PHY101 lecture 1 to 22

ORANGE MONKEY TEAM

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Lecture no 1

What is Physics?

The branch of science concerned with the nature and properties of matter and energy. The subject matter of physics includes mechanics, heat, light and other radiation, sound, electricity, magnetism, and the structure of atoms.

Define the category of Classical mechanics.

Classical mechanics is a physical theory describing the motion of macroscopic objects.

(OR)

Classical mechanics is the part of physics that describes how everyday things move and how their motion changes because of forces.

Define the category of electromagnetism.

Electromagnetism is a branch of physics involving the study of the electromagnetic force, a type of physical interaction that occurs between electrically charged particles.

Define the category of thermal physics.

Thermal physics is the combined study of thermodynamics, statistical mechanics, and kinetic theory of gases.

Define the category of quantum mechanics.

Science dealing with the behavior of matter and light on the atomic and subatomic scale is known as quantum mechanics

Points to be noted **imp quiz**

- Every physical quantity can be expressed in terms of three fundamental dimensions: Mass (M), Length (L), and Time (T).

- You cannot add quantities that have different dimensions. So force can be added
  to force, but force can never be added to energy.
- Do not confuse units and dimensions. We can use different units to measure the same physical quantity.

- Always check your equations to see if they have the same dimensions on the left side as on the right.
- MKS stand for Meter Kilogram-Second system

Formula

- **Area has dimensions of L^2**
- **Density has dimensions of M / L^3 (or ML^{-3})**
- **Frequency has dimensions of 1/T**
- **Speed has dimensions of L/T**

Lecture 2 – KINEMATICS I

Function

Position of a body at time t is denoted as x(t) x is called a function of t.

Displacement

In geometry and mechanics, a displacement is a vector whose length is the shortest distance from the initial to the final position of a point P undergoing motion.

Formula

If a body is moving with average speed v then in time t it will cover a distance d=vt

Speed and Velocity

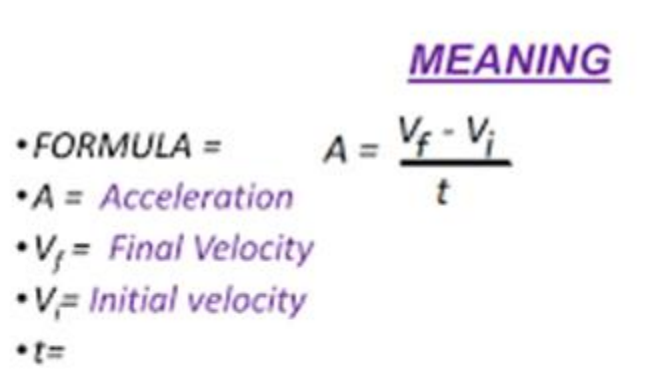
The velocity of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. Velocity is equivalent to a specification of an object's speed and direction of motion.

$$\text{Average velocity} = \frac{total\ distance\ traveled}{total\ time}$$

$$\textbf{Average velocity} = \frac{displacement}{total\ time} = \frac{x^2 - x^1}{t^2 - t^1} = \frac{\Delta x}{\Delta t}$$

Acceleration

In mechanics, acceleration is the rate of change of the velocity of an object with respect to time. Accelerations are vector quantities. The orientation of an object's acceleration is given by the orientation of the net force acting on that object.



Constant Acceleration

If **the velocity of the particle changes at a constant rate**, then this rate is called the constant acceleration.

Formula

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v = v_0 + at$$

Vectors

A vector is **an object that has both a magnitude and a direction**. Geometrically, we can picture a vector as a directed line segment, whose length is the magnitude of the vector and with an arrow indicating the direction. Both force and velocity are in a particular direction.

Velocity Vector

A velocity vector **represents the rate of change of the position of an object**. The magnitude of a velocity vector gives the speed of an object while the vector direction gives its direction.

Formula

$$r_x = x = r\cos\theta$$
$$r_y = y = r\sin\theta$$

$$where\ r = |\vec{r}| \quad and \quad \theta = \arctan(y/x)$$

Magnitude (length) of r is found by Pythagorean Theorem:

$$|\vec{r}| = r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan(C_y / C_x)$$

Vector Addition

To add vectors, **lay the first one on a set of axes with its tail at the origin. Place the next vector with its tail at the previous vector's head**. When there are no more vectors, draw a straight line from the origin to the head of the last vector. This line is the sum of the vectors.

Formula **C = A + B**

Lecture 3 – KINEMATICS II

Derivative

In mathematics, **the rate of change of a function with respect to a variable**. Geometrically, the derivative of a function can be interpreted as the slope of the graph of the function or, more precisely, as the slope of the tangent line at a point.

$$\frac{dx}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

How small should Δt be?

$$x(t) = t$$

$$\Delta x = x(t+\Delta t) - x(t)$$

$$= (t+\Delta t) - t = \Delta t$$

$$\Rightarrow \frac{dx}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = 1$$

$$x(t) = t^2$$

$$\Delta x = (t+\Delta t)^2 - t^2$$

$$= t^2 + (\Delta t)^2 + 2t\Delta t - t^2$$

$$\frac{\Delta x}{\Delta t} = \Delta t + 2t$$

$$\Rightarrow \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = 2t$$

$$x(t) = t^3$$

$$\Delta x = (t + \Delta t)^3 - t^3$$

$$= t^3 + 3t^2\Delta t + 3t\Delta t^2 + \Delta t^3 - t^3$$

$$\frac{\Delta x}{\Delta t} = (\Delta t)^2 + 3t^2 + 3t\Delta t$$

$$\Rightarrow \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = 3t^2$$

If $x(t) = t^n$

then:

$$\frac{dx}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = nt^{n-1}a$$

Unit vector

A unit vector is a vector that has magnitude 1 (no units). A unit vector is obtained by

Dividing a vector by its length.

Formula $\hat{A} = \frac{\vec{A}}{A}$

Scalar product

In mathematics, the dot product or scalar product is an algebraic operation that takes two equal-length sequences of numbers, and returns a single number. In Euclidean geometry, the dot product of the Cartesian coordinates of two vectors is widely used.

$$\vec{A} \cdot \vec{B} = AB\cos\theta, \quad 0 < \theta < \pi$$

$$\vec{A} \cdot \vec{B} = (A)(B\cos\theta)$$

$$= (\text{length of } \vec{A}) \times (\text{projection of } \vec{B} \text{ on } \vec{A})$$

- Acceleration along y is $a_y = -g$
- Acceleration along x is $a_x = 0$
- Velocity along x is *constant*

Lecture 4 – FORCE AND NEWTON'S LAWS

NEWTONS FIRST LAW

Isaac Newton (1643–1727) wrote *Principia Mathematical* in 1687 and proposed three "laws" of motion. An object will remain at rest or move with constant velocity unless acted upon by a net external force.

Inertial frame

A non-accelerating frame is called an inertial frame. Newton's first law holds only in inertial frames. In an accelerating frame we experience apparent forces. Inertia: resistance to change in motion *i.e.* resistance to acceleration. Mass is a measure of inertia.

Newton's First Law:

Newton's first law of motion is often stated as. **An object at rest stays at rest** and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

More force leads to more acceleration $\Rightarrow \quad a \propto F$

More mass leads to less acceleration $\Rightarrow \quad a \propto \frac{1}{m}$

Conclude that: $a \propto \frac{F}{m}$

Newton's Second Law:

Newton's second law is a quantitative description of the changes that a force can produce on the motion of a body. It states that **the time rate of change of the**

momentum of a body is equal in both magnitude and direction to the force imposed on it.

$$F = ma \quad (\text{or } a = F / m)$$

$$\text{where } F = F_1 + F_2 + F_3 + \cdots$$

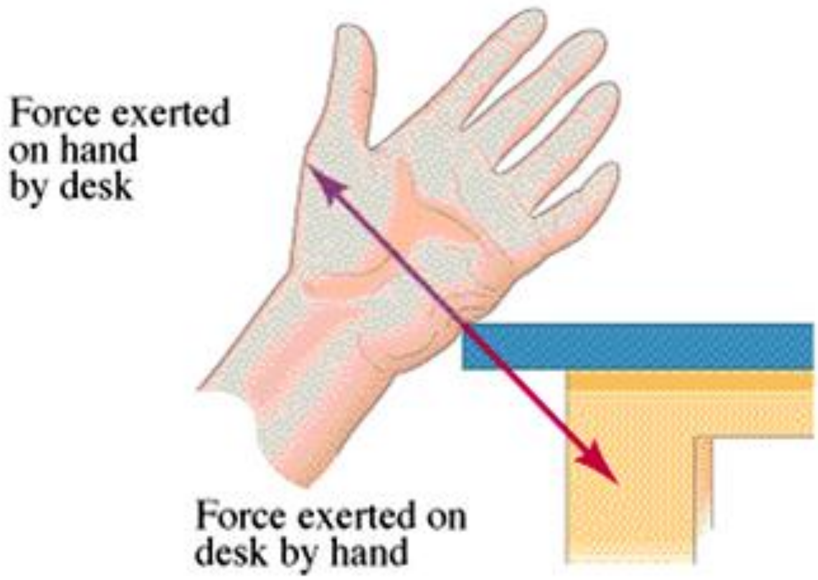
- Force has dimensions of [mass] × [acceleration] = M L T-2
- In the MKS system the unit of force is the Newton. It has the symbol N where:1 N = 1 **kg.m/s2**

The weight of a body is the force which gravity exerts upon it. Mass and weight are two completely different quantities.

Newton's Third Law:

Newton's third law states that when two bodies interact, they apply forces to one another that are equal in magnitude and opposite in direction. The third law is also known as **the law of action and reaction**. If a body has net force acting on it, it undergoes accelerated motion in accordance with the second law.

$F_{A \text{ on } B} = -F_{B \text{ on } A}$



CLAIM: If something is moving, there must be a net force on it.

FALSE. A body moving at constant velocity has no net force on it. An accelerating body must have a net force on it.

CLAIM: All equal and opposite forces are action-reaction pairs.

FALSE. The weight of a book sitting on a tabletop and the normal force of the table acting on the book are equal and opposite, but they are not an action-reaction pair!

CLAIM: If there is a force on an object, it must be accelerating.

FALSE. Only a *net* force on the object leads to acceleration.

Lecture 5 – APPLICATIONS OF NEWTON'S LAWS – I

EQUILIBRIUM

States in which opposing forces or actions are balanced so that one is not stronger or greater than the other Supply and demand were in equilibrium. Chemical equilibrium.

Or

A state of emotional balance or calmness It took me several minutes to recover my equilibrium.

Example :-Equilibrium is when **hot air and cold air are entering the room at the same time so that the overall temperature of the room does not change at all**.

An aircraft of mass m has position

vector that is measured to be:

$$\vec{r} = (at + bt^3)\hat{i} + (ct^2 + dt^4)\hat{j}$$

What force is acting upon it?

SOLUTION:

$$\vec{F} = m\frac{d^2x}{dt^2}\hat{i} + m\frac{d^2y}{dt^2}\hat{j}$$

$$= 6bmt\hat{i} + m(2c + 12dt^2)\hat{j}$$

Tension force

In physics, tension is described as the pulling force transmitted axially by the means of a string, a cable, chain, or similar object, or by each end of a rod, truss member, or similar three-dimensional object; tension might also be described as the action-reaction pair of forces acting at each end of said elements.

Consider a horizontal segment of rope:

By Newton's 2^{nd} Law:

$$S\ F = T_2 - T_1 = m\ a$$

So if $m = 0$ (*i.e.* the rope is light) then $T_1 = T_2$

The direction of the force provided by a rope is along the direction of the rope.

$$\Sigma\ F = T - mg = ma_y$$

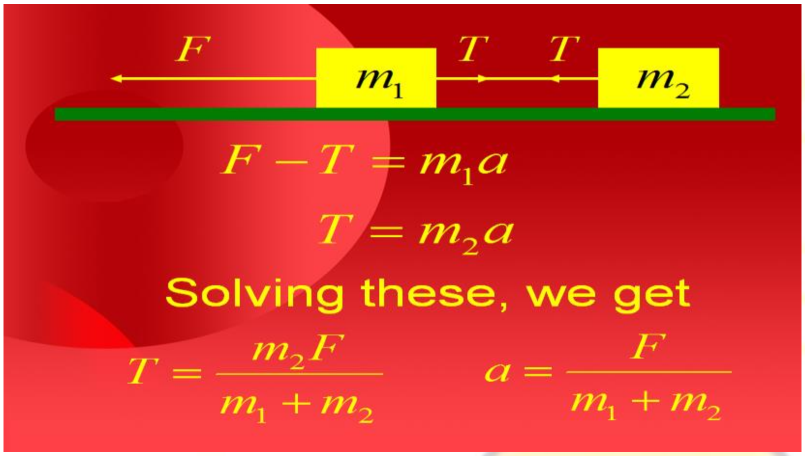
since $a_y = 0$ therefore:

$$T - mg = 0$$

$$T = mg$$

Masses connected by strings

If m_1 is pulled by a force F then a tension T develops in the string.



Friction

Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. There are several types of friction: Dry friction is a force that opposes the relative lateral motion of two solid surfaces in contact.

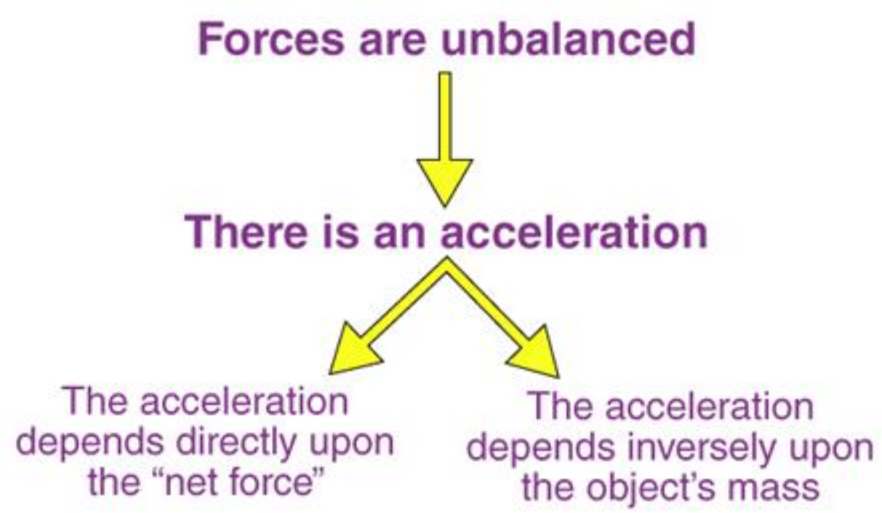
STATIC FRICTION

Static friction is **friction between two or more solid objects that are not moving relative to each other**.

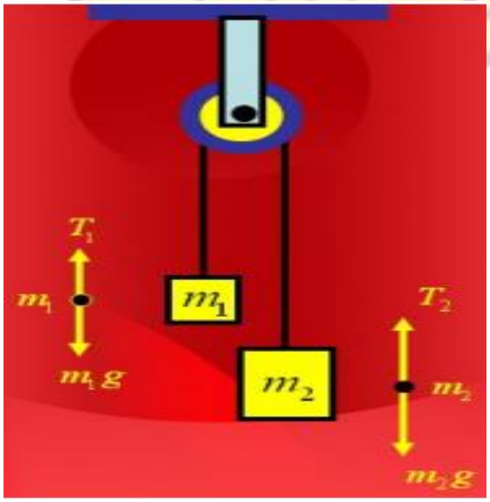
For example, static friction can prevent an object from sliding down a sloped surface. The coefficient of static friction, typically denoted as μ_s, is usually higher than the coefficient of kinetic friction.

Lecture 6 – APPLICATIONS OF NEWTON'S LAWS – II

Newton's second law states that the acceleration of an object depends upon two variables – the net force acting on the object and the mass of the object. The acceleration of the body is directly proportional to the net force acting on the body and inversely proportional to the mass of the body. This means that as the force acting upon an object is increased, the acceleration of the object is increased. Likewise, as the mass of an object is increased, the acceleration of the object is decreased.



a= $a = \frac{F_{net}}{m}$ the above equation can be rearranged to a familiar form as F=ma since force is a vector, Newton's second law can be written as F→=ma→ the equation shows that the direction of the total acceleration vector points in the same direction as the net force vector.



$m_2 > m_1$

$$T - m_1 g = m_1 a$$

$$a = \frac{T - m_1 g}{m_1} \qquad (i$$

$$m_2 g - T = m_2 a$$

$$a = \frac{m_2 g - T}{m_2} \qquad (ii$$

equating (i and (ii

$$\frac{T - m_1 g}{m_1} = \frac{m_2 g - T}{m_2}$$

$$m_2 T - m_1 m_2 g = m_1 m_2 g - m_1 T$$

$$m_2 T + m_1 T = m_1 m_2 g + m_1 m_2 g$$

$$T(m_2 + m_1) = 2 m_1 m_2 g$$

$$T = \frac{2 m_1 m_2 g}{(m_2 + m_1)}$$

$m_2 > m_1$

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$

$$m_2 g - m_1 g = (m_1 + m_2) a$$

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)} g$$

Lecture 7 – WORK AND ENERGY

Definition of work

In physics, work is the energy transferred to or from an object via the application of force along a displacement. In its simplest form, it is often represented as the product of force and displacement.

$$W = \vec{F} \cdot \vec{d}$$
$$= Fd\cos\theta$$

- Work is a *scalar*
- Work has dimensions: M L T^{-2} L = M L^{2} T^{-2}
- Work has units: Newton · Meter ≡ Joule (J)

$$\Delta W_1 = F_1 \Delta x$$
$$\Delta W_2 = F_2 \Delta x$$
$$\Delta W_3 = F_3 \Delta x$$

$$W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_N$$
$$= F_1 \Delta x + F_2 \Delta x + \cdots + F_N \Delta x$$

or

$$W = \sum_{n=1}^{N} F_n \Delta x$$

To get the exact result let $\Delta x \to 0$ and the number of intervals N $\to \infty$:

$$W = \lim_{\Delta x \to 0} \sum_{n=1}^{\infty} F_n \Delta x$$

Definition: $\lim_{\Delta x \to 0} \sum_{n=1}^{\infty} F_n \Delta x \equiv \int_{x_i}^{x_f} \mathrm{F(x)dx}$

is the integral of F with respect to x from x_i to x_f.

The total work done by F in moving a body from x_i and x_f is:

$$W = \int_{x_i}^{x_f} \mathrm{F(x)dx}$$

Energy is the capacity of a physical system to do work

- it comes in many forms
- it can be stored
- it can be converted into different forms
- it can never be *created* or *destroyed*

Important

Recall: $v_2^2 - v_1^2 = 2a(x_2 - x_1)$

where: v_2 = final velocity

x_2 = final position

v_1 = initial velocity

x_1 = initial position

$\therefore a = \frac{v_2^2 - v_1^2}{2d}$

Work Kinetic Energy Principle

Net work done on object= *Change* in *KE* of object

Work can be:

- Positive (KE increases)
- Negative (KE decreases)

Calculate work:

$W = F\,d$

$= ma\,d$

$= m\,\frac{v_2^2 - v_1^2}{2d}\,d$

$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

Energy has the same units as work: Joule = Newton Meter

$W = F\,\Delta x$

Work does not depend on time!

Define KINETIC ENERGY:

$KE = \frac{1}{2} m v^2$

- Time doesn't matter for *power* !
- Power is the "rate of doing work"

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

If the force does not depend on time:

$$\frac{\text{Work}}{\text{Time}} = \frac{F\,\Delta x}{\Delta t} = F\,\text{v}$$

$\therefore$ Power = F v

Units of power: *J/sec = Watts*

Old units: horsepower (hp)

1 hp = 746 W = 0.746 kW

The power is $P = F\,v = T\,v$

In the x direction: $T = mg\sin\theta$

Lecture 8 – CONSERVATION OF ENERGY

Potential energy

Potential energy **stored energy that depends upon the relative position of various parts of a system**. A spring has more potential energy when it is compressed or stretched. A steel ball has more potential energy raised above the ground than it has after falling to Earth.

The spring pulls/pushes with a *restoring force proportional to the extension x* :

$$F_{spring} = -k\,x$$

Work due to external force gives the *elastic potential energy:* $W = \int_0^x F dx = \int_0^x k\,x dx = \frac{1}{2}k\,x^2$

$$W = \int_0^x F dx = \int_0^x k\,x dx = \frac{1}{2}k\,x^2$$

Total mechanical energy is: $E_{mech} = KE + PE$

IF no friction then E_{mech} is conserved :

$\Delta(E_{mech}) = \Delta(KE) + \Delta(PE) = 0$

$E_{mech} = KE + PE$ is constant !!!

$$\frac{1}{2}mv_A^{\ 2} + mgh = \frac{1}{2}mv_B^{\ 2} + mgh$$
$$= \frac{1}{2}mv_C^{\ 2} + mg\frac{h}{2} = \frac{1}{2}mv_D^{\ 2}$$

$$v_A = v_B \ , \ v_C = \sqrt{v_0^{\ 2} + gh} \ , \ v_D = \sqrt{v_0^{\ 2} + 2gh}$$

$$\frac{1}{2}kx^2 = mgd\sin\theta \quad \Rightarrow \quad d = \frac{kx^2}{2mg\sin\theta}$$

CONSERVATIVE FORCE

Work does not depend on path take

- gravity
- electric force
- springs

Potential energy can be defined!

$$mgh = fx = \mu mgx \quad \Rightarrow \quad x = \frac{h}{\mu g}$$

Lecture 9 – MOMENTUM

Momentum

Momentum is the "quantity of motion" possessed by a body. More precisely, it is defined as: **Dimensions of momentum: MLT^{-1}**

Units of momentum: kg-m/s

Momentum = mass × velocity

Definition of linear momentum

The momentum of translation being **a vector quantity in classical physics equal to the product of the mass and the velocity of the center of mass.**

$$\vec{P} = m\vec{\mathbf{v}}$$

NEW FORM OF SECOND LAW

The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the direction of that force.

$$m\vec{\mathbf{a}} = \vec{\mathbf{F}} \text{ (old form)}$$

$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}} \quad \text{(new form of Newton's Law)}$$

They are the same:

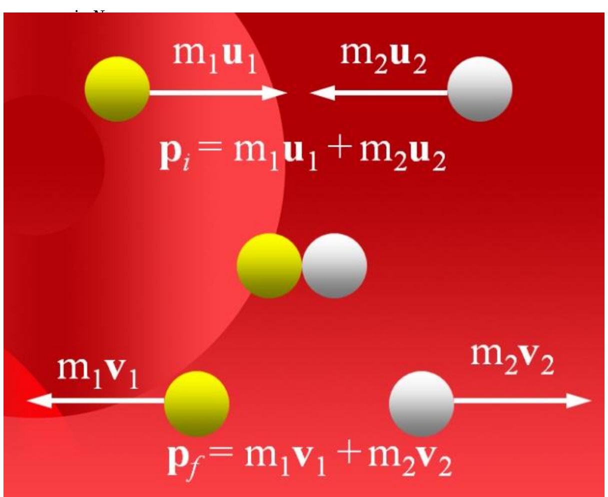
$$\frac{d\vec{\mathbf{p}}}{dt} = \frac{d(m\vec{\mathbf{v}})}{dt} = m\frac{d\vec{\mathbf{v}}}{dt} = m\vec{\mathbf{a}} = \vec{\mathbf{F}}$$

Newton's 2nd law for several particles

$$\vec{\mathbf{P}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \cdots \vec{\mathbf{p}}_N$$

$$\frac{d}{dt}\vec{\mathbf{P}} = \frac{d}{dt}\vec{\mathbf{p}}_1 + \frac{d}{dt}\vec{\mathbf{p}}_2 + \cdots \frac{d}{dt}\vec{\mathbf{p}}_N$$

$$= \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots \vec{\mathbf{F}}_N$$



Momentum is conserved:

Initial momentum = final momentum

$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

a) Linear momentum is conserved:

$$P_f = P_i$$

$$MV + m(\mathrm{v} + V) = 0$$

$$V = -\frac{m\mathrm{v}}{m+M} = -\frac{(72kg)(55m/s)}{1300kg + 72kg}$$

$$V = -2.9\ m/s$$

b) $$\mathrm{v_E} = \mathrm{v} + V$$

$$= 55m/s + (-2.9\ m/s)$$

$$\mathrm{v_E} = 52m/s$$

Suppose the total distance moved on the flat part before it comes to rest is x.

$$mgh = fx = \mu mgx \quad \Rightarrow \quad x = \frac{h}{\mu g}$$

Impulse and Momentum

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \Longrightarrow d\mathbf{p} = \mathbf{F}dt$$

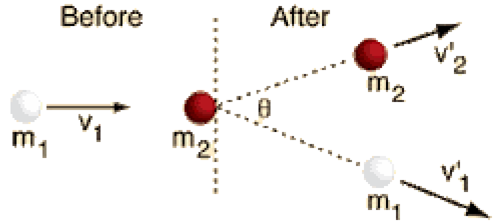
Define: $\mathrm{I} \equiv \int_{t_1}^{t_2} \mathrm{F}dt$ Since $\int_{t_1}^{t_2} \mathrm{F}dt = \int_{p_i}^{p_f} dp \quad \therefore \quad \mathrm{I} = \mathbf{p}_f - \mathbf{p}_i$

Lecture 10 – COLLISIONS

COLLISIONS

In physics, a collision is any event in which two or more bodies exert forces on each other in a relatively short time.

Elastic Collisions - Target Initially at Rest*



*This can actually apply to all elastic collisions since we can always choose a reference frame riding with m_2 before the collision.

General relationships:

a. Conservation of momentum: $m_1\vec{v}_1 = m_1\vec{v}'_1 + m_2\vec{v}'_2$

b. Conservation of kinetic energy: (elastic collision assumption) $\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 v'^2_1 + \frac{1}{2}m_2 v'^2_2$

c. For head-on collisions: $v'_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}v_1$; $v'_2 = \frac{2m_1}{(m_1 + m_2)}v_1$

d. For head-on collisions the velocity of approach is equal to the velocity of separation.

$$m_1 = m_2$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u$$

$$v_1 = u_2$$

$$v_2 = u_1$$

Massive target at rest

$$m_2 >> m_1, \quad u_2 = 0$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$\Rightarrow \quad v_1 = u_1 \text{ and } v_2 = 0$$

Lighter target at rest

$$m_2 << m_1 \quad u_2 = 0$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$\Rightarrow \quad v_1 = u_1 \text{ and } v_2 = 0$$

Elastic and inelastic collisions

The difference between elastic and an inelastic collision is **the loss or conservation of kinetic energy**. In an inelastic collision kinetic energy is not conserved, and will change forms into sound, heat, radiation, or some other form. In an elastic collision kinetic energy is conserved and does not change forms.

Fractional decrease in neutron K.E :

$$\frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{v_f^2}{v_i^2}$$

For a target at rest:

$$v_f = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_i$$

$$\frac{K_i - K_f}{K_i} = \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$

A car A of mass 1000 kg is traveling north at 15 m/s collides with another car B of mass 2000 kg traveling east at 10 m/s. After collision they move as one mass. Find the total momentum just after the collision.

$$mv = (m+M)V$$

$$v = \frac{(m+M)}{m} V$$

Conservation of energy gives,

$$\frac{1}{2}(m+M)V^2 = (m+M)gy$$

$$V = \sqrt{2gy}$$

$$v = \frac{(m+M)}{m}\sqrt{2gy}$$

$$P_x = p_{Ax} + p_{Bx} = m_A v_{Ax} + m_B v_{Bx}$$
$$= 2\times 10^4 \, kg\, m/s$$

$$P_y = p_{Ay} + p_{By} = m_A v_{Ay} + m_B v_{By}$$
$$= 1.5\times 10^4 \, kg\, m/s$$

$$P = \sqrt{P_x^2 + P_y^2} = 2.5\times 10^4 \, kg\, m/s$$

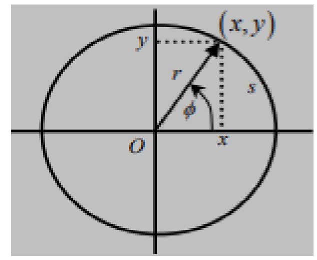
$$\tan\theta = \frac{P_y}{P_x} = 0.75 \quad \Rightarrow \quad \theta = 37^0$$

Lecture 11 – ROTATIONAL KINEMATICS

Definition of rotational kinematics

Kinematics is the description of motion. The kinematics of rotational motion describes **the relationships among rotation angle, angular velocity, angular acceleration, and time**.

$$\text{arc length} = \text{radius} \times \text{angular displacement}$$
$$s = r\phi$$
$$\text{one revolution} = 2\pi \text{ radians}$$
$$= 360 \text{ degrees}$$
$$1 \text{ radian} = 57.3^0$$
$$1 \text{ radian} = 0.159 \text{ revolution}$$



$$\bar{\omega} = \frac{\phi_2 - \phi_1}{t_2 - t_1} = \frac{\Delta\phi}{\Delta t}$$
$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\phi}{\Delta t}$$
$$\omega = \frac{d\phi}{dt} \quad s = 2\pi r = \text{total circumference}$$

To familiarize ourselves with the notion of angular speed, let us compute for a clock second, minute and hour hands:

$$\omega = \frac{2\pi}{T}$$
$$\omega_{\text{second}} = \frac{2\pi}{60} = 0.105\ rad/s$$
$$\omega_{\text{minute}} = \frac{2\pi}{60 \times 60} = 1.75 \times 10^{-3}\ rad/s$$
$$\omega_{\text{hour}} = \frac{2\pi}{60 \times 60 \times 12} = 1.45 \times 10^{-4}\ rad/s$$

Our sun is 2.3 x 10^4 light years away from the centre of our Milky Way galaxy. It moves in a circle around this centre at 250 km/s.

(a) How long does it take the sun to make one revolution about the galactic center?

(b) How many revolutions has the sun completed since it was formed about 4.5 x 10^9 years ago?

a) $1 \text{ Light Year} = 9.46\times10^{15} m$

$$\text{v} = R\omega = R\frac{\theta}{t} = R\frac{2\pi}{T}$$

$$\therefore \text{ for one revolution } T = \frac{2\pi R}{\text{v}}$$

$$T = 5.5\times10^{15} s = 1.74\times10^{8} \text{ years}$$

$$b)\ \frac{4.5\times10^9}{1.74\times10^8} = 26 \text{ revolutions}$$

$$\bar{\alpha} = \frac{\omega_2-\omega_1}{t_2-t_1} = \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \lim_{\Delta t\to0}\frac{\Delta\omega}{\Delta t}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\frac{d\phi}{dt} = \frac{d^2\phi}{dt^2}$$

$$s = r\phi$$

$$\frac{ds}{dt} = r\frac{d\phi}{dt}$$

$$\text{v} = r\omega$$

$$\frac{d\text{v}}{dt} = r\frac{d\omega}{dt}$$

$$\text{a}_T = r\alpha$$

Relationship between linear and angular variables

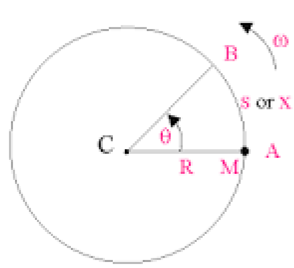
The Relations between Linear and Angular Variables: Each of the angular variables θ, ω, and α is related to its corresponding linear variable x, v, and a_t by factor R, the radius of rotation. **x = Rθ ; v = Rω ; a_t = Rα** . (a_t means tangential acceleration).

When mass M moves on the circle from A to B, the radius R sweeps angle θ that is called the "angular displacement" of mass M.

The linear displacement of mass M is arc S or X given by S = R θ as was discussed in Chapter 5 .

ω is the change in θ over time (t).

$\omega = \frac{\Delta\theta}{\Delta t}$ preferably in rd/s.



Translational Motion

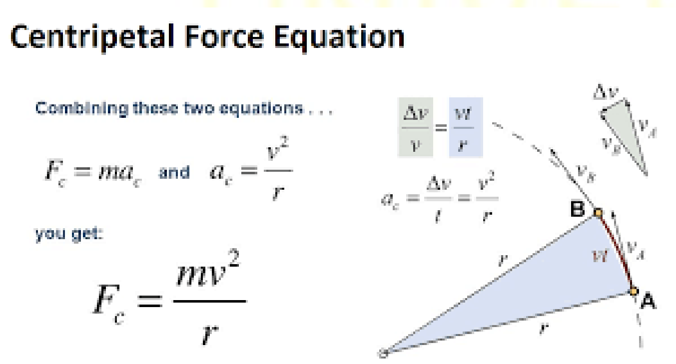
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

Rotational Motion

$$\omega = \omega_0 + \alpha t$$
$$\phi = \phi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha(\phi - \phi_0)$$

Centripetal acceleration

Centripetal acceleration, **the acceleration of a body traversing a circular path.** Because velocity is a vector quantity (that is, it has a magnitude, the speed, and a direction), when a body travels on a circular path, its direction constantly changes and thus its velocity changes, producing an acceleration.



$$\Delta v \approx v\theta$$
$$\bar{a} = \frac{\Delta v}{\Delta t} \approx \frac{v\theta}{r\theta / v} = \frac{v^2}{r}$$
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$
$$\vec{a}_R = -\frac{v^2}{r}\hat{r}$$

Cross product (vector product)
is defined as,

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

$\hat{n}$ is perpendicular to AB-plane

$$\hat{i}\times\hat{j}=\hat{k}$$

$$\hat{k}\times\hat{i}=\hat{j}$$

$$\hat{j}\times\hat{k}=\hat{i}$$

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

$$\vec{A}\times\vec{A}=0$$

$$(\vec{A}+\vec{B})\times\vec{c}=(\vec{A}\times\vec{B})+(\vec{A}\times\vec{c})$$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$$=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$$

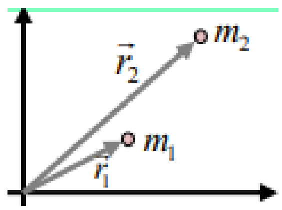
Lecture 12 – PHYSICS OF MANY PARTICLES

Centre

A body is made of a collection of particles. We would like to think of this body having a "Centre". For two masses the "center of mass" is defined as:

For two masses the centre of mass is:

$$\vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$



$$\vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\Rightarrow \quad x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

For N masses the obvious generalization of the Centre of mass position is the following:

For N masses the centre of mass is:

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N}{m_1 + m_2 + \cdots + m_N}$$

$$= \frac{1}{M}\left(\sum m_n\vec{r}_n\right)$$

Our definition of the cm allows Newton's Second Law to be written for entire collection of particles:

$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M}\left(\sum m_n\vec{v}_n\right)$$

$$\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{1}{M}\left(\sum m_n\vec{a}_n\right)$$

$$M\,\vec{a}_{cm} = \sum \vec{F}_n = \sum\left(\vec{F}_{ext} + \vec{F}_{int}\right)$$

$$\sum \vec{F}_{ext} = M\,\vec{a}_{cm}$$

Rotational Energy of Rigid Bodies

Consider rotational motion now for a rigid system of N particles. Rigid means that all particles have a fixed distance from the origin. The kinetic energy is,

Total kinetic energy:

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + \cdots$$

$$= \frac{1}{2}m_1 r_1^2 \omega^2 + \frac{1}{2}m_2 r_2^2 \omega^2 + \frac{1}{2}m_3 r_3^2 \omega^2 + \cdots$$

$$K = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

Rotational Inertia

$$K = \tfrac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$$\Rightarrow K = \tfrac{1}{2} I\omega^2, \text{where } I \equiv \sum m_i r_i^2$$

Compare with $K = \frac{1}{2}Mv^2$!!

Solid plate about cylinder axis

$$I = \int r^2 dm \quad (dm = 2\pi r dr \rho_0)$$

$$= \int_0^R 2\pi r^3 dr \rho_0$$

$$= \frac{1}{2}(\pi R^2 \rho_0) R^2$$

$$= \frac{1}{2} M R^2$$

Rotational Dynamics of Rigid Bodies

Torque:

In physics and mechanics, torque is the rotational equivalent of linear force. It is also referred to as the moment, moment of force, rotational force or turning effect, depending on the field of study. The concept originated with the studies by Archimedes of the usage of levers.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r F \sin\theta$$

Force

A force is **a push or pull upon an object resulting from the object's interaction with another object**. Whenever there is an interaction between two objects, there is a force upon each of the objects. Forces only exist as a result of an interaction.

Force (N) = mass (kg) × acceleration (m/s²).

Work done

Work is done **whenever a force moves something over a distance**. You can calculate the energy transferred, or work done, by multiplying the force by the distance moved in the direction of the force.

Energy transferred = work done = force x distance moved in the direction of the force.

$$
\begin{aligned}
dW &= \vec{F} \cdot d\vec{s} = F \cos\theta \, ds \\
&= \left(F \cos\theta\right)\left(r d\phi\right) \\
dW &= \tau d\phi \\
dW_{net} &= \left(F_1 \cos\theta_1\right) r_1 d\phi + \left(F_2 \cos\theta_2\right) r_2 d\phi + \\
&\qquad \cdots + \left(F_n \cos\theta_n\right) r_n d\phi \\
dW_{net} &= \left(\tau_1 + \tau_2 + \cdots + \tau_n\right) d\phi
\end{aligned}
$$

$$dW_{net} = \left(\sum \tau_{ext}\right) d\phi = \left(\sum \tau_{ext}\right) \omega dt$$

$$dK = d\left(\frac{1}{2} I \omega^2\right) = I \omega d\omega = (I\alpha)\omega dt$$

$$dW_{net} = dK \Rightarrow \sum \tau_{ext} = I\alpha$$

Like Newton's second law !!!

Translational	Rotational
x, M	ϕ, I
$\mathrm{v} = \frac{dx}{dt}$	$\omega = \frac{d\phi}{dt}$
$\mathrm{a} = \frac{d\mathrm{v}}{dt}$	$\alpha = \frac{d\omega}{dt}$
$F = M\mathrm{a}$	$\tau = I\alpha$
$W = \int F dx$	

Combined Rotational and Translational Motion

$$K = \frac{1}{2} M \mathrm{v}_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

WHY ?

$$K = \sum \frac{1}{2} m_i \mathrm{v}_{cm}^2 + \sum \frac{1}{2} m_i \mathrm{v}_i'^2$$

$$= \frac{1}{2} M \mathrm{v}_{cm}^2 + \sum \frac{1}{2} m_i r_i'^2 \omega^2$$

$$K = \frac{1}{2} M \mathrm{v}_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

Rolling without slipping

$$\mathrm{v}_{cm} = R\omega$$

$$K = \frac{1}{2} M\mathrm{v}_{cm}^2 + \frac{1}{2} I_{cm} \left(\frac{\mathrm{v}_{cm}^2}{R^2} \right)$$

$$K = \frac{1}{2} MR^2 \omega^2 + \frac{1}{2} I_{cm} \omega^2$$

$$K = \frac{1}{2} M\mathrm{v}_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$Mgh = \frac{1}{2} M\mathrm{v}_{cm}^2 + \frac{1}{2} \left(\frac{1}{2} MR^2 \right) \left(\frac{\mathrm{v}_{cm}^2}{R} \right)^2$$

$$\mathrm{v}_{cm} = \sqrt{\frac{4}{3} gh}$$

Lecture 13 – ANGULAR MOMENTUM

ANGULAR MOMENTUM

The quantity of rotation of a body, which is the product of its moment of inertia and its angular velocity

$$\vec{L} = \vec{r} \times \vec{p}$$

Angular Momentum

$$L = r p \sin\theta$$

$$L = (r \sin\theta) p = r_{\perp} p$$

$$L = r (p \sin\theta) = r p_{\perp}$$

Just different ways of writing L !!

Relation between torque and angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} + \Delta\vec{L} = \left(\vec{r} + \Delta\vec{r}\right) \times \left(\vec{p} + \Delta\vec{p}\right)$$

$$\vec{L} + \Delta\vec{L} = \vec{r} \times \vec{p} + \vec{r} \times \Delta\vec{p} + \Delta\vec{r} \times \vec{p} + \Delta\vec{r} \times \Delta\vec{p}$$

$$\Delta\vec{L} = \vec{r} \times \Delta\vec{p} + \Delta\vec{r} \times \vec{p}$$

$$\frac{\Delta \vec{L}}{\Delta t} = \frac{\vec{r} \times \Delta \vec{p} + \Delta \vec{r} \times \vec{p}}{\Delta t} = \vec{r} \times \frac{\Delta \vec{p}}{\Delta t} + \frac{\Delta \vec{r}}{\Delta t} \times \vec{p}$$

Take limit as $\Delta t \to 0$:

$$\because \quad \lim_{\Delta t \to 0} \frac{\Delta \vec{L}}{\Delta t} = \frac{d\vec{L}}{dt}$$

$$\therefore \quad \frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

But $\frac{d\vec{r}}{dt}$ is $\vec{v}$ and $\vec{p} = m\vec{v}$!

$$\frac{d\vec{r}}{dt} \times \vec{p} = \vec{v} \times m\vec{v} = m\left(\vec{v} \times \vec{v}\right) = 0$$

and we are left with only

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt}$$

Now use Newton's second law:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$$

$$\therefore \quad \frac{d\vec{L}}{dt} = \vec{\tau}$$

$$\frac{d\vec{L}}{dt} = \sum \vec{\tau}$$

The torque on a system of particles can come both from external and internal forces.

$\vec{\tau} = \vec{r} \times \vec{F}$

where $\vec{F} = m\vec{g}$

$\therefore \;\; \tau = Mgr\sin\theta$

$\vec{\tau}$ is perpendicular to $\vec{L}$.

$\therefore$ it cannot change the

magnitude of $\vec{L}$!!

$\Delta\vec{L} = \vec{\tau}\;\Delta t$

$$\Delta\phi = \frac{\Delta L}{L\sin\theta}$$

$$= \frac{\tau\;\Delta t}{L\sin\theta}$$

Precession speed ω_P is:

$$\omega_P = \frac{\Delta\phi}{\Delta t}$$

$$= \frac{\tau}{L\sin\theta} = \frac{Mgr\sin\theta}{L\sin\theta} = \frac{Mgr}{L}$$

Angular momentum for a system of particles

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \cdots + \vec{L}_N = \sum_{n=1}^{N} \vec{L}_n$$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{L}_1}{dt} + \frac{d\vec{L}_2}{dt} + \cdots + \frac{d\vec{L}_N}{dt} = \sum_{n=1}^{N} \frac{d\vec{L}_n}{dt}$$

Since $\frac{d\vec{L}_n}{dt} = \vec{\tau}_n$

$$\frac{d\vec{L}}{dt} = \sum_{n=1}^{N} \vec{\tau}_n$$

There are two sources of the torque acting on the system

1) **The torque exerted on the particles of the system by internal forces between the particles**

2) **The torque exerted on the particles of the system by external forces**

$$\sum \vec{\tau} = \sum \vec{\tau}_{\text{int}} + \sum \vec{\tau}_{ext}$$

If the forces between two particles not only are equal and opposite but are also directed along the line joining the two particles, then the total internal torque is zero.

$$\sum \vec{\tau}_{\text{int}} = 0$$
$$\sum \vec{\tau}_{\text{int}} = \vec{\tau}_1 + \vec{\tau}_2$$
$$= \vec{r}_1 \times \vec{F}_{12} + \vec{r}_2 \times \vec{F}_{21}$$
$$but$$
$$\vec{F}_{12} = -\vec{F}_{21} = F\hat{r}_{12}$$
$$\therefore \sum \vec{\tau}_{\text{int}} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{12} = \vec{r}_{12} \times (F\hat{r}_{12})$$
$$= F(\vec{r}_{12} \times \hat{r}_{12}) = 0$$
$$Hence$$
$$\sum \vec{\tau} = \sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

Conservation of Angular Momentum

If no net external torque acts on the system, then the angular momentum of the system does not change with the time.

$$\frac{d\vec{L}}{dt} = 0 \quad \Rightarrow \quad \vec{L} = \text{a constant}$$
$$\vec{F} = \frac{d\vec{p}}{dt} \Leftrightarrow \vec{\tau} = \frac{d\vec{L}}{dt}$$
$$\vec{p} = m\vec{\text{v}} \Leftrightarrow \vec{\text{L}} = \ ??$$
$\vec{L}$ depends on the choice of the origin:
$$\vec{L}' = \vec{r}' \times \vec{p} = (\vec{c} + \vec{r}) \times \vec{p} = \vec{c} \times \vec{p} + \vec{L}$$

Rotation of Rigid Bodies

When a rigid body is in pure rotational motion all particles in the body rotate through the same angle during the same time interval. Thus, all particles have the same angular velocity and the same angular acceleration.

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$v = \omega r \sin\theta$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\vec{\omega} \times \vec{r}\right)$$

$$= \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

$$\vec{a} = \vec{a}_T + \vec{a}_R$$

Linear and angular acceleration:

at=rΔωΔt a t = r Δ ω Δ t . These equations mean that linear acceleration and angular acceleration are directly proportional. The greater the angular acceleration is, the larger the linear (tangential) acceleration is, and vice versa.

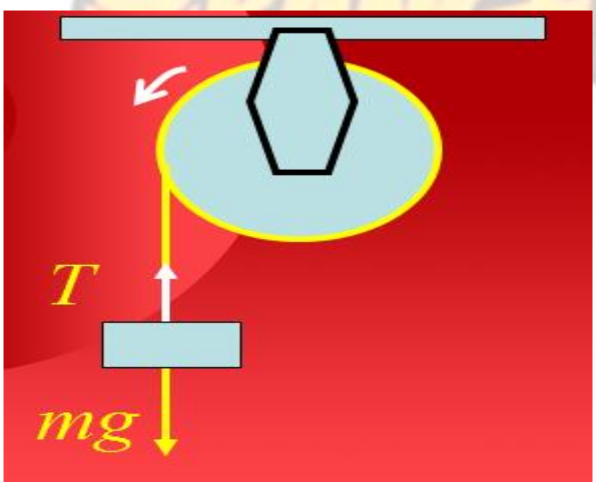
$$\text{minimum speed} = \frac{\text{length of the arrow}}{\text{time to pass one spoke}}$$

$$s = \text{distance traveled by one spoke} = \frac{2\pi r}{8}$$

$$\text{time to pass one spoke} = \frac{\text{distance traveled by one spoke}}{\text{speed of spoke}}$$

$$\text{time to pass one spoke} = \frac{s}{v} = \frac{2\pi r}{8r\omega}$$

$$\text{So minimum speed} = \frac{\ell \times 8r\omega}{2\pi r} = 4.8\,\text{m/s}$$



$$\sum F = mg - T = ma$$

$$\sum \tau = TR = \frac{1}{2} MR^2 \left(\frac{a}{R} \right)$$

$$T = \frac{1}{2} Ma$$

$$a = g \frac{2m}{M + 2m} = 4.8 \text{m/s}^2$$

$$T = mg \frac{M}{M + 2m} = 6.0 N$$

$$\alpha = \frac{a}{R} = 3.8 \text{rev/s}^2$$

Lecture 14 – EQUILIBRIUM OF RIGID BODIES

EQUILIBRIUM OF RIGID BODIES

Equilibrium is a defined as any **point where the total amount of external force or torque is zero**. This point may be anywhere near the center of mass.

where $\vec{F} = \sum \vec{F}_{ext}$ is net external force.

In equilibrium: $\frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{F} = 0$

(Must hold for all components !!).

The rotational motion of a rigid body is governed by:

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

where $\vec{\tau} = \sum \vec{\tau}_{ext}$ is net external torque.

In equilibrium $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{\tau} = 0$.

Conditions for Equilibrium

Conditions for equilibrium require that the sum of all external forces acting on the body is zero (first condition of equilibrium), and the sum of all external torques from external forces is zero (second condition of equilibrium). These two conditions must be simultaneously satisfied in equilibrium.

A rigid body is in mechanical equilibrium if both the linear momentum $\vec{P}$ and angular momentum $\vec{L}$ have a constant value.

i.e., $\frac{d\vec{P}}{dt} = 0$ and $\frac{d\vec{L}}{dt} = 0$

$\vec{P} = 0$ and $\vec{L} = 0 \Rightarrow$ static equilibrium

Angular momentum and torque depend on where you choose the origin of your coordinates for a body in equilibrium, the choice of origin for calculating torques is unimportant.

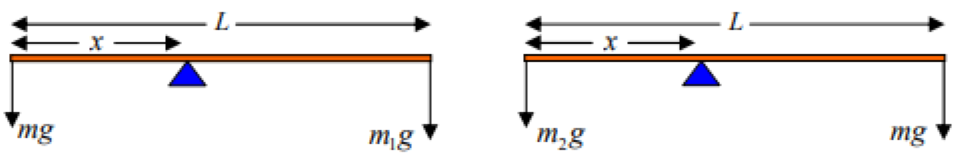
$$\vec{\tau}_O = \vec{\tau}_1 + \vec{\tau}_2 + \cdots + \vec{\tau}_N$$
$$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \cdots$$
$$\cdots + \vec{r}_N \times \vec{F}_N$$

$$\vec{\tau}_P = (\vec{r}_1 - \vec{r}_P) \times \vec{F}_1 + (\vec{r}_2 - \vec{r}_P) \times \vec{F}_1 + \cdots$$
$$\cdots + (\vec{r}_N - \vec{r}_P) \times \vec{F}_N$$
$$= \left[\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \cdots + \vec{r}_N \times \vec{F}_N\right]$$
$$- \left[\vec{r}_P \times \vec{F}_1 + \vec{r}_P \times \vec{F}_2 + \cdots + \vec{r}_P \times \vec{F}_N\right]$$
$$= \vec{\tau}_O - \left[\vec{r}_P \times \left(\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N\right)\right]$$
$$= \vec{\tau}_O - \left[\vec{r}_P \times \left(\sum \vec{F}_{ext}\right)\right]$$

but $\sum \vec{F}_{ext} = 0$, for a body in translational equilibrium

$\therefore \quad \vec{\tau}_P = \vec{\tau}_O$

Let us use the equilibrium conditions to do something of definite practical importance.



Taking the torques about the knife edge in the two cases, we have:

$$mgx = m_1g(L-x) \text{ and } m_2gx = mg(L-x)$$

$$\Rightarrow \frac{m}{m_2} = \frac{m_1}{m} \text{ or } m = \sqrt{m_1 m_2}.$$

Remarkably, we do not need the values of x or L.

Suppose the gravitational acceleration $\vec{g}$ has the same value at all points of a body.

Then:

1) The weight is equal to $M\vec{g}$, and

2) The center of gravity coincides with the centre of mass

The net force on the whole = sum over all individual particles

$$\sum \vec{F} = \sum m_i \vec{g}$$

Since $\vec{g}$ has the same value for every particle of the body $\therefore \sum \vec{F} = \vec{g} \sum m_i = M\vec{g}$

The net torque about the origin O:

$$\sum \vec{\tau} = \sum \left(\vec{r}_i \times m_i \vec{g} \right)$$

$$= \sum \left(m_i \vec{r}_i \times \vec{g} \right)$$

$$\therefore \sum \vec{\tau} = M\vec{r}_{cm} \times \vec{g} = \vec{r}_{cm} \times M\vec{g}$$

The torque due to gravity about the centre of mass of a body is zero !!

Center of gravity (COG)

Center of gravity is **the point where the mass of the body is concentrated**. The center of gravity (COG) of the human body is a hypothetical point around which the force of gravity appears to act. It is point at which the combined mass of the body appears to be concentrated. The center of gravity of an object is calculated by **taking the sum of its moments divided by the overall weight of the object**. The moment is the product of the weight and its location as measured from a set point called the origin.

$$CG = \frac{W_1 d_1 + W_2 d_2}{W}$$

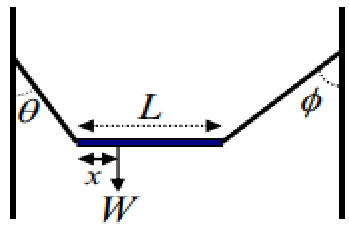
$$CG = \frac{(100)(9.8)(10) + (25)(9.8)(0)}{(100 + 25)9.8}$$

$$CG = 8\,m$$

Review: For two masses the CM is:

$$\vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Let's solve a problem of static equilibrium: A non-uniform bar of weight W is suspended at rest in a horizontal position by two light cords. Find the distance x from the left-hand end to the center of gravity.



Solution : Call the tensions T_1 and T_2 . Put the forces in both directions equal to zero,

a) $T_2 \sin\phi - T_1 \sin\theta = 0$ (horizontal)

b) $T_2 \cos\phi + T_1 \cos\theta - W = 0$ (vertical) $\Rightarrow T_2 = \dfrac{W}{\sin(\theta+\phi)}$

The torque about any point must vanish. Let us choose that point to be one end of the bar,

$$-Wx + (T_2\cos\phi)L = 0 \Rightarrow x = \frac{(T_2\cos\phi)L}{W} = \frac{L\cos\phi}{\sin(\theta+\phi)}$$

Suppose the gravitational acceleration $\vec{g}$ has the same value at all points of a body.

Then:

1) The weight is equal to $M\vec{g}$, and

2) The center of gravity coincides with the centre of mass

The net force on the whole = sum over all individual particles

$$\sum \vec{F} = \sum m_i \vec{g}$$

Since $\vec{g}$ has the same value for every particle of the body $\therefore \sum \vec{F} = \vec{g}\sum m_i = M\vec{g}$

The net torque about the origin O:

$$\sum \vec{\tau} = \sum(\vec{r}_i \times m_i\vec{g})$$

$$= \sum(m_i\vec{r}_i \times \vec{g})$$

$$\therefore \sum \vec{\tau} = M\vec{r}_{cm} \times \vec{g} = \vec{r}_{cm} \times M\vec{g}$$

The torque due to gravity about the centre of mass of a body is zero !!

Types of Equilibrium

a) Stable equilibrium: object returns to its original position if displaced slightly.
b) Unstable equilibrium: object moves farther away from its original position if displaced slightly
c) Neutral equilibrium: object stays in its new position if displaced slightly.

Lecture 15 – OSCILLATIONS: I

Oscillations are everywhere!

- **pendulum sitar and guitar strings**
- **boats bobbing at anchor**
- **quartz crystal in a watch**
- **masses on springs**

An oscillation is any self-repeating motion. This motion is characterized by:

A. The period T, this is the time for completing one full cycle.
B. The frequency F = 1/T, which is the number of per second. (Another frequently used symbol is).
C. The amplitude A, which is the maximum displacement from equilibrium (or the size of the oscillation).

Simple Harmonic Motion (SHM)

Simple Harmonic Motion or SHM is defined as a motion in which the restoring force is directly proportional to the displacement of the body from its mean position. ... The acceleration of a particle executing simple harmonic motion is given by, $\mathbf{a(t) = -\omega^2 x(t)}$. Here, ω is the angular velocity of the particle.
Displacement: x = -A
Acceleration: |a| = Max
Speed: |v| = 0

For there to be periodic motion, there must be:

- **an equilibrium position**
- **a restoring force**

- **energy transformation (kinetic $\Leftrightarrow$ potential)**

The restoring force depends on the displacement Δx

$$F_{\text{restore}} = -k\,\Delta x$$

$$F(x) = -kx$$

where k is the force constant and measures stiffness of the spring

$$U(x) = \frac{1}{2}kx^2$$

(stored energy)

$$F(x) = -kx$$

$$ma = F \Rightarrow$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

This is called equation of motion of the simple harmonic oscillator

Simple Harmonic Motion

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega^2 = \frac{k}{m}$$

How to calculate $\frac{\text{d}}{\text{dt}}\cos\omega t$?

$$x(t) = \cos\omega t$$

$$x(t+\Delta t) = \cos\omega(t+\Delta t)$$

$$x(t+\Delta t) - x(t) = \cos\omega(t+\Delta t) - \cos\omega t$$

$$= -\sin\omega\Delta t\sin(\omega t + \omega\Delta t/2)$$

$$\approx -\omega\Delta t\sin\omega t$$

$$\therefore \frac{\mathrm{d}}{\mathrm{dt}} \cos \omega t = -\omega \sin \omega t$$

Remember two important results:

$$\frac{d}{dt}(\sin \omega t) = \omega \cos \omega t$$

$$\frac{d}{dt}(\cos \omega t) = -\omega \sin \omega t$$

Remember two important results:

$$= \frac{d}{dt}(\sin \omega t) = \omega \cos \omega t$$

$$= \frac{d}{dt}(\cos \omega t) = -\omega \sin \omega t$$

Physical significance of constant ω

$$x = x_m \cos \omega \left(t + \frac{2\pi}{\omega} \right)$$

$$= x_m \cos(\omega t + 2\pi)$$

$$= x_m \cos \omega t$$

That is, the function merely repeats it self after a time $2\pi / \omega$

So $2\pi / \omega$ is the period of the motion T

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

The frequency ν of the oscillator is the number of complete vibrations per unit time:

$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega = 2\pi\nu = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

ω is called the angular frequency

$\dim[\omega] = T^{-1}$ Unit of ω is radian/second

Energy of simple harmonic motion

The total energy in simple harmonic motion is **the sum of its potential energy and kinetic energy**. Thus, the total energy in the simple harmonic motion of a particle is: Directly proportional to its mass. Directly proportional to the square of the frequency of oscillations and.

The quantity $\theta = \omega t + \phi$ is called the phase of the motion. The constant ϕ is called the phase constant.

Energy of simple harmonic motion

$$x = x_m \cos \omega t$$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 \cos^2 \omega t$$

$$K = \frac{1}{2}m\text{v}^2 = \frac{1}{2}m\omega^2 x_m^2 \sin^2 \omega t$$

$$= \frac{1}{2}kx_m^2 \sin^2 \omega t$$

$$E = K + U$$

$$= \frac{1}{2}kx_m^2 \cos^2 \omega t + \frac{1}{2}kx_m^2 \sin^2 \omega t$$

$$= \frac{1}{2}kx_m^2 \left(\cos^2 \omega t + \sin^2 \omega t\right)$$

$$= \frac{1}{2}kx_m^2$$

$$E = \frac{1}{2}m\text{v}^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2$$

$$\text{v} = \frac{dx}{dt} = \pm\sqrt{\frac{k}{m}\left(x_m^2 - x^2\right)}$$

speed is maximum at x=0

speed is zero at $x = \pm x_m$

Lecture 16 – OSCILLATIONS: II

Simple pendulum

A pendulum is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its resting, equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position.

For example

$$F = -mg\sin\theta$$

For small value of θ

$$\sin\theta \approx \theta$$

$$x = L\theta$$

$$F = -mg\theta = -mg\frac{x}{L}$$

$$= -\left(\frac{mg}{L}\right)x$$

$$F = m\frac{d^2x}{dt^2} = -\left(\frac{mg}{L}\right)x$$

$$\frac{d^2x}{dt^2} = -\left(\frac{g}{L}\right)x$$

Solution : $x = x_m \cos\omega t$

$$\omega = \sqrt{\frac{g}{L}}$$

The physical pendulum

A pendulum is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its resting, equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position.

$$\tau = -Mgd\sin\theta$$

For small θ, $\sin\theta \approx \theta$

$$\therefore \tau = -Mgd\theta$$

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} = -Mgd\theta$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{Mgd}{I}\right)\theta$$

Its time period will be:

$$\omega = \sqrt{\frac{Mgd}{I}}$$

$$\text{or } I = \frac{Mgd}{\omega^2}$$

The quantities on the right are all measurable. Using this formula we can determine the rotational inertia of anybody about an axis of rotation other than through the center of mass.

The physical pendulum includes the simple pendulum as a special case

$$d = L \text{ and } I = ML^2$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{ML^2}{MgL}}$$

$$= 2\pi\sqrt{\frac{L}{g}}$$

Center of gyration

The center of gyration with respect to the axis of a rotating body is a point at which if the entire mass of the body were concentrated its moment of inertia would remain unchanged. The distance of this point from the axis is the radius of gyration.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{I}{Mgd}} \Rightarrow L = \frac{I}{Md}$$

Simple harmonic motion and uniform circular motion

Uniform Circular Motion describes **the movement of an object traveling a circular path with constant speed**. The one-dimensional projection of this motion can be described as simple harmonic motion. A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion.

Acceleration of the particle is

$$\vec{a} = -\frac{v^2}{R}\hat{r} = -R\omega^2\hat{r}$$

acceleration along x direction is:

$$a_x = -R\omega^2\cos\theta$$

but

$$x = R\cos\theta$$

$$\therefore a_x = \frac{d^2x}{dt^2} = -\omega^2 x$$

Thus point P executes simple harmonic motion

Acceleration of the point Q is:

$$a_y = -R\omega^2\sin\theta$$

but

$$y = R\sin\theta$$

$$\therefore a_y = \frac{d^2y}{dt^2} = -\omega^2 y$$

Q also executes simple harmonic motion

Sum of two simple harmonic motions of the same period along the same line:

$x_1 = A_1\sin\omega t$ and $x_2 = A_2\sin(\omega t + \phi)$

The resultant displacement

$$x = x_1 + x_2$$
$$= A_1\sin\omega t + A_2\sin(\omega t + \phi)$$
$$= A_1\sin\omega t + A_2\sin\omega t\cos\phi + A_2\sin\phi\cos\omega t$$
$$= \sin\omega t(A_1 + A_2\cos\phi) + \cos\omega t(A_2\sin\phi)$$

Let $A_1 + A_2\cos\phi = R\cos\theta$

and $A_2\sin\phi = R\sin\theta$

We get $x = R\sin(\omega t + \theta)$

Thus the resultant motion is also simple harmonic motion along the same line and has the same time period. Its amplitude is

$R = \sqrt{A_1^2 + A_2^2 + A_1 A_2 \cos\phi}$

$\tan\theta = \dfrac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$

Special cases :

If $\phi = 0$ then

$R = \sqrt{A_1^2 + A_2^2 + A_1 A_2} = \sqrt{(A_1 + A_2)^2} = A_1 + A_2$

and

$\tan\theta = 0 \Rightarrow \theta = 0$

We get $x = (A_1 + A_2)\sin\omega t$

This is constructive interference

If $\phi = \pi$ then

$R = \sqrt{A_1^2 + A_2^2 - A_1 A_2} = \sqrt{(A_1 - A_2)^2} = A_1 - A_2$

and

$\tan\theta = 0 \Rightarrow \theta = 0$

We get $x = (A_1 - A_2)\sin\omega t$

This is destructive interference

Composition of two simple harmonic motions of the same period at right angles to each other

$x = A\sin\omega t$ and $y = B\sin(\omega t + \phi)$

$\sin\omega t = \dfrac{x}{A} =$ and $\cos\omega t = \sqrt{1 - x^2 / A^2}$

$\dfrac{y}{B} = \sin\omega t\cos\phi + \sin\phi\cos\omega t$

$= \dfrac{x}{A}\cos\phi + \sin\phi\sqrt{1 - x^2 / A^2}$

squaring and rearranging

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - 2\frac{xy}{AB}\cos\phi = \sin^2\phi =$$

This is the equation of an ellipse.

$Special\ cases:$

If $\phi = 0$ then

$$\frac{x^2}{A^2}+\frac{y^2}{B^2}-2\frac{xy}{AB}=0 \Rightarrow \left(\frac{x}{A}-\frac{y}{B}\right)^2=0$$

Lissajous Figures

A Lissajous curve, also known as Lissajous figure or Bowditch curve, is the graph of a system of parametric equations x=A\sin, \quad y=B\sin, which describe complex harmonic motion.

Or

If two oscillations of different frequencies at right angles are combined, the resulting motion is more complicated. It is not even periodic unless the two frequencies are in the ratio of integers. These resulting curves are called Lissajous figures.

$$x = A\sin\omega_x t \text{ and } y = B\sin\left(\omega_y t+\phi\right)$$

$$\frac{\omega_x}{\omega_y} = \text{integers} \Rightarrow \text{periodic motion}$$

Damped harmonic motion

Damped harmonic motion is **a real oscillation**, in which an object is hanging on a spring. Because of the existence of internal friction and air resistance, the system will over time experience a decrease in amplitude. The decrease of amplitude is due to the fact that the energy goes into thermal energy.

Damping force $= -b\dfrac{dx}{dt}$ where $b > 0$

From Newton's second law

$$\sum \vec{F} = m\vec{a}$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$\Rightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

Its solution for $\dfrac{k}{m} \geq \left(\dfrac{b}{2m}\right)^2$ is

$$x = x_m e^{-bt/2m}\cos\left(\omega' t + \phi\right)$$

where $\omega' = \sqrt{\dfrac{k}{m} - \left(\dfrac{b}{2m}\right)^2}$

$$m\frac{d^2x}{dt^2} + kx = F_0 \cos\omega t$$

Solution : $x = \dfrac{F_0}{m\left(\omega_0^2 - \omega^2\right)}\cos\omega t$

Check : $\text{LHS} = \dfrac{mF(-\omega^2) + kF}{m\left(\omega_0^2 - \omega^2\right)}\cos\omega t = \text{RHS}$

Here ω_0 is the natural frequency of the

system and is given by $\omega_0 = \sqrt{\dfrac{k}{m}}$

There is a characteristic value of the driving frequency ω at which the amplitude of oscillation is a maximum. This condition is called resonance. For negligible damping resonance occurs at $\omega = \omega_0$

Two body oscillations

$$m_1 \frac{d^2 x_1}{dt^2} = -kx \qquad m_2 \frac{d^2 x_2}{dt^2} = +kx$$

$$m_1 m_2 \frac{d^2 x_1}{dt^2} - m_1 m_2 \frac{d^2 x_2}{dt^2} = -m_2 kx - m_1 kx$$

$$\left(\frac{m_1 m_2}{m_1 + m_2}\right) \frac{d^2}{dt^2}(x_1 - x_2) = -kx$$

$$\text{let } m = \left(\frac{m_1 m_2}{m_1 + m_2}\right)$$

$$\frac{d}{dt}(x_1 - x_2) = \frac{d}{dt}(x + L) = \frac{dx}{dt}$$

$$\frac{d^2 x}{dt^2} + \frac{k}{m} x = 0$$

Lecture 17 – PHYSICS OF MATERIALS

Elasticity:

The property, by virtue of which a body tends to regain its original shape and size when external forces are removed, is called elasticity.

Perfectly elastic:

{If a body completely recovers its original shape and size, it is called perfectly elastic. 0r. If a body has no tendency to regain its original shape and size, it is called perfectly plastic.}Nobody in nature is perfectly elastic or perfectly plastic. Common plastics, kneaded dough, solid honey, etc are plastics.

Stress:

Stress characterizes the strength of the forces causing the stretch, squeeze, or twist, usually on a “force per unit area” basis.

Stress = Force/Area

SI unit of stress is N/m

There are three types of stresses

1. Longitudinal stress or tensile stress or compressive stress.
2. volume stress
3. tangential or shearing stress

Longitudinal stress or tensile stress or compressive stress

If the deforming force is applied along some linear dimension of a body, the corresponding stress is called longitudinal stress or tensile stress or compressive stress.

Volume stress

If the force acts normally and uniformly fromall sides, the stress is called volume stress.

Tangential or shearing stress

If the force is applied tangentially to one face of a rectangular body, keeping the other face fixed, the stress is called tangential or shearing stress.

Strain

When deforming forces are applied on a body, it undergoes a change in shape or size. The fractional (or relative) change in shape or size is called the strain.

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

Three types of strain

Longitudinal (linear) strain is the ratio of the change in length (ΔL) to original length (l)

$$\text{Linear strain} = \frac{\Delta l}{l}$$

Volume strain is the ratio of the change in volume (ΔV) to original volume (V)

$$\text{Volume strain} = \frac{\Delta V}{V}$$

Shearing strain: The angular deformation (θ) in radians is called shearing stress.

For small θ :

$$\text{Shearing strain: } = \theta \approx \tan\theta = \frac{\Delta x}{l}$$

Hooke's Law

This law states that for small deformations, stress is proportional to strain.

Strain = E (strain)

The constant E is called modulus of elasticity
E has the same units as stress
There are three moduli of elasticity:

Young's Modulus (Y)

Young's modulus (E or Y) is **a measure of a solid's stiffness or resistance to elastic deformation under load**. It relates stress (force per unit area) to strain (proportional deformation) along an axis or line. A high Young's modulus value means a solid is inelastic or stiff.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta l / l}$$

Bulk's Modulus (B)

It is defined as the ratio between pressure increase and the resulting decrease in a material's volume. Together with Young's modulus, the shear modulus, and Hooke's law, the bulk modulus describes a material's response to stress or strain. Usually, bulk modulus is indicated by K or B in equations and tables.

$$B = \frac{\text{Volume Stress}}{\text{Volume Strain}}$$

$$B \equiv -\frac{\Delta P}{\Delta V / V}$$

$1/B$ is called compressibility. A material having a small value of B can be compressed easily.

Shear Modulus (h)

The shear modulus is the earth's material response to the shear deformation. It is defined as the ratio of shear stress and shear strain. This valuable property tells us in advance how resistant a material is to shearing deformation.

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}}$$

Shearing modulusis also called modulus of rigidity

$$\eta = \frac{F/A}{\theta} = \frac{F}{A\tan\theta} = \frac{Fl}{A\Delta x}$$

Poisson's ratio

When a wire is stretched, its length increases and radius decreases. The ratio of the lateral strain to the longitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\Delta r / r}{\Delta l / l}$$ Its value lies between 0 and 0.5.

Work done in stretching a wire

$$F = \frac{YA}{l}x$$

The work done in extending the wire through Δl is given by

$$W = \int_0^{\Delta l} Fdx = \frac{YA}{l}\int_0^{\Delta l} xdx = \frac{YA}{l}\frac{(\Delta l)^2}{2}$$

$$W = \frac{YA}{l}\frac{(\Delta l)^2}{2} = \frac{1}{2}(Al)\left(\frac{Y\Delta l}{l}\right)\left(\frac{\Delta l}{l}\right)$$

$$= \frac{1}{2}\times \text{volume}\times\text{stress}\times\text{strain}$$

$$\text{Work / unit volume} = \frac{1}{2}\times\text{stress}\times\text{strain}$$

$$W = \frac{1}{2}\left(\frac{YA\Delta l}{l}\right)\Delta l = \frac{1}{2}\times\text{load}\times\text{extension}$$

Volumetric Stress

When the deforming force or applied force acts from all dimensions resulting in the change of volume of the object then such stress in called volumetric stress or Bulk stress. In short, when the **volume of body changes due to** the deforming force it is termed as Volume stress.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{\text{F}}{\text{A}} = \text{Pressure}$$

Volumetric Strain:

The volumetric strain is **the unit change in volume**, i.e. the change in volume divided by the original volume.

$$\text{Strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

Bulk Modulus

The relative change in the volume of a body produced by a unit compressive or tensile stress acting uniformly over its surface

$$\text{Bulk Modulus} = \frac{\text{Volumetric Stress}}{\text{Volumetric Strain}}$$

$$B = \frac{\Delta p}{\Delta V / V}$$

Fluid Statics

A fluid is a substance that can flow and does not have a shape of its own. Thus all liquids and gases are fluids. Solids possess all the three moduli of elasticity whereas fluid possesses only bulk modulus. A fluid at rest cannot sustain a tangential force. If such force is applied to a fluid, the different layers simply slide over one another. Therefore the forces acting on a fluid at rest have to be normal to the surface.

Fluid Pressure

The normal force per unit area is called pressure $P = \frac{\Delta F}{\Delta A}$.

Pressure is a scalar quantity

Its unit is N/m^2, or Pascal (Pa)

Typical Pressures

Centre of earth	4×10^{11} Pa
Ocean bottom	1×10^{8} Pa
Car tyre	2×10^{5} Pa
Atmosphere	1×10^{5} Pa
Blood pressure	1.6×10^{4} Pa
Lab vacuum	1×10^{-12} Pa

A measure of the amount of information on a storage medium $\rho = \frac{\Delta m}{\Delta V}$. Density has no directional property and is a scalar quantity. If density is uniform then $\rho = \frac{\Delta m}{\Delta V} = \frac{m}{V}$

Density of water is 1000 kg/m^3

Variation of pressure in fluid at rest

Consider a small element of form of disc of fluid volume submerged within the body of the fluid. The mass of this volume is

$$dm = \rho dV = \rho A dy$$
$$\text{and its weight is}$$
$$(dm)g = \rho g A dy$$

$$\sum F_y = 0$$
$$pA - (p + dp)A - \rho g A dy = 0$$
$$\Rightarrow \frac{dp}{dy} = -\rho g$$
$$\frac{dp}{dy} = -\rho g$$

Therefor as the elevation increases (dy positive),
the pressure decreases (dy negative) .The quantity ρg is called weight density of the fluid. It is the weight per unit volume of the fluid.For liquids, which are nearly incompressible, ρ is practically constant.

$$\therefore \rho g = \text{constant}$$
$$\Rightarrow \frac{dp}{dy} = \frac{\Delta p}{\Delta y} = \frac{p_2 - p_1}{y_2 - y_1} = -\rho g$$
$$p_2 - p_1 = -\rho g (y_2 - y_1)$$
$$\text{for a homogeneous liquid}$$

If liquid has a free surface, then this becomes the natural level from where we can measure the distances.

Therefore

$$p_2 - p_1 = -\rho g\left(y_2 - y_1\right)$$

becomes

$$p_0 - p = -\rho g\left(y_2 - y_1\right)$$

but $y_2 - y_1 = h,$ therefore $p = p_0 + \rho gh$

To what height should a cylindrical vessel be filled with a homogeneous liquid to make the force with which the liquid presses on the sides of the wall equal to the force exerted by the liquid on the bottom of the vessel?

Force exerted by the liquid

on the bottom $= \left(hg\rho\right)\pi r^2$

Mean pressure on the wall $= \frac{1}{2}\left(hg\rho\right)$

Area of the wall in contact

with the liquid $= 2\pi rh$

Force exerted by the liquid

on the wall $= \left(\frac{1}{2}hg\rho\right)2\pi rh$

Since the two forces are equal

$$\left(\frac{1}{2}hg\rho\right)2\pi rh = \left(hg\rho\right)\pi r^2$$

$$\Rightarrow h = r$$

Pascal's Principle

Pressure applied to an enclosed fluid is transmitted to every portion of the fluid and to the walls of the containing vessel

$$p = p_{ext} + \rho gh$$

$$\Delta p = \Delta p_{ext} + \Delta\left(\rho gh\right)$$

Since the liquid is incompressible, the density is constant

$$\therefore \Delta\left(\rho gh\right) = 0$$

$$\Rightarrow \Delta p = \Delta p_{ext}$$

Hydraulic Lever

The pressure on the Iiquid at the smaller piston due to our externally applied force is $p_i = \frac{F_i}{A_i}$.

According to Pascal's principle this "input pressure"must be equal to the "output pressure" $p_0 = \frac{F_0}{A_0}$

$$p_i = p_0 \quad \Rightarrow \quad \frac{F_i}{A_i} = \frac{F_0}{A_0}$$

or

$$F_i = F_0\left(\frac{A_i}{A_0}\right) = Mg\left(\frac{A_i}{A_0}\right)$$

$$\text{Q}\ A_i = A_0 \quad \Rightarrow \quad \frac{A_i}{A_0} = 1$$

$$\therefore F_i = F_0 = Mg$$

The downward movement of the smaller piston through a distance d_i displaces

a volume of fluid

$$V = d_i A_i$$

If the fluid is incompressible, then this volume must be equal to the volume displaced by the upward motion of the larger piston

$$V = d_i A_i = d_0 A_0$$

or

$$d_0 = d_i\left(\frac{A_i}{A_0}\right)$$

Now

$$\text{Q}\ \frac{A_i}{A_0} = 1 \quad \Rightarrow \quad d_0 = d_i$$

The price we pay for gaining the ability to lift a larger load is losing the ability to move it very far.

Also:

$$F_i = F_0\left(\frac{A_i}{A_0}\right) = F_0\left(\frac{d_0}{d_i}\right) \quad \Rightarrow F_i d_i = F_0 d_0$$

Therefore, work done by the external force on the smaller piston equals the work done by the fluid on the larger fluid

Gauge pressure

Gauge pressure = actual pressure – atmospheric pressure

Measuring Pressure

The space above the mercury column is in effect a vacuum containing only mercury vapor, whose pressure is so small at ordinary temperatures that it can be neglected.

i.e., $p_2 = 0$

$p_1 = p$ is the unknown pressure

Therefore

$$p_2 - p_1 = -\rho g\left(y_2 - y_1\right)$$

or

$$0 - p = -\rho gh$$

$$\because h = y_2 - y_1$$

$$\Rightarrow \quad p = \rho gh$$

Measuring the height of the column above the surface of the dish then gives the pressure.

Manometer

The open tube manometer measures gauge pressure. The U-shaped tube contains often mercury or water

The pressure at the bottom of the tube due to the fluid in the left column is $p_0 + \rho g y_2$

And the pressure at the bottom of the tube due to the fluid in the right column is $p + \rho g y_1$

These pressure are measured at the same point so they must be equal

$$p + \rho g y_1 = p_0 + +\rho g y_2$$

$p - p_0 = \rho g(y_2 - y_1) = \rho g h$

Thus the gauge pressure $p - p_0$ is proportional to the difference in height of the liquid column in the U-tube

Therefore equating the pressures at point C on each side, we obtain

$p_0 + \rho_w g(2a) = p_0 + \rho g(2a + d)$

$\Rightarrow \rho = \left(\frac{2a}{2a + d}\right)\rho_w$

Archimedes' principle

$\because p_{below} > p_{above}$

$\therefore \frac{F_{below}}{A} > \frac{F_{above}}{A}$

$\Rightarrow F_{below} > F_{above}$

$F_b = F_{below} - F_{above} > 0$

This net upward force is called the buoyant force or buoyancy.

This force acts through the center of gravity of the displaced fluid, called the center of buoyancy.

Thus a body appears to weigh less when immersed in a fluid Apparent Weight = True Weight - Upthrust

Suppose a body of the volume V and density ρ is fully immersed in a liquid of density ρ', Then

Weight of the body $= W = \rho g V$

Weight of the liquid displaced $= W' = \rho' g V$

$\therefore$ Net downward force, or, apparent weight

$W_a = (\rho - \rho')gV = \rho g V\left(1 - \frac{\rho'}{\rho}\right)$

$W_a = (\rho - \rho')gV = W\left(1 - \frac{\rho'}{\rho}\right)$

The following possibilities may occur :

1) If $\rho' < \rho$, $W_a > 0$. Therefore, the body will sink to the bottom.
2) If $\rho' = \rho$, $W_a = 0$. Therefore, the body will just float or remain hanging at whatever height it is left inside the liquid
3) If $\rho' > \rho$, the upthrust will be greater than the weight of the body. Therefore, the body will move partly out of the free surface of the liquid untill the upthrust becomes equal to W. The body will then float. Thus the principle of floatation is:

"For a body to float in a liquid, the weight of the liquid displaced by the immersed portion of the body must be equal to its own weight".

Lecture 18 – PHYSICS OF FLUIDS

Fluid

A fluid is matter that has no definite shape and adjusts to the container that it is placed in. Gases and liquids are both fluids. All fluids are made of molecules. Every molecules attracts other molecules around it.

Surface tension

Leaves and insect can be observed to float on the surface of a body of water. They are not partially submerged and thus not buoyed up because of Archimedes' principle. In this case the object is completely on the surface and none of it is submerged.

Quantitative measurement of surface tension

$w =$ weight of the sliding wire

$T =$ pull force

$T + w = F =$ net downward force

Since film has both front and back surfaces so the force F acts along a total length of $2L$

The surface tension in the film is defined as

$$\gamma = \frac{F}{2L} \qquad \Rightarrow F = 2\gamma L$$

In equilibrium

$$F = 2\gamma L = w + T$$

$$\Rightarrow \gamma = \frac{w+T}{2L}$$

$$\Delta U = F\Delta x = \gamma L \Delta x$$

where L is the length of the surface layer

$L\Delta x = \Delta A =$ change in area of the surface that occurs when we stretch it

Therefore

$$F = \frac{\Delta U}{\Delta A}$$

Surface tension is the surface potential energy per unit area !

Liquid in contact with a solid surface

A liquid surface in contact with a solid boundary, e.g. with the vertical wall of the container, will in general not meet it at right **angles**. The angle α between the bounding plane and the tangent to the liquid surface at the point of contact is called the angle of contact between the particular liquid and solid.

Angle of contact

The angle between the tangent planes to the liquid and the solid surfaces, measured through the liquid, is called the angle of contact (θ)

If $\theta < 90^0$ liquid wets the solid

If $\theta > 90^0$ liquid does not wet the solid

Pressure inside a bubble

Surface tension causes a pressure difference between the inside and outside of a soap bubble or a liquid drop. A soap bubble consists of two spherical surface films with a thin layer of liquid

$p =$ pressure exerted by the upper half

between $p_0 =$ external pressure

Surface tension force acts on the circumference of the bubble

$\therefore$ force exerted due to surface tension $= 2(2\pi r\gamma)$

$\because$ bubble has two surfaces

At equilibrium

$$\pi r^2 p = 2(2\pi r\gamma) + \pi r^2 p_0$$

Therefore

$$\text{Excess pressure} = p - p_0 = \frac{4\gamma}{r}(soap\ bubble)$$

A liquid drop has only one surface film

$$\text{Excess pressure} = p - p_0 = \frac{2\gamma}{r}(liquid\ drop)$$

capilliarity

The rise and fall of a liquid in a capillary tube dipped in the liquid is called capilliarity.

Surface tension causes the capillarity

If angle of contact is less than 90^0, liquid rises in the tube and if the angle of contact is greater than 90^0, the liquid is depressed

$$\text{Excess pressure} = p - p_0 = \frac{4\gamma}{R}$$

but $p - p_0 = \rho gh$

and $r = R\cos\theta$

Therefore

$$h = \frac{4\gamma\cos\theta}{r\rho g}$$

Thus narrower the tube, the greater the rise

Fluid flow can be steady or non-steady:

For steady flow, pressure density and velocity
are constant in time, e.g., a gently flowing stream
Example of non-steady flow is water fall

Fluid flow can be compressible or incompressible

If the density ρ of a fluid is a constant, independent of x, y ,z and t its flow is called incompressible flow.
Liquid can usually be considered as flowing incompressibly, but gases are considered to be incompressible.

Fluid flow can be viscous or non-viscous

Viscosity in fluid motion is the analogue of friction in the motion of solid

When a fluid flows such that no energy is dissipated through viscous forces, the flow is said to nonviscous.

Two main types of fluid flow:

1) laminar

Fluid "particles" follow fixed paths (*streamlines*)

Streamlines do not cross or meet

Velocity is tangent to streamlines

2) turbulent

Equation of continuity

Consider water flowing in a pipe with a changing diameter:

Volume of water flowing past here per second = Volume of water flowing past here per second

$$V_1 / t = V_2 / t \qquad \Rightarrow V_1 = V_2$$

$$V_1 = A_1 L_1 = A_1 v_1 t \qquad V_2 = A_2 L_2 = A_2 v_2 t$$
$$V_1 / t = V_2 / t \qquad \Rightarrow A_1 v_1 = A_2 v_2$$

Smaller area $\Rightarrow$ greater speed

$$\frac{dV}{dt} = Av = \text{constant}$$

$$\frac{dV}{dt} = Av = \text{constant} \quad \textit{Continuity Equation}$$

Human circulatory system

- Large arteries $\Rightarrow$ slow blood flow
- Small arteries $\Rightarrow$ fast blood flow

Work done due to $p_1 = p_1 A_1 \Delta l_1$

Work done due to $p_2 = -p_2 A_2 \Delta l_2$

Work done
by gravity
is associated
with lifting the
fluid element from y_1 to $y_2 = -\Delta mg\left(y_2 - y_1\right)$

$$W = p_1 A_1 \Delta l_1 - p_2 A_2 \Delta l_2 - \Delta mg\left(y_2 - y_1\right)$$

Now $A_1 \Delta l_1 = A_2 \Delta l_2 = \Delta V = \Delta m / \rho$

$$\therefore W = \left(p_1 - p_2\right)\left(\Delta m / \rho\right) - \Delta mg\left(y_2 - y_1\right)$$

The change in kinetic energy of the fluid element is

$$\Delta K = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2$$

From energy conservation,

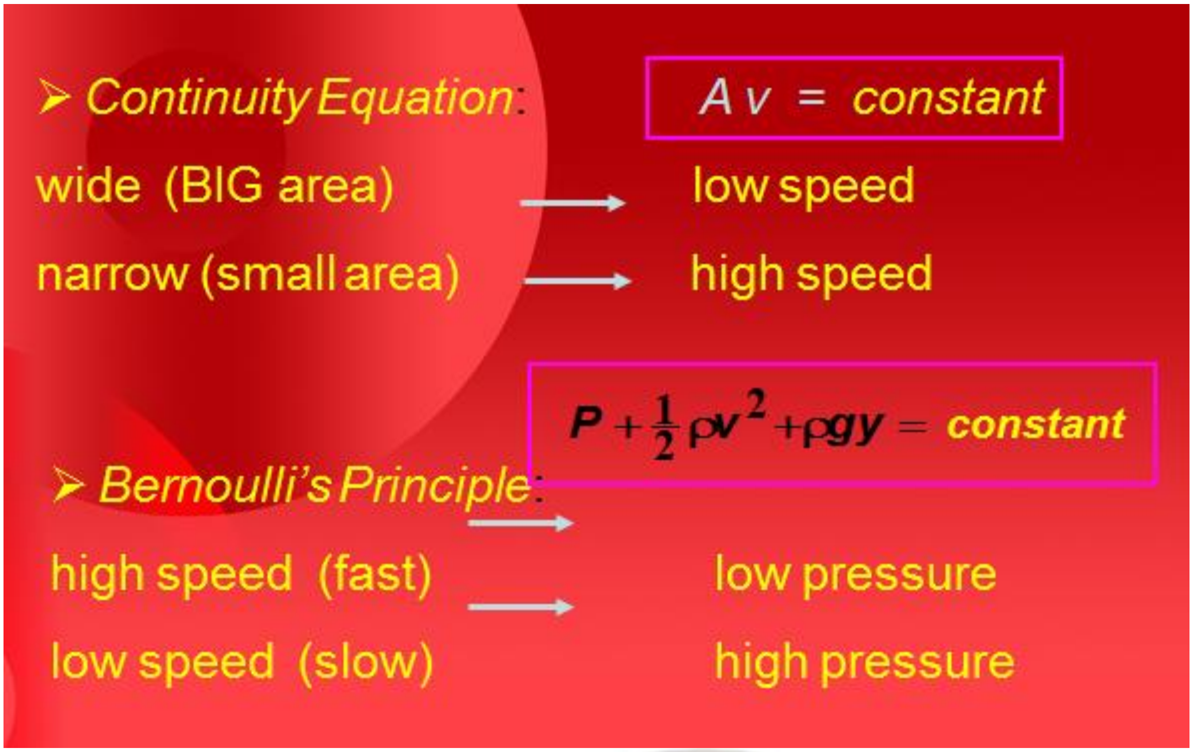
$$W = \Delta K$$

$$\left(p_1 - p_2\right)\left(\Delta m / \rho\right) - \Delta mg\left(y_2 - y_1\right) = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_2 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_1$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

This is called Bernoulli’s equation for steady incompressible, non-viscous and irrotational flow.



Lecture 19 – PHYSICS OF SOUND

Decibels (db): a relative measure to compare the intensity of different sounds with one another

$I_0 \equiv$ threshold of hearing

$= 10^{-12}$ watts per cm^3

$R \equiv$ relative intensity of sound I

$= \log_{10} \frac{I}{I_0}$

Sound	Intensity (W / m²)	Relative Intensity (I / I_0)	Sound Level (dB)
Threshold of hearing	10^{-12}	10^{0}	0
Rustle of leaves	10^{-11}	10^{1}	10
Whisper (at 1 m)	10^{-10}	10^{2}	20
City street, no traffic	10^{-9}	10^{3}	30
Conversation (at 1 m)	10^{-6}	10^{6}	60
Pressure horn (at 1 m)	10^{-3}	10^{9}	90
Ear damage	1	10^{12}	120
Jet engine (at 1 m)	10	10^{13}	130

At time $t = 0$,

$$y(x,0) = y_m \sin\frac{2\pi}{\lambda}x$$

At time t, in x direction;

$$y(x,t) = y_m \sin\frac{2\pi}{\lambda}(x - \text{v}t)$$

The period T of a wave is the time to undergo one complete cycle of motion $\lambda = \text{v}T$

The frequency n is the number of waves crossing a particular point every second:

$$\nu = \frac{1}{T}$$

$$y(x,t) = y_m \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

y at any given time, has the same value at $x, x+\lambda, x+2\lambda \cdots\cdots\cdots\cdots$

y at any given position, has the same value at $\text{t}, t+T, t+2T \cdots\cdots\cdots\cdots$

Introduce wavenumber k

and angular frequency ω,

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

Relative motion between source and observer causes *Doppler Effect.*

- *Moving Observer, Source at Rest*
- *Moving Source, Observer at Rest*

Number of waves received by an observer at rest is $\mathrm{v}t/\lambda$.

Additional number of waves received by an observer moving toward source with speed v_0 in the same time is $\mathrm{v}_0 t/\lambda$.

$$\text{frequency actually heard} = \frac{\text{number of waves received}}{\text{unit time}}$$

$$\nu' = \frac{\frac{\mathrm{v}t}{\lambda} + \frac{\mathrm{v}_0 t}{\lambda}}{t} = \frac{\mathrm{v} + \mathrm{v}_0}{\lambda}$$

$$= \frac{\mathrm{v} + \mathrm{v}_0}{\mathrm{v}/\nu} = \nu \frac{\mathrm{v} + \mathrm{v}_0}{\mathrm{v}}$$

$$\nu' = \nu\left(1 + \frac{\mathrm{v}_0}{\mathrm{v}}\right)$$

<u>Moving Source, Observer at Rest</u>

Each wavelength is reduced by $\frac{\mathrm{v}_s}{f}$.

$$\lambda' \neq \frac{\mathrm{v}}{\nu} \text{ but } \lambda' = \frac{\mathrm{v}}{\nu} - \frac{\mathrm{v}_s}{\nu}$$

$$\nu' = \frac{\mathrm{v}}{\lambda} = \frac{\mathrm{v}}{(\mathrm{v} - \mathrm{v}_s)/\nu} = \nu \frac{\mathrm{v}}{(\mathrm{v} - \mathrm{v}_s)}$$

If both source and observer are moving then:

$$\nu' = \nu \frac{\mathrm{v} + \mathrm{v}_o}{\mathrm{v} - \mathrm{v}_s}$$

Lecture 20 – WAVE MOTION

Wave motion is any kind of self-repeating (periodic, or oscillatory) motion that transports energy from one point to another. Waves are of two basic kinds:

- Longitudinal Waves: the oscillation Longitudinal Waves is parallel to the direction of wave travel. Examples: sound, spring, "P-type" earthquake waves.
- Transverse Waves: the oscillation is perpendicular to the direction of wave travel Transverse Waves. Examples: radio or light waves, string, "S-type" earthquake waves.

Waves transport energy, not matter.

Amplitude

The height of a wave is called the amplitude.

$$\text{Amplitude } \propto \frac{1}{r} \qquad \text{Power } \propto \frac{1}{r^2}$$

Spherically sound waves are emitted uniformly in all directions from a point source, the radiated power P being 25 W. What are the intensity and the sound level of the sound waves a distance r = 2.5 m from the source?

Let P be the radiated power

$$4\pi r_1^2 I_1 = P_1$$

$$4\pi r_2^2 I_2 = P_2$$

$$\text{But } \quad P_1 = P_2 = P$$

$$\therefore \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

P = 25 W, r = 2.5 m

$$I = \frac{P}{4\pi r^2} = .32 \text{ W/m}^2$$

$$SL = 10 \log \frac{I}{I_0} = 115 dB$$

Phase

If the displacement y is not zero at $x = 0$ at $t = 0$ then,

$y(x,t) = y_m \sin(kx - \omega t - \phi)$ $kx - \omega t - \phi$ is called the phase.

ϕ is called the phase constant.

The phase constant only moves the wave forward or backward in space or time:

$$y(x,t) = y_m \sin\left[k\left(x - \frac{\phi}{k}\right) - \omega t\right]$$

$$y(x,t) = y_m \sin\left[kx - \omega\left(t + \frac{\phi}{\omega}\right)\right]$$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

When two sources are present the total amplitude at any point is the sum of the two separate amplitudes.

$$\mathrm{A} = \mathrm{A}_1 + \mathrm{A}_2$$

$$\text{Power} \propto \mathrm{A}^2 = (\mathrm{A}_1 + \mathrm{A}_2)^2$$

$$y_1(x,t) = y_m \sin(kx - \omega t - \phi_1)$$

$$y_2(x,t) = y_m \sin(kx - \omega t - \phi_2)$$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$= y_m\left[\sin(kx - \omega t - \phi_1) + \sin(kx - \omega t - \phi_2)\right]$$

$$\sin B + \sin C = 2\sin\frac{1}{2}(B + C) \times \cos(B - C)$$

$$y(x,t) = y_m\left[\sin(kx - \omega t - \phi_1) + \sin(kx - \omega t - \phi_2)\right]$$

$$= \left[2y_m \cos\left(\frac{\Delta\phi}{2}\right)\right] \times \sin(kx - \omega t - \phi')$$

$$\Delta\phi = \phi_2 - \phi_1$$

$$\phi' = \frac{(\phi_1 + \phi_2)}{2}$$

For constructive interference,

$$\frac{\Delta\phi}{2} = 0, \pi, 2\pi, \cdots$$

For destructive interference,

$$\frac{\Delta\phi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \cdots$$

Two speakers, which are separated by a distance D of 2.3 m, emit a pure tone. The waves are in phase when they leave the speakers. For what wavelengths will the listener hear a minimum in the sound intensity?

$$D = 2.3m$$
$$x_2 = 1.2m, x_1 = ?$$
$$x_1 = \sqrt{x_2^2 + D^2} = 2.6m$$
$$x_1 = \sqrt{x_2^2 + D^2} = 2.6m$$
$$x_1 - x_2 = 1.4m$$
$$1.4m = \frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \cdots\cdots$$
$$\lambda = 2.8m, 0.93m, 0.56m, \cdots$$
$$y_1(x,t) = y_m \sin(kx - \omega t)$$
$$y_2(x,t) = y_m \sin(kx + \omega t)$$
$$y(x,t) = y_1(x,t) + y_2(x,t)$$
$$= y_m\left[\sin(kx - \omega t) + \sin(kx + \omega t)\right]$$
$$= [2y_m \sin kx]\cos \omega t$$
$$y(x,t) = [2y_m \sin kx]\cos \omega t$$
(remember that $k = 2\pi / \lambda$!)
$$kx = \pi, 2\pi, 3\pi, \cdots\cdots$$
$$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \cdots\cdots$$
$$\Delta p_1(t) = p_0 \sin \omega_1 t$$
$$\Delta p_2(t) = p_0 \sin \omega_2 t$$
$$\Delta p(t) = \Delta p_1(t) + \Delta p_2(t)$$
$$= p_0(\sin \omega_1 t + \sin \omega_2 t)$$
$$= \left[2p_0 \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t\right]\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t$$
$$\bar{\omega} = \frac{\omega_1 + \omega_2}{2}$$
$$\omega_{diff} = \frac{\omega_1 - \omega_2}{2}$$
$$\Delta p(t) = \left[2\Delta p_m \cos \omega_{diff} t\right]\sin \bar{\omega} t$$
$$\omega_{beat} = 2\omega_{diff} = \omega_1 - \omega_2$$

Medium	Speed (m/s)
Gases	
Air (0^0C)	331
Helium	965
Hydrogen	1284
Liquids	
Water (0^0C)	1402
Solids	
Aluminum	6420
Steel	5941

The speed of a pulse

He pulse wave in man travels in the arteries at a speed of **4 to 10 metres per second**.

At time $t = 0$,

$$y(x,0) = f(x)$$

At time t,

$$y(x,t) = f(x') = f(x - \text{v}t)$$

where, $x' = x - \text{v}t$

$$x - \text{v}t = \text{constant}$$

$$\frac{dx}{dt} - \text{v} = 0$$

$$\frac{dx}{dt} = \text{v}$$

phase velocity, independent of any property of the wave.

If $v_s > v$ then,

$$\sin\theta = \frac{v}{v_s}$$

Lecture 21 – GRAVITY

GRAVITY

Gravity, also called gravitation, in mechanics, **the universal force of attraction acting between all matters**.

Force of attraction between m_1 and m_2 is:

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2} \qquad \dim[G] = M^{-1}L^3T^{-2}$$

G is called the universal gravitational constant.

$$\vec{F}_{21} = \text{Force on } m_2 \text{ by } m_1$$
$$\vec{F}_{12} = \text{Force on } m_1 \text{ by } m_2$$
$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

Experiment to find G

Gravitational torque

$$= 2\left(\frac{GmM}{r^2}\right)\frac{L}{2}$$

Restoring torque $= \kappa\theta$

θ can be measured by observing the deflection of the beam of the light reflected from the small mirror.

In equilibrium both torques will be equal:

$$\frac{GmML}{r^2} = \kappa\theta \Rightarrow G = \frac{\kappa\theta r^2}{GmML}$$

κ can be found from

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \Rightarrow \kappa = \frac{4\pi^2 I}{T^2}$$

with $I = \frac{mL^2}{2}$

$$G = 6.67259\times10^{-11}\, N.m^2/kg^2$$

Mass of the Earth

The magnitude of the force with which Earth attracts the body towards its center is $F = \frac{GmM_E}{R_E^2}$

Apply law of motion:

$$F = mg = \frac{GmM_E}{R_E^2}$$

$$\Rightarrow M_E = \frac{gR_E^2}{G} = 5.97\times10^{24}\,\text{kg}$$

Density of the Earth

Considering Earth to a perfect sphere, we can find its density

$$\text{Volume of Earth} = V_E = \frac{4}{3}\pi R_E^3$$
$$= 1.08\times10^{21}\,m^3$$

$$\therefore \text{ Density of Earth} = \rho_E = \frac{V_E}{M_E}$$
$$= 5462\,\text{kg}\,m^{-3}$$

This is 5.5 times the density of the water

Gravitational Potential

It is the work done in moving a unit mass from infinity to a given point.

$$V(r) = \frac{U(r)}{m} = -\frac{GM}{r}$$

Proof: Conservation of energy says,

$$dV = -Fdr \Rightarrow \int_{V(R)}^{0} dV = -\int_{r}^{\infty} drF$$

$$0 - V(R) = GM\int_{R}^{\infty}\frac{dr}{r^2} = -GM\left[\frac{1}{r}\right]_R^{\infty}$$

$$\therefore V(R) = -\frac{GM}{R}$$

Potential energy near the surface of Earth

$$\Delta U = U(R+h) - U(R) = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= GMm\left(1 - \frac{1}{1+h/R}\right) = GMm\left(1 - \left(1+h/R\right)^{-1}\right)$$

For $h = \ R, \quad \left(1+h/R\right)^{-1} = 1 - h/R$

$$\therefore \Delta U = GMm\left(1 - \left(1 - h/R\right)\right) = m\left(\frac{GM}{R}\right)h = mgh$$

Escape Velocity

In celestial mechanics, escape velocity or escape speed is the minimum speed needed for a free, non-propelled object to escape from the gravitational influence of a primary body, thus reaching an infinite distance from it. It is typically stated as an ideal speed, ignoring atmospheric friction.

$$\left(KE + PE\right)_{r=R} = \left(KE + PE\right)_{r=\infty}$$

$$\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0 + 0$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Earth escape: $v_e = 11.2\,\text{km/s}$

Sun escape: $v_e = 618\,\text{km/s}$

Black Holes

A black hole is **a region of space time where gravity is so strong that nothing** no particles or even electromagnetic radiation such as light can escape from it. The theory of general relativity predicts that a sufficiently compact mass can deform space time to form a black hole.

$$v_e = \sqrt{\frac{2GM}{R}}$$

put $v_e = c$

$$\Rightarrow R = \sqrt{\frac{2GM}{c^2}}$$

is called the Schwarzchild radius of a black hole

Orbital Velocity

Orbital velocity is **the speed required to achieve orbit around a celestial body**, such as a planet or a star. This requires traveling at a sustained speed that: Aligns with the celestial body's rotational velocity. Is fast enough to counteract the force of gravity pulling the orbiting object toward the body's surface.

$$\frac{mv_o^2}{r} = \frac{GMm}{r^2} \Rightarrow v_o = \sqrt{\frac{GM}{r}}$$

If R is the radius of the planet and h is the height of the satellite above the surface, then $r = R + h$

$$v_o = \sqrt{\frac{GM}{R+h}}, \text{ for } h = R, \; v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

Time Period Of Revolution

The motion around the sun along its orbit is called a revolution. The amount of time it takes for a single trip around the sun is called a period of revolution. The period for the Earth to revolve around the sun is **365.24 days** or one year.

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v_o}$$

$$= 2\pi r\sqrt{\frac{r}{GM}} = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

or

$$T^2 = \frac{4\pi^2}{GM} r^3$$

i.e.,

$$T^2 \propto r^3$$

Energy of a Satellite

The total energy of a satellite is **just the sum of its gravitational potential and kinetic energies**. Assuming that mechanical energy is conserved the sum of the kinetic and potential energies of the satellite would remain constant.

$$E = KE + PE$$

$$= \frac{1}{2} mv_o^2 - \frac{GMm}{r}$$

but $v_o^2 = \frac{GM}{r}$

$$\therefore E = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$

Geostationary Satellite

An Earth satellite so positioned that it appears stationary to an observer on Earth is called a geostationary satellite. Its time period of revolution is I day.

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$\Rightarrow r^3 = \left(\frac{GM}{R}\right)\frac{RT^2}{4\pi^2} = \frac{gRT^2}{4\pi^2}$$

$$= 42.33 \times 10^6 m$$

$\therefore$ Height of the satellite $= r - R = 35.93 \times 10^6 m$

Kepler's laws of planetary motion

1) Law of Orbits

   All planets move in elliptic orbits with Sun at one focus.

2) Law of Areas

   The line joining a planet to the Sun sweeps out equal areas in equal intervals of time.

   $$\Delta A = \frac{1}{2} r (r\Delta\theta)$$

   $$\frac{dA}{dt} = \lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t}$$

   $$= \frac{1}{2} r^2 \left(\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}\right)$$

   $$= \frac{1}{2} r^2 \omega = \frac{L}{2m}$$

3) Law of Periods

   The square of the period of revolution of any planet is proportional to the cube of the planet's mean distance from the Sun.

   Let's prove this result for circular orbits.

   $$\frac{GMm}{r^2} = m\omega^2 r, \text{ where } \omega = \frac{2\pi}{T}$$

   $$\therefore \frac{GM}{r^2} = \left(\frac{2\pi}{T}\right)^2 r \text{ or } T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

   A similar result is obtained for elliptic orbits, with the radius r replaced by the semi major axis a.

A planet has a speed v_1 at a distance d_1 from the Sun. What will be its speed at distance d_2 ?

Using the law of conservation of angular momentum

$$mv_2 d_2 = mv_1 d_1 \Rightarrow v_2 = \frac{v_1 d_1}{d_2}$$

energy conservation :

$$-\frac{GMm}{d_1} + \frac{1}{2}mv_1^2 = -\frac{GMm}{d_2} + \frac{1}{2}mv_2^2$$

$$v_2 = \frac{v_1 d_1}{d_2}$$

Solving these equations, we get

$$v_1 = \sqrt{\frac{2GMd_2}{d_1(d_1 + d_2)}}$$

$$\therefore \text{Angular momentum } = mv_1 d_1 = m\sqrt{\frac{2GMd_1 d_2}{(d_1 + d_2)}}$$

Sling shot effect

The effect known as the gravity assist or slingshot effect is **a way of using the motion of a planet to accelerate a space probe on its journey towards the outer planets**. Sling shot effect is a non-impact collision used to give an extra boost to spacecraft.

Q M ? m

V will remain constant during the interaction

This is a one dimensional elastic collision !

speed of approach = speed of recession

$$v_i + V = -(v_f + V)$$

$$v_f = -(v + 2V)$$

Gravitational effect of a spherical distributional of matter

"A uniformly dense spherical shell attracts an external point mass as if the mass of the shell were concentrated at its center"

1) Variation of g

Value of g at the surface of the Earth. The magnitude of the force with which Earth attracts the body towards its center is

$$F = mg = \frac{GmM_E}{R_E^2}$$

$$\Rightarrow g = \frac{GM_E}{R_E^2}$$

Value of g will be constant all over the surface for a perfectly spherical Earth.

2) Variation of g with altitude

The magnitude of the force with which Earth attracts the body towards its center is

$$F = \frac{GmM_E}{(h+R_E)^2}.$$

According to Newton's second law

$$F = mg_h = \frac{GmM_E}{(h+R_E)^2} \Rightarrow g_h = \frac{GM_E}{(h+R_E)^2}$$

$$g_h = \frac{GM_E}{(h+R_E)^2} = \frac{GM_E}{R_E^2\left(1+\frac{h}{R_E}\right)^2} = \frac{g}{\left(1+\frac{h}{R_E}\right)^2}$$

For $h = R_E$, or $\frac{h}{R_E} = 1$

$$g_h = g\left(1+\frac{h}{R_E}\right)^{-2} ; \quad g\left(1-\frac{2h}{R_E}\right)$$

For $h = 0$, $g_h = g$

3) Variation of g depth below the surface of Earth

As the particle lies inside the shell of radius d, there is no force on the particle due to the shell. The only force exerted on the particle comes from the sphere of radius $R_E - d$.

Mass of the smaller sphere $= M'_E = \rho_E \left(\frac{4}{3} \pi \left(R_E - d \right)^3 \right)$

but $\rho_E = M_E / \left(\frac{4}{3} \pi R_E^3 \right)$

Now the force will be

$$F = mg_d = \frac{GmM'_E}{\left(R_E - d \right)^2}$$

$$\Rightarrow g_d = \frac{GM_E}{R_E^2} \left(1 - \frac{d}{R_E} \right)$$

$$= g \left(1 - \frac{d}{R_E} \right)$$

For $d = R_E, \; g_d = 0$

The value of g is zero at the center of the Earth.

where r is distance from the center of the Earth