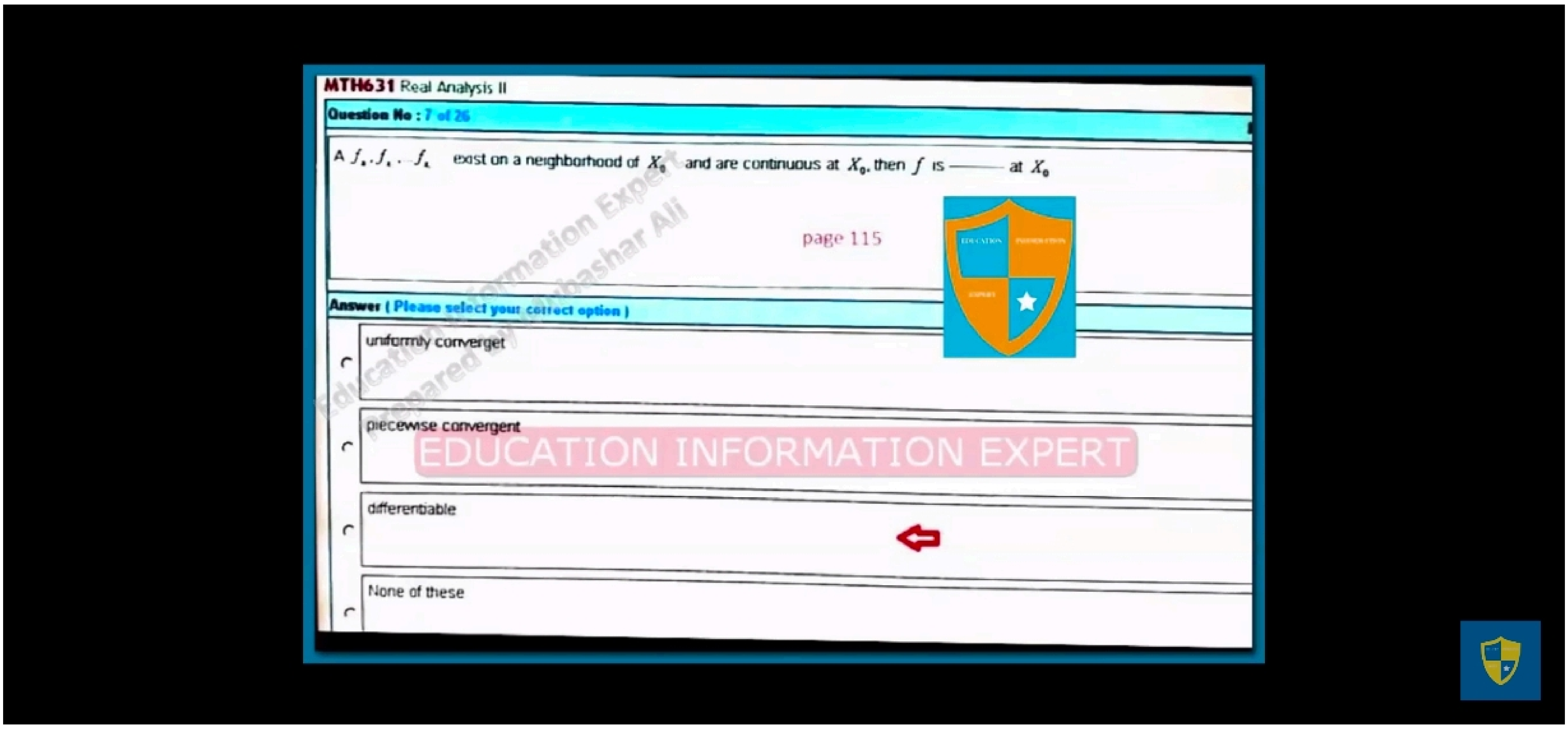
More specifically, $\mathbf{X}_0$ is a *local maximum point* if

$$f(\mathbf{X}) \leq f(\mathbf{X}_0)$$

or a *local minimum point* if

$$f(\mathbf{X}) \geq f(\mathbf{X}_0)$$

for all $\mathbf{X}$ in $S_\delta(\mathbf{X}_0) \cap D_f$.



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A $f_1, f_2, \dots f_k$ exist on a neighborhood of X_0 and are continuous at X_0, then f is ——— at X_0

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Answer (Please select your correct option)

- uniformly converget
- piecewise convergent
- differentiable
- None of these

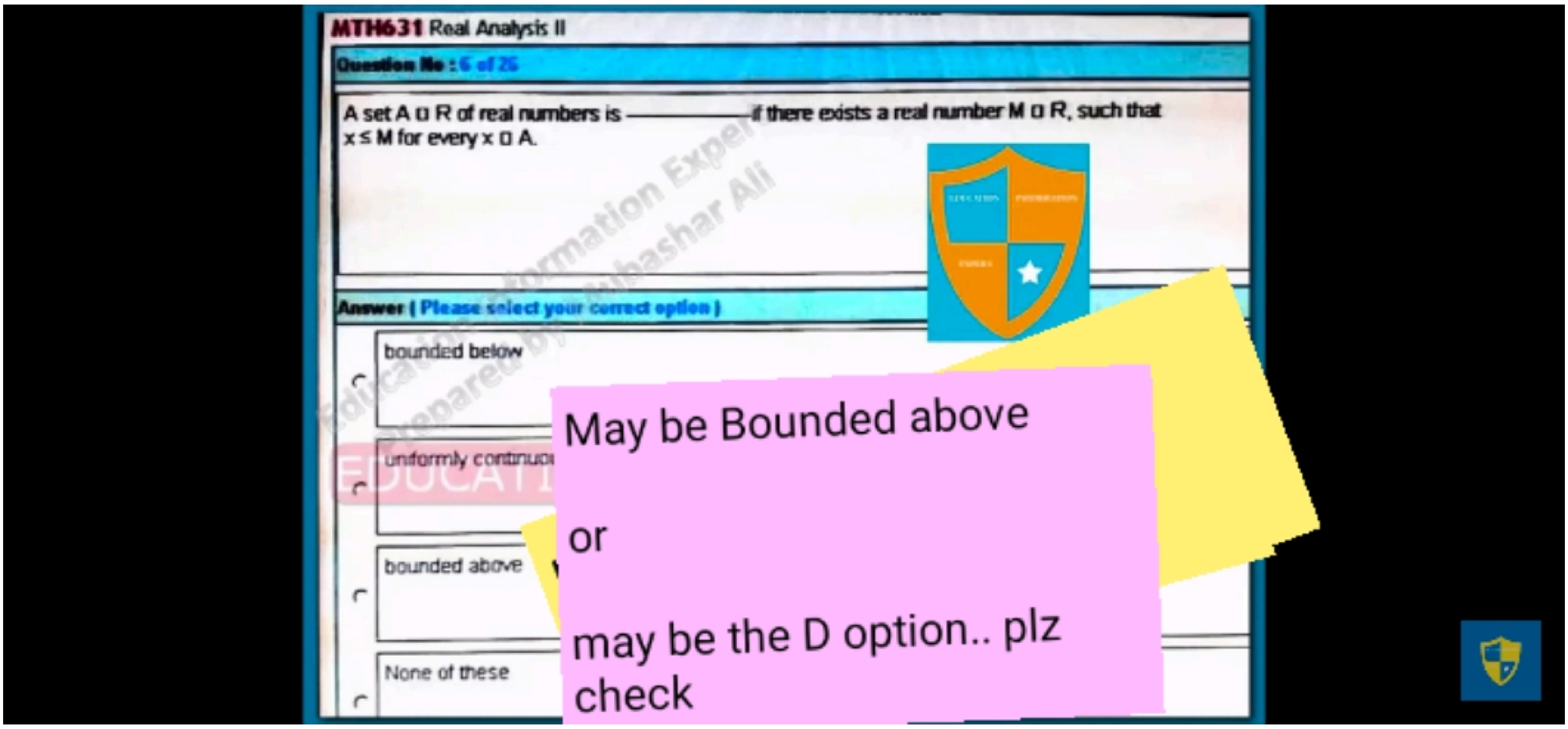
2.17.3 A sufficient Condition for Differentiability

Theorem: If $f_{x_1}, f_{x_2}, \ldots, f_{x_n}$ exist on a neighborhood of $\mathbf{X}_0$ and are continuous at $\mathbf{X}_0$, then f is differentiable at $\mathbf{X}_0$.

Proof: Let $\mathbf{X}_0 = (x_{10}, x_{20}, \ldots, x_{n0})$ and suppose that $\varepsilon > 0$. Our assumptions imply that there is a $\delta > 0$ such that $f_{x_1}, f_{x_2}, \ldots, f_{x_n}$ are defined in the n-ball

$$S_\delta(\mathbf{X}_0) = \{\mathbf{X} : |\mathbf{X} - \mathbf{X}_0| < \delta\}$$

and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A set A □ R of real numbers is ———— if there exists a real number M □ R, such that x ≤ M for every x □ A.
Answer (Please select your correct option)
bounded below
uniformly continuo
bounded above
None of these
May be Bounded above
or
may be the D option.. plz check

1.12 Pointwise and Uniform Bounded Functions

A sequence of functions $\{F_n\}$ on the set S is said to be pointwise bounded on S if the sequence of functions is bounded for every $x \in S$, that is, if there exists a finite valued function $\phi(x)$ defined on S such that

$$|F_n(x)| < \phi(x), \quad x \in S, n = 1, 2, 3, ...$$

We say that $\{F_n\}$ is uniformly bounded on S if there exist a number M such that

$$|F_n(x)| < M, \quad x \in S, n = 1, 2, 3, ...$$

conclude that $f(\mathbf{C}) = u$ for some $\mathbf{C}$ in S.

Theorem: A function f is uniformly continuous on a subset S of its domain in $\mathbb{R}^n$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that

$$|f(\mathbf{X}) - f(\mathbf{X}')| < \varepsilon$$

whenever

$$|\mathbf{X} - \mathbf{X}'| < \delta$$

and $\mathbf{X}, \mathbf{X}' \in S$.

Remark: We emphasize that δ must depend only on ε and S, and not on the particular points $\mathbf{X}$ and $\mathbf{X}'$.

Theorem: If f is continuous on a compact set S in $\mathbb{R}^n$, then f is uniformly continuous on S.

$|x - p_1| < \delta$.

Since $\{F_n\}$ is pointwise bounded, there exists $M_i < \infty$ such that

$$|F_n(p_i)| < M_i, n \in \mathbb{N}.$$

If we take

$$M = \max\{M_1, ..., M_r\},$$

then $|F_n(x)| < M + \varepsilon$ for every $x \in S$. This proves the first part of the theorem.

We say that $\{F_n\}$ is uniformly bounded on S if there exist a number M such that

$$|F_n(x)| < M, \quad x \in S, n = 1, 2, 3, ...$$

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In a vector valued function $G = (g_1, g_2, \ldots, g_n)$. Then $g_1, g_2, \ldots, g_n$ are called the ———— functions of G.

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Answer (**Please select your correct option**)

- bijective
- surjective
- component
- None of these

2.19 Differentiable Vector Valued Function

A vector-valued function $\mathbf{G} = (g_1, g_2, \ldots, g_n)$ is *differentiable* at

$$\mathbf{U}_0 = (u_{10}, u_{20}, \ldots, u_{m0})$$

if its component functions g_1, g_2, $\ldots$, g_n are differentiable at $\mathbf{U}_0$.

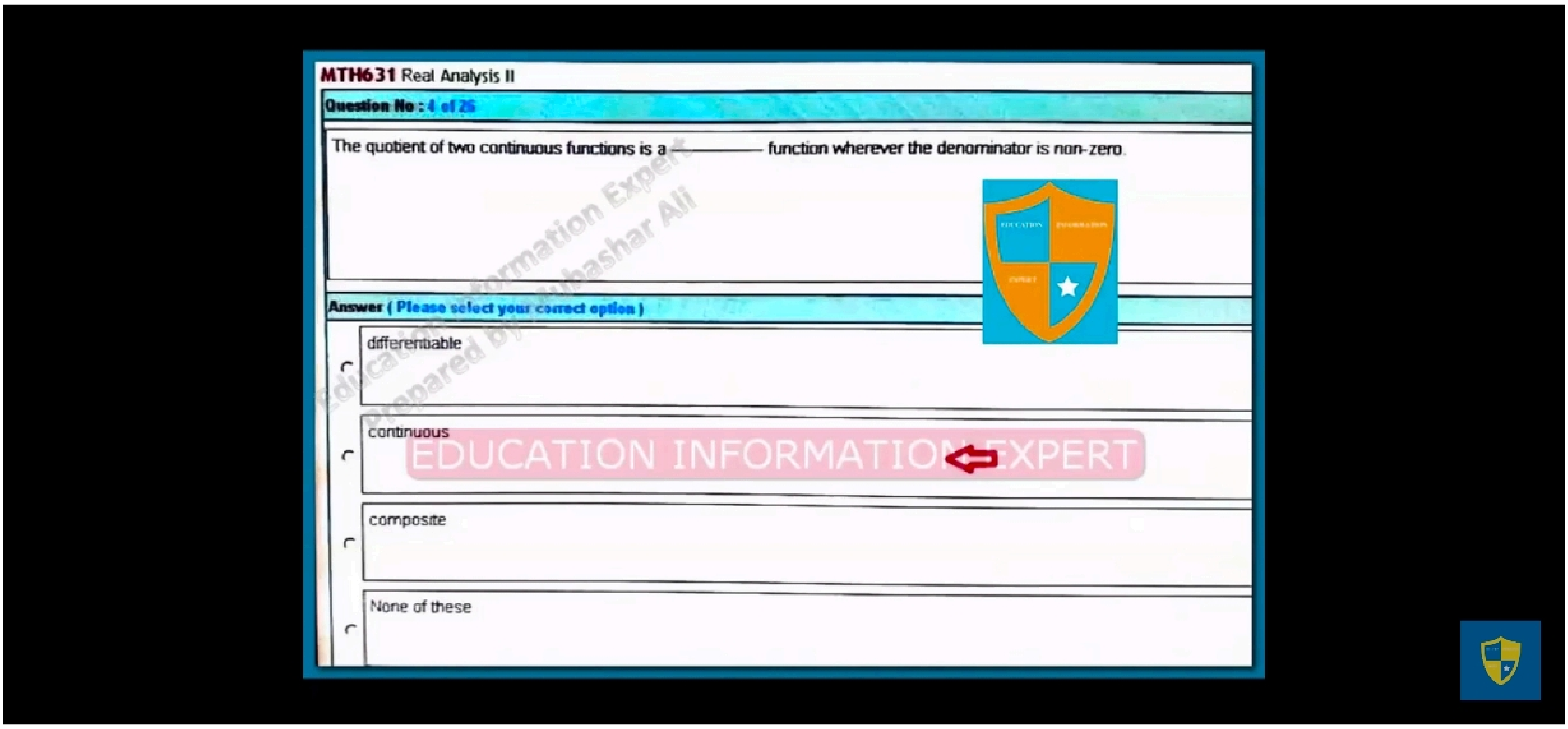
Lemma: Suppose that $\mathbf{G} = (g_1, g_2, \ldots, g_n)$ is differentiable at

$$\mathbf{U}_0 = (u_{10}, u_{20}, \ldots, u_{m0}),$$

and define

$$M = \left(\sum_{i=1}^{n} \sum_{j=1}^{m} \left(\frac{\partial g_i(\mathbf{U}_0}{\partial u_j} \right)^2 \right)^{1/2}.$$

Then, if $\epsilon > 0$, there is a $\delta > 0$ such that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The quotient of two continuous functions is a ________ function wherever the denominator is non-zero.

Answer (Please select your correct option)

- differentiable
- continuous
- composite
- None of these

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The set $S=\{(x,y), x^2+y^2\le 1 \text{ or } x^2+y^2\ge 4\}$ is not a region in R^2, since it is ________

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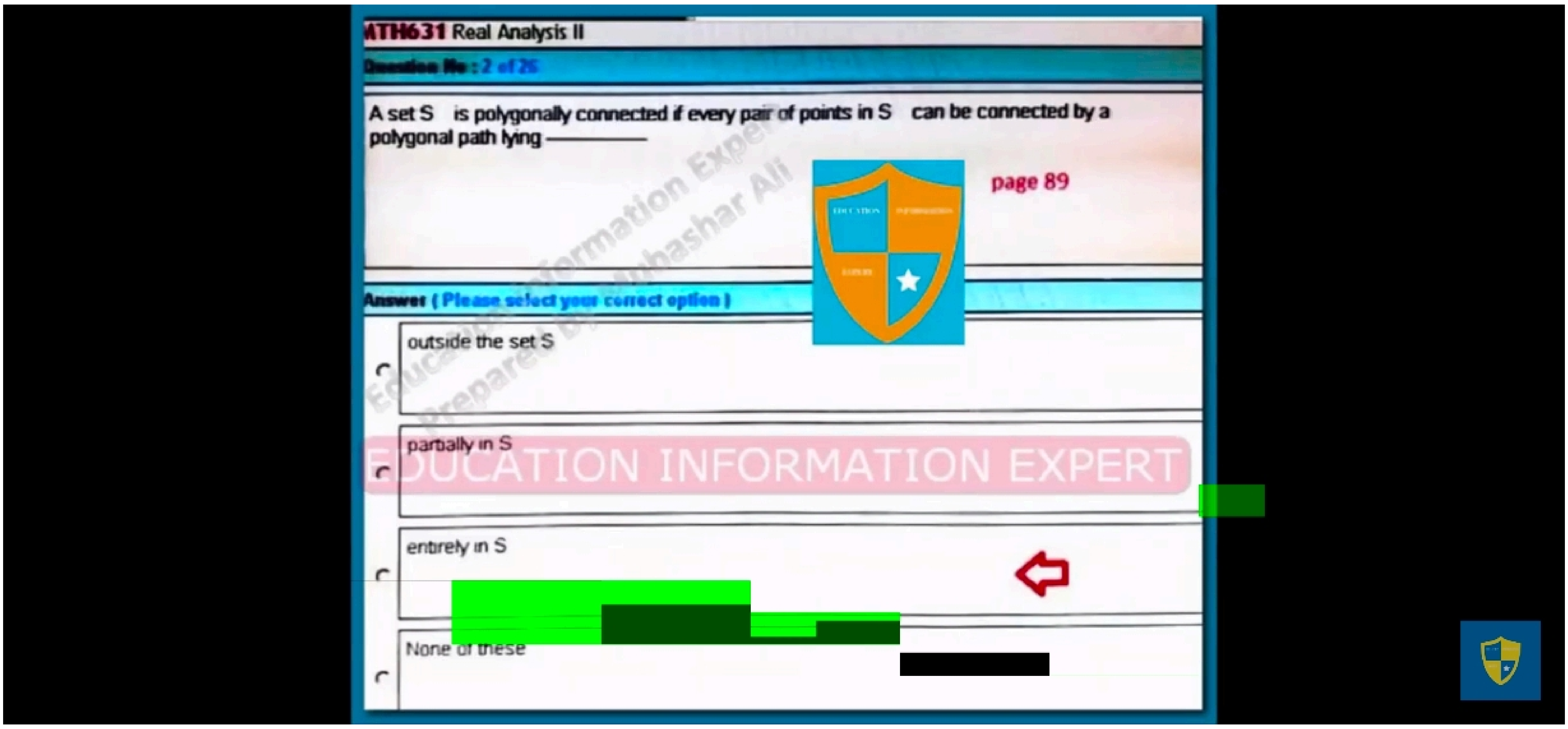
Answer (Please select your correct option)

- not connected
- connected
- polygonally connected
- None of these

Example: Intervals are the only regions in $\mathbb{R}$. The n-ball $B_r(\mathbf{X}_0)$ is a region in $\mathbb{R}^n$, as is its closure $\overline{S}_r(\mathbf{X}_0)$. The set $S = \{(x, y) : x^2 + y^2 \leq 1 \quad \text{or} \quad x^2 + y^2 \geq 4\}$ is not a region in $\mathbb{R}^2$, since it is not connected.

The set S_1 obtained by adding the line segment

$$L_1: \quad \mathbf{X} = t(0, 2) + (1 - t)(0, 1), \quad 0 < t < 1,$$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A set S is polygonally connected if every pair of points in S can be connected by a polygonal path lying ————
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Answer (Please select your correct option)
outside the set S
partially in S
entirely in S
None of these
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2.8 Polygonally Connected Set

A set S is *polygonally connected* if every pair of points in S can be connected by a polygonal path lying entirely in S.

Theorem: An open set S in $\mathbb{R}^n$ is connected if and only if it is polygonally connected.

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A set is closed if and only if it always contains all of its ——

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Answer (Please select your correct option)

- limit points
- neighbourhood points
- interior points
- None of these

limit $\mathbf{X}$. Since $\mathbf{X}$ is a limit point of every S_k and every S_k is closed, $\mathbf{X}$ is in e
S_k (**A set is closed if and only if it contains all its limit points**). There
$\overline{\mathbf{X}} \in I$, so $I \neq \emptyset$. Moreover, $\overline{\mathbf{X}}$ is the only point in I, since if $\mathbf{Y} \in I$, then

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The lower integral $\underline{\int_R} f(X)d(X)$ of f over R is the supremum of all the ——

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Answer (Please select your correct option)

- lower sums
- upper sums
- average sums
- None of these

$$s(\mathbf{P}) = \sum_{j=1}^{k} m_j V(R_j).$$

The *lower integral of* f *over* R, denoted by

$$\underline{\int_R} f(\mathbf{X})\, d\mathbf{X},$$

is the supremum of all lower sums.

Theorem: Let f be bounded on a rectangle R and le

Then

1. The upper sum $S(\mathbf{P})$ of f over $\mathbf{P}$ is the suprem
   sums of f over $\mathbf{P}$.

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If f and g are integrable on S and $f(X) \le g(X)$ for X in S, then ———

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Answer (Please select your correct option)

- $\int_S f(X)dX \ge \int_S g(X)dX$
- $\int_S f(X)dX \le \int_S g(X)dX$ ⇦
- $\int_S f(X)dX > \int_S g(X)dX$
- None of these

Theorem: If f and g are integrable on S and $f(\mathbf{X}) \leq g(\mathbf{X})$ for $\mathbf{X}$ in S, then

$$\int_S f(\mathbf{X})\, d\mathbf{X} \leq \int_S g(\mathbf{X})\, d\mathbf{X}.$$

Theorem: If f is integrable on S, then so is $|f|$, and

$$\left| \int_S f(\mathbf{X})\, d\mathbf{X} \right| \leq \int_S |f(\mathbf{X})|\, d\mathbf{X}.$$

Theorem: If f and g are integrable on S, then so is the product fg.

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The upper integral $\overline{\int_R} f(X)d(X)$ of f over R is the infimum of all the————

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Answer (Please select your correct option)

- lower sums
- upper sums
- average sums
- None of these

3.9 Upper and Lower Integrals

If f is bounded on a rectangle R in $\mathbb{R}^n$ and $\mathbf{P} = \{R_1, R_2, \ldots, R_k\}$ is a partition of R.

Let

$$M_j = \sup_{\mathbf{X} \in R_j} f(\mathbf{X}), \quad m_j = \inf_{\mathbf{X} \in R_j} f(\mathbf{X}).$$

The *upper sum* of f over $\mathbf{P}$ is

$$S(\mathbf{P}) = \sum_{j=1}^{k} M_j V(R_j).$$

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A function f is said to be absolutely integrable on $[a,b)$ if f is locally integrable on $[a,b)$ and ———

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Answer (Please select your correct option)

- $\int_a^b |f(x)|dx < \infty$
- $\int_a^b |f(x)|dx = \infty$
- $\int_a^b |f(x)|dx \geq \infty$
- None of these

3.2 Absolute integrability

We say that f is *absolutely integrable on* $[a, b)$ if f is locally integrable on $[a, b)$ and $\int_a^b |f(x)|\, dx < \infty$. In this case we also say that $\int_a^b f(x)\, dx$ *converges absolutely* or *is absolutely convergent*.

Remark: If f is nonnegative and integrable on $[a, b)$, then f is absolutely integrable on $[a, b)$, since $|f| = f$.

Example: The empty set and singleton sets are connected, because they cannot represented as the union of two disjoint nonempty sets.

2.6 Heine-Borel Theorem

We are going to state and prove the Heine-Borel theorem for $\mathbb{R}^n$.

This theorem concerns *compact* sets. As in $\mathbb{R}$, a compact set in $\mathbb{R}^n$ is a closed and bounded set.

of *supremum* and *infimum* of a function on a set S.

Theorem: If f is continuous on a compact set S in $\mathbb{R}^n$, then f is bounded on S.