

Q-1

Let $W = \text{span}(x_1, x_2)$, where $x_1 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$
 $x_2 = \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix}$, construct an orthogonal

basis (v_1, v_2) for W ? M.M

Solution:-

Let $v_1 = x_1$ and compute $v_2 = x_2 - p$

$$\therefore p = \frac{x_2 \cdot x_1}{x_1 \cdot x_1} x_1$$

then,

$$v_2 = \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} - \frac{\begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}}{\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}} x_1$$

$$v_2 = \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} - \frac{30}{10} \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -6 \end{bmatrix} - 3 \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 8-9 \\ 5-0 \\ -6+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix}$$

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Thus, the orthogonal basis are,

$$(v_1, v_2) = \left(\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix} \right) \text{ Ans.}$$

Math-Sol

Q.2

Find A^2 , given that $A = PDP^{-1}$ where

$$A = \begin{pmatrix} 2 & 6 \\ -4 & 12 \end{pmatrix}, P = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}, P^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$\text{and } D = \begin{pmatrix} 6 & 0 \\ 0 & 8 \end{pmatrix} ?$$

Solution:-

since,

$$A = PDP^{-1}$$

$$A^2 = PDP^{-1} \times PDP^{-1} = PDP^{-1}PDP^{-1} \quad \because PP^{-1} = I = 1$$

$$A^2 = PDDP^{-1} = PD^2P^{-1}$$

therefore,

$$A^2 = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 6^2 & 0 \\ 0 & 8^2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 36 & 0 \\ 0 & 64 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 108+0 & 0+64 \\ 72+0 & 0+64 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 108 & 64 \\ 72 & 64 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 108-128 & -108+192 \\ 72-128 & -72+192 \end{pmatrix}$$

$$= \begin{pmatrix} -20 & 84 \\ -56 & 120 \end{pmatrix} \quad \text{Ans}$$

Q.3

Find the dominant eigen pair (i.e. the Eigenvalue and Eigenvector) by using the power method for the following matrix (Perform only 1st iteration)

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Solution:-

since 1st iteration is

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$$u_1 = Ax_0$$

$$u_1 = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5+0 \\ 1+0 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Now we find the eigenvalue and eigenvector

$$u_1 = 5 \begin{bmatrix} 1 \\ \frac{1}{5} \end{bmatrix} = \lambda_1 x_1$$

here " λ_1 " is eigenvalue and eigenvector is " x_1 "

thus

$$\lambda_1 = \text{eigenvalue} = 5$$

$$x_1 = \text{eigenvector} = \begin{bmatrix} 1 \\ \frac{1}{5} \end{bmatrix}$$

Q.4

Determine whether the set of vectors are orthogonal or not, $\begin{bmatrix} 5 \\ -4 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 1 \\ -3 \\ 8 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ 5 \\ 1 \end{bmatrix}$

Solution:-

$$\text{Let } a = \begin{bmatrix} 5 \\ -4 \\ 0 \\ 3 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 1 \\ -3 \\ 8 \end{bmatrix}, c = \begin{bmatrix} 3 \\ 3 \\ 5 \\ 1 \end{bmatrix}$$

a, b, c will be orthogonal if

$$a^t b = 0 \rightarrow \text{i)}$$

$$a^t c = 0 \rightarrow \text{ii)}$$

$$b^t c = 0 \rightarrow \text{iii)}$$

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by (i)

$$a^t b = \begin{bmatrix} 5 & -4 & 0 & 3 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \\ -3 \\ 8 \end{bmatrix}$$

$$= -20 - 4 + 0 + 24 = 0$$

thus, the vector "a" and "b" are orthogonal.

same as we can find " $a^t c = 0$ " and " $b^t c = 0$ " which are not orthogonal.

Q.5

$$\text{If } A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix} \text{ and } \det(A^t) = 6$$

Find the determinant of the matrix

Solution:-

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 4 & 7 \\ 4 & 5 & 8 \\ 5 & 6 & 8 \end{bmatrix}$$

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$$\det(A^t) = \begin{vmatrix} 1 & 4 & 7 \\ 4 & 5 & 8 \\ 5 & 6 & 8 \end{vmatrix}$$

$$\det(A^t) = 1(40 - 48) - 4(32 - 40) + 7(24 - 25)$$

$$= -8 + 32 - 7$$

$$= 32 - 15$$

$$= 17 \text{ Ans}$$

Q.6

Is the following set of vectors is orthogonal with respect to the Euclidean inner product on \mathbb{R}^3 ?

$$\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right), \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$$

Let

$$v_1 = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right), v_2 = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$$

v_1 and v_2 will be orthogonal w.r.t the euclidean inner product if.

$$\langle v_1, v_2 \rangle = 0$$

$$\left\langle \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right) \cdot \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right) \right\rangle = 0$$

$$\left\langle \frac{1}{\sqrt{6} \times \sqrt{2}} - \frac{1}{\sqrt{6} \times \sqrt{2}} + 0 \right\rangle = 0$$

$$\langle 1 - 1 + 0 \rangle = 0$$

$$\langle 0 \rangle = 0$$

$$0 = 0$$

Thus v_1 and v_2 are orthogonal.

Q. 7

Determine whether the vector $y = \begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix}$ and

$z = \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$ are orthogonal?

Solution:-

The vectors "y" and "z" will be orthogonal

if $y^t z = 0$ or $z^t y = 0$

Proof:-

$$y^t z = [-3 \quad 7 \quad 4 \quad 0] \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$$

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$$y^t z = -3 - 56 + 60 + 0$$

$$y^t z = 60 - 59$$

$$y^t z = 1$$

here

$$y^t z \neq 0$$

therefore,

"y" and "z" vectors are not orthogonal.

Q.8

$$A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix} \text{ and } B = \{b_1, b_2\} \text{ where}$$

$$b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \text{ Find } B \text{ matrix}$$

for the transformation $x \rightarrow Ax$?

since,

$$M = \left[[T(b_1)]_c \quad [T(b_2)]_c \quad \dots \quad [T(b_n)]_c \right]$$

here

$$[T(b_1)]_c = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$[T(b_2)]_c = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

therefore

$$B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

Ans.

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Q.9

Find a distance between $x = \begin{bmatrix} 10 \\ -3 \end{bmatrix}$ and $y = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$?

Solution:-

Since,

$$\text{dist}(x, y) = \|x - y\| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

therefore

$$\text{dist}(x, y) = \sqrt{(10 + 1)^2 + (-3 + 5)^2}$$

$$= \sqrt{121 + 4}$$

$$= \sqrt{125}$$

Ans.

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