Math 104
Practice Midterm Exam 3B

Name _____

Write all answers (A, B, C, D, E, F) in the spaces provided below!

1. _____	7. _____
2. _____	8. _____
3. _____	9. _____
4. _____	10. _____
5. _____	11. _____
6. _____	12. _____

1. The testing booklet contains 12 multiple choice questions and 3 free response questions.
2. No calculators are permitted.
3. One piece of paper (8.5 in. by 11 in.) is permitted, with writing on both sides allowed.
4. There is no penalty for guessing.
5. No partial credit will be given on the multiple choice questions.

1. The curve $r = 2\cos(3\theta)$ is a calculus flower with 3 petals. Find the area contained in the flower.

A.) $\frac{\pi}{6}$

B.) $\frac{\pi}{4}$

C.) $\frac{\pi}{3}$

D.) $\frac{\pi}{2}$

E.) π

F.) 2π

2. Determine whether the sequence converges or diverges. If it converges find its sum.

$$\sum_{n=0}^{\infty} \frac{1+2^n}{6^n}$$

A.) 1/3

B.) 1/2

C.) 7/10

D.) 3/2

E.) 27/10

F.) Series is divergent

3. How many of the following series converge

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{2}^n} \quad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \quad \sum_{n=1}^{\infty} (\frac{\pi}{e})^n \quad \sum_{n=2}^{\infty} \frac{3^n - 1}{4^n} \quad \sum_{n=2}^{\infty} e^{1/n}$$

A.) none

B.) one

C.) two

D.) three

E.) four

F.) five

4. Which statement is true of the following series

$$(I)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^2}{n^2+1}=\frac{1}{2}-\frac{4}{5}+\frac{9}{10}-\dots$$

$$(II)\sum_{n=1}^{\infty}\frac{n}{2+n^2}=\frac{1}{3}+\frac{1}{3}+\frac{3}{11}+\dots$$

$$(III)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n\ln(n)}{1+n^2}=\frac{2\ln(2)}{5}-\frac{3\ln(3)}{10}+\frac{3\ln(3)}{17}-\dots$$

A.) (I) converges, (II) and (III) diverge.

B.) (II) converges, (I) and (III) diverge.

C.) (III) converges, (I) and (II) diverge.

D.) (I) and (II) converges, (III) diverge.

E.) (I) and (III) converges, (II) diverge.

F.) (II) and (III) converges, (I) diverge.

5. Consider the following series.

$$(I)\sum_{n=1}^{\infty}\frac{1}{n^{5/3}} \quad (II)\sum_{n=1}^{\infty}\frac{1}{5^{n/3}} \quad (III)\sum_{n=1}^{\infty}\frac{1}{n^{3/5}} \quad (IV)\sum_{n=1}^{\infty}\frac{1}{5^{3/n}}$$

Which of the following is true?

A.) All are convergent

B.) I, II converge, III, IV diverge

C.) II, III converge, I, IV diverge

D.) I, IV converge, II, III diverge

E.) I converge, II, III, IV diverge

F.) All are divergent

6. A curve is defined by the parametric equation $x = t^3 - 4t$, $y = 2\sqrt{3}(t^2 - 4)$. This curve goes through the origin twice and hence has a loop. Find the length of the loop.

A.) 4

B.) $4\sqrt{2}$

C.) $8\sqrt{2}$

D.) 16

E.) $16\sqrt{2}$

F.) 32

7. Find the equation for the line tangent to the curve defined by the parametric equation $x = e^t$, $y = \sin(3t)$ at the point $(x, y) = (1, 0)$

A.) $y = 3x - 3$

B.) $y = -3x + 3$

C.) $y = \frac{1}{3}x - \frac{1}{3}$

D.) $y = -x + 1$

E.) $y = x + 1$

F.) $y = -\frac{1}{3}x + \frac{1}{3}$

8. Consider the cartesian coordinate $(x, y) = (-2\sqrt{3}, 2)$. Find the polar coordinate (r, θ) where $r > 0$, and $0 \leq \theta < 2\pi$

A.) $(r, \theta) = (2, 7\pi/6)$

B.) $(r, \theta) = (4, 7\pi/6)$

C.) $(r, \theta) = (2, \pi/6)$

D.) $(r, \theta) = (2, 5\pi/6)$

E.) $(r, \theta) = (4, 5\pi/6)$

F.) $(r, \theta) = (4, 11\pi/6)$

9. Find the limit of the sequence $\{n\cos(\frac{1}{n})\}$

A.) 2

B.) 1

C.) -1

D.) $\pi/2$

E.) 0

F.) diverges

10. Define a sequence $a_1 = 3,\ a_2 = 3^{1+\frac{2}{3}},\ a_3 = 3^{1+\frac{2}{3}+\frac{4}{9}},\ a_n = 3^{1+\frac{2}{3}+\cdots+\frac{2^{n-1}}{3^{n-1}}}$. Find $\lim_{n\to\infty} a_n$

A.) 0

B.) 1

C.) $\sqrt{3}$

D.) 9

E.) 27

F.) $+\infty$

11. Find the area inside one leaf (i.e. one loop) of the graph $r = \sin(4\theta)$

A.) $\frac{\pi}{16}$

B.) $\frac{\pi}{8}$

C.) $\frac{\pi}{4}\pi$

D.) $\frac{\pi}{2}$

E.) π

F.) 2π

12. Find the limit of the sequence $\{\ln(2n+1) - \frac{1}{2}\ln(n^2+1)\}$

A.) -2

B.) $-\ln(2)$

C.) 0

D.) $\ln(2)$

E.) 2

F.) Sequence Diverges

Free Response 1. (A) Does the series $\sum_{n=1}^{\infty} \frac{4+\sin(2n)}{3^n}$ converge or diverge? Justify your answer.

(B) Does the series $\sum_{n=1}^{\infty} \frac{n^2+2n+1}{n^2\sqrt{n}}$ converge or diverge? Justify your answer.

Free Response 2. Set up (but don't solve) an equation for the for the length of the curve $x = 2\sin(t)$, $y = 3\cos(t)$, $0 \leq t \leq 2\pi$

Free Response 3. Find the area between the two curves $r = \sin(\theta)$, $r = \cos(\theta)$.