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FINALTERM EXAMINATION
Spring 2010
MTH603- Numerical Analysis (Session - 2)

Question No: 1 (Marks: 1) - Please choose one

Symbol used for forward differences is

- ∇
- Δ
- δ
- μ

Question No: 2 (Marks: 1) - Please choose one

The relationship between central difference operator and the shift operator is given by

- $\delta = \mathrm{E} - \mathrm{E}^{-1}$
- $\delta = \mathrm{E} + \mathrm{E}^{-1}$
- $\delta = \mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}}$
- $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}}$

Question No: 3 (Marks: 1) - Please choose one

Muller's method requires --------starting points

- 1
- 2
- 3
- 4

Question No: 4 (Marks: 1) - Please choose one

If S is an identity matrix, then

- $S^{-1} = S$
- $S^{t} = S$
- *All are true*

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- $S^{-1} = S^{t}$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_0, x_1, \ldots, x_r$

- r+2
- r+1
- r
- r-1

Question No: 6 (Marks: 1) - Please choose one

P in Newton's forward difference formula is defined as

- $p = (\frac{x - x_0}{h})$
- $p = (\frac{x + x_0}{h})$
- $p = (\frac{x + x_n}{h})$
- $p = (\frac{x - x_n}{h})$

Question No: 7 (Marks: 1) - Please choose one

Octal number system has the base ----------

- 2
- 8
- 10
- 16

Question No: 8 (Marks: 1) - Please choose one

Newton's divided difference interpolation formula is used when the values of the independent variable are

- Equally spaced
- Not equally spaced

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► Constant
► None of the above

Question No: 9 (Marks: 1) - Please choose one

Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

► 1.5

► 3
► 2

► 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

► $\varepsilon(x) = y(x) + P_n(x)$

► $\varepsilon(x) = y(x) - P_n(x)$

► $\varepsilon(x) = P_n(x) - y(x)$

► $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11 (Marks: 1) - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I.

► n-1
► n+2
► n
► n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

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$$D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}+\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}+...)$$

►

$$D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2}+\frac{\Delta^3}{3}+\frac{\Delta^4}{4}+\frac{\Delta^5}{5}+...)$$

►

$$D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\frac{\Delta^5}{5}-...)$$

►

$$D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}-\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}-...)$$

►

Question No: 13 (Marks: 1) - Please choose one

Finding the first derivative of $f(x)$ at x=0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

► Forward difference operator
► Backward difference operator
► Central difference operator
► None of the given choices

Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

► -0.5
► 0.5

- 0.75
- -0.75

Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

- $ax+b$
- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question No: 16 (Marks: 1) - Please choose one

$$I=\int_a^b f(x)dx$$

While integrating , h, width of the interval, is found by the formula----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

Question No: 17 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

- 7
- 8
- 5
- 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7

$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as ……

- ► 17.5
- ► 12.5
- ► 7.5
- ► -12.5

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

- ► 2
- ► 3
- ► 5
- ► 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

- ► 10
- ► 11
- ► 12
- ► 13

Question No: 21 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- ► $\frac{(a+b)}{2}$
- ► $\frac{(a-b)}{2}$
- ► $\frac{(b-a)}{2}$
- ► None of the given choices

Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- ► Bisection Method
- ► Regula Falsi Method
- ► Secant Method
- ► None of the given choices

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

- ► (1, 3)
- ► (1, 2)
- ► (0, 1)
- ► (1, 2)

Question No: 24 (Marks: 1) - Please choose one

Rate of change of any quantity with respect to another can be modeled by

- ► An ordinary differential equation
- ► A partial differential equation
- ► A polynomial equation
- ► None of the given choices

Question No: 25 (Marks: 1) - Please choose one

If

$$\frac{dy}{dx}=f(x,y)$$

Then the integral of this equation is a curve in

- ► None of the given choices
- ► xt-plane
- ► yt-plane
- ► xy-plane

Question No: 26 (Marks: 1) - Please choose one

In solving the differential equation

$$y'=x+y \;;\; y(0.1)=1.1$$

$h=0.1$, By Euler's method $y(0.2)$ is calculated as

- ► 1.44

- 1.11
- 1.22
- 1.33

Question No: 27 (Marks: 1) - Please choose one

In second order Runge-Kutta method
k_1
is given by

- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

Question No: 28 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

- $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
- $k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

Question No: 29 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

- $k_3 = hf(x_n + 2h, y_n + 2k_3)$
- $k_3 = hf(x_n - h, y_n - k_3)$
- $k_3 = hf(x_n + h, y_n + k_3)$
- None of the given choices

Question No: 30 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

- ► Newton's backward difference interpolation formula
- ► Newton's forward difference interpolation formula
- ► Newton's divided difference interpolation formula
- ► None of the given choices

Question No: 31 (Marks: 2)

If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32 (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2) dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

Evaluate the integral

$$\int_3^5 (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36 (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37 (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38 (Marks: 5)

Evaluate the integral

$$\int_0^3 (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39 (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$$

taking $h = 0.1$

Virtual University of Pakistan

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MTH603

Numerical Analysis

File Version v11.02.02 *Prepared For: Final Term*

Note: Use Table Of Content to view the Topics, In PDF(Portable Document Format) format , you can check Bookmarks menu

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File Version updated on ***7/24/2011***

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Introduction To Numerical Analysis

Course Content:

Number systems, Errors in computation, Methods of solving non-linear equations, Solution of linear system of equations and matrix inversion, Eigen value problems, power method, Jacobi's method, Different techniques of interpolation, Numerical differentiation and integration, Numerical integration formulas, different methods of solving ordinary differential equations.

Each Topic One Solved Question with Steps:Compiled from Past assignment

Preliminary material:

Representation of numbers

Binary Number System (Base 2)=0,1

Used in Computer Internal Operations and Calculation

Octal Number System (Base 8)=0,1,2,3,4,5,6,7

ERROR

Types of error are,

- Inherent errors,
- Local round-off errors
- Local truncation errors

Decimal Number System (Base 10)=0,1,2,3,4,5,6,7,8,9

World Wide used and the most common system

Hexa Number System (Base 16)=0,1,2,3,4,5,6,78,9,A,B,C,D,E,F

Used in Processor Register Addressing

Errors in computations

The initial approximation

It may be found by two methods either by graphical method or
Analytical method
Graphical method

Non-Linear Equations

Bisection Method

Procedure in Detail:

Step,Take two Initial Approximation such that f(x1).F(x2)<0.Means both must have opposite signs.Take their mean by x3=(x1+x2)/2
Next Take two element from which 1 will be x3 and another from x1 or x2 such that both x3 and the other element should have opposite sign.
Repeat the above process to the required numbers of iterations.

Question 1

Find the root of the equation given below by bisection method.

$x^3 - x^2 + x - 7 = 0$

(Note: accuracy up to three decimal places is required.) **Marks: 10**

SOLUTION

Let

$$f(x) = x^3 - x^2 + x - 7$$

Now $f(1) = (1)^3 - (1)^2 + 1 - 7 = -6 < 0$

$f(2) = (2)^3 - (2)^2 + 2 - 7 = -1 < 0$

and $f(3) = (3)^3 - (3)^2 + 3 - 7 = 14 > 0$

As $f(2) f(3) = -14 < 0$

Therefore a real root lies between 2 *and* 3

Iteration 1

Let

$x_0 = 2$ *and* $x_1 = 3$

then

$$x_2 = \left(\frac{x_0 + x_1}{2}\right)$$

$$= \left(\frac{2+3}{2}\right) = 2.5$$

Now $f(2.5) = (2.5)^3 - (2.5)^2 + 2.5 - 7 = 4.875 > 0$

As $f(2) f(2.5) = -1(4.875) = -4.875 < 0$

Therefore a real root lies between 2 *and* 2.5

Iteration 2

$x_0 = 2$ *and* $x_2 = 2.5$

then

$$x_3 = \left(\frac{x_0 + x_2}{2}\right)$$

$$= \left(\frac{2+2.5}{2}\right) = 2.25$$

Now $f(2.25) = (2.25)^3 - (2.25)^2 + 2.25 - 7 = 1.578125 > 0$

As $f(2) f(2.25) = -1(1.578125) = -1.578125 < 0$

Therefore a real root lies between 2 *and* 2.25

Iteration 3

$x_0 = 2 \text{ and } x_3 = 2.25$

then

$$x_4 = \left(\frac{x_0 + x_3}{2}\right)$$

$$= \left(\frac{2 + 2.25}{2}\right) = 2.125$$

$Now\ f(2.125) = (2.125)^3 - (2.125)^2 + 2.125 - 7 = 0.205078 > 0$

$As\ f(2) f(2.125) = -1(0.205078) = -0.205078 < 0$

Therefore a real root lies between 2 *and* 2.125

Iteration 4

$x_0 = 2 \text{ and } x_4 = 2.125$

then

$$x_5 = \left(\frac{x_0 + x_4}{2}\right)$$

$$= \left(\frac{2 + 2.125}{2}\right) = 2.0625$$

$Now\ f(2.0625) = (2.0625)^3 - (2.0625)^2 + 2.0625 - 7 = -0.417725 < 0$

$As\ f(2.0625) f(2.125) = -0.417725(0.205078) = -0.085666 < 0$

Therefore a real root lies between 2.0625 *and* 2.125

Iteration 5

$x_5 = 2.0625 \text{ and } x_4 = 2.125$

then

$$x_6 = \left(\frac{x_5 + x_4}{2}\right)$$

$$= \left(\frac{2.0625 + 2.125}{2}\right) = 2.09375$$

$Now\ f(2.09375) = (2.09375)^3 - (2.09375)^2 + 2.09375 - 7 = -0.111481 < 0$

$As\ f(2.09375) f(2.125) = -0.111481(0.205078) = -0.022862 < 0$

Therefore a real root lies between 2.09375 *and* 2.125

Iteration 6

$x_6 = 2.09375 \text{ and } x_4 = 2.125$

then

$$x_7 = \left(\frac{x_6 + x_4}{2}\right)$$

$$= \left(\frac{2.09375 + 2.125}{2}\right) = 2.109375$$

$Now\ f(2.109375) = (2.109375)^3 - (2.109375)^2 + 2.109375 - 7 = 0.0455 > 0$

$As\ f(2.09375) f(2.109375) = -0.111481(0.0455) = -0.00507 < 0$

Therefore a real root lies between 2.09375 *and* 2.109375

Iteration 7

$x_6 = 2.09375 \, and \, x_7 = 2.109375$

then

$$x_8 = (\frac{x_6 + x_7}{2})$$

$$= (\frac{2.09375 + 2.109375}{2}) = 2.1015625$$

Now $f(2.1015625) = (2.1015625)^3 - (2.1015625)^2 + 2.1015625 - 7 = -0.0333 < 0$

As $f(2.1015625) f(2.109375) = -0.0333(0.0455) = -0.01515 < 0$

Therefore a real root lies between 2.1015625 *and* 2.109375

Iteration 8

$x_8 = 2.1015625 \, and \, x_7 = 2.109375$

then

$$x_9 = (\frac{x_8 + x_7}{2})$$

$$= (\frac{2.1015625 + 2.109375}{2}) = 2.10546875$$

Now $f(2.10546875) = (2.10546875)^3 - (2.10546875^2 + 2.10546875 - 7 = 0.006 > 0$

As $f(2.1015625) f(2.10546875) = -0.0333(0.006) = -0.0002 < 0$

Therefore a real root lies between 2.1015625 *and* 2.10546875

Iteration 9

$x_8 = 2.1015625 \, and \, x_9 = 2.10546875$

then

$$x_{10} = (\frac{x_8 + x_9}{2})$$

$$= (\frac{2.1015625 + 2.10546875}{2}) = 2.103515625$$

Now $f(2.103515625) = (2.103515625)^3 - (2.103515625)^2 + 2.103515625 - 7 = -0.01367 < 0$

As $f(2.103515625) f(2.10546875) = -0.01367(0.006) = -0.00008 < 0$

Therefore a real root lies between 2.103515625 *and* 2.10546875

Iteration 10

$x_{10} = 2.103515625 \, and \, x_9 = 2.10546875$

then

$$x_{11} = (\frac{x_{10} + x_9}{2})$$

$$= (\frac{2.103515625 + 2.10546875}{2}) = 2.104492188$$

Now $f(2.104492188) = (2.104492188)^3 - (2.104492188)^2 + 2.104492188 - 7 = -0.0038 < 0$

As $f(2.104492188) f(2.10546875) = -0.0038(0.006) = -0.0000228 < 0$

Therefore a real root lies between 2.104492188 *and* 2.10546875

Iteration 11

$x_{11} = 2.104492188 \, and \, x_9 = 2.10546875$

then

$$x_{12} = (\frac{x_{11} + x_9}{2})$$

$$= (\frac{2.104492188 + 2.10546875}{2}) = 2.104980469$$

$Now \; f(2.104980469) = (2.104980469)^3 - (2.104980469)^2 + 2.104980469 - 7 = 0.0011 > 0$

$As \; f(2.104492188) \, f(2.104980469) = -0.0038(0.0011) = -0.00000418 < 0$

Therefore a real root lies between 2.104492188 *and* 2.104980469

As the next root lies between 2.104492188 *and* 2.104980469 and these roots are equal up to three decimal places.

So, the required root up to three decimal places is 2.104

Example From Handout at page # 4

Solve $x^3 - 9x + 1 = 0$ for the root between x=2 and x=4 by bisection method

Solution:

Here we are given the interval (2,4) so we need not to carry out intermediate value property to locate initial approximation.

Here $f(x) = x^3 - 9x + 1 = 0$

Now f(2)=-9 and f(4)=29.

Here f(2).f(4)<0 So the root lies between 2 and 4.

$So, x_o = 2, x_1 = 4$

and $x_2 = \frac{x_o + x_1}{2}$ =3 (This formula predicts next iteration)

Now f(3)=1,Here f(2).f(3)<0 So the root lies between 2 and 3.

Repeat above process to required number of iteration.

Step,Take two Initial Approximation such that f(x1).F(x2)<0.Means both must have opposite signs.Take their mean by x3=(x1+x2)/2

Next Take two element from which 1 will be x3 and another from x1 or x2 such that both x3 and the other element should have opposite sign.

Repeat the above process to the required numbers of iterations.

Regula Falsi Method

Formula for the Regula Falsi Method is $x3 = x2 - \frac{x2 - x1}{f(x2) - f(x1)} f(x2)$ or

$$x_{(n+1)} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

Steps: If the interval is given , check whether the root lies in between the interval or not by the principle that states if both number have opposite sign , then the root lies in between interval.

Find the next approximation with the help of the formula.

Question#2 **Marks 10**

Use Regula-Fasli method to compute the root of the equation $f(x)=\cos x-xe^x$ In the interval [0, 1] after third iteration.

Solution:

As $f(x)=\cos x-xe^x$

$f(0)=\cos 0-0e^0=1\succ 0$

$f(1)=\cos 1-1e^1=0.5403-2.718=-2.1779\prec 0$

So root of the eq. lie between 0 *and* 1. *Let* $x_0=0$ *and* $x_1=1$

Thus $f(x_0)=1$ *and* $f(x_1)=-2.1779$

The formula for finding the root of the function f(x) by Regula-Fasli is given by.

$$x_{(n+1)}=x_n-\frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})}f(x_n)$$

For n=1 we have,

$$x_2=x_1-\frac{x_1-x_o}{f(x_1)-f(x_o)}f(x_1)=1-\frac{1-0}{\text{-}2.17797-1}(\text{-}2.17797)=1-\frac{2.17797}{3.17797}=1-0.68533=0.31467$$

$$f(x_2)=\cos(0.31467)-(0.31467)e^{0.31467}=0.950899-(0.31467)(1.369806)=0.519863$$

Since $f(x_1)$ and $f(x_2)$ are of opposite signs, the root lies between x_1 and x_2.
Therefore, for n=2 we have,

$$x_3=x_2-\frac{x_2-x_1}{f(x_2)-f(x_1)}f(x_2)=0.31467-\frac{0.31467-1}{0.519863-(\text{-}2.17797)}(0.519863)$$

$$=0.31467+\frac{0.68533}{2.697833}(0.519863)=0.44673$$

$$f(x_3)=\cos(0.44673)-(0.44673)e^{0.44673}=0.901864-(0.44673)(1.563191)=0.20353$$

Since $f(x_1)$ and $f(x_3)$ are of opposite signs, the root lies between x_1 and x_3.
Therefore, for n=3 we have,

$$x_4=x_3-\frac{x_3-x_1}{f(x_3)-f(x_1)}f(x_2)=0.44673-\frac{0.44673-1}{0.20353-(\text{-}2.17797)}(0.20353)$$

$$=0.44673+\frac{0.54327}{2.3815}(0.20353)=0.493159$$

The required root after 3rd iteration using Regula-Falsi method is 0.493159

Question 2

Use the Regula Falsi (method of false position) to solve the equation $x^3-4x-9=0$

(Note: accuracy up to four decimal places is required) **Marks: 10**

SOLUTION

Let

$f(x) = x^3 - 4x - 9$

Now

$f(0) = (0)^3 - 4(0) - 9 = 0 - 0 - 9 = -9 < 0$

$f(1) = (1)^3 - 4(1) - 9 = 1 - 4 - 9 = -12 < 0$

$f(2) = (2)^3 - 4(2) - 9 = 8 - 8 - 9 = -9 < 0$

$f(3) = (3)^3 - 4(3) - 9 = 27 - 12 - 9 = 6 > 0$

Now, Since $f(2)$ *and* $f(3)$ *are of opposite signs, therefore, the real root lies between* 2 *and* 3

Now,

let $x_1 = 2$ *and* $x_2 = 3$

First iteration

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$= 3 - \frac{3-2}{6-(-9)}(6)$$

$$= 3 - \frac{1}{15}(6)$$

$$= 3 - \frac{2}{5}$$

$$= 2.6$$

$$f(x_3) = f(2.6) = (2.6)^3 - 4(2.6) - 9$$

$$= -1.824$$

Second iteration

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3)$$

$$= 2.6 - \frac{2.6-3}{-1.824-6}(-1.824)$$

$$= 2.69325$$

$$f(x_4) = f(2.69325) = (2.69325)^3 - 4(2.69325) - 9$$

$$= -0.23725$$

Third iteration

$$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_4)$$

$$= 2.69325 - \frac{2.69325 - 2.6}{-0.23725 - (-1.824)}(-0.2061)$$

$$= 2.70536$$

$$f(x_5) = f(2.70536) = (2.70536)^3 - 4(2.70536) - 9$$

$$= -0.02098$$

Fourth iteration

$$x_6 = x_5 - \frac{x_5 - x_4}{f(x_5) - f(x_4)} f(x_5)$$

$$= 2.70536 - \frac{2.70536 - 2.69325}{-0.02098 - (-0.23725)}(-0.02098)$$

$$= 2.70653$$

$$f(x_6) = f(2.70653) = (2.70653)^3 - 4(2.70653) - 9$$

$$= 0.0000368$$

Fifth iteration

$$x_7 = x_6 - \frac{x_6 - x_5}{f(x_6) - f(x_5)} f(x_6)$$

$$= 2.70653 - \frac{2.70653 - 2.70536}{0.0000368 - (-0.02098)}(0.0000368)$$

$$= 2.70653$$

Hence,

The required root of the given equation correct to 4 decimal places is 2.7065

Method of Iterations

Question#1 **Marks 10**

Use the method of iterations to determine the real root of the equation $e^{-x} = 10x$ in the interval [0, 1], correct to four decimal places after four Iterations.

Solution:

Since $e^{-x} = 10x$. *Let* $f(x) = e^{-x} - 10x$. *we see that* $f(0) = 1$ *and* $f(1) = -9.6321$

As $e^{-x} = 10x$, *we can easily find the value of* x, *thus*

$x = \frac{e^{-x}}{10}$, *Let* $\phi(x) = \frac{e^{-x}}{10}$

By taking its derivative we get

$\phi'(x) = -\frac{e^{-x}}{10}$, *we see that* $|\phi'(x)| < 1$ *for all values in* $[0,1]$.

Therefore we can apply method of iterations to the given function.

Take any value within $[0,1]$. *Let*

$x_0 = 0.5$

$x_1 = \phi(0.5) = -\frac{e^{-0.5}}{10} = 0.0607 \quad , \quad f(x_1) = -0.09516$

$x_2 = \phi(0.0607) = -\frac{e^{-0.0607}}{10} = 0.0914 \quad , \quad f(x_2) = 0.00869$

$x_3 = \phi(0.0914) = -\frac{e^{-0.0914}}{10} = 0.0941 \ , \quad f(x_3) = -0.000790477 = -7.90477x10^{-4}$

$x_4 = \phi(0.0941) = -\frac{e^{-0.0941}}{10} = 0.0913 \ , \quad f(x_4) = 0.0000275784 = 2.75784x10^{-5}$

$x_4 = 0.0913$

Example From the Handout at page #

Example

Find a real root of the equation $\cos x = 3x - 1$ correct to seven places of decimal.

Solution

Here it is a transcendental function and all the calculation must be done in the radians mode and value of pi should be 3.14

$f(x) = \cos x - 3x + 1$

$f(0) = \cos 0 - 3(0) + 1 = 1 > 0$

$f(\pi/2) = \cos(1.57) - 3(1.57) + 1 = 0.0007963 - 4.71 + 1 = -3.7092037 < 0$

so a real root lies between 0 *and* $\pi/2$

here $\phi(x) = \frac{1}{3}(\cos x + 1)$

we have $\phi'(x) = -\frac{1}{3}\sin x$

it is clearly less than 1 *as* $\sin$ *is a bounded function and it's values lies between* -1 *and* 1

hence iteration method can be applied

let $x_0 = 0.5$ *be the inital approximation then*

$x_1 = \phi(x_0) = \frac{1}{3}[\cos(0.5) + 1] = 0.6258608$

$x_2 = \phi(x_1) = \frac{1}{3}[\cos(0.6258608) + 1] = 0.6034863$

$x_3 = \phi(x_2) = \frac{1}{3}[\cos(0.6034863) + 1] = 0.6077873$

$x_4 = \phi(x_3) = \frac{1}{3}[\cos(0.6077873) + 1] = 0.6069711$

$x_5 = \phi(x_4) = \frac{1}{3}[\cos(0.6069711) + 1] = 0.6071264$

$x_6 = \phi(x_5) = \frac{1}{3}[\cos(0.6071264) + 1] = 0.6070969$

$$x_7 = \phi(x_6) = \frac{1}{3}[\cos(0.6070969) + 1] = 0.6071025$$

$$x_8 = \phi(x_7) = \frac{1}{3}[\cos(0.6071025) + 1] = 0.6071014$$

$$x_9 = \phi(x_8) = \frac{1}{3}[\cos(0.6071014) + 1] = 0.6071016$$

$$x_{10} = \phi(x_9) = \frac{1}{3}[\cos(0.6071016) + 1] = 0.6071016$$

Newton-Raphson Method

Procedural Detail:
Find the limit if not provided by starting from x=0 to ,…
Find two consecutive numbers for f(x) should have opposite sign.
Use the Newton Raphson Formula to find next approximation .

Question#3 **Marks 10**

Find the real root of the equation $x^3 - 3x - 5 = 0$ using Newton-Raphson method in the interval [2,3] after third iteration.

Solution:

As $f(x) = x^3 - 3x - 5$

$f(2) = 2^3 - 3.2 - 5 = -3 \prec 0$

$f(3) = 3^3 - 3.3 - 5 = 13 \succ 0$

So root of the eq. will lie in $[2,3]$

now $f'(x) = 3x^2 - 3$ *and* $f''(x) = 6x$

$f'(2) = 3.2^2 - 3 = 9$ *and* $f'(3) = 3.3^2 - 3 = 24$

$f''(2) = 6.2 = 12$ *and* $f''(3) = 6.3 = 18$

Since $f(3)$ *and* $f''(3)$*are of same sign.So we choose* $x_0 = 3$

so by Newton's method we have

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{13}{24} = 2.4583$$

$$f(x_1) = (2.4583)^3 - 3(2.4583) - 5 = 14.8561 - 7.3749 - 5 = 2.4812$$

$$f'(x_1) = f'(2.4583) = 3(2.4583)^2 - 3 = 18.1297 - 3 = 15.1297$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.4583 - \frac{2.4812}{15.1297} = 2.2943$$

$$f(x_2) = (2.2943)^3 - 3(2.2943) - 5 = 12.0767 - 6.8829 - 5 = 0.1938$$

$$f'(x_2) = f'(2.2943) = 3(2.2943)^2 - 3 = 15.7914 - 3 = 12.7914$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.2943 - \frac{0.1938}{12.7914} = 2.2791$$

Example From the Handout at page #

Value of e=2.7182

Secant Method

Question 1

Do Four iterations of Secant method, with an accuracy of 3 decimal places to find the root of .

$f(x) = x^3 - 3x + 1 = 0, \quad x_0 = 1, x_1 = 0.5$ **Marks: 10**

Solution:

FORMULA OF SECANT METHOD

$$x_{n+1} = \frac{x_{n-1}(fx_n) - x_n f(x_n - 1)}{(fx_n) - f(x_n - 1)}$$

ITERATION 1

$n = 1$

$f(x_0) = f(1) = -1$

$f(x_1) = f(0.5) = -0.375$

$$x_2 = \frac{x_o(fx_1) - x_1 f(x_o)}{(fx_1) - f(x_o)}$$

$$x_2 = \frac{(1)(-0.375) - (0.5)(-1)}{(-0.375) - (-1)}$$

$$x_2 = \frac{(1)(-0.375) + (0.5)(1)}{(-0.375) + (1)}$$

$$x_2 = \frac{(-0.375) + (0.5)}{0.625}$$

$$x_2 = \frac{0.125}{0.625}$$

$x_2 = 0.2$

ITERATION 2

$$n = 2$$

$$f(x_2) = f(0.2) = 0.408$$

$$x_3 = \frac{x_1(fx_2) - x_2 f(x_1)}{(fx_2) - f(x_1)}$$

$$x_3 = \frac{(0.5)(0.408) - (0.2)(-0.375)}{(0.408) - (-0.375)}$$

$$x_3 = \frac{(0.204) + (0.075)}{(0.408) + (0.375)}$$

$$x_3 = \frac{0.279}{0.783}$$

$$x_3 = 0.3563$$

ITERATION 3

$$n = 3$$

$$f(x_3) = f(0.3563) = -0.02367$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$x_4 = \frac{(0.2) f(0.3563) - (0.3563) f(0.408)}{f(0.3563) - f(0.408)}$$

$$x_4 = \frac{(0.2)(-0.02367) - (0.3563)(0.408)}{(-0.02367) - (0.408)}$$

$$x_4 = \frac{-0.004737 - 0.1453}{-0.43167}$$

$$x_4 = \frac{-0.150034}{-0.43167}$$

$$x_4 = 0.3477$$

ITERATION 4

$n = 4$

$f(x_3) = f(0.3477) = -0.02367$

$x_4 = 0.3477 \quad f(x_4) = -0.00107$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$x_5 = \frac{(0.3563)(-0.00107) - (0.3563)(-0.02367)}{(-0.00107) - (-0.02367)}$$

$$x_5 = \frac{-0.000381 + 0.0823}{-0.00107 + 0.02367}$$

$$x_5 = \frac{0.7849}{0.0226}$$

$x_5 = 0.3473$

$x_5 = 0.3473 \quad f(x_5) = -0.0000096$

Hence, the root after four iterations is **0.347**

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Question 2

Use the secant method to solve the equation $e^x - 3x^2$ for $0 \le x \le 1$. (Perform only 3 iterations.)

Solution

$f(x) = e^x - 3x^2$, $x_0 = 0$, $x_1 = 1$

both are the initial approximations

$f(0) = e^0 - 3(0) = 1$

$f(1) = e^1 - 3(1) = -0.281$

now we calculate the second approximation

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_0) - f(x_1)} = \frac{(0)(-0.281) - (1)1}{-0.281 - 1} = 0.7806$$

$f(0.7806) = e^{0.7806} - 3(0.7806)^2 = 2.1827 - 1.82800 = 0.3546$

now $x_1 = 1$ *and* $x_2 = 0.786$

$f(1) = -0.281 \quad f(0.786) = 0.3546$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \frac{1(0.3546) - (0.786)(-0.281)}{0.3546 + 0.281} = 0.9052$$

now

$x_3 = 0.9052 \qquad f(0.9052) = 2.71^{0.9052} - 3(0.9052)^2 = 0.0074$

$x_2 = 0.786 \qquad f(0.786) = 0.3546$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} = \frac{0.7806(0.0074) - (0.9052)(0.3546)}{0.0074 + 0.3546} = 0.9076$$

so root is 0.9076

Muller's Method

Question 1

Solve the equation $x^3 - 7x^2 + 14x - 6$ by using Muller's method only perform three Iterations.(

$x_0 = 0.5, x_1 = 1, x_2 = 0$)

Solution

1st iteration

$x_0 = 0.5$, $x_1 = 1$ and $x_2 = 0$

$$f(x_0) = f_0 = f(0.5) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6 = -0.625$$
$$f(x_1) = f_1 = f(1) = 1^3 - 7(1)^2 + 14(1) - 6 = 2$$
$$f(x_2) = f_2 = f(0) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$$

$$c = f_0 = -0.625$$
$$h_1 = x_1 - x_0 = 1 - 0.5 = 0.5$$
$$h_2 = x_0 - x_2 = 0.5 - 0 = 0.5$$

$$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$$
$$a = \frac{(0.5)(2) - (0.5 + 0.5)(-0.625) + (0.5)(-6)}{(0.5)(0.5)(0.5 + 0.5)} = \frac{1 - (1)(-0.625) + (0.5)(-6)}{(0.5)(0.5)(1)} = -5.5$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1}$$
$$b = \frac{2 - (-0.625) - (-5.5)(0.5)^2}{0.5} = \frac{2 + 0.625) + (5.5)(0.25)^2}{0.5} = 8$$

$$x = x_0 - \frac{2c}{b - \sqrt{b^2 - 4ac}}$$
$$x = 0.5 - \frac{2(-0.625)}{8 - \sqrt{(8)^2 - (4)(-5.5)(-0.625)}}$$
$$x = 1.8721$$

2nd iteration

$x_0 = 0.5$, $x_1 = 1.8721$ and $x_2 = 1$

$$f(x_0) = f_0 = f(0.5) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6 = -0.625$$
$$f(x_1) = f_1 = f(1) = 1.8721^3 - 7(1.8721)^2 + 14(1.8721) - 6 = 2.228$$

$$f(x_2) = f_2 = f(0) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$$

$$c = f_0 = -0.625$$
$$h_1 = x_1 - x_0 = 1.8721 - 0.5 = 1.3721$$
$$h_2 = x_0 - x_2 = 0.5 - 1 = -0.5$$

$$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$$
$$a = \frac{(-0.5)(2.228) - (1.3721 - 0.5)(-0.625) + (1.3721)(-6)}{(1.3721)(-0.5)(1.3721 - 0.50.5)} = 14.70$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1}$$
$$b = \frac{2.228 - (-0.625) - (14.70)(1.3721)^2}{0.5} = -17.72$$

$$x = x_0 - \frac{2c}{b - \sqrt{b^2 - 4ac}}$$
$$x = 0.5 + \frac{1.25}{12.62 - \sqrt{(17.72)^2 - (4)(14.71)(-0.625)}} = 0.29$$
$$x = 1.8721$$

For 3rd **iteration**

$x_0 = 0.29$, $x_1 = 1.8721$, $x_2 = 1$

For third iteration we will proceed in the same manner.

Graeffe's Root Square Method

Linear Equations

Gaussian Elimination Method

Question 2

Using Gaussian Elimination Method, solve the following system of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$x_1 + 3x_2 + 2x_3 = 13$$
$$3x_1 + x_2 + 3x_3 = 14$$

Solution:

Marks: 10

The Augmented Matrix of the given system of equations is

$$A_b = \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad by \quad R_2 - R_1 \; and \; R_3 - 3R_1$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad by \quad \frac{1}{2}R_2$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \quad by \quad R_3 + (-2)R_2$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad by \quad \frac{-1}{3}R_3$$

Which shows that from the third and second rows

Z=3, y = 2
And from the first row

X+y+2z=9

Using the values of y and z, we get x = 1
Hence the solution of the given system is
X = 1, y = 2, z = 3
Question 1
Using Gaussian Elimination Method, solve the following system of equations

$$2x - y + 2z = 2$$
$$x + 10y - 3z = 5$$
$$x - y - z = 3$$

Marks: 10

Solution:
The Augmented Matrix of the given system of equations is

$$A_b = \begin{bmatrix} 2 & -1 & 2 & 2 \\ 1 & 10 & -3 & 5 \\ 1 & -1 & -1 & 3 \end{bmatrix}$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 2 & -1 & 2 & 2 \\ 1 & -1 & -1 & 3 \end{bmatrix} \quad by \quad R_{12}$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & -21 & 8 & -8 \\ 0 & -11 & 2 & -2 \end{bmatrix} \quad by \quad R_2 - 2R_1, R_3 - R_1$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & 1 & -\frac{8}{21} & \frac{8}{21} \\ 0 & -11 & 2 & -2 \end{bmatrix} \quad by \ \frac{-1}{21} R_2$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & 1 & -\frac{8}{21} & \frac{8}{21} \\ 0 & 0 & -\frac{46}{21} & \frac{46}{21} \end{bmatrix} \quad by \quad R_3 + 11R_2$$

Using Gaussian Elimination method, by backward substitution, we get as follows
From the third row, we get

$$-\frac{46}{21} z = \frac{46}{21}$$

$$\Rightarrow z = -1$$

From the second row, we get

$$y - \frac{8}{21} z = \frac{8}{21}$$

$$\Rightarrow y = \frac{8}{21} + \frac{8}{21} z$$

Putting the value of z, *we get*

$$y = \frac{8}{21} + \frac{8}{21}(-1)$$

$$= \frac{8}{21} - \frac{8}{21}$$

$$= 0$$

And finally from the first row, we get

$$x + 10y - 3z = 5$$

Putting the values of y and z, we get

$$x + 10(0) - 3(-1) = 5$$

$$\Rightarrow x + 3 = 5$$

$$\Rightarrow x = 2$$

So, the solution is

$$x = 2, y = 0, z = -1$$

Guass-Jordan Elimination Method

Question#1 **Marks 10**

By using Gauss –Jordan elimination method, solve the following system of equations,

$$x + y + z = 7$$
$$3x + 3y + 4z = 24$$
$$2x + y + 3z = 16$$

Solution:

The given system in matrix form is

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 24 \\ 16 \end{bmatrix}$$

$$A \quad X = B$$

$$\therefore [A \mid B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 7 \\ 3 & 3 & 4 & 24 \\ 2 & 1 & 3 & 16 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 7 \\ 2 & 1 & 3 & 16 \\ 3 & 3 & 4 & 24 \end{array}\right] \quad R_{23}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 7 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 2 & 9 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to R_1 + R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to R_1 - 2R_3, R_2 \to R_2 - R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad -R_2$$

So,

$x = 3, y = 1, z = 3$

Jacobi's Iterative Method

Solve the following system of equations

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

By Jacobi's iterative method taking the initial starting of solution vector as $(0,0,0)^T$ and perform the first three iterations.

Solution:

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$
$$x = \frac{17 - y + 2z}{20}$$
$$y = \frac{-18 - 3x + z}{20}$$
$$z = \frac{25 - 2x + 3y}{20}$$

starting with $(0,0,0)$

Iteration#01

$$x = \frac{17 - 0 + 2(0)}{20} = \frac{17}{20} = 0.85$$
$$y = \frac{-18 - 3(0) + 0}{20} = \frac{-18}{20} = -0.9$$
$$z = \frac{25 - 2(0) + 3(0)}{20} = \frac{25}{20} = 1.25$$

Iteration#02

$x = 0.85,\ y = -0.9,\ z = 1.25$

$$x = \frac{17 - (-0.9) + 2(1.25)}{20} = \frac{20.4}{20} = 1.02$$
$$y = \frac{-18 - 3(0.85) + 1.25}{20} = \frac{-19.3}{20} = -0.965$$
$$z = \frac{25 - 2(0.85) + 3(-0.9)}{20} = \frac{20.6}{20} = 1.03$$

Iteration#03

$x = 1.02,\ y = -0.965,\ z = 1.03$

$$x=\frac{17-(-0.965)+2(1.03)}{20}=\frac{20.025}{20}=1.00125$$
$$y=\frac{-18-3(1.02)+1.03}{20}=\frac{-20.03}{20}=-1.0015$$
$$z=\frac{25-2(1.02)+3(-0.965)}{20}=\frac{20.065}{20}=1.00325$$

Gauss-Seidel Iteration Method

Question#3 **Marks 10**

Solve Question No. #2 by Gauss-Seidel iterative method and perform first three iterations. What you see the difference after solving the same question by two different iterative methods? Give your comments.

Solution:

The above system of linear equations is diagonally dominant; therefore, Gauss-Seidel iterative method could be applied to find out real roots

$$20x+y-2z=17$$
$$3x+20y-z=-18$$
$$2x-3y+20z=25$$

The above system of equations could be written in the form

$$x=\frac{17-y+2z}{20}$$
$$y=\frac{-18-3x+z}{20}$$
$$z=\frac{25-2x+3y}{20}$$

starting with $(0,0,0)$

Iteration#01

$$x=\frac{17-0+2(0)}{20}=\frac{17}{20}=0.85$$
$$y=\frac{-18-3(0.85)+0}{20}=\frac{-20.55}{20}=-1.0275$$
$$z=\frac{25-2(0.85)+3(-1.0275)}{20}=\frac{20.2175}{20}=1.010875$$

Iteration#02

$x=0.85, y=-1.0275, z=1.010875$

$$x=\frac{17-(-1.0275)+2(1.010875)}{20}=\frac{20.04925}{20}=1.0024625$$
$$y=\frac{-18-3(1.0024625)+1.010875}{20}=\frac{-19.9965125}{20}=-0.999825625$$
$$z=\frac{25-2(1.0024625)+3(-0.999825625)}{20}=\frac{19.995598125}{20}=0.99977990625$$

Iteration#03

$x=1.0024625,\ y=-0.999825625,\ z=0.99977990625$

$$x=\frac{17-(-0.999825625)+2(0.99977990625)}{20}=\frac{19.9904160625}{20}=0.9999708$$
$$y=\frac{-18-3(0.9999708)+0.99977990625}{20}=\frac{-20.00013249375}{20}=-1.0000066247$$
$$z=\frac{25-2(0.9999708)+3(-1.0000066247)}{20}=\frac{20.000038526}{20}=1.0000019263$$

In Gauss Siedal Method, the newly computed values in each iteration are directly involved to find the other value of the system of equation and save memory for computation and hence results are more accurate.

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Question 2

Do five iterations to solve the following system of equations by Gauss-Seidal iterative method

$$10x-2y-3z=305$$
$$-2x+10y-2z=154$$
$$-2x-y+10z=120$$

Marks: 10

Solution:

Since, the given system is diagonally dominant; hence we can apply here the Gauss-Seidal method.

From the given system of equations

$$x^{r+1}=\frac{1}{10}\left[305+2y^{r}+3z^{r}\right]$$
$$y^{r+1}=\frac{1}{10}\left[154+2x^{r+1}+2z^{r}\right]$$
$$z^{r+1}=\frac{1}{10}\left[120+2x^{r+1}+y^{r+1}\right]$$

ITERATION 1

For r =0

Taking y=z=0 on right hand side of first equation. In second equation we take z=0 and current value of x. In third equation we take current value of both x and y.

$$x^1 = \frac{1}{10}[305 + 2(0) + 3(0)] = \frac{305}{10} = 30.5$$

$$y^1 = \frac{1}{10}[154 + 2(30.5) + 2(0)] = \frac{1}{10}[154 + 61] = \frac{215}{10} = 21.5$$

$$z^1 = \frac{1}{10}[120 + 2(30.5) + (21.5)] = \frac{1}{10}[120 + 61 + 21.5] = \frac{202.5}{10} = 20.25$$

ITERATION 2

Similar procedure as used in Iteration 1 will be used for Iterations 2, 3, 4 and 5.

$$x^2 = \frac{1}{10}[305 + 2(21.5) + 3(20.25)] = \frac{408.75}{10} = 40.875$$

$$y^2 = \frac{1}{10}[154 + 2(40.875) + 2(20.25)] = \frac{276.25}{10} = 27.625$$

$$z^2 = \frac{1}{10}[120 + 2(40.875) + (27.625)] = \frac{229.375}{10} = 22.938$$

ITERATION 3

$$x^3 = \frac{1}{10}[305 + 2(27.625) + 3(22.938)] = \frac{429.064}{10} = 42.906$$

$$y^3 = \frac{1}{10}[154 + 2(42.906) + 2(22.938)] = \frac{285.688}{10} = 28.569$$

$$z^3 = \frac{1}{10}[120 + 2(42.906) + (28.569)] = \frac{234.381}{10} = 23.438$$

ITERATION 4

$$x^4 = \frac{1}{10}[305 + 2(28.569) + 3(23.438)] = \frac{432.452}{10} = 43.245$$

$$y^4 = \frac{1}{10}[154 + 2(43.245) + 2(23.438)] = \frac{287.366}{10} = 28.737$$

$$z^4 = \frac{1}{10}[120 + 2(43.245) + (28.737)] = \frac{235.227}{10} = 23.523$$

ITERATION 5

$$x^5 = \frac{1}{10}[305 + 2(28.737) + 3(23.523)] = \frac{433.043}{10} = 43.304$$

$$y^5 = \frac{1}{10}[154 + 2(43.304) + 2(23.523)] = \frac{287.654}{10} = 28.765$$

$$z^5 = \frac{1}{10}[120 + 2(43.304) + (28.765)] = \frac{235.373}{10} = 23.537$$

Above Results are summarized in tabular form as

	Variables		
Iterations	**x**	**y**	**z**
1	30.5	21.5	20.25
2	40.875	27.625	22.938
3	42.906	28.569	23.438

4	43.245	28.737	23.523
5	43.304	28.765	23.537

Hence the solution of the given system of equations, after five iterations, is

$x = 43.304$

$y = 28.765$

$z = 23.537$

Relaxation Method

Matrix Inversion

Eigen Value Problems

Power Method

Question 1

Find the largest eigen value of the matrix

$$\begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix}$$

MARKS 10

And the corresponding eigenvector, by Power Method after fourth iteration starting with the initial vector $v^{(0)} = (0,0,1)^T$

SOLUTION

Let $A = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix}$

Choosing an initial vector as $v^{(0)} = (0,0,1)^T$ then

ITERATION 1

$$u^{(1)} = [A]v^{(0)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$$

Now we normalize the resultant vector to get

$$u^{(1)} = 4 \begin{pmatrix} 0 \\ \frac{1}{4} \\ 1 \end{pmatrix} = q_1 v^{(1)}$$

Continuing this procedure for subsequent Iterations , we have

ITERATION 2

$$u^{(2)} = [A]v^{(1)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ 6 \\ 4 \end{pmatrix}$$

$$u^{(2)} = 6 \begin{pmatrix} \frac{1}{24} \\ 1 \\ \frac{2}{3} \end{pmatrix} = q_2 v^{(2)}$$

ITERATION 3

$$u^{(3)} = [A]v^{(2)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} \frac{1}{24} \\ 1 \\ \frac{2}{3} \end{pmatrix} = \begin{pmatrix} 1.167 \\ 20.708 \\ 2.667 \end{pmatrix}$$

$$u^{(3)} = 20.708 \begin{pmatrix} 0.056 \\ 1 \\ 0.129 \end{pmatrix} = q_3 v^{(3)}$$

ITERATION 4

$$u^{(4)} = [A]v^{(3)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0.056 \\ 1 \\ 0.129 \end{pmatrix} = \begin{pmatrix} 1.224 \\ 20.185 \\ 0.516 \end{pmatrix}$$

$$u^{(4)} = 20.185 \begin{pmatrix} 0.061 \\ 1 \\ 0.026 \end{pmatrix} = q_4 v^{(4)}$$

Therefore, the largest eigen value and the corresponding eigen vector accurate to three decimals places are

$\lambda = 20.185$

$$(X) = \begin{pmatrix} 0.061 \\ 1 \\ 0.026 \end{pmatrix}$$

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Question 1 **marks 10**

Find the largest eigen value of the matrix

$$\begin{bmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{bmatrix}$$

And the corresponding eigen vector, by Power Method after fourth iteration starting with the initial vector $v^{(0)} = (0,1,0)^T$

SOLUTION

Let $A = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix}$

Choosing an initial vector

$v^{(0)} = (0,1,0)^T$ then

ITERATION 1

$$u^{(1)} = [A]v^{(0)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 10 \end{pmatrix}$$

Now we normalize the resultant vector to get

$$u^{(1)} = 10\begin{pmatrix} 0.5 \\ 0.1 \\ 1 \end{pmatrix} = q_1 v^{(1)}$$

Continuing this procedure for subsequent Iterations, we have

ITERATION 2

$$u^{(2)} = [A]v^{(1)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5.5 \\ 5.6 \\ 4.5 \end{pmatrix}$$

$$u^{(2)} = 5.6\begin{pmatrix} \frac{5.5}{5.6} \\ \frac{5.6}{5.6} \\ \frac{4.5}{5.6} \end{pmatrix} = \begin{pmatrix} 0.982 \\ 1 \\ 0.804 \end{pmatrix} = q_2 v^{(2)}$$

ITERATION 3

$$u^{(3)} = [A]v^{(2)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix} \begin{pmatrix} 0.982 \\ 1 \\ 0.804 \end{pmatrix} = \begin{pmatrix} 12.5 \\ 7.162 \\ 13.394 \end{pmatrix}$$

$$u^{(3)} = 13.394 \begin{pmatrix} 0.933 \\ 0.535 \\ 1 \end{pmatrix} = q_3 v^{(3)}$$

ITERATION 4

$$u^{(4)} = [A]v^{(3)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix} \begin{pmatrix} 0.933 \\ 0.535 \\ 1 \end{pmatrix} = \begin{pmatrix} 10.273 \\ 7.334 \\ 9.283 \end{pmatrix}$$

$$u^{(4)} = 10.273 \begin{pmatrix} 1 \\ 0.714 \\ 0.904 \end{pmatrix} = q_4 v^{(4)}$$

Therefore, the largest eigen value and the corresponding eigen vector accurate to four decimals places are

$\lambda = 10.273$

and

$$(X) = \begin{pmatrix} 1 \\ 0.714 \\ 0.904 \end{pmatrix}$$

Jacobi's Method

Question 1

Using Jacobi's method, find all the eigenvalues and the corresponding eigenvectors of the following matrix,

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Marks: 10

Note: Give results at the end of third rotation.

Solution.

$$Let\ A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$Then,\ A^t = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Hence,

Matrix A is real and Symmetric and Jacobi's method can be applied.

Rotation 1

In Matrix A, all the off-diagonal elements are found be 2.
So, the largest off-diagonal element is found to be
$a_{12} = a_{13} = a_{21} = a_{23} = a_{31} = a_{32} = 2$.

Therefore, we can choose any one of them as the largest element.

Suppose, we choose a_{12} as the largest element

Then, we compute the rotation angle as,

$$\tan 2\theta = \frac{2a_{12}}{a_{11} - a_{22}} = \frac{2 \times 2}{1-1} = \infty$$

Therefore

$$\tan 2\theta = \infty$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \boldsymbol{\theta = \frac{\pi}{4}}$$

Therefore, we construct an orthogonal matrix S_1 such that,

$$S_1 = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now the first rotation gives,

$$D_1 = S_1^{-1} A S_1$$

Here, $S_1^{-1} = = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So,

$D_1 = S_1^{-1} A S_1$

i.e.

$$D_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 2.828 \\ 0 & -1 & 0 \\ 2.828 & 0 & 1 \end{bmatrix}$$

To check that we are right in our calculations, we can see that the sum of the diagonal elements is, $3+(-1)+(1)=3$, which is same as the sum of the diagonal elements of the original matrix A.

Rotation 2

For the second rotation we choose the largest off-diagonal element $d_{13} = d_{31} = 2.828$, then

$$\tan 2\theta = \frac{2d_{13}}{d_{11} - d_{33}} = \frac{2(2.828)}{3-(1)} = \frac{5.657}{2} = 2.828$$

So,

$2\theta = \tan^{-1}(2.828) = 70.526$

$\theta = 35.263$

Therefore, we construct an orthogonal matrix S_2 such that,

$$S_2 = \begin{bmatrix} \cos(35.263) & 0 & -\sin(35.263) \\ 0 & 1 & 0 \\ \sin(35.263) & 0 & \cos(35.263) \end{bmatrix}$$

$$= \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$S_2^{-1} = = \begin{bmatrix} 0.817 & 0 & 0.577 \\ 0 & 1 & 0 \\ -0.577 & 0 & 0.817 \end{bmatrix}$$

Now the rotation 2 gives,

$$D_2 = S_2^{-1} D_1 S_2$$

$$D_2 = \begin{bmatrix} 0.817 & 0 & 0.577 \\ 0 & 1 & 0 \\ -0.577 & 0 & 0.817 \end{bmatrix} \begin{bmatrix} 3 & 0 & 2.828 \\ 0 & -1 & 0 \\ 2.828 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Again to check that we are right in our calculations, we can see that the sum of the diagonal elements is $5+(-1)+(-1)=3$ which is same as the sum of the diagonal elements of the original matrix A.

Rotation 3

We can see that in above iteration that D_2 is a diagonal matrix, so we stop here and take the ***Eigen Values*** as

$$\lambda_1 = 5 \ , \ \lambda_2 = -1 \ , \ \lambda_3 = -1$$

Now the Eigenvectors are the columns vectors of the matrix $S = S_1 S_2$, which are,

$$S = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$= \begin{bmatrix} 0.577 & -0.707 & -0.408 \\ 0.577 & 0.707 & -0.408 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

Therefore, the corresponding ***EigenVectors*** are

$$X_1 = \begin{bmatrix} 0.577 \\ 0.577 \\ 0.577 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -0.707 \\ 0.707 \\ 0 \end{bmatrix}, \quad X_3 = \begin{bmatrix} -0.408 \\ -0.408 \\ 0.817 \end{bmatrix}$$

Interpolation

For a given table of values (,), 0,1,2,..., k k x y k= nwith equally spaced abscissas of a function y= f(x),we define the forward difference operator Δ as follows,

$\Delta y_i = y_{i+1} - y_i, \qquad i = 0,1,\ldots,(n-1)$

These differences are called first differences of the function y and are denoted by the symbol Δy_i Here, Δ is called the first difference operator.

Similarly, rth Difference operator would be $\Delta^r y_i = \Delta^{r-1} y_{i+1} - \Delta^{r-1} y_i$

Leading term =y_o

Leading Diffenrence=Δy

Backward Difference Operators:

$\nabla y_i = y_i - y_{i-1} i = n, \quad (n-1),\ldots,1$

Central Difference is given by,

$\delta y_i = y_{i+(1/2)} - y_{i-(1/2)}$

$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\vdots \quad \vdots \quad \vdots$$
$$\Delta y_{n-1} = y_n - y_{n-1}$$

$$\nabla y_i = y_i - y_{i-1} i = n, \quad (n-1), \ldots, 1$$

OR

$$\nabla y_1 = y_1 - y_0$$
$$\nabla y_2 = y_2 - y_1$$
$$\vdots \quad \vdots \quad \vdots$$
$$\nabla y_n = y_n - y_{n-1}$$

$$\delta y_{1/2} = y_1 - y_0, \qquad \delta y_{3/2} = y_2 - y_1,$$

In General,

$$\delta y_i = y_{i+(1/2)} - y_{i-(1/2)}$$

Higher order differences are defined as follows:

$$\delta^2 y_i = \delta y_{i+(1/2)} - \delta y_{i-(1/2)}$$

$$\delta^n y_i = \delta^{n-1} y_{i+(1/2)} - \delta^{n-1} y_{i-(1/2)}$$

Shift operator, E

Let y = f (x) be a function of x, and let x takes the consecutive values x, x + h, x + 2h, etc. We then define an operator having the property

E f(x) = f(x+h)

Thus, when E operates on f (x), the result is the next value of the function. Here, E is called the shift operator. If we apply the operator E twice on f (x), we get

$E^2 f(x) = E[E f(x)]$

$= E[f(x+h)] = f(x+2h)$

Thus, in general, if we apply the operator 'E' n times on f (x), we get

$E^n f(x) = f(x+nh)$

The inverse operator E is defined as

$$E^{-1} f(x) = f(x-h)$$

Similarly

$$E^{-n} f(x) = f(x-nh)$$

Average Operator, μ;

it is defined as

$$\mu f(x) = \frac{1}{2}\left[f\left(x + \frac{h}{2}\right) + f\left(x - \frac{h}{2}\right)\right]$$

$$= \frac{1}{2}\left[y_{x+(h/2)} + y_{x-(h/2)}\right]$$

Differential Operator, D

it is defined as

$$Df(x) = \frac{d}{dx} f(x) = f'(x)$$
$$D^2 f(x) = \frac{d^2}{dx^2} f(x) = f''(x)$$

$$\Rightarrow \Delta = E - 1$$
$$\Rightarrow \nabla = 1 - E^{-1} = \frac{E-1}{E}$$
$$\delta = E^{1/2} - E^{-1/2}$$
$$\Rightarrow \mu = \frac{1}{2}(E^{1/2} + E^{-1/2})$$
$$hD = \log E$$

Newton Forward Difference Interpolation.

for any real number p, we have the operator E such that

$$E^p f(x) = f(x + ph).$$
$$f(x_0 + ph) = E^p f(x_0) = (1+\Delta)^p f(x_0)$$
$$= \left[1 + p\Delta + \frac{p(p-1)}{2!}\Delta^2 + \frac{p(p-1)(p-2)}{3!}\Delta^3 + \cdots\right] f(x_0)$$
$$f(x_0 + ph) = f(x_0) + p\Delta f(x_0)$$
$$+ \frac{p(p-1)}{2!}\Delta^2 f(x_0) + \frac{p(p-1)(p-2)}{3!}\Delta^3 f(x_0)$$
$$+ \cdots + \frac{p(p-1)\cdots(p-n+1)}{n!}\Delta^n f(x_0) + \text{Error}$$

Forward Differences

Question 2

Construct a forward difference table from the following values of x and y.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y=f(x)	2.0	3.5	-1.7	2.3	4.2	6.5	5.7

Marks:

10

Solution.

Forward-difference table

x	**y**	**Δy**	**Δ^2y**	**Δ^3y**	**Δ^4y**	**Δ^5y**	**Δ^6y**
1.0	2						
		1.5					

1.2	3.5		-6.7				
		-5.2		15.9			
1.4	-1.7		9.2		-27.2		
		4		-11.3		41	
1.6	2.3		-2.1		13.8		-60.8
		1.9		2.5		-19.8	
1.8	4.2		0.4		-6		
		2.3		-3.5			
2.0	6.5		-3.1				
		-0.8					
2.2	5.7						

Backward Differences

Question 2

Construct a backward difference table from the following values of x and y .

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y=f(x)	2.0	3.5	-1.7	2.3	4.2	6.5	5.7

Marks:

10

Solution.

Backward-difference table

x	**y**	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	2						
		1.5					
1.2	3.5		-6.7				
		-5.2		15.9			
1.4	-1.7		9.2		-27.2		
		4		-11.3		41	
1.6	2.3		-2.1		13.8		-60.8
		1.9		2.5		-19.8	
1.8	4.2		0.4		-6		
		2.3		-3.5			

2.0	6.5		-3.1				
		-0.8					
2.2	5.7						

==========

Question 2 **marks 10**

Form a table of forward and backward differences of the function

$f(x) = x^3 - 3x^2 - 5x - 7$ For $x = -1, 0, 1, 2, 3, 4, 5$

SOLUTION

For the given function, the values of y for the given values of x are calculated as

$for\ x = -1$

$f(-1) = (-1)^3 - 3(-1)^2 - 5(-1) - 7 = -6$

$for\ x = 0$

$f(0) = (0)^3 - 3(0)^2 - 5(0) - 7 = -7$

$for\ x = 1$

$f(1) = (1)^3 - 3(1)^2 - 5(1) - 7 = -14$

$for\ x = 2$

$f(2) = (2)^3 - 3(2)^2 - 5(2) - 7 = -21$

$for\ x = 3$

$f(3) = (3)^3 - 3(3)^2 - 5(3) - 7 = -22$

$for\ x = 4$

$f(4) = (4)^3 - 3(4)^2 - 5(4) - 7 = -11$

$for\ x = 5$

$f(5) = (5)^3 - 3(5)^2 - 5(5) - 7 = 18$

So, the table of values of x and y is

x	-1	0	1	2	3	4	5
$y = f(x)$	-6	-7	-14	-21	-22	-11	18

Forward difference table

Forward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
-1	-6						
		-1					

0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Backward difference table

Backward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Divided Differences

Question 1 **marks 10**

For the following table of values, estimate $f(2.5)$ using Newton's forward difference interpolation formula.

x	1	2	3	4	5	6	7	8
$y = f(x)$	1	8	27	64	125	216	343	512

SOLUTION

Forward difference table

Forward difference table for the given values of x and y is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1	1						
		7					
2	8		12				
		19		6			
3	27		18		0		
		37		6		0	
4	64		24		0		0
		61		6		0	
5	125		30		0		
		91		6			
6	216		36				
		127					
7	343		42				
		169					
8	512						

Newton's forward difference interpolation formula is given by

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$
$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots + \frac{p(p-1)(p-2).....(p-n+1)}{n!}\Delta^n y_0$$

Here

$$p = \frac{x - x_0}{h} = \frac{2.5-1}{1} = \frac{1.5}{1} = 1.5$$

And

$$y_0 = 1, \Delta y_0 = 7, \Delta^2 y_0 = 12, \Delta^3 y_0 = 6, \Delta^4 y_0 = \Delta^5 y_0 = \Delta^6 y_0 = 0$$

So, by putting the above values in Newton's forward difference interpolation formula, We have

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$
$$= 1 + 1.5(7) + \frac{1.5(1.5-1)}{2!}(12) + \frac{1.5(1.5-1)(1.5-2)}{3!}(6)$$
$$= 1 + 10.5 + 4.5 - 0.375$$
$$= 15.625$$

i.e.

$y_{2.5} = 15.625$

==========

Question 2 **marks 10**

Compute $f(1.5)$ for the following data by using Newton's divided difference interpolation formula.

x	1	2	4	5	8
$f(x)$	5	14	28	45	92

SOLUTION

The divided difference table for the given data is given by

x	y	1st D.D.	2nd D.D.	3rd D.D.	4th D.D.
1	5				
		9			
2	14		-2/3		
		7		1	
4	28		10/3		-29/126
		17		-11/18	
5	45		-1/3		
		47/3			
8	92				

Newton's Divided Difference formula is

$$f(x) = y_0 + (x-x_0)y[x_0,x_1] + (x-x_0)(x-x_1)y[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)y[x_0,x_1,x_2,x_3]$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)y[x_0,x_1,x_2,x_3,x_4]$$

$$f(1.5) = 5 + (1.5 - 1)\,9 + (1.5\text{-}1)(1.5\text{-}2)(-\frac{2}{3}) + (1.5\text{-}1)\,(1.5\text{-}2)(1.5\text{-}4)1$$
$$+(1.5\text{-}1)\,(1.5\text{-}2)(1.5\text{-}4)(1.5\text{-}5)(-\frac{29}{126})$$
$$=5+4.5+0.1667+0.625+0.5035$$
$$=10.7952$$

Question 3 **marks 10**

Find $y'(6)$ *and* $y''(6)$ from the following table of values.

x	1	2	3	4	5	6
y	3	9	17	27	40	55

SOLUTION

Backward difference table

Backward difference table for the given values of x and y is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1	3					
		6				
2	9		2			
		8		0		
3	17		2		1	
		10		1		-3
4	27		3		-2	
		13		-1		
5	40		2			
		15				
6	55					

By backward difference formula for first derivative, we have

$$y_n' = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4}\right)$$

$$y_n' = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \frac{\nabla^5 y_n}{5}\right)$$

$$y'(6) = \frac{1}{1}\left(15 + \frac{2}{2} + \frac{-1}{3} + \frac{-2}{4} + \frac{-3}{5}\right)$$

$$= 15 + 1 - 0.333 - 0.5 - 0.6$$

$$= 14.567$$

By backward difference formula for second derivative, we have

$$y_n'' = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n\right)$$

Therefore

$$y''(6) = \frac{1}{1^2}[2 + (-1) + \frac{11}{12}(-2) + \frac{5}{6}(-3)]$$

$$= 2 - 1 - 1.833 - 2.5$$

$$= -3.333$$

Question#1 **Marks 10**

Find an equation of a cubic curve which passes through the points (4,-43), (7, 83), (9,327) and (12, 1053) using Divided Difference Formula.

Solution:

Newton's divided difference table is

X	Y	1st D.D	2nd D.D	3rd D.D
4	-43			
7	83	42		
9	327	122	16	
12	1053	242	24	1

Newton's Divided Difference formula is

$$y = f(x) = y_0 + (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2] + ...$$
$$+ (x - x_0)(x - x_1)....(x - x_{n-1})y[x_0, x_1, ...x_n]$$

now putting values in formula

$$y = f(x) = -43 + (x-4)(42) + (x-4)(x-7)(16) + (x-4)(x-7)(x-9)(1)$$
$$= -43 + (x-4)\{42 + 16x - 112 + x^2 - 16x + 63\}$$
$$= -43 + (x-4)(x^2 - 7)$$
$$= -43 + x^3 - 7x - 4x^2 + 28$$
$$= x^3 - 4x^2 - 7x - 15$$

Hence it is required polinomial

Question 2 **marks 10**

Form a table of forward and backward differences of the function

$f(x) = x^3 - 3x^2 - 5x - 7$ For $x = -1, 0, 1, 2, 3, 4, 5$

SOLUTION

For the given function, the values of y for the given values of x are calculated as

for $x = -1$

$f(-1) = (-1)^3 - 3(-1)^2 - 5(-1) - 7 = -6$

for $x = 0$

$f(0) = (0)^3 - 3(0)^2 - 5(0) - 7 = -7$

for $x = 1$

$f(1) = (1)^3 - 3(1)^2 - 5(1) - 7 = -14$

for $x = 2$

$f(2) = (2)^3 - 3(2)^2 - 5(2) - 7 = -21$

for $x = 3$

$f(3) = (3)^3 - 3(3)^2 - 5(3) - 7 = -22$

for $x = 4$

$f(4) = (4)^3 - 3(4)^2 - 5(4) - 7 = -11$

for $x = 5$

$f(5) = (5)^3 - 3(5)^2 - 5(5) - 7 = 18$

So, the table of values of x and y is

x	-1	0	1	2	3	4	5
$y = f(x)$	-6	-7	-14	-21	-22	-11	18

Forward difference table

Forward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Backward difference table

Backward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11					

				6			
4	-11		18				
		29					
5	18						

Question 1 **Marks 10**

Find the interpolating polynomial for the following data by

a) Newton's Forward Difference Formula

b)Lagrange's Formula

Hence show that both the methods give raise to the same polynomial.

x	0	1	2	3
y	1	3	7	13

Solution:

By Newton's Forward Difference Formula

The Newton's Forward difference table is given as

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		2		
1	3		2	
		4		0
2	7		2	
		6		
3	13			

We have Newton's forward difference interpolation formula as

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots \qquad (1)$$

Since, the third and higher order differences are zero; Therefore, Newton's forward difference Interpolation formula reduces to

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 \qquad (2)$$

Here,

$x_0 = 0,\ y_0 = 1,\ \Delta y_0 = 2,\ \Delta^2 y_0 = 2,\ h = 1$

And p is given by

$$p=\frac{x-x_0}{h}=\frac{x-0}{1}=x$$

Substituting these values in Eq.(2), we have

$$y=1+x(2)+\frac{x(x-1)}{2}(2)$$
$$=1+2x+x(x-1)$$
$$=1+2x+x^2-x$$
$$=x^2+x+1$$

Hence, the required polynomial is

$$y=x^2+x+1.$$

b) By Lagrange's Formula

Lagrange's Interpolation formula is given by

$$y=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$
$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

By putting the values in the formula, we get

$$y=\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}(1)+\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}(3)$$
$$+\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}(7)+\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}(13)$$

$$y=\frac{x^3-6x^2+11x-6}{(-1)(-2)(-3)}+\frac{3x^3-15x^2+18x}{(1)(-1)(-2)}$$
$$+\frac{7(x^3-4x^2+3x)}{(2)(1)(-1)}+\frac{13(x^3-3x^2+2x)}{(3)(2)(1)}$$

$$y=\frac{x^3-6x^2+11x-6}{-6}+\frac{3x^3-15x^2+18x}{2}$$
$$+\frac{7x^3-28x^2+21x}{-2}+\frac{13x^3-39x^2+26x}{6}$$

$$y=\frac{x^3-6x^2+11x-6}{-6}+\frac{3x^3-15x^2+18x}{2}+\frac{7x^3-28x^2+21x}{-2}+\frac{13x^3-39x^2+26x}{6}$$
$$=\frac{-x^3+6x^2-11x+6+9x^3-45x^2+54x-21x^3+84x^2-63x+13x^3-39x^2+26x}{6}$$

$$y = \frac{-x^3 + 9x^3 - 21x^3 + 13x^3 + 6x^2 - 45x^2 + 84x^2 - 39x^2 - 11x + 54x - 63x + 26x + 6}{6}$$

$$= \frac{6x^2 + 6x + 6}{6}$$

$$= \frac{6(x^2 + x + 1)}{6}$$

$$= x^2 + x + 1$$

Thus we have the equation

$$y = x^2 + x + 1$$

This is the same polynomial as obtained in Newton's Forward difference interpolating formula. Hence it is proved that both the methods give rise to the same polynomial

Langrange's Interpolation

Find interpolation polynomial by Lagrange's Formula, with the help of following table,

x	1	2	3	5
f(x)	0	7	26	124

And hence find value of f (6).

Solution:

Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$y = f(x) = \frac{(x-2)(x-3)(x-5)}{(1-2)(1-3)(1-5)}(0) + \frac{(x-1)(x-3)(x-5)}{(2-1)(2-3)(2-5)}(7) + \frac{(x-1)(x-2)(x-5)}{(3-1)(3-2)(3-5)}(26)$$
$$+ \frac{(x-1)(x-2)(x-3)}{(5-1)(5-2)(5-3)}(124)$$

$$= 0 + \frac{7\left(x^3-9x^2+23x-15\right)}{3} - \frac{26\left(x^3-8x^2+17x-10\right)}{4} + \frac{124\left(x^3-6x^2+11x-6\right)}{24}$$

$$= \frac{7\left(x^3-9x^2+23x-15\right)}{3} - \frac{13\left(x^3-8x^2+17x-10\right)}{2} + \frac{31\left(x^3-6x^2+11x-6\right)}{6}$$

$$= \frac{14\left(x^3-9x^2+23x-15\right) - 39\left(x^3-8x^2+17x-10\right) + 31\left(x^3-6x^2+11x-6\right)}{6}$$

$$= \frac{1}{6}\left(14x^3-126x^2+322x-210-39x^3+312x^2-663x+390+31x^3-186x^2+341x-186\right)$$

$$= \frac{1}{6}\left(6x^3-6\right)$$

$y = f(x) = x^3 - 1$

now we have to find $f(6)$ *so*

$$f(6) = x^3 - 1$$
$$= (6)^3 - 1$$
$$= 216 - 1$$
$$= 215$$

Differentiation Using Difference Operators

Question#3 **Marks 10**

Find $f'(2.5)$ *and* $f''(2.5)$ from the following table,

x	1.3	1.6	1.9	2.2	2.5
f(x)	2.4	2.9	3.2	4.7	6.4

Solution:

Since x=2.5 occur at the end of the table, it is appropriate to use the backward difference formula for the derivation.

Backward difference table is as

x	$f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.3	2.4				
1.6	2.9	0.5			
1.9	3.2	0.3	-0.2		
2.2	4.7	1.5	1.2	1.4	
2.5	6.4	1.7	0.2	-1	-2.4

using the backward difference formula for $y'(x)$ and $y''(x)$, we have

$$y'_n = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4}\right)$$

now putting value from backward diff .table we have

$$y'_{2.5} = \frac{1}{0.3}\left(1.7 + \frac{0.2}{2} + \frac{(-1)}{3} + \frac{(-2.4)}{4}\right)$$

$$y'_{2.5} = \frac{1}{0.3}(1.7 + 0.1 - 0.3333 - 0.6)$$

$$y'_{2.5} = \frac{1}{0.3}(0.8667)$$

$$y'_{2.5} = 2.8890$$

$$y''_n = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n\right)$$

$$y''_{2.5} = \frac{1}{(0.3)^2}\left(0.2 - 1 + \frac{11}{12}(-2.4)\right)$$

$$y''_{2.5} = \frac{1}{(0.09)}(0.2 - 1 - 2.2)$$

$$y''_{2.5} = \frac{1}{(0.09)}(-3)$$

$$y''_{2.5} = -33.33$$

Numerical Integration

Trapezoidal Rule

Question 1
Marks 10

Evaluate the integral $\int_0^1 e^{2x}dx$
Using (I) Trapezoidal rule
(2) Simpson's 1/3 rule
By dividing the interval of integration into eight equal parts. Also calculate the percentage error from its true values in both cases.

Solution:

a) By Trapezoidal Rule

Since the no. of intervals are eight, so,
$n=8$
And $a=0, b=1$
So, the width of the interval is given by

$$h=\frac{b-a}{n}$$
$$=\frac{1-0}{8}$$
$$=\frac{1}{8}$$
$$=0.125$$

Since the integrand is given by
$y=f(x)=e^{2x}$,
Therefore, the table of values will be

x	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
$y=f(x)$	1	1.2840	1.6487	2.1170	2.7183	3.4903	4.4817	5.7546	7.3891

Now applying the Trapezoidal rule, we have

$$\int_0^1 e^{2x}dx=\frac{h}{2}\left[y_0+2\left(y_1+y_2+y_3+y_4+y_5+y_6+y_7\right)+y_8\right]$$
$$=\frac{0.125}{2}\left[1+2\left(1.2840+1.6487+2.1170+2.7183+3.4903+4.4817+5.7546\right)+7.3891\right]$$
$$=0.0625\left[8.3891+2\left(21.4946\right)\right]$$
$$=0.0625\left(8.3891+42.9892\right)$$
$$=0.0625\left(51.3783\right)$$
$$=3.2111$$

Therefore
$$\int_0^1 e^{2x}dx=3.2111$$

b) By Simpson's Rule

Applying Simpson's rule, we have

$$\int_0^1 e^{2x}\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6) + y_8\right]$$

$$= \frac{0.125}{3}\left[1 + 4[1.2840 + 2.1170 + 3.4903 + 5.7546] + 2[1.6487 + 2.7183 + 4.4817] + 7.3891\right]$$

$$= \frac{0.125}{3}\left[8.3891 + 4(12.6459) + 2(8.8487)\right]$$

$$= \frac{0.125}{3}(8.3891 + 50.5836 + 17.6974)$$

$$= \frac{0.125}{3}(76.6701)$$

$$= 3.1946$$

Therefore

$$\int_0^1 e^{2x}\,dx = 3.1946$$

True value

Bu integration, we have

$$\int_0^1 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^1$$

$$= \frac{1}{2}\left[e^{2x}\right]_0^1$$

$$= \frac{1}{2}\left[e^2 - e^0\right]$$

$$= \frac{1}{2}\left[7.389056099 - 1\right]$$

$$= 3.1945$$

Therefore the true value is

$$\int_0^1 e^{2x}\,dx = 3.1945$$

Percentage error in Trapezoidal rule

$$\text{Error} = |Exact\ value - Computed\ value|$$

So,

$$Error = |3.1945 - 3.2111|$$

$$= |-0.0166|$$

$$= 0.0166$$

And % error is given by

$$\%\,error = \frac{error}{exact\ value} \times 100\%$$
$$= \frac{0.0166}{3.1945} \times 100\%$$
$$= 0.52\%$$

Percentage error in Simpson's 1/3 rule

Error = $|Exact\ value - Computed\ value|$

$$Error = |3.1945 - 3.1946|$$
$$= |-0.0001|$$
$$= 0.0001$$

And % error is given by

$$\%\,error = \frac{error}{exact\ value} \times 100\%$$
$$= \frac{0.0001}{3.1945} \times 100\%$$
$$= 0.002\%$$

Question #2 **Marks 10**

Evaluate the following double integral,

$$\int_1^2 \int_1^2 \frac{1}{x^3 + y^3} dxdy$$

By using trapezoidal rule, with h = k = 0.25.

Solution:

Taking X = 1, 1.25, 1.50, 1.75, 2.00 and y = 1, 1.25, 1.50, 1.75, 2.00, the following table is obtained using the integrand

$$f(x, y) = \frac{1}{x^3 + y^3}$$

X , Y	1.00	1.25	1.50	1.75	2.00
1.00	0.5	0.3386	0.2286	0.1572	0.1111
1.25	0.3386	0.2560	0.1877	0.1368	0.1005
1.50	0.2286	0.1877	0.1481	0.1145	0.0879
1.75	0.1572	0.1368	0.1145	0.0933	0.0749
2.00	0.1111	0.1005	0.0879	0.0749	0.0625

Now using trapezoidal, keeping variable x fixed and changing the variable y.

$$\int_1^2 f(1, y)dy = \frac{0.25}{2}[0.5 + 2(0.3386 + 0.2286 + 0.1572) + 0.1111]$$

= 0.2575

$$\int_1^2 f(1.25, y)dy = \frac{0.25}{2}[0.3386 + 2(0.2560 + 0.1877 + 0.1368) + 0.1005]$$

= 0.02

$$\int_1^2 f(1.50, y)dy = \frac{0.25}{2}[0.2286 + 2(0.1877 + 0.1481 + 0.1145) + 0.0879]$$

= 0.1521

$$\int_1^2 f(1.75, y)dy = \frac{0.25}{2}[0.1572 + 2(0.1368 + 0.1145 + 0.0933) + 0.0749]$$

= 0.1152

$$\int_1^2 f(2.00, y)dy = \frac{0.25}{2}[0.1111 + 2(0.1005 + 0.0879 + 0.0749) + 0.0625]$$

= 0.0875

$$\int_1^2 \int_1^2 \frac{1}{x^3 + y^3} dxdy = \frac{h}{2}[f(1, y) + 2\{f(1.25, y) + f(1.50, y) + f(1.75, y)\} + f(2.00, y)]$$

$$= \frac{0.25}{2}[2575 + 2(0.2 + 0.1521 + 0.1152) + 0.0875]$$

= 0.1600

Simpson's 1/3 and 3/8 rules

A Simpson's 1/3 Rule:

Question: Evaluate the following integral, $\int_{1.0}^{1.8} \frac{e^x + e^{-x}}{2} dx$

Using Simpson's 1/3 rule by taking h=0.2 and e=2.7183

Solution:

First of all table of the function is created,

x	$y = f(x) = \frac{e^x + e^{-x}}{2}$
1.0	1.543
1.2	1.811
1.4	2.151
1.6	2.577
1.8	3.107

Simpson's $\frac{1}{3}$*rd rule for the given function is*

$$I = \frac{h}{3}\left[f_0 + 4(f_1 + f_3) + 2f_2 + f_4\right]$$

$$I = \frac{0.2}{3}\left[1.543 + 4(1.811 + 2.577) + 2(2.151) + 3.107\right]$$

$$I = \frac{0.2}{3}\left[1.543 + 17.552 + 4.302 + 3.107\right]$$

$$I = \frac{0.2}{3}\left[26.504\right] = 1.7669$$

Question 1 **marks 10**

A river is 80 feet wide. Depth d in feet at a distance of x feet from one bank is given by the following table.

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross-section by Simpson's 1/3 rule.

SOLUTION

Let $y = d$, depth in feet, then the table is given by

x	0	10	20	30	40	50	60	70	80
$y = d$	0	4	7	9	12	15	14	8	3

Here $h = 10$

Let A be the area of the cross-section, then the area of the cross-section is given by

$$A = \int_0^{80} y dx$$

And the area of cross-section by Simpson's 1/3 rule is given by

$$A = \int_0^{80} y dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4 + y_6) + y_8]$$

$$= \frac{10}{3}[0 + 4(4 + 9 + 15 + 8) + 2(7 + 12 + 14) + 3]$$

$$= \frac{10}{3}[0 + 4(36) + 2(33) + 3]$$

$$= \frac{10}{3}[0 + 144 + 66 + 3]$$

$$= \frac{10}{3}(213)$$

$$= 710\, sq.ft$$

A Simpson's 3/8 Rule:
Use Simpson's 3/8 rule to estimate the number of square feet of hand in given lots when x and y are measured in feet (Use accuracy up to three places of decimal)

x	0	100	200	300	400	500	600	700	800	900	1000
y	125	125	120	112	90	90	95	88	75	35	0

Solution

Simpson's 3/8th rule may be written as

$$I=\frac{3h}{8}[f_0+3(f_1+f_2)+2f_3+3(f_4+f_5)+2f_6+3(f_7+f_8)+2f_9+f_{10}]$$

$$I=\frac{3(100)}{8}[125+3(125+120)+2(112)+3(90+90)+2(95)+3(88+75)+2(35)+0]$$

$$I=\frac{3(100)}{8}[125+3(245)+2(112)+3(180)+2(95)+3(163)+2(35)]$$

$$I=(37.5)[125+735+224+540+190+489+70]$$

$$I=(37.5)[2375]=88987.5$$

Differential Equations

Taylor Series Method

Question 1 **marks 10**

Apply Taylor's Series algorithm of order 2 on the initial value problem

$$y'=x^2+y^2 \;;\quad y(0)=1 \;;\quad h=0.05$$

in the interval [0,0.1].

Sol.

As $y'=x^2+y^2$ *Since* $x_0=0$ *and* $y_0=1$

so $y'_0=x_0^2+y_0^2=0+1=1$

Now $y_0''=2x_0+2y_0y_0'=2(0)+2(1)(1)=2$

As Taylor's Series method algorithm of order 2 is

$$y(t)=y(t_0)+(t-t_0)y'(t_0)+\frac{(t-t_0)^2}{2!}y''(t_0)$$

$$y(t)=y(t_0)+hy'(t_0)+\frac{h^2}{2!}y''(t_0)$$

$$y(.05)=y_1=1+.05(1)+\frac{(.05)^2}{2!}(2)=1.0525$$

Now we find y_2, *so* $y'_1=x_1^2+y_1^2=(.05)^2+(1.0525)^2=.0025+1.1077=1.11025$

Now $y_1''=2x_1+2y_1y_1=2(.05)+2(1.0525)(1.11025)=.1+2.337=2.437$

Now $y(t)=y_2=y(t_1)+hy'(t_1)+\frac{h^2}{2!}y''(t_1)$

$$y(0.1)=y_2=1.0525+(.05)(1.11025)+\frac{(.05)^2}{2!}(2.437)$$

$$=1.0525+.0555+.003=1.111$$

Question 2 **marks 10**

Given $y'=3x^2+2y$ and $y(0)=2$, find by Taylor's series $y(0.1)$ and $y(0.2)$ taking $h=0.1$.

SOLUTION

First few derivatives from the given differential equation are as follows

$$y' = 3x^2 + 2y$$
$$y'' = 6x + 2y'$$
$$y''' = 6 + 2y''$$
$$y^{(iv)} = 2y'''$$

The initial condition is given as

$x_0 = 0$, $y_0 = 2$

So, using the given initial condition, we have

$$y_0' = 3x_0^2 + 2y_0 = 3(0)^2 + 2(2) = 4$$
$$y_0'' = 6x_0 + 2y_0' = 6(0) + 2(4) = 8$$
$$y_0''' = 6 + 2y_0'' = 6 + 2(8) = 22$$
$$y_0^{(iv)} = 2y_0''' = 2(22) = 44$$
$$y_0^{(v)} = 2y_0^{(iv)} = 2(44) = 88$$

Now, using Taylor's series method up to the fifth term, we have

$$y(x) = y_0 + (x - x_0)y_0' + \frac{(x - x_0)^2}{2}y_0'' + \frac{(x - x_0)^3}{6}y_0''' + \frac{(x - x_0)^4}{24}y_0^{IV} + \frac{(x - x_0)^5}{120}y_0^{V}$$

Substituting the above values of the derivatives, and the initial condition, we obtain

$$y(0.1) = 2 + (0.1 - 0_0)(4) + \frac{(0.1 - 0)^2}{2}(8) + \frac{(0.1 - 0)^3}{6}(22) + \frac{(0.1 - 0)^4}{24}(44) + \frac{(0.1 - 0)^5}{120}(88)$$
$$y(0.1) = 2 + 0.4 + 0.04 + 0.0037 + 0.00018 + 0.00000067$$
$$y(0.1) = 2.44388067 \cong 2.4439$$

So, $y(0.1) = 2.4439$

Now, we have

$x_1 = 0.1$, $y_1 = 2.4439$

Using the above condition derivatives are calculated as follows

$$y_1' = 3x_1^2 + 2y_1 = 3(0.1)^2 + 2(2.4439) = 4.9178$$
$$y_1'' = 6x_1 + 2y_1' = 6(0.1) + 2(4.9178) = 10.4356$$
$$y_1''' = 6 + 2y_1'' = 6 + 2(10.4356) = 26.8712$$
$$y_1^{(iv)} = 2y_1''' = 2(26.8712) = 53.7424$$
$$y_1^{(v)} = 2y_1^{(iv)} = 2(53.7424) = 107.4848$$

Substituting the value of y_1 and its derivatives into Taylor's series expansion we get, after retaining terms up to fifth derivative only, we have

$$y(0.2) = y_1 + (x - x_1)y_1' + \frac{(x-x_1)^2}{2}y_1'' + \frac{(x-x_1)^3}{6}y_1''' + \frac{(x-x_1)^4}{24}y_1^{IV} + \frac{(x-x_1)^5}{120}y_1^{V}$$

$$y(0.2) = 2.4439 + (0.2-0.1)(4.9178) + \frac{(0.2-0.1)^2}{2}(10.4356) + \frac{(0.2-0.1)^3}{6}(26.8712) + \frac{(0.2-0.1)^4}{24}(53.7424) + \frac{(0.2-0.1)^5}{120}(107.4848)$$

$$y(0.2) = 2.4439 + 0.49178 + 0.052178 + 0.004785 + 0.000224 + 0.000009$$

$$= 2.992876 \cong 2.9929$$

So, $y(0.2) = 2.9929$

Euler Method

Question 2 **marks 10**

Use Euler's method to approximate y when x=1, given that

$$\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1, taking\ h = 0.2$$

Sol.

$$\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1, taking\ h = 0.2$$

$$here\ y_{m+1} = y_m + hf(x_m, y_m)$$

$$f(x,y) = \frac{y-x}{y+x}$$

$$x_0 = 0 \ , \ y_0 = 1$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + (0.2)\frac{1-0}{1} = 1.2$$

$x_1 = 0.2 \quad , \quad y_1 = 1.2$

$y_2 = y_1 + hf(x_1, y_1)$

$= 1.2 + (0.2)\dfrac{1.2 - 0.2}{1.2 + 0.2} = 1.2 + (0.2)(0.7142) = 1.2 + 0.14284 = 1.34284$

$x_2 = 0.4 \quad , \quad y_2 = 1.34284$

$y_3 = y_2 + hf(x_2, y_2)$

$= 1.34284 + (0.2)\dfrac{1.34284 - 0.4}{1.34284 + 0.4} = 1.34284 + (0.2)(0.10819) = 1.34284 + 0.10819 = 1.45103$

$x_3 = 0.6 \quad , \quad y_3 = 1.45103$

$y_4 = y_3 + hf(x_3, y_3)$

$= 1.45103 + (0.2)\dfrac{1.45103 - 0.6}{1.45103 + 0.6} = 1.45103 + (0.2)(0.40327) = 1.45103 + 0.08065 = 1.531684$

$x_4 = 0.8 \quad , \quad y_4 = 1.531684$

$y_5 = y_4 + hf(x_3, y_3)$

$= 1.531684 + (0.2)\dfrac{1.531684 - 0.8}{1.531684 + 0.8} = 1.531684 + (0.2)(0.31380) = 1.531684 + 0.06276 = 1.594444$

$x_5 = 1 \quad , \quad y_5 = 1.594444$

$y_6 = y_5 + hf(x_5, y_5)$

$= 1.594444 + (0.2)\dfrac{1.594444 - 1}{1.594444 + 1} = 1.594444 + (0.2)(0.22912) = 1.594444 + 0.045824 = 1.640268$

Euler Modified Method:

Question 1 **marks 10**

Given that $y' = \log(x + y)$ with $y(0) = 1$, Use Modified Euler's method to find $y(0.2)$, taking $h = 0.2$

SOLUTION

The given differential equation is

$$\frac{dy}{dx} = y' = \log(x + y), \quad y(0) = 1$$

,

So, here $x_0 = 0,\ y_0 = 1$,
$h = 0.2$

At first, we use Euler's method to get

$$y_1^{(1)} = y_0 + hf(x_0, y_0)$$
$$= y_0 + h\log(x_0 + y_0)$$
$$= 1 + 0.2[\log(0+1)]$$
$$= 1 + 0.2\log(1)$$
$$= 1 + 0.2(0)$$
$$= 1 + 0$$
$$= 1$$

So $x_1 = 0.2,\ y_1^{(1)} = 1$

Then, we use modified Euler's method to find

$$y(0.2) = y_1 = y_0 + h\frac{f(x_0, y_0) + f\left(x_1, y_1^{(1)}\right)}{2}$$
$$= 1 + (0.2)\frac{\log(0+1) + \log(0.2+1)}{2}$$
$$= 1 + (0.2)\frac{\log(1) + \log(1.2)}{2}$$
$$= 1 + (0.1)[\log(1) + \log(1.2)]$$
$$= 1 + (0.1)(0 + 0.0792)$$
$$= 1 + 0.1(0.0792)$$
$$= 1 + 0.00792$$
$$= 1.00792$$

Runge-Kutta Method

Question 2 **marks 10**

Given $y' = x + 2y + 1, y(0) = 2$,

Find $y(1)$ and $y(2)$ using the Runge-Kutta method of fourth order taking $h = 1$

SOLUTION

Given differential equation is

$y' = x + 2y + 1, y(0) = 2$

Here $h = 1$ and $x_0 = 0, y_0 = 2$

The Fourth-Order Runge-Kutta Method is described as :-

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where,

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

First Iteration:-

$$x_0 = 0 \quad , \quad y_0 = 2$$

$$k_1 = hf(x_0, y_0)$$
$$= 1f(0,2)$$
$$= 1[0 + 2(2) + 1]$$
$$= 1(5)$$
$$= 5$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$
$$= 1f(0 + \frac{1}{2}, 2 + \frac{5}{2})$$
$$= f(0.5, 4.5)$$
$$= 0.5 + 2(4.5) + 1$$
$$= 10.5$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$= 1f(0 + \frac{1}{2}, 2 + \frac{10.5}{2})$$
$$= f(0.5, 7.25)$$
$$= 0.5 + 2(7.25) + 1$$
$$= 16$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$
$$= 1f(0 + 1, 2 + 16)$$
$$= f(1,18)$$
$$= 1 + 2(18) + 1$$
$$= 38$$

Now

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 2 + \frac{1}{6}(5 + 2(10.5) + 2(16) + 38)$$

$$= 2 + \frac{1}{6}(5 + 21 + 32 + 38)$$

$$= 2 + \frac{1}{6}(96)$$

$$= 18$$

Second Iteration:-

$$x_1 = 1,\ y_1 = 18, h = 1$$

$$k_1 = hf(x_1, y_1)$$

$$= 1 f(1,18)$$

$$= 1[1 + 2(18) + 1]$$

$$= 1(38)$$

$$= 38$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 1 f(1 + \frac{1}{2}, 18 + \frac{38}{2})$$

$$= f(1.5, 37)$$

$$= 1.5 + 2(37) + 1$$

$$= 76.5$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 1 f(1 + \frac{1}{2}, 18 + \frac{76.5}{2})$$

$$= f(1.5, 56.25)$$

$$= 1.5 + 2(56.25) + 1$$

$$= 115$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= 1 f(1 + 1, 18 + 115)$$

$$= f(2, 133)$$

$$= 2 + 2(133) + 1$$

$$= 269$$

So,

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 18 + \frac{1}{6}(38 + 2(76.5) + 2(115) + 269)$$

$$= 18 + \frac{1}{6}(38 + 153 + 230 + 269)$$

$$= 18 + \frac{1}{6}(690)$$

$$= 18 + 115$$

$$= 133$$

So,

$$y(1) = 18$$

$$y(2) = 133$$

Milne's Predictor Corrector Method

Question 2
Marks 10

Find the solution of

$$y' = y(x + y), \quad y(0) = 1$$

Using Milne's P-C method at $x = 0.4$ given that $y(0.1) = 1.11689$, $y(0.2) = 1.27739$ and $y(0.3) = 1.50412$

Solution:

Here,

$x_0 = 0 \quad , \quad x_1 = 0.1 \quad , \quad x_2 = 0.2 \quad , \quad x_3 = 0.3 \quad , \quad x_4 = 0.4$

and $y_0 = 1, y_1 = 1.11689, y_2 = 1.27739, y_3 = 1.50412$

$y' = (x + y)y$

$y_1' = (x_1 + y_1)y_1 = (0.1 + 1.11689)(1.11689) = 1.3591323$

$y_2' = (x_2 + y_2)y_2 = (0.2 + 1.27739)(1.27739) = 1.8872032$

$y_3' = (x_3 + y_3)y_3 = (0.2 + 1.50412)(1.50412) = 2.713613$

Now, using Predictor Formula

$$y_4 = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$y_4 = 1 + \frac{4*0.1}{3}[2(1.3591323) - 1.8872032 + 2(2.713613)]$$

$$y_4 = 1.8344383$$

Adam Moultan's Predictor Corrector Method

Question **marks 10**

Using Adam-Moulton Predictor-Corrector Formula find f(0.4) from Ordinary Differential Equation

$$y' = 1 + 2xy \quad ; \quad y(0) = 0 \; ; \quad h = 0.1$$

with the help of following table.

X	0	0.1	0.2	0.3
Y	0	0.1007	0.2056	0.3199

Solution: Here,

$h = 0.1 \qquad f(x,y) = 1 + 2xy$

$y_0' = 1 + 2x_0y_0 = 1 + 2(0)(0) = 1$

$y_1' = 1 + 2x_1y_1 = 1 + 2(0.1)(0.1007) = 1.02014$

$y_2' = 1 + 2x_2y_2 = 1 + 2(0.2)(0.2056) = 1.08224$

$y_3' = 1 + 2x_3y_3 = 1 + 2(0.3)(0.3199) = 1.19194$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24}(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$

$$y_4 = 0.3199 + \frac{0.1}{24}(55(1.19194) - 59(1.08224) + 37(1.02014) - 9(1))$$

$y_4 = 0.446773833$

Computing y'_4 *for the Corrector Formula*;

$y'_4 = 1 + 2x_4y_4 = 1 + 2(0.4)(0.446773833)$

$y'_4 = 1.3574190664$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})$$

$$y_4 = y_3 + \frac{h}{24}(9y'_4 + 19y'_3 - 5y'_2 + y'_1)$$

$$y_4 = 0.3199 + \frac{0.1}{24}(9(1.3574190664) + 19(1.19194) - 5(1.08224) + 1.02014)$$

$y_4 = 0.446869048$

FAQ updated version.

Question: What is Bracketing method?

Answer: Methods such as bisection method and the false position method of finding roots of a nonlinear equation f(x) = 0 require bracketing of the root by two guesses. Such

methods are called bracketing methods. These methods are always convergent since they are based on reducing the interval between the two guesses to zero in on the root.

Question: What is an Open method?

Answer: In the Newton-Raphson method, the root is not bracketed. Only one initial guess of the root is needed to get the iterative process started to find the root of an equation. Hence, the method falls in the category of open methods.

Question: Explain Muller's method briefly.

Answer: In Muller's method, f (x) = 0 is approximated by a second degree polynomial; that is by a quadratic equation that fits through three points in the vicinity of a root. The roots of this quadratic equation are then approximated to the roots of the equation f (x) 0.This method is iterative in nature and does not require the evaluation of derivatives as in Newton-Raphson method. This method can also be used to determine both real and complex roots of f (x) = 0.

Question: Explain the difference between the linear and non-linear equations.

Answer: Linear Equation An algebraic equation is said to be linear in which each term is either a constant or the product of a constant and the first power of a single variable. One or more variables can be involved in the linear equations. e.g. x+3y+z=0 2x-y+4z=7 etc. Non-Linear Equation An equation is said to Non-Linear equation if it is not linear. Equations involving the power of the variable 2 or higher, transcendental, logarithmic and trigonometric equations etc lie in the category of Non-Linear equations. e.g. x2+5x+3=0 sinx+3y+9=0 xlogx-7x+4y=2 etc.

Question: Explain which value is to be choosed as X0 in N-R method.

Answer: If, for a given function, f(a)*f(b)<0, then any value between a and b inclusive can be chosen as x0.

Question: Define iterative method of solving linear equations with two examples.

Answer: Under iterative methods, the initial approximate solution is assumed to be known and is improved towards the exact solution in an iterative way. We consider Jacobi, Gauss-Seidel and relaxation methods under iterative methods.

Question: Define Pivoting.

Answer: The Gaussian elimination method fails if any one of the pivot elements becomes zero. In such a situation, we rewrite the equations in a different order to avoid zero pivots. Changing the order of equations is called pivoting.

Question: Write the two steps of solving the linear equations using Gaussian Elimination method.

Answer: In this method, the solution to the system of equations is obtained in two stages.

i) the given system of equations is reduced to an equivalent upper triangular form using elementary transformations

ii) the upper triangular system is solved using back substitution procedure

Question: Describe Gauss-Jordan elimination method briefly

Answer: This method is a variation of Gaussian elimination method. In this method, the elements above and below the diagonal are simultaneously made zero. That is a given system is reduced to an equivalent diagonal form using elementary transformations. Then the solution of the resulting diagonal system is obtained. Sometimes, we normalize the pivot row with respect to the pivot element, before elimination. Partial pivoting is also used whenever the pivot element becomes zero.

Question: Describe briefly Crout's reduction method.

Answer: Here the coefficient matrix [A] of the system of equations is decomposed into the product of two matrices [L] and [U], where [L] is a lower-triangular matrix and [U] is an upper-triangular matrix with 1's on its main diagonal.

Question: Describe briefly the Jacobi's method of solving linear equations.

Answer: This is an iterative method, where initial approximate solution to a given system of equations is assumed and is improved towards the exact solution in an iterative way.

Question: What is the difference between Jacobi's method and Gauss Seidal method?

Answer: The difference between jacobi's method and gauss Seidel method is that in jacobi's method the approximation calculated are used in the next iteration for next approximation but in Gauss-seidel method the new approximation calculated is instantly replaced by the previous one.

Question: What is the basic idea of Relaxation method?

Answer: We can improve the solution vector successively by reducing the largest residual to zero at that iteration. This is the basic idea of relaxation method.

Question: How the fast convergence in the relaxation method is achieved?

Answer: To achieve the fast convergence of the procedure, we take all terms to one side and then reorder the equations so that the largest negative coefficients in the equations appear on the diagonal.

Question: Which matrix will have an inverse?

Answer: Every square non-singular matrix will have an inverse.

Question: What are the popular methods available for finding the inverse of a matrix?

Answer: Gauss elimination and Gauss-Jordan methods are popular among many methods available for finding the inverse of a matrix.

Question: Explain Gaussian Elimination Method for finding the inverse of a matrix.

Answer: In this method, if A is a given matrix, for which we have to find the inverse; at first, we place an identity matrix, whose order is same as that of A, adjacent to A which we call an augmented matrix. Then the inverse of A is computed in two stages. In the first stage, A is converted into an upper triangular form, using Gaussian elimination method In the second stage, the above upper triangular matrix is reduced to an identity matrix by row transformations. All these operations are also performed on the adjacently placed identity matrix. Finally, when A is transformed into an identity matrix, the adjacent matrix gives the inverse of A. In order to increase the accuracy of the result, it is essential to employ partial pivoting

Question: What are the steps for finding the largest eigen value by power method.

Answer:

Procedure Step 1: Choose the initial vector such that the largest element is unity.
Step 2: The normalized vector is pre-multiplied by the matrix [A].
Step 3: The resultant vector is again normalized.

Question: What is the method for finding the eigen value of the least magnitude of the matrix [A]?

Answer: For finding the eigen value of the least magnitude of the matrix [A], we have to apply power method to the inverse of [A].

Question: What is interpolation?

Answer: The process of estimating the value of y, for any intermediate value of x, lying inside the table of values of x is called interpolation.

Question: What is extrapolation?

Answer: The method of computing the value of y, for a given value of x, lying outside the table of values of x is known as extrapolation.

Question: What happens when shift operator E operates on the function.

Answer: When shift operator E operates on the function it results in the next value of the function.Ef(x)=f(x+h)

Question: What is the basic condition for the data to apply Newton's interpolation methods?

Answer: To apply Newton's interpolation methods, data should be equally spaced.

Question: When is the Newton's forward difference interpolation formula used?

Answer: Newton's forward difference interpolation formula is mainly used for interpolating the values of y near the beginning of a set of tabular values and for extrapolating values of y, a short distance backward from y0.

Question: When is the Newton's forward difference interpolation formula used?

Answer: Newton's forward difference interpolation formula is mainly used for interpolating the values of y near the beginning of a set of tabular values and for extrapolating values of y, a short distance backward from y0.

Question: When the Newton's backward difference interpolation formula is used?

Answer: For interpolating the value of the function y = f (x) near the end of table of values, and to extrapolate value of the function a short distance forward from yn, Newton's backward Interpolation formula is used.

Question: What is the formula for finding the value of p in Newton's forward difference interpolation formula?

Answer: P=(x-x0)/h

Question: What is the formula for finding the value of p in Newton's backward difference interpolation formula?

Answer: P=(x-xn)/h

Question: If the values of the independent variable are not equally spaced then which formula should be used for interpolation?

Answer: If the values of the independent variable are not given at equidistant intervals, then the Lagrange's interpolation formula should be used.

Question: To use Newton's divided difference interpolation formula, what should the values of independent variables be?

Answer: To use Newton's divided difference interpolation formula, the values of independent variables should not be equally spaced.

Question: Which difference formula is symmetric function of its arguments?

Answer: Divided difference formula is symmetric function of its arguments.

Question: Is the interpolating polynomial found by Lagrange's and Newton's divided difference formulae is same?

Answer: Yes. The interpolating polynomial found by Lagrange's and Newton's divided difference formulae is one and the same.

Question: Which formula involves less number of arithmetic operations? Newton or Lagrange's?

Answer: Newton's formula involves less number of arithmetic operations than that of Lagrange's.

Question: When do we need Numerical Methods for differentiation and integration?

Answer: If the function is known and simple, we can easily obtain its derivative (s) or can evaluate its definite integral However, if we do not know the function as such or the function is complicated and is given in a tabular form at a set of points x0,x1,…,xn, we use only numerical methods for differentiation or integration of the given function.

Question: If the value of the independent variable at which the derivative is to be found appears at the beginning of the table of values, then which formula should be used?

Answer: If the value of the independent variable at which the derivative is to be found appears at and near the beginning of the table, it is appropriate to use formulae based on forward differences to find the derivatives.

Question: If the value of the independent variable at which the derivative is to be found occurs at the end of the table of values, then which formula should be used?

Answer: If the value of the independent variable at which the derivative is to be found occurs at the end of the table of values, it is appropriate to use formulae based on backward differences to find the derivatives.

Question: Why we need to use RICHARDSON'S EXTRAPOLATION METHOD?

Answer: To improve the accuracy of the derivative of a function, which is computed by starting with an arbitrarily selected value of h, Richardson's extrapolation method is often employed in practice.

Question: To apply Simpson's 1/3 rule, what should the number of intervals be?

Answer: To apply Simpson's 1/3 rule, the number of intervals must be even.

Question: To apply Simpson's 3/8 rule, what should the number of intervals be?

Answer: To apply Simpson's 3/8 rule, the number of intervals must be multiple of 3.

Question: What is the order of global error in Simpson's 1/3 rule?

Answer: The global error in Simpson's 1/3 rule is of the order of 0(h4).

Question: Is the order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule?

Answer: Yes. The order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule.

Question: What is the order of global error in Trapezoidal rule?

Answer: The global error in Trapezoidal rule is of the order of 0(h2).

Question: What is the formula for finding the width of the interval?

Answer: Width of the interval, h, is found by the formula h=(b-a)/n

Question: What type of region does the double integration give?

Answer: Double integration gives the area of the rectangular region.

Question: Compare the accuracy of Romberg's integration method to trapezoidal and Simpson's rule.

Answer: Romberg's integration method is more accurate to trapezoidal and Simpson's rule

Question: What is the order of global error in Simpson's 3/8 rule?

Answer: The global error in Simpson's 3/8 rule is of the order of 0(h4).

Question: Which equation models the rate of change of any quantity with respect to another?

Answer: An ordinary differential equation models the rate of change of any quantity with respect to another.

Question: By employing which formula, Adam-Moulton P-C method is derived?

Answer: Adam-Moulton P-C method is derived by employing Newton's backward difference interpolation formula.

Question: What are the commonly used number systems in computers?

Answer: Binary Octal Decimal Hexadecimal

Question: If a system has the base M, then how many different symbols are needed to represent an arbitrary number? Also name those symbols.

Answer: M 0, 1, 2, 3, … , M-1

Question: What is inherent error and give its example.

Answer: It is that quantity of error which is present in the statement of the problem itself, before finding its solution. It arises due to the simplified assumptions made in the mathematical modeling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the problem. e.g. if writing 8375 instead of 8379 in a statement lies in the category of inherent error.

Question: What is local round-off error?

Answer: At the end of computation of a particular problem, the final results in the computer, which is obviously in binary form, should be converted into decimal form-a form understandable to the user-before their print out. Therefore, an additional error is committed at this stage too. This error is called local round-off error.

Question: What is meant by local truncation error?

Answer: Retaining the first few terms of the series up to some fixed terms and truncating the remaining terms arise to local truncation error.

Question: What is transcendental equation and give two examples.

Answer: An equation is said to be transcendental equation if it has logarithmic, trigonometric and exponential function or combination of all these three. For example it is a transcendental equation as it has an exponential function These all are the examples of transcendental equation.

Question: What is meant by intermediate value property?

Answer: If f(x) is a real valued continuous function in the closed interval if f(a) and f(b) have opposite signs once; that is f(x)=0 has at least one root such that Simply If f(x)=0 is a polynomial equation and if f(a) and f(b) are of different signs ,then f(x)=0 must have at least one real root between a and b.

Question: What is direct methods of solving equations?

Answer: Those methods which do not require any information about the initial approximation of root to start the solution are known as direct methods. The examples of

direct methods are Graefee root squaring method, Gauss elimination method and Gauss Jordan method. All these methods do not require any type of initial approximation.

Question: What is Iterative method of solving equations?

Answer: These methods require an initial approximation to start. Bisection method, Newton raphson method, secant method, jacobi method are all examples of iterative methods

Question: If an equation is a transcendental, then in which mode the calculations should be done?

Answer: All the calculations in the transcendental equations should be done in the radian mode.

Question: What is the convergence criterion in method of iteration?

Answer: If be a root of f(x) =0 which is equivalent to I be any interval containing the point x= and will converge to the root provided that the initial approximation is chosen in I

Question: When we stop doing iterations when Toll is given?

Answer: Here if TOL is given then we can simply find the value of TOL by subtracting both the consecutive roots and write it in the exponential notation if the required TOL is obtained then we stop.

Question: How the value of h is calculated in interpolation?

Answer: There are two types of data in the interpolation one is equally spaced and other is unequally spaced. In equally spaced data we need to calculate the value of h that is calculated by subtracting any two consecutive values and taking their absolute value.

Question: What is an algebraic equation?

Answer: An algebraic equation is an equation which is purely polynomial in any variable. Supposed x2+3x+2=0, x4+3x2=0, y3+6y2=0 all are algebraic equations as these are purely polynomial in x and y variable.

Question: What is Descartes rule of signs?

Answer: The number of positive roots of an algebraic equation f(x) =0 can not exceed the no of changes in signs. Similarly the no of negative roots of negative roots of and algebraic equation can not exceed the no of changes in sign of equation f (-x) =0.

Question: What are direct methods?

Answer: The numerical methods which need no information about the initial approximation are known as direct methods like Graffee's root squaring method.

Question: What is meant by iterative methods?

Answer: The methods which need one or more iterations are known as iterative methods like bisection method, Newton raphson method, and many other methods.

Question: What is graphically meant by the root of the equation?

Answer: If the graph of a function f(x) =0 cuts the x-axis at a point a then a is known as the root of the equation.

Question: Q. What is the difference between open and bracketing method?

Answer: In open methods we need only one initial approximation of the root that may be any where lying and if it is not very close then we have to perform more iteration and the example of open method is Newton raphson method. In bracketing method we bracket the root and find that interval in which root lies means we need two initial

approximations for the root finding. Bisection method is an example of the bracketing method.

Question: Condition for the existence of solution of the system of equations.

Answer: If the |A| is not equal to zero then the system will have a unique solution if |A|=0 then the system will have no solution

Question: Should the system be diagonally dominant for gauss elimination method?

Answer: The system of equation need not to be diagonally dominant for Gauss elimination method and gauss Jordan method for both the direct method it is not necessary for the system to be diagonally dominant .it should be diagonally dominant for iterative methods like jacobie and gauss seidel method.

Question: What is meant by diagonally dominant system?

Answer: A system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 is said to be diagonally dominant if the following condition holds. |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3| => (greater or equal) |a3|+|b3|

Question: State the sufficient condition for the convergence of the system of equation by iterative methods.

Answer: A sufficient condition for convergence of iterative solution to exact solution is |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3| => (greater or equal) |a3|+|b3| For the system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 Similarly for the system with more variables we can also construct the same condition

Question: The calculation for numerical analysis should be done in degree or radians.

Answer: All the calculation for numerical analysis should be done in radians not in degrees set your calculator in radians mode and suppose the value of pi=3.14.

Question: How we can identify that Newton forward or backwards interpolation formula is to be used.

Answer: If the value at which we have to interpolate is in the start of the table then we will use Newton's forward interpolation formula if it is at the end of the table then we will use the Newton's backward interpolation formula.

Question: What is meant precision and accuracy?

Answer: Precision and accuracy are two terms which are used in numerical calculations by precision we mean that how the values in different iterations agree to each other or how close are the different values in successive iterations. For example you have performed 3 different iterations and result of all the iteration are 1.32514,1.32516,31.32518 these three values are very precise as these values agree with each other . Accuracy means the closeness to the actual value. Suppose that you have calculated an answer after some iteration and the answer is 2.718245 and the actual answer is 2.718254 and the answer calculated is very accurate but if this answer is 2.72125 then it is not accurate.

Question: What is the condition that a root will lie in an interval.

Answer: Suppose that you have a function f(x) and an interval [a,b]and you calculate both f(a)and f(b)if f(a)f(b)<0 then there must exist a root for this function in this interval.In simple words f(a) and f(b)must have opposite signs.

Question: How the divided difference table is constructed?

Answer: x y First difference Second difference Third difference 1 0.2 y0 y1-y0/x1-x0=sy0 sy1- sy0/x2-x0=s2y0 s2y0-s2y1/x3-x0=s3y0 s3y1-s3y0/x4x0 =s4y0 2 0.69 y1 y2-y1/x2-x1=sy1 sy2- sy1/x3-x1=s2y1 s2y2-s2y1/x4-x1=s3y1 3 0.985 y2 y3-y2/x3-x2=sy2 sy3- sy2/x4-x1=s2y2 4 1.2365 y3 y4-y3/x4-x3=sy3 5 2.3651 y4 This is the complete table showing the central differences . Here s stands for difference

Question: What is Gauss-seidel Method

Answer: It is also an iterative method and in this we also check either the system is diagonally dominant and, if so then we proceed on the same lines as in the Jacobi's method. The difference in both is when we calculate a value of any variable that is instantly replaced by the previous value. If we have initial values like x1=0,x2=0 and x3=0 in the case of three variables in the first iteration we will use all the three values to calculate x1 but to calculate the value of x2 we will use the recent value of x1 which is calculated in the first iteration.

Question: What is partial and full pivoting?

Answer: Partial and full pivoting, In gauss elimination method when you have any of diagonal element aii zero it means the solution does not exist to avoid this we change the equation so that a non zero pivot is achieved .Now you have an argument matrix in which he elements in the first column are 1,3,4 respectively here in case of partial pivoting we will replace first element with the last element it is done by replacing the first equation with last equation it is known as partial pivoting. In full pivoting we change rows and columns but that is not implemented manually it is used in computers.

Question: How the initial vector is choose in power method?

Answer: Choose the initial vector such that the largest element is unity. But no problem with this you will precede with this vector as it is and will not change it. In the given question the initial vector is provided so you have to proceed with the given vector and will normalize this vector and normalizing this vector you will keep in mind that the greatest value should be unity. An eigenvector Vis said to be normalized if the coordinate of largest magnitude is equal to unity Actually this condition is for the normalization of the vectors when you have to normalize the vector you keep in mind that the larget entery in the vector must be 1 and so you take the largest element and divide the remaining values by the greatest value.

Question: what is the relation ship between p=0 and non zero p in interpolation.

Answer: fp=fo+pDfo+1/2(p2-p)D2fo+1/6(p3-3p2+2p)D3fo+1/24(p4-6p3+11p2-6p)D4fo+… This is the interpolation formula For the derivative you will have to take the derivative of this formula w.r.t p and you will get f'p=1/h{Dfo+1/2(2p-1)D2fo+1/6(3p2-6p+2)D3fo+…} Now put here p=0 f'p=1/h{Dfo+1/2(0-1)D2fo+1/6(0-0+2)D3fo+…} So the formula becomes f'p=1/h{Dfo+D2fo/2+D3fo/3+…} so this is the relation ship between non zero and zero p no when you have to calculate p the you use formula p=x-xo/h so this is the impact of x.

Question: What is chopping and Rounding off?

Answer: Chopping and rounding are two different techniques used to truncate the terms needed according to your accuracy needs. In chopping you simply use the mentioned number of digits after the decimal and discard all the remaining terms. Explanation (1/3 - 3/11) + 3/20=(0.333333…-0.27272727..)+0.15=(0.333-0.272)+0.15 This is the three digit chopping.

Question: When the forward and backward interpolation formulae are used?

Answer: In interpolation if we have at the start then we use the forward difference formula and the formula to calculate p is x-x0/h. If the value of x lies at the end then we use Newton's backward formula and formula to calculate the value of p is x-xn/h. Now I come to your question as in this case the value lies at the end so 6 will be used as the xn. This procedure has been followed by the teacher in the lectures. But some authors also use another technique that is if you calculate the value of p and that is negative then the origin is shifted to that value for which the value of p becomes positive. And then according to that origin the values of differences are used and you need not follow this procedure.

Question: What is forward and backward difference operator and the construction of their table.

Answer: For forward Dfr =fr+1 –fr Df0 = f1-f0 In terms of y Dyr+1=yr+1-yr D stands for the forward difference operator For backward Dfr =fr –fr-1 Df1 = f1-f0 In terms of y Dy1=y1-y0 Here D stands for backwards operator Now the construction of the difference table is based on X Y 1st forward 2nd forward 3rd forward x1 Y1 Y2-Y1=Dy0 x2 Y2 Y3-Y2=Dy1 x3 Y3 Y4-Y3=Dy2 x4 Y4 Now consider the construction of table for the backward table X Y 1st forward 2nd forward 3rd forward x1 Y1 Y2-Y1=Dy1 x2 Y2 Y3-Y2=Dy2 x3 Y3 Y4-Y3=Dy3 x4 Y4

Dear student this is the main difference in the construction of the forward and backwards difference table when you proceed for forward difference table you get in first difference the value Dy0 but in the construction of backwards difference table in the first difference you get Dy1 and in the second difference in the forward difference table you get D2 y0 and in the backward difference table the first value in the second difference is D2 y1. I think so you have made it clear.

Question: What is Jacobi's method?

Answer: Jacobi's Method It is an iterative method and in this method we first of all check either the system is diagonally dominant and, if the system is diagonally dominant then we will calculate the value of first variable from first equation in the form of other variables and from the second equation the value of second variable in the form of other variables and so on. We are provided with the initial approximations and these approximations are used in the first iteration to calculate first approximation of all the variables. The approximations calculated in the first iteration are used in the second iteration to calculate the second approximations and so on.

Question: what is Simpson's 3/8th rule.

Answer: The general formula for Simpson's 3/8th rule is 3h/8[f0+3(f1+f2)+2f3+(3f4+f5)+2f6+…+3(fn-2+fn-1)+fn] Now if we have to calculate the integral by using this rule then we can simply proceed just write first and last vale and distribute all the remaining values with prefix 3 and 2 Like you have f0,f1,f2,f3,f4 Then the integral can be calculated as 3h/8[f0+f4+3(f1+f2)+2f4] If we have values like f0,f1,f2,f3,f4,f5 Then integral can be calculated as 3h/8[f0+3(f1+f2)+2f3+3f4+f5] Similarly proceeding in this fashion we can calculate the integral in this fashion

Question: what is classic runge-kutta method

Answer: The fourth order Runge-Kutta method is known as the classic formula of classic Runge-Kutta method. yn+1=yn+1/6(k1+2k2+2k3+k4) Where k1=hf(xn,yn) k2=hf(xn+h/2+yn+k1/2) k3=hf(xn+h/2,yn+k'2/2) k4=hf(xn+h,yn+k3)

Question: What is meant by TOL?

Answer: The TOL means the extent of accuracy which is needed for the solution. If you need the accuracy to two places of decimal then the TOL will be 10-2 .similarly the 10-3 means that the accuracy needed to three places of decimal. Suppose you have the root from last iteration 0.8324516 and 0.8324525 if we subtract both and consider absolute value of the difference 0.0000009 now it can be written as 0.09*10-5 so the TOL in this case is 10-5.similarly if we have been provided that you have to for the TOL 10-2 you will check in the same way. In the given equation you will solve the equation by any method and will consider some specific TOL and try to go to that TOL. Some time no TOL is provided and you are asked to perform to some specific no of iterations.

Question: what is meant by uniqueness of LU method.

Answer: An invertible (whose inverse exists) matrix can have LU factorization if and only if all its principal minors(the determinant of a smaller matrix in a matrix) are non zero .The factorization is unique if we require that the diagonal of L or U must have 1's.the matrix has a unique LDU factorization under these condition . If the matrix is singular (inverse does not exists) then an LU factorization may still exist, a square matrix of rank (the rank of a matrix in a field is the maximal no of rows or column) k has an LU factorization if the first k principal minors are non zero. These are the conditions for the uniqueness of the LU decomposition.

Question: how the valu of h is calculated from equally spaced data.

Answer: Consider the following data x y 1 1.6543 2 1.6984 3 2.4546 4 2.9732 5 3.2564 6 3.8765 Here for h=2-1=3-2=1 x y 0.1 1.6543 0.2 1.6984 0.3 2.4546 0.4 2.9732 0.5 3.2564 0.6 3.8765 Here for the calculation of h =0.2-0.1=0.3-0.2=0.1 I think so that you may be able to understand .

Glossary (Updated Version)

Absolute Error :

The absolute error is used to denote the actual value of a quantity less it's rounded value if x and x* are respectively the rounded and actual values of a quantity, then absolute error is defined by AE=|x-x*|

Accuracy :

The Extent of the closeness between the actual value and estimated value is known as accuracy. Suppose you have taken readings like 2.1234,2.1236and 2.1238 and 2.45,2.52,2.63 Now if the actual root is 2.65 then the second values are more accurate and not precise but first set of values is precise but not accurate as it differs from the actual value.

Algebraic equation :

The equation f(x)=0 is known as algebraic equation if it is purely a polynomial in x. Like f(x)=3 ,$f(x)=x^3-3x-4$ are some examples of algebraic equations and the if variable is changed to y x or any one then under the same conditions it will be algebraic.

Bisection method :

It is a bracketing method which is used to locate the root of an equation and it make use of intermediate value property to locate the interval in which the root lies, formula which is used is (a+b)/2 where a and b are points such that f(a)f(b)<0.

Bracketing Method :

The iterative methods which require two initial approximation for it's first iteration are known as bracketing methods. Bisection method is the example of the bracketing method as it requires a interval for the approximation of the root.

Crout's Method :

This method is used to solve the system of the equations by decomposing the system into two matrices L and U where L is a lower triangular matrix and U is an upper triangular matrix. In U all the elements in the main diagonal are 1.

Descartes rule of signs : The no of positive roots of an algebraic equation f(x)=0 with real coefficients cannot exceed the no of changes in the in sign of the coefficients in the polynomial f(x)=0,similarly the no of negative roots of f(x)=0 can not exceed the no of changes in the sign of the coefficients of f(-x)=0

Direct methods :

These are the methods which do not need the knowledge of the initial approximation are known as direct methods.

Gauss Elimination Method :

It is a direct method which is used to solve a system of equations

Gauss Seidel iterative method :

It is an iterative method in which an initial approximation is given. First of all system should be checked either it is diagonally dominant or not if it is not then it is made diagonally dominant. Secondly the first variable is calculated in terms of other variables from first equation and Second variable from second equation and so on .the previous value of the variable is replaced by new value instantly as it is obtained. This is the difference in the gauss seidel and jacobie iterative method.

Graeffee's root squaring method :

This method is used to find all the roots of the polynomial equation.

Intermediate value property :

If for an equation f(x)=0 for two values a and b we have such that $f(a)f(b)<0$ then there must exist a root between a and b in the interval [a,b].

Inverse of a matrix :

The matrix B is said to be inverse of a matrix A if the product of and B is I identity matrix.

Iterative methods :

Iterative methods are those type of methods which always require an initial approximation to start an iteration

Jacobie's iterative method: :

It is an iterative method in which an initial approximation is given. First of all system should be checked either it is diagonally dominant or not if it is not then it is made diagonally dominant. Secondly the first variable is calculated in terms of other variables from first equation and Second variable from second equation and so on after first iteration in the second iteration all the variables are replaced by the previous one.

Muller's Method :

In Muller's method f(x)=0 is approximated by a second degree polynomial, that is by quadratic equation that fits through three points in the vicinity of a root .Then roots of this quadratic equation are then approximated to the roots of equation f(x)=0

Newton Raphson Method. :

It is an open method which is used to locate a root of the equation, it needs only one initial approximation for it's first iteration. xn+1=xn-(fxn/f'xn)

Non singular matrix :

A matrix is said to be non singular if the determinant of the matrix is non zero. |A| is not equal to zero and the inverse of non singular exists.

Open methods :

The methods which require only one initial approximation to start the first iteration, for example the Newton's Raphsom method is known as the open method as it requires only one initial approximation.

Pivoting :

If any of the pivot elements in gauss elimination become zero then this method fails so to Avoid such type of situation equation are rearranged to get rid of zero pivot element, this procedure is known as pivoting.

Polynomial:

An expression of the form $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ where n is a positive integer and $a_0, a_1, a_2 + \ldots . a_n$ are real constants, such type of expression is called an nth degree polynomial in x if $a_o \neq 0$

Precision :

The extent of closeness of different measurement taken to estimate a certain value. Suppose you have done different iterations to measure the root of an equation and take different values as 2.1234, 2.1236, and 2.1238 these all the values are very close to each other so these are very precise.

Regula –Falsi method :

It is also an iterative method and is a bracketing method and use intermediate value property to get it's initial guess. The formula used for this is xn+1=xn-(xn-xn-1/f(xn)-f(xn-1))

Relative Error : It is the ratio of the absolute error to the actual value of the quantity. Thus RE=AE=AE\|x*|

Relaxation method :

It is also an iterative method and in this method you solve the system of equation by making the greatest residual to zero.

Root : If you have an equation f(x)=0 then the no a is said to be the root of the equation if f(a)=0 Suppose you have an equation f(x)=x2-4 the 2 is a root of the equation as f(2)=4-4=0

Secant Method :

The secant method is also an open method and it takes two initial values for it's first approximation, the formula used for this is known as xn+1={xn-1f(xn)-xnf(xn-1)}/{f(xn)-f(xn-1)}

Significant digits :

A significant digit in an approximate no is a digit, which gives reliable information about the size of the number. In other words a significant digit is used to express accuracy, that Is how many digits in the no have meaning.

Singular matrix :

A matrix s said to be a singular matrix if the determinant of the matrix is zero |A|=0,the inverse of the singular matrix do not exist.

Transcendental equation :

An equation f(x) =0 is said to be transcendental equation if it contains trigonometric, logarithmic and exponential functions

Truncation Error :

It is defined as the replacement of one series by another series with fewer terms. The error arising by this approximation is known as truncation error.

Important Formula For MTH603

Bisection Method

$$x2 = \frac{x0 + x1}{2}$$

Muller Method

$$x = \frac{x0 - 2c}{+b - \sqrt{b^2 - 4ac}}$$

in this formula x0,x1, x2 will be and U will just put the values according to the given formulas.

$$a = \frac{h2\,f1 - (h1 + h2)\,f0 + h1\,f2}{h1h2(h1 + h2)}$$

$$b = \frac{f1 = f0 - ah1^2}{h1}$$

$$c = f(x0)$$

$$h1 = x1 - x0$$

$$h2 = x0 - x2$$

Regula Falsi Method (Method of False position)

$$x3 = x2 - \frac{x2 - x1}{f(x2) - f(x1)}\,f(x2)$$

Newton Rophson method

$$x1 = x0 - \frac{f(x0)}{f'(x0)}$$

Secant Method

$$x2 = \frac{x0\,f(x1) - x1\,f(x0)}{f(x1) - f(x0)}$$

Newton's Formula

$$x1 = \frac{1}{2}(x0 + {}^{n}\!/_{x0})$$

in this formula
x0= 2 perfect square near to 12 such like 9 and 16

Graffee root squaring method

$$f(x)\,f(-x) = a_3^{\,2}x^6 - (a_2^{\,2} - 2a_1a_3)x^4 + (a_1^{\,2} - 2a_0a_2)x^2 - x_0^2$$

the truncation error (TE) is given by

$$\text{TE} \le \frac{x^{2n+2}}{(2n+2)!}$$

The TE is independent of the computer used.

Short Questions Paper

Set-01

Approximate the integral $\int_{1}^{1.6} \frac{2x}{x^2+1}\,dx$ using Simpson's 1/3 rule and calculate the percentage error. (Take result up to 4 decimal places)

Note: In order to get full marks do all necessary steps.

Construct a forward difference table for the following values

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

Note : In order to get full marks do all necessary steps.

Solve the system

$$4x+3y=24$$
$$3x+4y-z=30$$
$$-y+4z=-24$$

by Gauss Seidal Method, taking $(0, 0, 0)^t$ as initial approximation(Two iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Let $f(x)=x+\frac{2}{x}$, use cubic Lagrange interpolation based on the nodes $x_0=0.5, x_1=1, x_2=2 \text{ and } x_3=2.5$ to approximate *f(1.5)* and *f(1.3)*.

Note : In order to get full marks do all necessary stepsSolution

Approximate the Dominant Eigenvalue and corresponding Eigenvector for the matrix

$$\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

by using Power Method. Start with $X_0=(1,1,1)^t$. (Five iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

==========

Set-02

Question No: 31 (Marks: 2)

If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32 (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2)dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

Evaluate the integral

$$\int_3^5 (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36 (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37 (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38 (Marks: 5)

Evaluate the integral

$$\int_0^3 (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39 (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$ taking $h = 0.1$

==========

Set-03

Question:

Answer:

==========

Multiple Choice Question

Set-01

Question No: 1 (Marks: 1) - Please choose one

Symbol used for forward differences is

- ∇

- Δ

- δ

- μ

Question No: 2 (Marks: 1) - Please choose one

The relationship between central difference operator and the shift operator is given by

- $\delta = \mathrm{E} - \mathrm{E}^{-1}$

- $\delta = \mathrm{E} + \mathrm{E}^{-1}$

- $\delta = \mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}}$

- $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}}$

Question No: 3 (Marks: 1) - Please choose one

Muller's method requires --------starting points

- 1

- 2

- 3

- 4

Question No: 4 (Marks: 1) - Please choose one

If S is an identity matrix, then

- ▶ $S^{-1} = S$
- ▶ $S^t = S$
- ▶ $All\ are\ true$
- ▶ $S^{-1} = S^t$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_0, x_1, ..., x_r$

- ▶ r+2
- ▶ r+1
- ▶ r
- ▶ r-1

Question No: 6 (Marks: 1) - Please choose one

P in Newton's forward difference formula is defined as

- ▶ $p = (\frac{x - x_0}{h})$
- $p = (\frac{x + x_0}{h})$
- ▶

$$p = (\frac{x + x_n}{h})$$

►

$$p = (\frac{x - x_n}{h})$$

►

Question No: 7 (Marks: 1) - Please choose one

_____ Octal number system has the base ----------

► 2

► 8

► 10

► 16

Question No: 8 (Marks: 1) - Please choose one

_____ Newton's divided difference interpolation formula is used when the values of the independent variable are

► Equally spaced

► Not equally spaced

► Constant

► None of the above

Question No: 9 (Marks: 1) - Please choose one

_____ Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

► 1.5

► 3

► 2

► 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

► $\varepsilon(x) = y(x) + P_n(x)$

► $\varepsilon(x) = y(x) - P_n(x)$

► $\varepsilon(x) = P_n(x) - y(x)$

► $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11 (Marks: 1) - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I.

- n-1
- n+2
- n
- n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

- $D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}+\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}+...)$
- $D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2}+\frac{\Delta^3}{3}+\frac{\Delta^4}{4}+\frac{\Delta^5}{5}+...)$
- $D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\frac{\Delta^5}{5}-...)$
- $D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}-\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}-...)$

Question No: 13 (Marks: 1) - Please choose one

Finding the first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

► Forward difference operator

► **Backward difference operator**

► Central difference operator

► None of the given choices

Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

► -0.5

► 0.5

► 0.75

► **-0.75**

Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

► $ax+b$

► ax^2+bx+c

- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question No: 16 (Marks: 1) - Please choose one

While $I=\int_a^b f(x)dx$ integrating , h , width of the interval, is found by the formula-----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

Question No: 17 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

- 7
- 8
- **5**
- 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as ……

► 17.5

► 12.5

► **7.5**

► -12.5

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

► 2

► 3

► **5**

► 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

► 10

► 11

► **12**

► 13

Question No: 21 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- $\frac{(a+b)}{2}$
- $\frac{(a-b)}{2}$
- $\frac{(b-a)}{2}$
- None of the given choices

Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- Bisection Method
- Regula Falsi Method
- Secant Method
- None of the given choices

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

- (1, 3)

► (1, 2)

► (0, 1)

► (1, 2)

Question No: 24 (Marks: 1) - Please choose one

_____ Rate of change of any quantity with respect to another can be modeled by

► An ordinary differential equation

► A partial differential equation

► A polynomial equation

► None of the given choices

Question No: 25 (Marks: 1) - Please choose one

_____ If

$$\frac{dy}{dx} = f(x, y)$$

Then the integral of this equation is a curve in

► None of the given choices

► xt-plane

► yt-plane

► xy-plane

Question No: 26 (Marks: 1) - Please choose one

In solving the differential equation

$$y' = x + y \;;\; y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

- 1.44
- 1.11
- 1.22
- 1.33

Question No: 27 (Marks: 1) - Please choose one

In second order Runge-Kutta method

k_1 is given by

- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

Question No: 28 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

▶ $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

▶ $k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$

▶ $k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$

▶ $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

Question No: 29 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

▶ $k_3 = hf(x_n + 2h, y_n + 2k_3)$

▶ $k_3 = hf(x_n - h, y_n - k_3)$

▶ $k_3 = hf(x_n + h, y_n + k_3)$

▶ None of the given choices

Question No: 30 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

▶ Newton's backward difference interpolation formula

▶ Newton's forward difference interpolation formula

▶ Newton's divided difference interpolation formula

► None of the given choices

==========

Set-02

/wEPDwUKMTY2I

/wEWBgK+3MqI/

Question # 1 of 8 (Start time: 01:24:49 PM) **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

- antiderivative
- derivatives

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLy4ZCpA

Time Left 59 sec(s)

Question # 2 of 8 (Start time: 01:26:18 PM) **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLlwO3cC

Time Left 79 sec(s)

Question # 3 of 8 (Start time: 01:27:35 PM) **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

diverge

converge

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKMu86b

Time Left 85 sec(s)

Question # 5 of 8 (Start time: 01:30:21 PM) **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas
- constant

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgK6oIaxB

Time Left 83 sec(s)

Question # 6 of 8 (Start time: 01:31:18 PM) **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

Select correct option:

- TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgKPg8yyI

Time Left 84 sec(s)

Question # 7 of 8 (Start time: 01:32:33 PM) **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

Select correct option:

FALSE

TRUE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKK6eLRI

Time Left 84 sec(s)

Question # 8 of 8 (Start time: 01:33:57 PM) **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

Select correct option:

- constant
- finite
- infinity
- zero

Click here to Save Answer & Move to Next Question

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Set-03

No.	Questions	Mks
1	Find value of given data by Adam Moultan's method	10
2	Find value of given data by Dividend Difference Composite method	10
3	Draw backward difference tables for given Data	5+5

x			
y			

x			
y			

4	Write Simpson's 1/3 formula	2
5	Find value by Euler's Method	3
6	Find value of K1 by 2nd Order R-K method	2
7	Convergence is used when ----------	1
8	Bisection method is …….. method • Bracketing Method • Open • Random • none http://vustudents.ning.com/	1
9	Newton Raphson method is …….. method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real • Vector • odd • even	1
11	Find value of y'(1) by Euler's Method taking h=1	2
12	Find value of y'(3) from given table. x | | | | y | | | |	2
13	Find value of y'(0.3) by Lagrange's Method x | | | | y | | | |	3
14	For Simpson's 1/3 rule no.of intervals must be • 1 • 3 • 5 • 8	1

15	For Simpson's 1/3 rule valid no.of intervals are • 1 • 3 • 5 • 8	1
16	For Simpson's 3/8 rule no.of intervals must be • 10 • 11 • 12 • 14 http://vustudents.ning.com/	1
17	Find the value of y'(1) from given forward difference table	2

x	y	Δy	Δy^2	Δy^3

Numerical analysis mth603 paper

Numerical Analysis numerical paper 2009
The paper was very easy.
The mcq's were really easy.
Most of the mcq's were from the last 5 lecture.
and from jacobi's method and other's.
then the logical mcq's like
s inverse * s = I
it was repeated twice.
one 5 mark question was from newton's rapson method
one 10 mark question was from lecture 11 page 69 example
best of luck

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Set-04

/w EPDw UKMTY2
/w EWBgKPm46n

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 1 of 10 (Start time: 04:40:08 PM) Total Marks: 1

The determinant of a diagonal matrix is the product of the diagonal elements.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWBgKin4OcE

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 2 of 10 (Start time: 04:40:58 PM) Total Marks: 1

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWBgKx743v

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 3 of 10 (Start time: 04:41:55 PM) Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY
/w EWBgKR8tba

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 4 of 10 (Start time: 04:42:30 PM) Total Marks: 1

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY
/w EWBgKjyL19A

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 5 of 10 (Start time: 04:42:58 PM) Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

Select correct option:

- no
- atleast one

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY
/w EWBgKw w uK

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 6 of 10 (Start time: 04:43:20 PM) Total Marks: 1

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

Select correct option:

- unity
- zero

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY
/w EWBgLdv/PBE

Time Left 55 sec(s)

Quiz Start Time: 04:40 PM

Question # 7 of 10 (Start time: 04:43:39 PM) Total Marks: 1

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWCgL0zNjNE

Time Left 90 sec(s)

Quiz Start Time: 04:40 PM

Question # 8 of 10 (Start time: 04:45:08 PM) Total Marks: 1

The determinant of a _____ matrix is the product of the diagonal elements.

Select correct option:

- diagonal
- upper triangular
- lower triangular
- scalar

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWCgKw hsW

Time Left 89

sec(s)

Quiz Start Time: 04:40 PM

Question # 9 of 10 (Start time: 04:46:35 PM) Total Marks: 1

Eigenvalues of a symmetric matrix are all _____.

Select correct option:

- real
- zero
- positive
- negative

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWBgLX34ivA

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 10 of 10 (Start time: 04:47:33 PM) Total Marks: 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

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Set-05

/w EPDw UKMTY2

/w EWBgKqiMqh0

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 1 of 10 (Start time: 04:50:37 PM) Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWCgLw ndfn

Time Left 90 sec(s)

Quiz Start Time: 04:50 PM

Question # 2 of 10 (Start time: 04:50:51 PM) Total Marks: 1

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

Select correct option:

- (x-1)^3

- (x+1)^3
- x^3-1
- x^3+1

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKJ5b6A

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 3 of 10 (Start time: 04:51:41 PM) Total Marks: 1

For differences methods we require the set of values.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKd3KGC

Time Left 88 sec(s)

Quiz Start Time: 04:50 PM

Question # 4 of 10 (Start time: 04:51:57 PM) Total Marks: 1

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY:

/w EWBgLC663Y

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 5 of 10 (Start time: 04:52:22 PM) Total Marks: 1

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgL71Ojw

Time Left 88 sec(s)

Quiz Start Time: 04:50 PM

Question # 6 of 10 (Start time: 04:53:38 PM) Total Marks: 1

Central difference method seems to be giving a better approximation, however it requires more computations.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgLTr6GgI

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 7 of 10 (Start time: 04:54:56 PM) Total Marks: 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY

/w EWBgKr97ef

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 8 of 10 (Start time: 04:55:31 PM) Total Marks: 1

Iterative algorithms can be more rapid than direct methods.

Select correct option:

FALSE

TRUE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgL/r4KyC

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 9 of 10 (Start time: 04:55:49 PM) Total Marks: 1

Central Difference method is the finite difference method.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgLonf2RE

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 10 of 10 (Start time: 04:56:20 PM) Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

==========

Set-06

/wEPDwUKMTY2I

/wEWBgK+3MqI/

Question # 1 of 8 (Start time: 01:24:49 PM) Total Marks: 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

antiderivative

derivatives

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLy4ZCpA

Time Left 59 sec(s)

Question # 2 of 8 (Start time: 01:26:18 PM) **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLlwO3cC

Time Left 79 sec(s)

Question # 3 of 8 (Start time: 01:27:35 PM) **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

- diverge
- converge

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKMu86b

Time Left 85 sec(s)

Question # 5 of 8 (Start time: 01:30:21 PM) **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas

constant

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgK6oIaxB

Time Left 83 sec(s)

Question # 6 of 8 (Start time: 01:31:18 PM) **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgKPg8yyI

Time Left 84 sec(s)

Question # 7 of 8 (Start time: 01:32:33 PM) **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

Select correct option:

- FALSE
- TRUE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKK6eLRI

Time Left 84 sec(s)

Question # 8 of 8 (Start time: 01:33:57 PM) **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

Select correct option:

- constant
- finite
- infinity
- zero

Click here to Save Answer & Move to Next Question

==========

Set-07

Question # 1 of 10 (Start time: 10:22:24 AM) Total Marks: 1

Eigenvalues of a symmetric matrix are all _____.
Select correct option:

real
zero
positive
negative

Question # 2 of 10 (Start time: 10:23:07 AM) Total Marks: 1

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.
Select correct option:

TRUE
FALSE

Question # 3 of 10 (Start time: 10:23:55 AM) Total Marks: 1

Exact solution of 2/3 is not exists.
Select correct option:

TRUE
FALSE

Question # 4 of 10 (Start time: 10:24:53 AM) Total Marks: 1

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric _____ definite matrices A.
Select correct option:

positive
negative

Question # 5 of 10 (Start time: 10:26:04 AM) Total Marks: 1

Differences methods find the _____ solution of the system.

Select correct option:

numerical
Analytical

Question # 6 of 10 (Start time: 10:26:49 AM) Total Marks: 1

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.
Select correct option:

(x-1)^3
(x+1)^3 **i m not sure about this answer**
x^3-1
x^3+1

Question # 7 of 10 (Start time: 10:28:08 AM) Total Marks: 1

The Power method can be used only to find the eigen value of A that is largest in absolute value—----------we call this eigen value the dominant eigen value of A.
Select correct option:

TRUE
FALSE

Question # 8 of 10 (Start time: 10:29:33 AM) Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.
Select correct option:

main diagonal
last column
last row
first row **i m not sure about this answer**

Question # 9 of 10 (Start time: 10:30:33 AM) Total Marks: 1

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.
Select correct option:

TRUE
FALSE

Question # 10 of 10 (Start time: 10:31:28 AM) Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.
Select correct option:

TRUE
FALSE

==========

Set-08

Differences methods find the _____ solution of the system.

Select correct option:

- **numerical**
- Analytical

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

- **TRUE**
- FALSE

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

Select correct option:

- **TRUE**
- FALSE

Eigenvalues of a _____ matrix are all real.

Select correct option:

- **symmetric**
- antisymmetric
- rectangular
- triangular

By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.

Select correct option:

- **TRUE**
- FALSE

The dominant eigenvector of a matrix is an eigenvector corresponding to the eigenvalue of largest magnitude (for real numbers, smallest absolute value) of that matrix.

Select correct option:

- **TRUE**
- FALSE

Eigenvalues of a symmetric matrix are all _____ .

Select correct option:

- **real**
- complex
- zero
- positive

Below are all the finite difference methods EXCEPT _____.

Select correct option:

- jacobi's method
- newton's backward difference method
- **Stirlling formula**
- Forward difference method

Quiz#10

The central difference method is finite method.

Select correct option:

True

False

Power method is applicable if the eigen vectors corresponding to eigen values are linearly _____.

Select correct option:

- **independent**
- dependent

Central difference method seems to be giving a better approximation, however it requires more computations.

Select correct option:

- **TRUE**
- FALSE

The Jacobi iteration _____, if A is strictly diagonally dominant.

Select correct option:

- **converges**
- diverges

Power method is applicable if the eigen values are _____.

Select correct option:

- **real and distinct**
- real and equal
- positive and distinct
- negative and distinct

determinant of a diagonal matrix is the product of the diagonal elements.

Select correct option:

- **TRUE**
- FALSE

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

Select correct option:

- **TRUE**
- FALSE

If n x n matrices A and B are similar, then they have the _____ eigenvalues (with the same multiplicities).

Select correct option:

- **same**
- different

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

Select correct option:

- **unity**
- zero

Set-09

Exact solution of 2/3 is not exists.

TRUE

FALSE

The Jacobi's method is

a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

no

atleast one

A 3 x 3 identity matrix have three and _____eigen values.

same

different

Eigenvalues of a symmetric matrix are all _____ .

real

complex

zero

positive

The Jacobi iteration converges, if A is strictly diagonally dominant.

TRUE

FALSE

Below are all the finite difference methods EXCEPT _____.

jacobi's method

newton's backward difference method

Stirlling formula

Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).

TRUE

FALSE

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

TRUE

FALSE

The characteristics polynomial of a 3x 3

identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

(x-1)^3

(x+1)^3

x^3-1

x^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE

FALSE

==========

Set-10

/wEPDwUKMTY2I

/wEWBgLDz9rUC

MC090400760 : Imran Shahzad

Time Left 67 sec(s)

Quiz Start Time: 05:13 PM

Question # 1 of 10 (Start time: 05:13:27 PM) **Total Marks:** 1

The determinant of a _____ matrix is the product of the diagonal elements.

Select correct option:

- diagonal
- upper triangular
- lower triangular
- scalar

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBALlstSEAv

MC090400760 : Imran Shahzad

Time Left 44 sec(s)

Quiz Start Time: 05:13 PM

Question # 2 of 10 (Start time: 05:14:53 PM) **Total Marks:** 1

The absolute value of a determinant (|detA|) is the product of the absolute values of the eigenvalues of matrix A

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAL85ba6D

MC090400760 : Imran Shahzad

Time Left 78 sec(s)

Quiz Start Time: 05:13 PM

Question # 3 of 10 (Start time: 05:16:13 PM) **Total Marks:** 1

Central difference method seems to be giving a better approximation, however it requires more computations.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBALvyM2J(

MC090400760 : Imran Shahzad

Time Left 66 sec(s)

Quiz Start Time: 05:13 PM

Question # 4 of 10 (Start time: 05:17:36 PM) **Total Marks:** 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

- (•) TRUE
- ( ) FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLv7oHdI

MC090400760 : Imran Shahzad

Time Left 67 sec(s)

Quiz Start Time: 05:13 PM

Question # 5 of 10 (Start time: 05:18:12 PM) **Total Marks:** 1

Let A be an n ×n matrix. The number x is an eigenvalue of A if there exists a non-zero vector v such that _____.

Select correct option:

- ( ) Av = xv
- ( ) Ax = xv

- Av + xv=0
- Av = Ax

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAKOofDw

MC090400760 : Imran Shahzad

Time Left 40 sec(s)

Quiz Start Time: 05:13 PM

Question # 6 of 10 (Start time: 05:19:22 PM) **Total Marks:** 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

Select correct option:

- TRUE
- /wEPDwUKMTY2I

/wEWBAKyoPefC

MC090400760 : Imran Shahzad

Time Left 78 sec(s)

Quiz Start Time: 05:13 PM

Question # 8 of 10 (Start time: 05:21:26 PM) **Total Marks:** 1

The Jacobi iteration converges, if A is strictly diagonally dominant.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgK0iY31C

MC090400760 : Imran Shahzad

Time Left 74 sec(s)

Quiz Start Time: 05:13 PM

/wEPDwUKMTY2I

/wEWBAKG4seDI

MC090400760 : Imran Shahzad

Time Left 47 sec(s)

Quiz Start Time: 05:13 PM

Question # 9 of 10 (Start time: 05:22:47 PM) Total Marks: 1

Central Difference method is the finite difference method.

Total Marks: 1

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAKw0/X2E

MC090400760 : Imran Shahzad

Time Left 34 sec(s)

Quiz Start Time: 05:13 PM

Question # 10 of 10 (Start time: 05:23:47 PM) Total Marks: 1

By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

(Start time: 05:20:47 PM)

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

Select correct option:

- (x-1)^3
- (x+1)^3
- x^3-1
- x^3+1

Click here to Save Answer & Move to Next Question

==========

Set-11

Contents:

1. mth603 compiled mcq.doc
2. MTH603_Final.doc
3. NAPaperFinalTerm.doc
4. NAPaperFinalTermmth603.doc
5. Spring 2010 FinalTerm OPKST MTH603 mc080408680.doc

Contents:

1. mid -2006.doc
2. mid-2006a.doc
3. mth603 quiz.doc
4. mth603.doc
5. MTH603 MID FALL2005.doc
6. my MTH603 quiz.doc
7. my.mth603.doc
8. quizz mth603(2).doc
9. quizz mth603.doc

/w EPDw UKMTY:

/w EWBgKPm46n

Question # 1 of 10 (Start time: 04:40:08 PM) Total Marks: 1

The determinant of a diagonal matrix is the product of the diagonal elements.

1. True
2. False

Question # 2 of 10 (Start time: 04:40:58 PM) Total Marks: 1

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

1. True
2. false

A 3 x 3 identity matrix have three and different eigen values.

1. True
2. False

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

1. True

2. False

The Jacobi's method is a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

1. No
2. At least one

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

1. Unity
2. zero

/w EPDw UKMTY2

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

1. True
2. False

/w EWBgKin4OcB

/w EPDw UKMTY2

The determinant of a _____ matrix is the product of the diagonal elements.

1. Diagonal
2. Upper triangular
3. Lower triangular
4. Scalar

Waisay main nay is ka answer Diagnol keea tha….par yeh charon options theek hain….
You can confirm it from internet…

Jab main yeh MCQ kar raha tha tou tab hi mujhay is par shak ho raha tha….kyun k upper aur lower triangular matrices tou linear algebra mein bhi bahut ziada bataye gaye tou yeh property wahan se hi yaad thi…

Eigenvalues of a symmetric matrix are all _____.

1. Real
2. Zero
3. Positive
4. Negative /w EPDw UKMTY2

/w EWBgKR8tbaA

The Power method can be used only to find the eigen value of A that is largest in absolute value—we call this eigen value the dominant eigen value of A.

1. True
2. False

/w EPDw UKMTY2

/w EWBgKjyL19A

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

1. (x-1)^3
2. (x+1)^3
3. x^3-1
4. x^3+1

For differences methods we require the set of values.

1. True
2. False

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

1. True
2. False

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

1. True
2. False

a ko agar aap lambda se replace kar dain tou baat clear ho jaye gi….labda ki jagah a use keea gaya hai tou is liay yeh working thora sa confuse karti hai…

Central difference method seems to be giving a better approximation, however it requires more computations.

1. True
2. False

Iterative algorithms can be more rapid than direct methods.

1. True (main nay true hi keea tha, aap isay dekh lena)
2. False

Central Difference method is the finite difference method.

1. True
2. False

/w EPDw UKMTY2

MTH603 Numerical Analysis

Mid Term Examination - Spring 2006
Time Allowed: 90 Minutes

Question No. 1

Mar

ks : 10

Use bisection method to find the solution for
(Perform only three iterations.)

Question No. 2

$2 + \cos(e^{x} - 2) - e^{x} = 0$

on interval [0.5, 1.5]

Marks : 2

Bisection and false position methods are also known as bracketing method and are always

- Divergent
- Convergent

Question No. 3

Mar

ks : 10

Use Gauss Elimination method to solve the following system.

$4x_1 - x_2 + x_3 = 8$

$$2x_1 + 5x_2 + 2x_3 = 3$$
$$x_1 + 2x_2 + 4x_3 = 11$$

Question No. 4 **Mar**

ks : 2

The Inverse of a matrix can only be found if the matrix is

- Singular
- Non singular
- Scalar
- Diagonal

Question No. 5 **Mar**

ks : 2

If f (x) contains trigonometric, exponential or logarithmic functions then this equation is known as

- Transcendental equation
- Algebraic
- Polynomial
- Linear

Question No. 6 **Mar**

ks : 2

In interpolation is used to represent the δ

- Forward difference
- Central difference
- Backward difference

Question No. 7 **Mar**

ks : 2

The base of the decimal system is _____

- 10
- 0
- 2
- 8
- None of the above.

Question No. 8

Use Newton's Raphson Method to find the solution for (Perform only three iterations.)

Question No. 9

Marks : 10

$x^3 + 3x^2 - 1 = 0$ on [-3,-2].

Marks : 10

Approximate f(0.05) by using any of the interpolation technique.

x	0.0	0.2	0.4	0.6	0.8
F(x)	1.000	1.22140	1.49182	1.82212	2.22554

$$[(f_0 + f_n) + 2\sum_{i=1}^{n-1} f_i]$$

is known as

- Simpson's 1/3 rd Rule
- Simpson's 3/8 rule
- Trapezoidal rule

Question No: 2 (Marks: 2) - Please choose one

Bisection method is …….. method

- Open Method
- Bracketing Method

Question No: 3 (Marks: 2) - Please choose one

Which method is not used to solve problems related to integration?

- Runge-Kutta Method
- Simpson's 1/3rd rule
- Trapezoidal rule.

Question No: 7 (Marks: 10)

Interpolate the value of 0.25 using Newton's forward difference formula.

x	0.2	0.3	0.4	0.5	0.6
F(x)	0.2304	0.2788	0.3222	0.3617	0.3979

(Perform all the necessary calculation missing calculation and steps may deduct marks.)

Exact solution of 2/3 is not exists.
TRUE
FALSE

The Jacobi's method is
a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

no
atleast one

A 3 x 3 identity matrix have three and _____eigen values.
same
different

Eigenvalues of a symmetric matrix are all _____ .
real
complex
zero
positive

The Jacobi iteration converges, if A is strictly diagonally dominant.
TRUE
FALSE

Below are all the finite difference methods EXCEPT _____.

jacobi's method
newton's backward difference method
Stirlling formula
Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).
TRUE
FALSE

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

TRUE
FALSE

The characteristics polynomial of a 3x 3
identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

(x-1)^3
(x+1)^3
x^3-1
x^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE
FALSE

Top

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i have attempt mth 603 today..... my paper was power method 5 marks , from vactor 3 marks, Newton_Repshon's forwrd distnce formula 5 marks.,define extra polation 2 marks ,, and most of MCQ's from , Newton_Repshon's method ,or 18 to 22 lec's

question 29: Distinguish between Related and unrelated diversification with example 5marks
question 30 Five porters Model 5 marks
question 31: conecntric diversification two example 3marks
question 32 significant of R&D for an organization 3marks

Which of the following period strategic management was considered to be cure for all problems?
Mid 1950s to mid 1960s
Mid 1960s to mid 1970s
Mid 1970s to mid 1980s
Mid 1980s to mid 1990s

Which of the following is not a pitfall an organization should avoid in strategic planning?
Select correct option:
Failing to involve key employees in all phases of planning
Involving all managers rather than delegating planning to a planner
Top managers not actively supporting the strategic planning process
Doing strategic planning only to satisfy accreditation or regulatory requirements

which of the following are the factors that concern the nature and direction of the economy in which a firm operates?
Select correct option:
Technological
Ecological
Social
Economic

Which of the following best describes this statement; “a Systematic and ethical process for gathering and analyzing information about the competition’s activities and general business trends to further a business’ own goals”?
Select correct option:
External assessment

Industry analysis
Competitive intelligence program
Business ethics

According to Porter, which strategy offers products or services to a small range of customers at the lowest price available on the market?
Select correct option:
Low cost
Best value
Cost focus
Differentiation

Long-term objectives includes all of the following EXCEPT:

Measurable

Reasonable

Varying

Consistent

Which one of the following is NOT is a basic mission of a competitive intelligence program?

To provide a general understanding of an industry

To provide a general understanding of a company's competitors

To identify industry executives who could be hired by the firm

To identify potential moves a competitor might make that would endanger a firm

While preparing an External Factor Evaluation Matrix, a total score of 0.8 indicates that:

Firm is taking advantages of strengths and avoiding threats

Firm is taking no advantage of opportunities and is avoiding threats

Firm is not taking advantages of opportunities and is not avoiding threats

Firm is taking advantage of opportunities and is avoiding the threats

Top

Use the Regula-Falsi method to compute a real root of the equation

$x^3 - 9x + 1 = 0$, **(Perform two iterations for each part)**

(i) if the root lies between 2 and 4
(ii) if the root lies between 2 and 3.

Solve the following system of equations by Crout's method

$$x_1 + 2x_2 + x_3 = 2$$
$$2x_1 + x_2 - 10x_3 = 0.182$$
$$2x_1 + 3x_2 - x_3 = 2$$

Find the numerical solution of

$$55x - 25z = -200$$
$$-37y - 4z = -250$$
$$-25x - 4y + 29z = 100$$

Using Gauss-Siedel Mehtod with $(1,1,1)$ as the initial value. Perform two iterations only.

Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$ by the bisection method.
(Perform four iterations only)

Top

Question # 1 of 8 (Start time: 08:34:31 PM) **Total Marks:** 1

Adams – Bashforth is a multistep method.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 2 of 8 (Start time: 08:35:01 PM)

Generally, Adams methods are superior if output at _____ points is needed.

Select correct option:

- many

- two
- single
- at most

Click here to Save Answer & Move to Next Question

Question # 3 of 8 (Start time: 08:36:02 PM) **Total Marks:** 1

In Trapezoidal rule, the integral is computed on each of the sub-intervals by using linear interpolating formula, i.e. for n = 1 and then summing them up to obtain the desired integral.

Total Marks: 1

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

Question # 4 of 8 (Start time: 08:36:52 PM)

Euler's Method numerically computes the approximate _____ of a function.

Select correct option:

- antiderivative
- derivative
- value
- error

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 5 of 8 (Start time: 08:37:25 PM)

Multistep method does not improves the accuracy of the answer at each step.

Select correct option:

- FALSE
- TRUE

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 6 of 8 (Start time: 08:37:56 PM)

The trapezoidal rule is a numerical method that approximates the value of a _____.

Select correct option:

- indefinite integral
- definite integral
- improper integral
- function

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 7 of 8 (Start time: 08:38:47 PM)

Simpson's rule is a numerical method that approximates the value of a definite integral by using _____ polynomials.

Select correct option:

- quadratic
- linear
- cubic
- quartic

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 8 of 8 (Start time: 08:39:23 PM)

In Simpson's Rule, we use parabolas to approximate each part of the curve. This proves to be very efficient as compared to Trapezoidal rule.

Select correct option:

- TRUE
- FALSE

Question # 3 of 8 (Start time: 09:46:11 PM) Total Marks: 1

The first langrange polynomial with equally spaced nodes produced the formula for _____.

Select correct option:

- Simpson's rule
- Trapezoidal rule
- Newton's method
- Richardson's method

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2 /wEWBgLgsdHjB

MC090406505 : Sumera Naz

Time Left 12 sec(s)

Quiz Start Time: 09:44 PM

Question # 4 of 8 (Start time: 09:46:48 PM) Total Marks: 1

The need of numerical integration arises for evaluating the indefinite integral of a function that has no explicit antiderivative or whose antiderivative is not easy to obtain.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2 /w EWCgKl9pWZE

MC090406505 : Sumera Naz

Time Left 73 sec(s)

Quiz Start Time: 09:44 PM

Question # 5 of 8 (Start time: 09:48:13 PM) Total Marks: 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas
- constant

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2 /w EWCgLo7tKAA

MC090406505 : Sumera Naz Time Left 69 sec(s)

Quiz Start Time: 09:44 PM

Question # 6 of 8 (Start time: 09:48:35 PM) Total Marks: 1

The Euler method is numerically unstable because of _____ convergence of error.

Select correct option:

- slow
- fast
- moderate
- no

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2 /w EWBgLyjI72BA

MC090406505 : Sumera Naz Time Left 74 sec(s)

Quiz Start Time: 09:44 PM

Question # 8 of 8 (Start time: 09:49:41 PM) Total Marks: 1

Adams – Bashforth is a multistep method.

Select correct option:

- TRUE
- FALSE

Click here to Save Answ er & Move to Next Question

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/wEPDwUKMTY2N /wEWBgK+3MqIA

Question # 1 of 8 (Start time: 01:24:49 PM) **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

- antiderivative
- derivatives

/wEPDwUKMTY2N /wEWBgLy4ZCpA

Question # 2 of 8 (Start time: 01:26:18 PM) **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

- TRUE
- FALSE

/wEPDwUKMTY2N /wEWBgLlwO3cC

Question # 3 of 8 (Start time: 01:27:35 PM) **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

- diverge
- converge

/wEPDwUKMTY2N /wEWCgKMu86bD

Question # 5 of 8 (Start time: 01:30:21 PM) **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas
- constant

/wEPDwUKMTY2N /wEWBgK6oIaxB

Question # 6 of 8 (Start time: 01:31:18 PM) **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or ∞ or -∞ or, in some cases, as both endpoints approach limits.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N

/wEWBgKPg8yyD

Question # 7 of 8 (Start time: 01:32:33 PM) **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

Select correct option:

- FALSE
- TRUE

/wEPDwUKMTY2N /wEWCgKK6eLRD

Question # 8 of 8 (Start time: 01:33:57 PM) **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

Select correct option:

- constant
- finite
- infinity
- zero

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/wEPDwUKMTY2N /wEWBgK+3MqIA

Question # 1 of 8 (Start time: 01:24:49 PM) **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

- antiderivative
- derivatives

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWBgLy4ZCpA

Time Left 59 sec(s)

Question # 2 of 8 (Start time: 01:26:18 PM) **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWBgLlwO3cC

Time Left 79 sec(s)

Question # 3 of 8 (Start time: 01:27:35 PM) **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

- diverge
- converge

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWCgKMu86bD

Time Left 85 sec(s)

Question # 5 of 8 (Start time: 01:30:21 PM) **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas
- constant

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWBgK6oIaxB

Time Left 83 sec(s)

Question # 6 of 8 (Start time: 01:31:18 PM) **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWBgKPg8yyD

Time Left 84 sec(s)

Question # 7 of 8 (Start time: 01:32:33 PM) **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

Select correct option:

FALSE

TRUE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWCgKK6eLRD

Time Left 84 sec(s)

Question # 8 of 8 (Start time: 01:33:57 PM) **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

Select correct option:

constant

finite

- infinity
- zero

Click here to Save Answer & Move to Next Question

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MTH603-Numerical Analysis
SEMESTER FALL 2005
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Approximate the integral $\int_{1}^{1.6} \frac{2x}{x^2+1} dx$ using Simpson's 1/3 rule and calculate the percentage error. (Take result up to 4 decimal places)

Note: In order to get full marks do all necessary steps.

Construct a forward difference table for the following values

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

Note : In order to get full marks do all necessary steps.

Solve the system

$4x+3y=24$

$3x+4y-z=30$

$-y+4z=-24$

by Gauss Seidal Method, taking $(0, 0, 0)^t$ as initial approximation(Two iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Let $f(x) = x + \frac{2}{x}$, use cubic Lagrange interpolation based on the nodes $x_0 = 0.5, x_1 = 1, x_2 = 2$ *and* $x_3 = 2.5$ to approximate *f(1.5)* and *f(1.3)*.

Note : In order to get full marks do all necessary stepsSolution

Approximate the Dominant Eigenvalue and corresponding Eigenvector for the matrix

$$\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

by using Power Method. Start with $X_0 = (1,1,1)^t$. (Five iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

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NUMERICAL ANALYSIS

Paper Final Term (Held: 25th Feb 2010)

No.	Questions	Mks
1	Find value of given data by Adam Moultan's method	10
2	Find value of given data by Dividend Difference Composite method	10
3	Draw backward difference tables for given Data x | | | | y | | | | x | | | | y | | | |	5+5
4	Write Simpson's 1/3 formula	2
5	Find value by Euler's Method	3
6	Find value of K1 by 2nd Order R-K method	2
7	Convergence is used when ----------	1
8	Bisection method is …….. method • Bracketing Method • Open • Random	1

	• none http://vustudents.ning.com/	
9	Newton Raphson method is …….. method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real • Vector • odd • even	1
11	Find value of y'(1) by Euler's Method taking h=1	2
12	Find value of y'(3) from given table. x | | | | y | | | |	2
13	Find value of y'(0.3) by Lagrange's Method x | | | | y | | | |	3
14	For Simpson's 1/3 rule no.of intervals must be • 1 • 3 • 5 • 8	1
15	For Simpson's 1/3 rule valid no.of intervals are • 1 • 3 • 5 • 8	1
16	For Simpson's 3/8 rule no.of intervals must be • 10 • 11 • 12 • 14 http://vustudents.ning.com/	1
17	Find the value of y'(1) from given forward difference table x | y | Δy | Δy^2 | Δy^3 | | | | | | | | | | | | | | | | | | | |	2

Numerical analysis mth603 paper
Numerical Analysis numerical paper 2009

The paper was very easy.
The mcq's were really easy.
Most of the mcq's were from the last 5 lecture.
and from jacobi's method and other's.
then the logical mcq's like
s inverse * s = I
it was repeated twice.
one 5 mark question was from newton's rapson method
one 10 mark question was from lecture 11 page 69 example
best of luck

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9	Newton Raphson method is …….. method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real	1

<table>
<tr><td></td><td>• Vector
• odd
• even</td><td></td></tr>
<tr><td>11</td><td>Find value of y'(1) by Euler's Method taking h=1</td><td>2</td></tr>
<tr><td>12</td><td>Find value of y'(3) from given table.
<table><tr><td>x</td><td></td><td></td><td></td></tr><tr><td>y</td><td></td><td></td><td></td></tr></table></td><td>2</td></tr>
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<table><tr><td>x</td><td></td><td></td><td></td></tr><tr><td>y</td><td></td><td></td><td></td></tr></table></td><td>3</td></tr>
<tr><td>14</td><td>For Simpson's 1/3 rule no.of intervals must be
• 1
• 3
• 5
• 8</td><td>1</td></tr>
<tr><td>15</td><td>For Simpson's 1/3 rule valid no.of intervals are
• 1
• 3
• 5
• 8</td><td>1</td></tr>
<tr><td>16</td><td>For Simpson's 3/8 rule no.of intervals must be
• 10
• 11
• 12
• 14
http://vustudents.ning.com/</td><td>1</td></tr>
<tr><td>17</td><td>Find the value of y'(1) from given forward difference table
<table><tr><td>x</td><td>y</td><td>Δy</td><td>Δy^2</td><td>Δy^3</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></table></td><td>2</td></tr>
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best of luck

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FINALTERM EXAMINATION
Spring 2010
MTH603- Numerical Analysis (Session - 2)

Ref No: 1508683
Time: 90 min
Marks: 60

Question No: 1 (Marks: 1) - Please choose one

Symbol used for forward differences is

- ∇
- Δ
- δ
- μ

Question No: 2 (Marks: 1) - Please choose one

The relationship between central difference operator and the shift operator is given by

- $\delta = \mathrm{E} - \mathrm{E}^{-1}$
- $\delta = \mathrm{E} + \mathrm{E}^{-1}$
- $\delta = \mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}}$
- $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}}$

Question No: 3 (Marks: 1) - Please choose one

Muller's method requires --------starting points

- 1
- 2
- 3
- 4

Question No: 4 (Marks: 1) - Please choose one

If S is an identity matrix, then

- $S^{-1} = S$
- $S^t = S$
- $All\ are\ true$
- $S^{-1} = S^t$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with

y_x at $x_{0,}x_1,...,x_r$

- ► r+2
- ► r+1
- ► r
- ► r-1

Question No: 6 (Marks: 1) - Please choose one

_____ P in Newton's forward difference formula is defined as

- ► $p = (\frac{x - x_0}{h})$
- ► $p = (\frac{x + x_0}{h})$
- ► $p = (\frac{x + x_n}{h})$
- ► $p = (\frac{x - x_n}{h})$

Question No: 7 (Marks: 1) - Please choose one

_____ Octal number system has the base ----------

- ► 2
- ► 8
- ► 10
- ► 16

Question No: 8 (Marks: 1) - Please choose one

_____ Newton's divided difference interpolation formula is used when the values of the independent variable are

- ► Equally spaced
- ► Not equally spaced
- ► Constant
- ► None of the above

Question No: 9 (Marks: 1) - Please choose one

_____ Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

- 1.5
- 3
- 2
- 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

- $\varepsilon(x) = y(x) + P_n(x)$
- $\varepsilon(x) = y(x) - P_n(x)$
- $\varepsilon(x) = P_n(x) - y(x)$
- $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11 (Marks: 1) - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I .

- n-1
- n+2
- n
- n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

- $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + ...)$
- $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + ...)$
- $D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - ...)$
- $D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} - ...)$

Question No: 13 (Marks: 1) - Please choose one

Finding the

first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

- Forward difference operator
- Backward difference operator
- Central difference operator
- None of the given choices

Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

- -0.5
- 0.5
- 0.75
- -0.75

Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

- $ax+b$
- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question No: 16 (Marks: 1) - Please choose one

While integrating $I=\int_a^b f(x)dx$, h, width of the interval, is found by the formula-----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

Question No: 17 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

- 7
- 8
- 5
- 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f^{//}(0.2)$, using three-point equation will be calculated as ……

- 17.5
- 12.5
- 7.5
- -12.5

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

- 2
- 3
- 5
- 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

- 10
- 11
- 12
- 13

Question No: 21 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- $\frac{(a+b)}{2}$
- $\frac{(a-b)}{2}$
- $\frac{(b-a)}{2}$
- None of the given choices

Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- Bisection Method
- Regula Falsi Method
- Secant Method
- None of the given choices

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

- (1, 3)
- (1, 2)
- (0, 1)
- (1, 2)

Question No: 24 (Marks: 1) - Please choose one

Rate of change of any quantity with respect to another can be modeled by

- An ordinary differential equation
- A partial differential equation
- A polynomial equation
- None of the given choices

Question No: 25 (Marks: 1) - Please choose one

If $\frac{dy}{dx} = f(x, y)$

Then the integral of this equation is a curve in

- None of the given choices
- xt-plane
- yt-plane
- xy-plane

Question No: 26 (Marks: 1) - Please choose one

In solving the differential equation

$$y' = x + y \;;\; y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

- 1.44
- 1.11
- 1.22
- 1.33

Question No: 27 (Marks: 1) - Please choose one

In second order Runge-Kutta method

k_1 is given by

- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

Question No: 28 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

- $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
- $k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

Question No: 29 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

- $k_3 = hf(x_n + 2h, y_n + 2k_3)$
- $k_3 = hf(x_n - h, y_n - k_3)$
- $k_3 = hf(x_n + h, y_n + k_3)$
- None of the given choices

Question No: 30 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

- Newton's backward difference interpolation formula
- Newton's forward difference interpolation formula
- Newton's divided difference interpolation formula
- None of the given choices

Question No: 31 (Marks: 2)

If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32 (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2)dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

Evaluate the integral

$$\int_3^5 (\log x + 2) dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36 (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37 (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38 (Marks: 5)

Evaluate the integral

$$\int_0^3 (x^2 + x) dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39 (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$ taking $h = 0.1$

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Numerical Analysis

MTH603



Virtual University of Pakistan

Knowledge beyond the boundaries

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Numerical Analysis

Course Contents

Solution of Non Linear Equations
Solution of Linear System of Equations
Approximation of Eigen Values
Interpolation and Polynomial Approximation
Numerical Differentiation
Numerical Integration
Numerical Solution of Ordinary Differential Equations

Introduction

We begin this chapter with some of the basic concept of representation of numbers on computers and errors introduced during computation. Problem solving using computers and the steps involved are also discussed in brief.

Number (s) System (s)

In our daily life, we use numbers based on the decimal system. In this system, we use ten symbols 0, 1,…,9 and the number 10 is called the base of the system.
Thus, when a base N is given, we need N different symbols 0, 1, 2, …,$(N-1)$ to represent an arbitrary number.
The number systems commonly used in computers are

Base, N	Number
2	Binary
8	Octal
10	Decimal
16	Hexadecimal

An arbitrary real number, a can be written as

$$a = a_m N^m + a_{m-1} N^{m-1} + \cdots + a_1 N^1 + a_0 + a_{-1} N^{-1} + \cdots + a_{-m} N^{-m}$$

In binary system, it has the form,

$$a = a_m 2^m + a_{m-1} 2^{m-1} + \cdots + a_1 2^1 + a_0 + a_{-1} 2^{-1} + \cdots + a_{-m} 2^{-m}$$

The decimal number 1729 is represented and calculated

$$(1729)_{10} = 1\times 10^3 + 7\times 10^2 + 2\times 10^1 + 9\times 10^0$$

While the decimal equivalent of binary number 10011001 is

$$1\times 2^0 + 0\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4} + 0\times 2^{-5} + 0\times 2^{-6} + 1\times 2^{-7}$$

$$= 1 + \frac{1}{8} + \frac{1}{16} + \frac{1}{128}$$

$$= (1.1953125)_{10}$$

Electronic computers use binary system whose base is 2. The two symbols used in this system are 0 and 1, which are called *binary digits* or simply *bits*.
The internal representation of any data within a computer is in binary form. However, we prefer data input and output of numerical results in decimal system. Within the computer, the arithmetic is carried out in binary form.
Conversion of decimal number 47 into its binary equivalent
Sol.

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2	47	Remainder
2	23	1
2	11	1
2	5	1
2	2	1
2	1	0
	0	1 Most significant bit

$(47)_{10} = (101111)_2$

Binary equivalent of the decimal fraction 0.7625

Sol.

		Product	Integer
0.7625	x2	1.5250	1
0.5250	x2	1.0500	1
0.05	x2	0.1	0
0.1	x2	0.2	0
0.2	x2	0.4	0
0.4	x2	0.8	0
0.8	x2	1.6	1
0.6	x2	1.2	1
0.2	x2	0.4	0

$(0.7625)_{10} = (0.11....11(0011))_2$

Conversion $(59)_{10}$ into binary and then into octal.

Sol.

2	29	1
2	14	1
2	7	0
2	3	1
2	1	1
	0	1

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$$(59)_{10} = (11011)_2$$

$$(111011)_2 = 111011 = (73)_8$$

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Errors in Computations

Numerically, computed solutions are subject to certain errors. It may be fruitful to identify the error sources and their growth while classifying the errors in numerical computation. These are

Inherent errors,
Local round-off errors
Local truncation errors

Inherent errors

It is that quantity of error which is present in the statement of the problem itself, before finding its solution. It arises due to the simplified assumptions made in the mathematical modeling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the problem.

Local round-off errors

Every computer has a finite word length and therefore it is possible to store only a fixed number of digits of a given input number. Since computers store information in binary form, storing an exact decimal number in its binary form into the computer memory gives an error. This error is computer dependent.

At the end of computation of a particular problem, the final results in the computer, which is obviously in binary form, should be converted into decimal form-a form understandable to the user-before their print out. Therefore, an additional error is committed at this stage too.

This error is called *local round-off* error.

$$(0.7625)_{10} = (0.110000110011)_2$$

If a particular computer system has a word length of 12 bits only, then the decimal number 0.7625 is stored in the computer memory in binary form as 0.110000110011. However, it is equivalent to 0.76245.

Thus, in storing the number 0.7625, we have committed an error equal to 0.00005, which is the round-off error; inherent with the computer system considered.

Thus, we define the *error* as

$$\text{Error} = \text{True value} - \text{Computed value}$$

Absolute error, denoted by |Error|,

While, the *relative error* is defined as

$$\text{Relative error} = \frac{|\text{Error}|}{|\text{True value}|}$$

Local truncation error

It is generally easier to expand a function into a power series using Taylor series expansion and evaluate it by retaining the first few terms. For example, we may approximate the function $f(x) = \cos x$ by the series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$$

If we use only the first three terms to compute $\cos x$ for a given x, we get an approximate answer. Here, the error is due to truncating the series. Suppose, we retain the first n terms, the truncation error (TE) is given by

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$$\text{TE} \le \frac{x^{2n+2}}{(2n+2)!}$$

The TE is independent of the computer used.
If we wish to compute *cos x* for accurate with five significant digits, the question is, how many terms in the expansion are to be included? In this situation

$$\frac{x^{2n+2}}{(2n+2)!} < .5\times 10^{-5} = 5\times 10^{-6}$$

Taking logarithm on both sides, we get

$$(2n+2)\log x - \log[(2n+2)!]$$
$$< \log_{10} 5 - 6\log_{10} 10 = 0.699 - 6 = -5.3$$

or

$$\log[(2n+2)!] - (2n+2)\log x > 5.3$$

We can observe that, the above inequality is satisfied for $n = 7$. Hence, seven terms in the expansion are required to get the value of *cos x*, with the prescribed accuracy
The truncation error is given by

$$\text{TE} \le \frac{x^{16}}{16!}$$

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Polynomial

An expression of the form $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n$ where n is a positive integer and $a_0, a_1, a_2 + ...a_n$ are real constants, such type of expression is called an nth degree polynomial in x if $a_0 \neq 0$

Algebraic equation:

An equation f(x)=0 is said to be the algebraic equation in x if it is purely a polynomial in x.

For example

$x^5 + x^4 + 3x^2 + x - 6 = 0$ It is a fifth order polynomial and so this equation is an algebraic equation.

$x^3 - 6 = 0$

$x^6 - 7x = 0$

$y^4 - 4y^3 + 3y^2 - y - 2 = 0$ *polynomial in y*

$t^4 - 6t^2 - 21 = 0$ *polynomail in t*

These all are the examples of the polynomial or algebraic equations.

Some facts

1. Every equation of the form f(x)=0 has at least one root ,it may be real or complex.
2. Every polynomial of nth degree has n and only n roots.
3. If f(x) =0 is an equation of odd degree, then it has at least one real root whose sign is opposite to that of last term.
4. If f(x)=0 is an equation of even degree whose last term is negative then it has at least one positive and at least one negative root .

Transcendental equation

An equation is said to be transcendental equation if it has logarithmic, trigonometric and exponential function or combination of all these three.

For example

$e^x - 5x - 3 = 0$ it is a transcendental equation as it has an exponential function

$e^x - \sin x = 0$

$\ln x - \sin x = 0$

$2\sec^2 x - \tan x - e^x = 0$

These all are the examples of transcendental equation.

Root of an equation

For an equation f(x) =0 to find the solution we find such value which satisfy the equation f(x)=0,these values are known as the roots of the equation .

A value a is known as the root of an equation f(x) =0 if and only if f (a) =0.

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Properties of an Algebraic equation

1. Complex roots occur in the pairs. That is ,If (a+ib) is a root of f(x)=0 then (a-ib) is also a root of the equation
2. if x=a is a root of the equation f(x)=0 a polynomial of nth degree ,then (x-a) is a factor of f(x) and by dividing f(x) by (x-a) we get a polynomial of degree n-1.

Descartes rule of signs

This rule shows the relation ship between the signs of coefficients of an equation and its roots.

"The number of positive roots of an algebraic equation f(x) =0 with real coefficients can not exceed the number of changes in the signs of the coefficients in the polynomial f(x) =0.similarly the number of negative roots of the equation can not exceed the number of changes in the sign of coefficients of f (-x) =0"

Consider the equation $x^3 - 3x^2 + 4x - 5 = 0$ here it is an equation of degree three and there are three changes in the signs
First +ve to –ve second –ve to +ve and third +ve to –ve so the tree roots will be positive
Now $f(-x) = -x^3 - 3x^2 - 4x - 5$ so there is no change of sign so there will be no negative root of this equation.

Intermediate value property

If f(x) is a real valued continuous function in the closed interval $a \le x \le b$ if f(a) and f(b) have opposite signs once; that is f(x)=0 has at least one root β such that $a \le \beta \le b$
Simply
If f(x)=0 is a polynomial equation and if f(a) and f(b) are of different signs ,then f(x)=0 must have at least one real root between a and b.

Numerical methods for solving either algebraic or transcendental equation are classified into two groups

Direct methods

Those methods which do not require any information about the initial approximation of root to start the solution are known as direct methods.
The examples of direct methods are Graefee root squaring method, Gauss elimination method and Gauss Jordan method. All these methods do not require any type of initial approximation.

Iterative methods

These methods require an initial approximation to start.

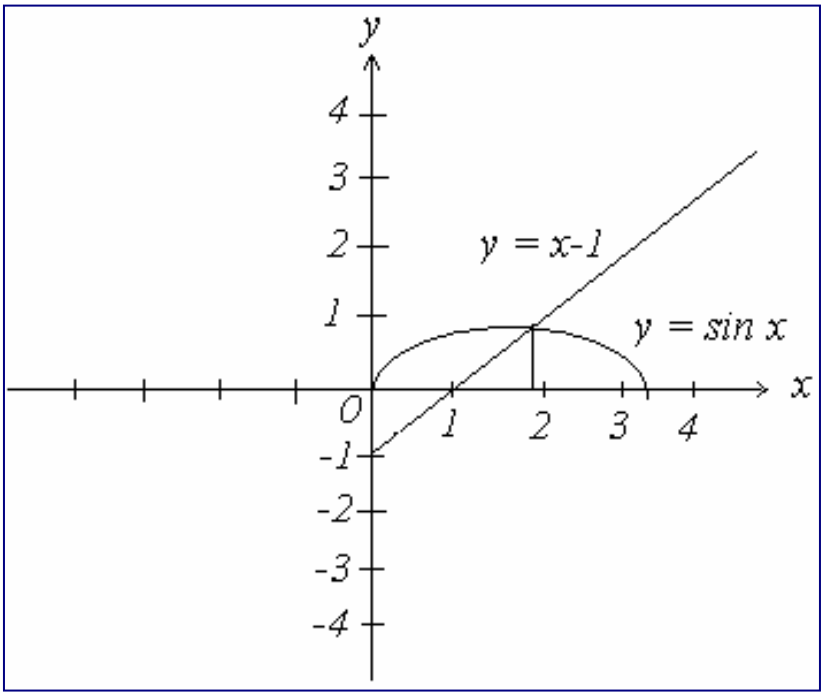
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Bisection method, Newton raphson method, secant method, jacobie method are all examples of iterative methods.

How to get an initial approximation?

The initial approximation may be found by two methods either by graphical method or analytical method
Graphical method

The equation f(x)=0 can be rewritten as $f_1(x) = f_2(x)$ and initial approximation of f(x) may be taken as the abscissa of the point of intersection of graphs of
$y = f_1(x)$ *and* $y = f_2(x)$
for example $f(x) = x - \sin x - 1 = 0$
so this may be written as $x - 1 = \sin x$ Now we shall draw the graphs of
$y = x - 1$ *and* $y = \sin x$



Here both the graphs cut each other at 1.9 so the initial approximation should be taken as 1.9

Analytical method

This method is based on the intermediate value property in this we locate two values a and b such that f(a) and f(b) have opposite signs then we use the fact that the root lies

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between both these points ,this method can be used for both transcendental and algebraic equations.
Consider the equation

$f(0) = -1$
$f(x) = 3x - \sqrt{1+\sin x} = 0$
$f(1) = 3 - \sqrt{1+\sin(1\times\frac{180}{\pi})} = 3 - \sqrt{1+0.84147} = 1.64299$

Here f(0) and f(1) are of opposite signs making use of intermediate value property we infer that one root lies between 0 and 1 .
So in analytical method we must always start with an initial interval (a,b) so that f(a) and f(b) have opposite signs.

Bisection method (Bolzano)

Suppose you have to locate the root of the equation f(x)=0 in an interval say (x_0, x_1) ,let $f(x_0)$ and $f(x_1)$ are of opposite signs such that $f(x_0)f(x_1) < 0$

Then the graph of the function crossed the x-axis between x_0 *and* x_1 which exists the existence of at least one root in the interval (x_0, x_1).

The desired root is approximately defined by the mid point $x_2 = \frac{x_0 + x_1}{2}$ if $f(x_2) = 0$ then x_2 is the root of the equation otherwise the root lies either between $x_0 and\ x_2$ or $x_1 and\ x_2$

Now we define the next approximation by $x_3 = \frac{x_0 + x_2}{2}$ provided $f(x_0)f(x_2) < 0$ then root may be found between $x_0 and\ x_2$

If provided $f(x_1)f(x_2) < 0$ then root may be found between $x_1 and\ x_2$ by $x_3 = \frac{x_1 + x_2}{2}$

Thus at each step we find the desired root to the required accuracy or narrow the range to half the previous interval.
This process of halving the intervals is continued in order to get smaller and smaller interval within which the desired root lies. Continuation of this process eventually gives us the required root.

NOTE: In numerical analysis all the calculation are carried out in radians mode and the value of pi is supposed to be 3.14

Example
Solve $x^3 - 9x + 1 = 0$ for the root between x=2 and x=4 by bisection method

Solution:
Here we are given the interval (2,4) so we need not to carry out intermediate value property to locate initial approximation.
Here

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$f(x) = x^3 - 9x + 1 = 0$

now $f(2) = 2^3 - 9(2) + 1 = 8 - 18 + 1 = 9$

$f(4) = 4^3 - 9(4) + 1 = 64 - 36 + 1 = 29$

here $f(2)\ f(3) < 0$ *so root lies between* 2 *and* 4

$x_0 = 2 \quad x_1 = 4$

$$x_2 = \frac{2+4}{2} = 3$$

$f(3) = 3^3 - 9(3) + 1 = 27 - 27 + 1 = 1$

here $f(2) f(3) < 0$ *so the root lies between* 2 *nad* 3

$$x_3 = \frac{2+3}{2} = 2.5$$

$f(2.5) = 2.5^3 - 9(2.5) + 1 = 15.625 - 22.5 + 1 = -5.875 < 0$

so the root lies between 2.5 *and* 3 *as* $f(2.5) f(3) < 0$

$$x_4 = \frac{2.5+3}{2} = 2.75$$

now

similarly $x_5 = 2.875$ *and* $x_6 = 2.9375$ *and the process is continued untill the desired accuracy is obtained*.

n	*xn*	*f (xn)*
2	3	1.0
3	2.5	-5.875
4	2.75	-2.9531
5	2.875	-1.1113
6	2.9375	-0.0901

When to stop the process of iteration?

Here in the solved example neither accuracy is mentioned and nor number of iteration is mentioned by when you are provided with the number of iteration then you will carry those number of iteration and will stop but second case is accuracy like you are asked to

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find root with an accuracy of 10^{-3} then you will check the accuracy by subtracting two consecutive root like 2.135648 and 2.135769

2.135769-2.135648=0.000121

So the accuracy of 10^{-3} is achieved so you will stop here.

Example:

Carry out the five iterations for the function $f(x) = 2x\cos(2x) - (x+1)^2$

Note: All the calculations should be done in the radians.

Solution:

$f(x) = 2x\cos(2x) - (x+1)^2$

$f(-1) = 2(-1)\cos(-2) - (-1+1)^2 = -2(-0.4161) = +0.8322 > 0$

$f(0) = 2(0)\cos(0) - (0+1)^2 = -1 = -1 < 0$

so the root lies between 0 *and* -1 *as* $f(0)f(-1) < 0$

$$x_2 = \frac{0-1}{2} = -0.5$$

$f(-0.5) = 2(-0.5)\cos(-1) - (-0.5+1)^2 = -0.5403 - 0.25 = -0.7903 < 0$

so root lies between -1 *and* -0.5 *as* $f(-1)f(-0.5)$

$$x_3 = \frac{-0.5-1}{2} = -0.75$$

$f(-0.75) = 2(-0.75)\cos(-1.5) - (-0.75+1)^2 = -0.106 - 0.0625 = -0.1686 < 0$

so root lies between -1 *and* -0.75 *as* $f(-1)f(-0.75)$

$$x_4 = \frac{-0.75-1}{2} = -0.875$$

$f(-0.875) = 2(-0.875)\cos(-1.75) - (-0.875+1)^2 = 0.3119 - 0.015625 = 0.296275 > 0$

so root lies between -0.875 *and* -0.75 *as* $f(-0.75)f(-0.875)$

$$x_5 = \frac{-0.75-0.875}{2} = -0.8125$$

$f(-0.8125) = 2(-0.8125)\cos(-1.625) - (-0.8125+1)^2 = 0.0880 - 0.0351 = 0.052970 > 0$

so root lies between -0.8125 *and* -0.75 *as* $f(-0.75)f(-0.8125)$

$$x_5 = \frac{-0.75-0.8125}{2} = -0.78125$$

Example :

Carry out the first five iterations $f(x) = x\cos x - 2x^2 + 3x - 1$, $1.2 \le x \le 1.3$

Note: All the calculations should be done in the radians.

Solution:

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$$f(x) = x\cos x - 2x^2 + 3x - 1$$

$$f(1.2) = 1.2\cos 1.2 - 2(1.2)^2 + 3(1.2) - 1$$
$$= 1.2(0.3623) - 2(1.44) + 3.6 - 1 = 0.1548 > 0$$

$$f(1.3) = 1.3\cos 1.3 - 2(1.3)^2 + 3(1.3) - 1$$
$$= 1.3(0.2674) - 2(1.69) + 9.3 - 1 = -0.1322 < 0$$

as $f(1.2)f(1.3) < 0$ *so the root lies between both*

$$x_2 = \frac{1.2 + 1.3}{2} = 1.25$$

$$f(1.25) = 1.25\cos 1.25 - 2(1.25)^2 + 3(1.25) - 1$$
$$= 1.25(0.3153) - 2(1.5625) + 3.75 - 1 = 0.0191 > 0$$

as $f(1.25)f(1.3) < 0$ *so the root lies between both*

$$x_3 = \frac{1.25 + 1.3}{2} = 1.275$$

$$f(1.275) = 1.275\cos 1.275 - 2(1.275)^2 + 3(1.275) - 1$$
$$= 1.275(0.2915) - 2(1.6256) + 3.825 - 1 = -0.0545 < 0$$

as $f(1.25)f(1.275) < 0$ *so the root lies between both*

$$x_4 = \frac{1.25 + 1.275}{2} = 1.2625$$

$$f(1.2625) = 1.2625\cos 1.2625 - 2(1.2625)^2 + 3(1.2625) - 1$$
$$= 1.275(0.3034) - 2(1.5939) + 3.7875 - 1 = -0.0172 < 0$$

as $f(1.25)f(1.265) < 0$ *so the root lies between both*

$$x_5 = \frac{1.25 + 1.2625}{2} = 1.25625$$

$$f(1.25625) = 1.25625\cos 1.25625 - 2(1.25625)^2 + 3(1.25625) - 1$$
$$= 1.25625(0.3093) - 2(1.5781) + 3(1.25625) - 1 = 0.00108 > 0$$

as $f(1.25625)f(1.265) < 0$ *so the root lies between both*

$$x_6 = \frac{1.25625 + 1.2625}{2} = 1.259375$$

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Regula-Falsi method (Method of false position)

Here we choose two points x_n and x_{n-1} such that $f(x_n)$ *and* $f(x_{n-1})$ have opposite signs. Intermediate value property suggests that the graph of the y=f(x) crosses the x-axis between these two points and therefore, a root lies between these two points.
Thus to find the real root of f(x)=0 using Regula-Falsi method ,we replace the part of the curve between the points $A(x_n, f(x_n))$ *and* $B(x_{n-1}, f(x_{n-1}))$ by a chord in the interval and we take the point of intersection of this chord with x-axis as initial approximation.
Now, the equation of the chord joining the points A and B is,

$$\frac{y - f(x_n)}{f(x_{n-1}) - f(x_n)} = \frac{x - x_n}{x_{n-1} - x_n}$$

Setting y=0 in the above equation we get

$$x = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

Hence the first approximation to the root is given by

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

We observe that $f(x_{n-1})$ *and* $f(x_{n+1})$ are of opposite signs thus it is possible to apply the above procedure, to determine the line through B and A_1 and so on.
Hence for successive approximation to the root above formula is used.

Example

Use the Regula-Falsi method to compute a real root of the equation $x3 – 9x + 1 = 0$,
(i) if the root lies between 2 and 4
(ii) if the root lies between 2 and 3.
Comment on the results.

Solution

Let
f (x) = x3 - 9x + 1

f (2) = 2^3 - 9(2) + 1=8 – 18+1= – 9and f (4) = 4^3 - 9(4) + 1=64 – 36+1=29.

Since $f(2)$ and $f(4)$ are of opposite signs, the root of $f(x) = 0$ lies between 2 and 4.
Taking $x1 = 2$, $x2 = 4$ and using Regula-Falsi method, the first approximation is given by

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = 4 - \frac{4-2}{29-(-9)}(29) = 4 - \frac{2(29)}{38}$$

$$= 4 - \frac{58}{38} = 4 - 1.5263 = 2.4736$$

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Now

f (x3) $= 2.4736^3 - 9(2.4736) + 1 = 15.13520 - 22.2624 + 1 = -6.12644$.

Since $f(x2)$ and $f(x3)$ are of opposite signs, the root lies between $x2$ and $x3$.

The second approximation to the root is given as

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3) = 2.4736 - \frac{2.4736 - 4}{-6.12644 - 29}(-6.12644)$$

$$= 2.4736 - \frac{-1.5264}{-35.12644}(-6.12644) = 2.4736 - (0.04345)(-6.12644)$$

$$= 2.4736 + 0.26619 = 2.73989$$

Therefore

f (x4) $= 2.73989^3 - 9(2.73989) + 1 = 20.5683 - 24.65901 + 1 =$ $=- 3.090707$.

Now, since $f(x2)$ and $f(x4)$ are of opposite signs, the third approximation is obtained from

$$x_5 = x_4 - \frac{x_4 - x_2}{f(x_4) - f(x_2)} f(x_4) = 2.73989 - \frac{2.73989 - 4}{-3.090707 - 29}(-3.090707) = 2.86125$$

$$= 2.73989 - \frac{-1.26011}{-32.090707}(-3.090707) = 2.73989 + 0.039267(3.090707) = 2.73989 + 0.121363 = 2.86125$$

Now

f (x5) $= 2.86125^3 - 9(2.86125) + 1 = 23.42434 - 25.75125 + 1 = -1.326868$.

(ii)

Here

f (x) = x3 - 9x + 1

f (2) $= 2^3 - 9(2) + 1 = 8 - 18 + 1 = -9$ and f (3) $= 3^3 - 9(3) + 1 = 27 - 27 + 1 = 1$.

Since $f(2)$ and $f(3)$ are of opposite signs, the root of $f(x) = 0$ lies between 2 and 3.

Taking $x1 = 2$, $x2 = 3$ and using Regula-Falsi method, the first approximation is given by

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = 3 - \frac{3 - 2}{1 + 9}(1)$$

$$= 3 - \frac{1}{10} = 2.9$$

f (x_3) $= 2.9^3 - 9(2.9) + 1 = 24.389 - 26.1 + 1 = -0.711$

Since $f(x2)$ and $f(x3)$ are of opposite signs, the root lies between $x2$ and $x3$.

The second approximation to the root is given as

$$x_4 = 2.9 - \frac{2.9 - 3}{-0.711 - 1}(-0.711) = 2.9 - \frac{-0.1}{-1.711}(-0.711) = 2.9 - \frac{-0.1}{-1.711}(-0.711)$$

$$= 2.9 - (0.05844)(-0.711) = 2.9 _ 0.04156 = 2.94156$$

$$f(x_4) = -0.0207$$

f (x_4) $= 2.94156^3 - 9(2.94156) + 1 = 25.45265 - 26.47404 + 1 = -0.0207$

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Now, we observe that $f(x2)$ and $f(x4)$ are of opposite signs; the third approximation is obtained from

$$x_5 = 2.94156 - \frac{2.94156 - 3}{-0.0207 - 1}(-0.0207) = 2.94156 - \frac{-0.05844}{-1.0207}(-0.0207)$$

$$= 2.94156 - (-0.05725)(-0.0207) = 2.94275$$

$$f(x_5) = 2.94275^3 - 9(2.94275) + 1 = 25.48356 - 26.48475 + 1 = -0.0011896$$

We observe that the value of the root as a third approximation is evidently different in both the cases, while the value of $x5$, when the interval considered is (2, 3), is closer to the root.

***Important observation*: The initial interval ($x1$, $x2$) in which the root of the equation lies should be sufficiently small.**

Example

Use Regula-Falsi method to find a real root of the equation $lnx - cos\ x = 0$ accurate to four decimal places after three successive approximations.

Note: here is a transcendental equation all the calculation should be done in the radians mode

Sol:

$f(x) = \ln x - \cos x$

we have

f(1)=ln1-cos(1)=0-0.540302=-0.540302<0

f(2)=ln2-cos(2)=0.69315-0.41615=1.109

As f(1)f(2)<0 so the root lies between 1 and 2

the first approximation is *obtained form*

$$x_3 = 2 - \frac{2-1}{1.109 + 0.540302}(1.109)$$

$$= 2 - \frac{1.1093}{1.6496} = 1.3275$$

$$f(x_3) = \ln 1.3275 - \cos 1.3275 = 02833 - 0.2409 = 0.0424$$

Now, since $f(x1)$ and $f(x3)$ are of opposite signs, the second approximation is obtained as

$$x_4 = 1.3275 - \frac{(.3275)(.0424)}{0.0424 + 0.5403}$$

$$= 1.3037$$

$$f(x_4) = \ln 1.3037 - \cos 1.3037 = 1.24816 \times 10^{-3}$$

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Similarly, we observe that $f(x1)$ and $f(x4)$ are of opposite signs, so, the third approximation is given by

$$x_5 = 1.3037 - \frac{(0.3037)(0.001248)}{0.001248 + 0.5403}$$
$$= 1.3030$$
$$f(x_5) = 0.6245 \times 10^{-4}$$

The required real root is 1.3030

Example:

Use method of false position to solve $e^{-x} + 2^{-x} + 2\cos x - 6 = 0 \quad 1 \le x \le 2$

Solution:

$$f(x) = e^x + 2^{-x} + 2\cos x - 6$$
$$x_0 = 1 \quad , \quad x_1 = 2$$
now
$$x_{n+1} = \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$
$$f(1) = e^1 + 2^{-1} + 2\cos 1 - 6 = 2.7182 + 0.5 + 2(0.5403) - 6 = -1.7011$$
$$f(2) = e^2 + 2^{-2} + 2\cos 2 - 6 = 7.3886 + 0.25 + 2(-0.4161) - 6 = 0.8068$$
now for $n = 1$
$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1) = 2 - \frac{2-1}{0.8068 + 1.7011}(0.8068)$$
$$x_2 = 2 - \frac{1}{2.5079}(0.8068) = 1.6783$$
$$f(1.6783) = e^{1.6783} + 2^{-1.6783} + 2\cos(1.6783) - 6 = -0.5457$$
now for $n = 2$
$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = 1.6783 - \frac{1.6783 - 2}{(-0.5457) - 0.8068}(-0.5457)$$
$$x_3 = 1.6783 - \frac{(-0.3217)}{(-1.3525)}(-0.5457) = 1.6783 + 0.12979 = 1.8081$$

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$f(1.8081) = e^{1.6783} + 2^{-1.8081} + 2\cos(1.8081) - 6 = -0.8575$

now for $n = 3$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3) = 1.8081 - \frac{1.8081 - 1.6783}{(-0.08575) + 0.5457}(-0.08575)$$

$$x_3 = 1.8081 - \frac{0.1298}{0.45995}(-0.08575) = 1.6783 + 0.12979 = 1.8323$$

$f(1.8323) = e^{1.8323} + 2^{-1.8323} + 2\cos(1.8323) - 6 = 0.1199$

now for $n = 4$

$$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_4) = 1.8323 - \frac{1.8323 - 1.8081}{0.01199 + 0.08575}(0.01199)$$

$$x_5 = 1.8323 - \frac{0.0242}{0.09774}(0.01199) = 1.8323 - 0.00296 = 1.8293$$

$f(1.8293) = e^{1.8293} + 2^{-1.8293} + 2\cos(1.8293) - 6 = -0.000343$

now for $n = 5$

$$x_6 = x_5 - \frac{x_5 - x_4}{f(x_5) - f(x_4)} f(x_5) = 1.8293 - \frac{1.8293 - 1.8323}{-0.000343 - 0.01199}(-0.000343)$$

$$x_6 = 1.8293 - \frac{(-0.003)}{-0.012333}(-0.000343) = 1.8293$$

Example:

Solve the equation $2x\cos 2x - (x-2)^2 = 0 \quad 2 \le x \le 3$ Perform only three iterations.

Solution

$f(x) = 2x\cos 2x - (x-2)^2$

here $x_0 = 2$ *and* $x_1 = 3$

so $f(2) = 2(2)\cos 4 - (2-2)^2 = 4\cos 4 = -2.6146$

$f(3) = 2(3)\cos 2(3) - (3-2)^2 = 6\cos 6 - 1 = 4.7610$

here $x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$

for $n = 1$

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1) = 3 - \frac{3-2}{4.7610 - 2.4146}(4.7610)$$

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$$= 3 - \frac{1}{7.3756}(4.7610) = 3 - 0.6455 = 2.3545$$

$$f(2.3545) = 2(2.3545)\cos 2(2.3545) - (2.3545 - 2)^2 = 4.709\cos 4.709 - 0.1257 = -0.1416$$

for $n = 2$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = 2.3545 - \frac{2.3545 - 3}{-0.1416 - 4.7610}(-0.1416)$$

$$= 2.3731$$

$$f(2.3713) = 2(2.3713)\cos 2(2.3713) - (2.3713 - 2)^2 = 4.7462\cos 4.7462 - 0.1392 = -0.1392$$

for $n = 3$

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3) = 2.3713 - \frac{2.3731 - 2.3545}{0.0212 + 0.1416}(0.0212)$$

$$= 2.3707$$

$$f(2.3707) = 2(2.3707)\cos 2(2.3707) - (2.3707 - 2)^2 = 4.7414\cos 4.7412 - 0.1374 = 0.00013$$

for $n = 4$

$$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_4) = 2.3707 - \frac{2.3707 - 2.3731}{0.00013 - 0.0212}(0.00013)$$

$$= 2.3707$$

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Example

Using Regula-Falsi method, find the real root of the following equation correct, to three decimal places: x log 10 x =1.2

Solution:

Let f (x) = x $\log_{10}$x – 1.2

f (2) = 2 $\log_{10}$2 – 1.2 = – 0.5979,

f (3) = 3 $\log_{10}$3 – 1.2 = 0.2314.

Since f (2) and f (3) are of opposite signs, the real root lies betweenx1 = 2, x2 = 3.

The first approximation is obtained from

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = 3 - \frac{3-2}{0.2314 + 0.5979}(0.2314)$$

$$= 3 - \frac{0.2314}{0.8293} = 2.72097$$

$f(x_3)$ = Let f (x) = 2.72097$\log_{10}$2.72097 – 1.2 = – 0.01713.

Since $f(x2)$ and $f(x3)$ are of opposite signs, the root of $f(x) = 0$ lies between $x2$ and $x3$. Now, the second approximation is given by

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3) = 2.72097 - \frac{2.72097 - 3}{-0.1713 - 0.2314}(-0.1713) = 2.7402$$

$f(x_4) = 2.7402 \log_{10} 2.7402 - 1.2 - 3.8905 \times 10^{-4}$

Thus, the root of the given equation correct to three decimal places is 2.740

NOTE: Here if TOL is given then we can simply find the value of TOL by subtracting both the consecutive roots and write it in the exponential notation if the required TOL is obtained then we stop.

Method of Iteration

Method of iterations can be applied to find a real root of the equation $f(x) = 0$ by rewriting the same in the form.

$x = \phi(x)$

Let $x = x_0$ be the initial approximation to the actual root, say, α of the equation .then the first approximation is $x_1 = \phi(x_0)$ and the successive approximation are $x_2 = \phi(x_1)$

$x_3 = \phi(x_2), x_4 = \phi(x_3), ..., x_n = \phi(x_{n-1})$ if sequence of the approximate roots, $x_1, x_2, x_3, ...x_n$ converges to α it is taken as the root of the equation f(x)=0.

For convergence purpose the initial approximation is to be done carefully.the choice of the x_0 is done according to the theorem.

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Theorem

If α be a root of f(x) =0 which is equivalent to $x = \phi(x)$ I be any interval containing the point x= α and $|\phi'(x)| < 1 \forall\ x \varepsilon I$, *then the sequence of approximations* $x_1, x_2, x_3, ...x_n$ will converge to the root α provided that the initial approximation x_0 is chosen in I

Example,

f (x) = cos x - 2x + 3 = 0.

It can be

Rewritten as $x = \frac{1}{2}(\cos x + 3) = \phi(x)$

$\phi(x) = \frac{1}{2}(\cos x + 3)$

f (x) = cos x - 2x + 3 = 0.

f(1)=cos 1 - 2(1) + 3 =1.54030>0

f(2)=cos 1 - 2(2) + 3 =-0.041614-4+3=-1.41614<0

so root lies between 1 and 2

$\phi'(x) = -\frac{1}{2}(\sin x)$

both $\phi'(1)$ *and* $\phi'(2) < 1$ *so the method of iterations can be applied*

let $x_0 = 1.5$

$x_1 = \frac{1}{2}(\cos x_0 + 3) = \frac{1}{2}(\cos(1.5) + 3) = 1.999825$

$x_2 = \frac{1}{2}(\cos x_1 + 3) = \frac{1}{2}(\cos(1.999825) + 3) = 1.999695$

$x_3 = \frac{1}{2}(\cos x_2 + 3) = \frac{1}{2}(\cos(1.999625) + 3) = 1.999695$

So this is the required root correct up to 5 places of decimal.

Example

Find the real root of the equation $x^3 + x^2 - 1 = 0$ by method of iterations

Solution

let $f(x) = x^3 + x^2 - 1$

now $f(0) = 0^3 + 0^2 - 1 = -1 < 0$

$f(1) = 1^3 + 1^2 - 1 = 1 > 0$

hence a real root lies between 0 *and* 1

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here

$x^3 + x^2 - 1 = 0$

$x^2(x+1) = 1$

$x^2 = \frac{1}{(x+1)} \Rightarrow x = \frac{1}{\sqrt{x+1}} = \phi(x)$

here $\phi'(x) = -1/2[1/(x+1)^{\frac{3}{2}}]$

$\phi'(0) = 1/2 < 1$ *and* $\phi'(1) = 1/2^{\frac{5}{2}} < 1$

so $\phi'(x) < 1$ *for all the values in the* int *erval*

let $x_0 = 0.65$

$x_1 = \phi(x_0) = \frac{1}{\sqrt{x_0+1}} = \frac{1}{\sqrt{1.65}} = 0.7784989$

$x_2 = \phi(x_1) = \frac{1}{\sqrt{x_1+1}} = \frac{1}{\sqrt{1.7784989}} = 0.7498479$

$x_3 = \phi(x_2) = \frac{1}{\sqrt{x_2+1}} = \frac{1}{\sqrt{1.7498479}} = 0.7559617$

$x_4 = \phi(x_3) = \frac{1}{\sqrt{x_3+1}} = \frac{1}{\sqrt{1.7559617}} = 0.7546446$

$x_5 = \phi(x_4) = \frac{1}{\sqrt{x_4+1}} = \frac{1}{\sqrt{1.7546446}} = 0.7549278$

$x_6 = \phi(x_5) = \frac{1}{\sqrt{x_5+1}} = \frac{1}{\sqrt{1.7549278}} = 0.7548668$

$x_7 = \phi(x_6) = \frac{1}{\sqrt{x_6+1}} = \frac{1}{\sqrt{1.7548668}} = 0.7548799$

$x_8 = \phi(x_7) = \frac{1}{\sqrt{x_7+1}} = \frac{1}{\sqrt{1.7548799}} = 0.7548771$

$x_9 = \phi(x_8) = \frac{1}{\sqrt{x_8+1}} = \frac{1}{\sqrt{1.7548771}} = 0.7548777$

$x_{10} = \phi(x_9) = \frac{1}{\sqrt{x_9+1}} = \frac{1}{\sqrt{1.7548777}} = 0.7548776$

$x_{11} = \phi(x_{10}) = \frac{1}{\sqrt{x_{10}+1}} = \frac{1}{\sqrt{1.7548776}} = 0.7548776$

hence root is 0.7548776

Note: In this question the accuracy up to 7 places is acquires or here the TOL is 10^{-7}

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Example

Find a real root of the equation $\cos x = 3x - 1$ correct to seven places of decimal.

Solution

Here it is a transcendental function and all the calculation must be done in the radians mode and value of pi should be 3.14

$f(x) = \cos x - 3x + 1$

$f(0) = \cos 0 - 3(0) + 1 = 1 > 0$

$f(\pi/2) = \cos(1.57) - 3(1.57) + 1 = 0.0007963 - 4.71 + 1 = -3.7092037 < 0$

so a real root lies between 0 *and* $\pi/2$

here $\phi(x) = \frac{1}{3}(\cos x + 1)$

we have $\phi'(x) = -\frac{1}{3}\sin x$

it is clearly less than 1 *as* $\sin$ *is a bounded function and it's values lies between* -1 *and* 1

hence iteration method can be applied

let $x_0 = 0.5$ *be the inital approximation then*

$x_1 = \phi(x_0) = \frac{1}{3}[\cos(0.5) + 1] = 0.6258608$

$x_2 = \phi(x_1) = \frac{1}{3}[\cos(0.6258608) + 1] = 0.6034863$

$x_3 = \phi(x_2) = \frac{1}{3}[\cos(0.6034863) + 1] = 0.6077873$

$x_4 = \phi(x_3) = \frac{1}{3}[\cos(0.6077873) + 1] = 0.6069711$

$x_5 = \phi(x_4) = \frac{1}{3}[\cos(0.6069711) + 1] = 0.6071264$

$x_6 = \phi(x_5) = \frac{1}{3}[\cos(0.6071264) + 1] = 0.6070969$

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$$x_7 = \phi(x_6) = \frac{1}{3}[\cos(0.6070969) + 1] = 0.6071025$$

$$x_8 = \phi(x_7) = \frac{1}{3}[\cos(0.6071025) + 1] = 0.6071014$$

$$x_9 = \phi(x_8) = \frac{1}{3}[\cos(0.6071014) + 1] = 0.6071016$$

$$x_{10} = \phi(x_9) = \frac{1}{3}[\cos(0.6071016) + 1] = 0.6071016$$

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Newton -Raphson Method

This method is one of the most powerful method and well known methods, used for finding a root of f(x)=0 the formula many be derived in many ways the simplest way to derive this formula is by using the first two terms in Taylor's series expansion of the form,

$f(x_{n+1}) = f(x_n) + (x_{n+1} - x_n) f'(x_n)$

setting $f(x_{n+1}) = 0$ *gives*,

$f(x_n) + (x_{n+1} - x_n) f'(x_n) = 0$

thus on simplification, we get ,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \text{ for } n = 0,1,2...$$

Geometrical interpretation

Let the curve f(x)=0 meet the x-axis at x=α meet the x axis at x=α .it means that α is the original root of the f(x)=0.Let x_0 be the point near the root α of the equation f(x)=0 then the equation of the tangent $P_0[x_0, f(x_0)]$ is

$$y - f(x_0) = f'(x_0)(x - x_0)$$

This cuts the x-axis at $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

This is the first approximation to the root α .if $P_1[x_1, f(x_1)]$ is the point corresponding to x_1 on the curve then the tangent at P_1 is

$$y - f(x_1) = f'(x_1)(x - x_1)$$

This cuts the x-axis at $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

This is the second approximation to the root α .Repeating this process we will get the root α with better approximations quite rapidly.

Note:

1. When $f'(x)$ very large .i.e. is when the slope is large, then h will be small (as assumed) and hence, the root can be calculated in even less time.
2. If we choose the initial approximation x_0 close to the root then we get the root of the equation very quickly.
3. The process will evidently fail if $f'(x) = 0$ is in the neighborhood of the root. In such cases the regula falsi method should be used.

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4. If the initial approximation to the root is not given, choose two say, a and b, such that f(a) and f(b) are of opposite signs. If |f(a)|<|f(b)| then take a as the initial approximation.
5. Newton's raphson method is also referred to as the method of tangents.

Example

Find a real root of the equation $x3 - x - 1 = 0$ using Newton - Raphson method, correct to four decimal places.

Solution

$f(x)=x^3 - x - 1$

$f(1)=1^3 - 1 - 1 =-1<0$

$f(2)=2^3 - 2 - 1 = 8 - 2 - 1 = 5 > 0$

so the root lies between 1 *and* 2

here $f'(x) = 3x^2 - 1$ *and* $f''(x) = 6x$

$f'(1) = 3*1^2 - 1 = 2$

$f'(x) = 3*2^2 - 1 = 11$

here

$f''(1) = 6$

$f''(2) = 6(2) = 12$

here $f(2)$ *and* $f''(2)$ *have the same signs so* $x_0 = 2$

The second approximation is computed using Newton-Raphson method as

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$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{5}{11} = 1.54545$$

f(1.54545)=1.54545^3 - 1.54541 - 1 =3.691177-1.54541-1=1.14576

$$f'(x) = 3x^2 - 1 = 3(1.54541)^2 - 1 = 3(2.38829) - 1 = 7.16487 - 1 = 6.16487$$

$$x_2 = 1.54545 - \frac{1.14576}{6.16525} = 1.35961$$

f(1.35961)=1.35961^3 - 1.35961 - 1 =3.691177-1.54541-1=1.14576

$$f'(x) = 3x^2 - 1 = 3(1.54541)^2 - 1 = 3(2.38829) - 1 = 7.16487 - 1 = 6.16487$$

$$x_2 = 1.54545 - \frac{1.14576}{6.16525} = 1.35961$$

$$x_3 = 1.35961 - \frac{0.15369}{4.54562} = 1.32579, \qquad f(x_3) = 4.60959 \times 10^{-3}$$

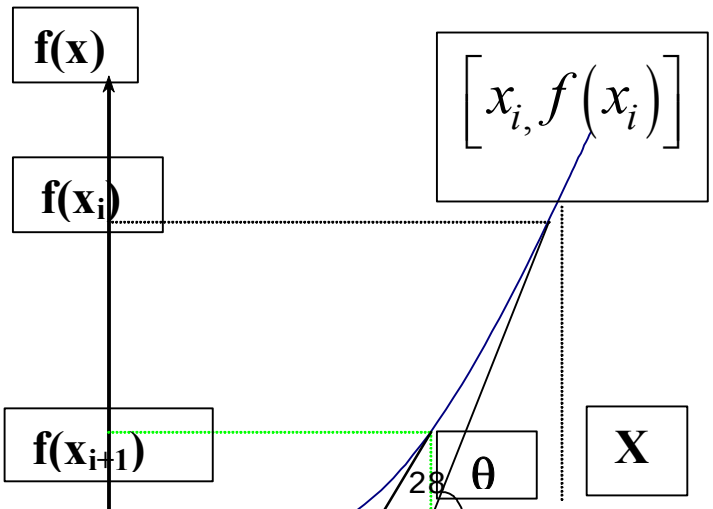
$$x_4 = 1.32579 - \frac{4.60959 \times 10^{-3}}{4.27316} = 1.32471, \qquad f(x_4) = -3.39345 \times 10^{-5}$$

$$x_5 = 1.32471 + \frac{3.39345 \times 10^{-5}}{4.26457} = 1.324718, \qquad f(x_5) = 1.823 \times 10^{-7}$$

Hence, the required root is 1.3247

Note!

Methods such as bisection method and the false position method of finding roots of a nonlinear equation *f(x) = 0* require bracketing of the root by two guesses. Such methods are called bracketing methods. These methods are always convergent since they are based on reducing the interval between the two guesses to *zero in* on the root.
In the Newton-Raphson method, the root is not bracketed. Only one initial guess of the root is needed to get the iterative process started to find the root of an equation. Hence, the method falls in the category of open methods.
Newton - Raphson method is based on the principle that if the initial guess of the root of *f(x) = 0* is at xi, then if one draws the tangent to the curve at *f(xi)*, the point xi+1 where the tangent crosses the x-axis is an improved estimate of the root



Draw backs of N-R Method

Divergence at inflection points:

If the selection of a guess or an iterated value turns out to be close to the inflection point of $f(x)$,
[where $f''(x) = 0$],
the roots start to diverge away from the root.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Division of zero or near zero:

If an iteration, we come across the division by zero or a near-zero number, then we get a large magnitude for the next value, xi+1.

Root jumping:

In some case where the function $f(x)$ is oscillating and has a number of roots, one may choose an initial guess close to a root. However, the guesses may jump and converge to some other root.

Oscillations near local maximum and minimum:

Results obtained from N-R method may oscillate about the local max or min without converging on a root but converging on the local max or min. Eventually, it may lead to division to a number close to zero and may diverge.

Convergence of N-R Method

Let us compare the N-R formula

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

with the general iteration formula

$x_{n+1} = \phi(x_n),$

$$\phi(x_n) = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\phi(x) = x - \frac{f(x)}{f'(x)}$$

The iteration method converges if

$|\phi'(x)| < 1.$

Therefore, N-R formula converges, provided

$|f(x)f''(x)| < |f'(x)|^2$

in the interval considered.

Newton-Raphson formula therefore converges, provided the initial approximation $x0$ is chosen sufficiently close to the root and are continuous and bounded in any small interval containing the root.

Definition

Let $x_n = \alpha + \varepsilon_n, x_{n+1} = \alpha + \varepsilon_{n+1}$

where α is a root of f (x) = 0.

If we can prove that $\varepsilon_{n+1} = K\varepsilon_n^p$,

where K is a constant and ε_n is the error involved at the n - th step, while finding the root by an iterative method, then the rate of convergence of the method is *p*.

The N-R method converges quadratically

$x_n = \alpha + \varepsilon_n,$

$x_{n+1} = \alpha + \varepsilon_{n+1}$

where α is a root of $f(x) = 0$ and ε_n is the error involved at the *n*-th step, while finding the root by N-R formula

$$\alpha + \varepsilon_{n+1} = \alpha + \varepsilon_n - \frac{f(\alpha + \varepsilon_n)}{f'(\alpha + \varepsilon_n)}$$

$$\varepsilon_{n+1} = \varepsilon_n - \frac{f(\alpha + \varepsilon_n)}{f'(\alpha + \varepsilon n)} = \frac{\varepsilon_n f'(\alpha + \varepsilon_n) - f(\alpha + \varepsilon_n)}{f'(\alpha + \varepsilon_n)}$$

Using Taylor's expansion, we get

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$$\varepsilon_{n+1} = \frac{1}{f'(\alpha) + \varepsilon_n f''(\alpha) + \cdots} \left\{ \varepsilon_n [f'(\alpha) + \varepsilon_n f''(\alpha) + \cdots] - \left[f(\alpha) + \varepsilon_n f'(\alpha) + \frac{\varepsilon_n^2}{2} f''(\alpha) + \cdots \right] \right\}$$

Since α is a root, $f(\alpha) = 0.$ Therefore, the above expression simplifies to

$$\varepsilon_{n+1} = \frac{\varepsilon_n^2}{2} f''(\alpha) \frac{1}{f'(\alpha) + \varepsilon_n f''(\alpha)}$$

$$= \frac{\varepsilon_n^2}{2} \frac{f''(\alpha)}{f'(\alpha)} \left[1 + \varepsilon_n \frac{f''(\alpha)}{f'(\alpha)} \right]^{-1}$$

$$= \frac{\varepsilon_n^2}{2} \frac{f''(\alpha)}{f'(\alpha)} \left[1 - \varepsilon_n \frac{f''(\alpha)}{f'(\alpha)} \right]$$

$$= \frac{\varepsilon_n^2}{2} \frac{f''(\alpha)}{f'(\alpha)} \left[1 - \varepsilon_n \frac{f''(\alpha)}{f'(\alpha)} \right]$$

$$\varepsilon_{n+1} = \frac{\varepsilon_n^2}{2} \frac{f''(\alpha)}{f'(\alpha)} + O(\varepsilon_n^3)$$

On neglecting terms of order ε_n^3 and higher powers, we obtain

$$\varepsilon_{n+1} = K \varepsilon_n^2$$

Where

$$K = \frac{f''(\alpha)}{2f'(\alpha)}$$

It shows that N-R method has second order convergence or converges quadratically.

Example

Set up Newton's scheme of iteration for finding the square root of a positive number *N*.

Solution

The square root of *N* can be carried out as a root of the equation

$x^2 - N = 0.$

Let

$f(x) = x^2 - N.$

By Newton's method, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

In this Problem

$f(x) = x^2 - N, f'(x) = 2x.$

Therefore

$$x_{n+1} = x_n - \frac{x_n^2 - N}{2x_n} = \frac{1}{2}\left(x_n + \frac{N}{x_n} \right)$$

Example

Evaluate $\sqrt{12}$, by Newton's formula.

Solution

Since

$\sqrt{9} = 3, \sqrt{16} = 4,$

We take

$x_0 = (3+4)/2 = 3.5.$

$$x_1 = \frac{1}{2}\left(x_0 + \frac{N}{x_0} \right) = \frac{1}{2}\left(3.5 + \frac{12}{3.5} \right) = 3.4643$$

$$x_2 = \frac{1}{2}\left(3.4643 + \frac{12}{3.4643} \right) = 3.4641$$

$$x_3 = \frac{1}{2}\left(3.4641 + \frac{12}{3.4641} \right) = 3.4641$$

Hence

$\sqrt{12} = 3.4641.$

Here in this solution we use the iterative scheme developed in the above example and simply replace the previous value in the next iteration and finally come to the result.

Example

Find the first three iteration of the equation $f(x) = x - 0.8 - 0.2\sin x$ in the interval $[0, \pi/2]$.

Solution

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$$f(0) = 0 - 0.8 - 0.2\sin(0) = 0 - 0.8 - 0.2(0) = -0.8$$

$$f(1.57) = 1.57 - 0.8 - 0.2\sin(1.75)$$
$$= 1.57 - 0.8 - 0.2(0.99999)$$
$$= 1.57 - 0.8 - 0.199998 = 0.570002$$

$$f'(x) = 1 - 0.2\cos x$$

$$f'(0) = 1 - 0.2\cos(0) = 1 - 0.2 = 0.8$$

here $|f(0)|$ *is greater then* $x_0 = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{-0.8}{0.8} = 1$$

now

$$f(1) = 1 - 0.8 - 0.2\sin(1)$$
$$= 1 - 0.8 - 0.1683$$
$$= 0.0317$$

$$f'(x) = 1 - 0.2\cos x$$

$$f'(1) = 1 - 0.2\cos(1) = 1 - 0.1081 = 0.8919$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{0.0317}{0.8919} = 1 - 0.0355 = 0.9645$$

$$f(0.9645) = 0.9645 - 0.8 - 0.2\sin(0.9645)$$
$$= 0.9645 - 0.8 - 0.1645$$
$$= 0.0002$$

$$f'(x) = 1 - 0.2\cos x$$

$$f'(0.9645) = 1 - 0.2\cos(0.9645) = 1 - 0.11396 = 0.88604$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.9645 - \frac{0.0002}{0.88604} = 0.9645 - 0.00022 = 0.9643$$

NOTE: In the above question the function is a transcendental function and we have to perform all the calculation in the radians mode and the value of pi should be taken as 3.14

Example

Consider the equation $f(x) = 4x\cos x - (x-2)^2$ find the root of the equation in the range $0 \le x \le 8$

Solution

$f(x) = 4x\cos x - (x-2)^2$

here

$f(0) = 4(0)\cos 2(0) - (0-2)^2 = -4$

$f(8) = 4(8)\cos 2(8) - (8-2)^2$

$= 32\cos 16 - (6)^2 = -30.6451 - 36 = -66.6451$

$f'(x) = 4\cos 2x - 8x\sin 2x - 2(x-2)$

$= 4\cos 2x - 8x\sin 2x - 2(x-2)$

$f'(8) = 4\cos 16 - 64\sin 16 - 2(8-2)$

$= -3.8306 + 18.4250 - 12 = 2.5952$

$\sin ce\,|f(8)|\,is\ greater\ so\ x_0 = 8$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 8 - \frac{(-66.6451)}{2.5952} = 33.6801$$

$f(33.6801) = 4(33.6801)\cos 2(33.6801) - (33.6801-2)^2$

$= -24.6545 - 1003.6 = -1028.25$

$f'(33.6801) = 4\cos 2(33.6801) - 8(33.6801)\sin 2(33.6801) - 2(33.6801-2)$

$= -0.7320 + 264.89 = -63.3602$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 33.6801 + \frac{1028.25}{200.79} = 38.8011$$

$f(38.8011) = 4(38.8011)\cos 2(38.8011) - (38.8011-2)^2$

$= -91.8361 - 1354.3 = 1446.14$

$f'(38.8011) = 4\cos 2(38.8011) - 8(38.8011)\sin 2(38.8011) - 2(38.8011-2)$

$= -2.3668 - 250.236 - 73.6022 = -326.205$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 38.8011 + \frac{1446.14}{-326.205} = 38.8011 + 4.4332 = 43.2343$$

Example

Perform three iteration of the equation $\ln(x-1) + \cos(x-1) = 0\ when\ 1.2 \le x \le 2$ .Use Newton Raphson method to calculate the root.

Solution

Here

$\ln(x-1) + \cos(x-1) = 0\ when\ 1.2 \le x \le 2$

$f(x) = \ln(x-1) + \cos(x-1)$

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$$f(x) = \ln(x-1) + \cos(x-1)$$
$$f(1.2) = \ln(1.2-1) + \cos(1.2-1)$$
$$= -1.6094 + 0.9801 = -0.6293$$
$$f(2) = \ln(2-1) + \cos(2-1)$$
$$= 0 + 0.5403 = 0.5403$$
now $$f(x) = \ln(x-1) + \cos(x-1)$$
$$f'(x) = \frac{1}{x-1} - \sin(x-1)$$
$$f'(1.2) = \frac{1}{1.2-1} - \sin(1.2-1)$$
$$= 5 - 0.1986 = 4.8014$$
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.2 - \frac{-0.6293}{4.8014} = 1.2 + 0.1311 = 1.3311$$
$$f(1.311) = \ln(1.3311-1) + \cos(1.3311-1)$$
$$= -1.1053 + 0.9457 = -0.1596$$
$$f'(1.3311) = \frac{1}{1.3311-1} - \sin(1.3311-1)$$
$$= 3.0202 - 0.3251 = 2.6951$$
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.3311 - \frac{-0.1596}{2.6951} = 1.3311 + 0.0592 = 1.3903$$
$$f(1.3903) = \ln(1.3903-1) + \cos(1.3903-1)$$
$$= -0.9408 + 0.9248 = -0.016$$
$$f'(1.3903) = \frac{1}{1.3903-1} - \sin(1.3903-1)$$
$$= 2.5621 - 0.3805 = 2.1816$$
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3903 - \frac{-0.016}{2.1816} = 1.3903 + 0.0073 = 1.3976$$

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Secant Method

The secant method is modified form of Newton-
Raphson method .If in Newton-Raphson method; we replace the derivative $f'(x_n)$ by the difference ratio, i.e,

$$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

Where x_n and x_{n-1} are two approximations of the root we get

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$
$$= \frac{x_n f(x_n) - x_n f(x_{n-1}) - f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$
$$= \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

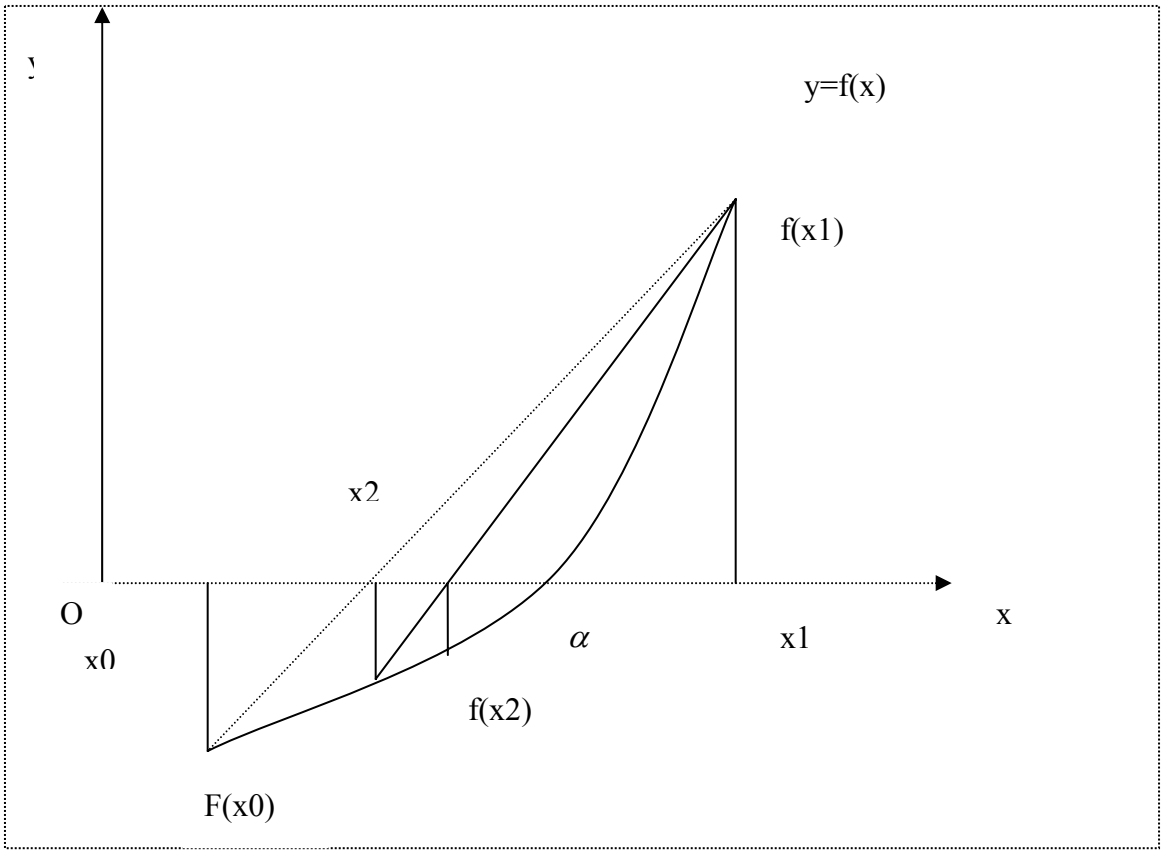
Provided $f(x_n) \neq f(x_{n-1})$

This method requires two starting values x_0 *and* x_1; values at both these points are calculated which gives two points on the curve the next point on the curve are obtained by using the derived formula. We continue this procedure until we get the root of required accuracy.

Geometrical Interpretation

Geometrically, the secant method corresponds to drawing secants rather than tangents to obtain various approximations to root α ;
To obtain x_2 we find the intersection between the secant through the points $(x_0, f(x_0))$ And $(x_1, f(x_1))$ and the x-axis.
It is experienced that the secant method converges rather quickly .there is a possibility of the divergence if the two roots lie on the same side of the curve .the order of the convergence of the decant method equal to $\frac{(1+\sqrt{5})}{2} = 1.618$,which shows that this method has the order of convergence slightly inferior that of Newton-Raphson method, In this method f(x) is not required to change signs between the estimate.

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Some more description

We choose x0, x1 close to the root 'a' of $f(x) = 0$ such that $f(x0)$ $f(x1)$

As a next approximation x2 is obtained as the point of intersection of y = 0 and the chord passing through the points

$(x0, f(x0))$, $(x1\ f(x1))$.

$$y - f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0),$$

Putting y = 0 we get

$$x_2 = x = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Convergence of Secant Method

Here the sequence is generated by the rule

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Starting with x0 and x1 as $\{x_0, x_1 \ldots\}$

It will converge to 'a' , that is $f(a) = 0$

NOTE

The Secant method converges faster than linear and slower than Newton's quadratic

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Example

Do three iterations of secant method to find the root of

$f(x) = x^3 - 3x + 1 = 0,$

Taking

$x_0 = 1,\ x_1 = 0.5$

$n = 1,$

$f(x_0) = f(1) = 1^3 - 3(1) + 1 = -1$

$f(x_1) = f(0.5) = 0.5^3 - 3(0.5) + 1 = 0.125 - 1.5 + 1 = -0.375$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{(1)(-0.375) - (0.5)(-1)}{-0.375 - (-1)} = 0.2$$

$n = 2,$

$f(x_2) = f(0.2) = 0.2^3 - 3(0.2) + 1 = 0.008 - 0.6 + 1 = 0.408$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{(0.5)(0.408) - 0.2(-0.375)}{0.408 - (-0.375)} = 0.3563$$

$n = 3,$

$f(x_3) = f(0.3563) = 0.3563^3 - 3(0.3563) + 1 = 0.04523 - 1.0689 + 1 = -0.02367$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{(0.2) f(0.3563) - 0.3563 f(0.2)}{f(0.3563) - f(0.2)}$$

$x_5 = 0.3473, f(x_5) = -0.0000096$

and $|x_5 - x_4| = 0.0004.$

Though *X*5 is not the true root, yet this is good approximation to the root and convergence is faster than bisection.

Example

Find the root of $2\cosh x \sin x = 1$ using the secant method, with accuracy of 4 decimal point .Take 0.4 and 0.5 as the two starting values

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$$f(x) = 2\cosh x \sin x - 1$$

$$now\ f(x_0) = 2\cosh x_0 \sin x_0 - 1$$

$$= 2\cosh 0.4 \sin 0.4 - 1$$

$$= 2 \times 1.081 \times 0.3894 - 1 = -0.1580$$

$$f(x_1) = 2\cosh x_1 \sin x_1 - 1$$

$$= 2\cosh 0.5 \sin 0.5 - 1$$

$$= 2 \times 1.1276 \times 0.4794 - 1 = 0.0811$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{0.4 \times 0.0811 - 0.5 \times -0.1580}{0.0811 + 0.1580} = \frac{0.03244 + 0.979}{0.2391} = 0.4661$$

$$f(x_2) = 2\cosh x_2 \sin x_2 - 1$$

$$= 2\cosh 0.5 \sin 0.5 - 1$$

$$= 2 \times 1.1106 \times 0.4494 - 1 = -0.0811$$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{0.5 \times -0.018 - 0.4661 \times 0.0811}{-0.0018 - 0.081} = \frac{0.009 - 0.0378}{-0.0828} = 0.4668$$

$$f(x_3) = 2\cosh x_3 \sin x_3 - 1$$

$$= -0.00009$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{0.4661 \times -0.00009 - 0.4668 \times -0.0018}{-0.00009 + 0.0018} = \frac{-0.000042 + 0.00048}{-0.00171} = 0.4667$$

Comparison:

In secant method, we do not check whether the root lies in between two successive approximates *X*n-1, and *X*n.
This checking was imposed after each iteration, in Regula –Falsi method.

Muller's Method

In *Muller's method*, f (*x*) = 0 is approximated by a second degree polynomial; that is by a quadratic equation that fits through three points in the vicinity of a root. The roots of this quadratic equation are then approximated to the roots of the equation *f* (*x*) 0.This method is iterative in nature and does not require the evaluation of derivatives as in Newton-

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Raphson method. This method can also be used to determine both real and complex roots of f (x) = 0.

Suppose, x_{i-2}, x_{i-1}, x_i

be any three distinct approximations to a root

Of f (x) = 0.

$f(x_{i-2}) = f_{i-2}, f(x_{i-1}) = f_{i-1}$

$f(x_i) = f_i$.

Noting that any three distinct points in the (x, y)-plane uniquely; determine a polynomial of second degree.

A general polynomial of second degree is given by

Noting that any three distinct points in the (x, y)-plane uniquely; determine a polynomial of second degree.

A general polynomial of second degree is given by

$f(x) = ax^2 + bx + c$

Suppose, it passes through the points

$(x_{i-2}, f_{i-2}), (x_{i-1}, f_{i-1}), (x_i, f_i)$

Then the following equations will be satisfied

$$ax_{i-2}^2 + bx_{i-2} + c = f_{i-2}$$
$$ax_{i-1}^2 + bx_{i-1} + c = f_{i-1}$$
$$ax_i^2 + bx_i + c = f_i$$

Eliminating *a, b, c*, we obtain

$$\begin{vmatrix} x^2 & x & 1 & f \\ x_{i-2}^2 & x_{i-2} & 1 & f_{i-2} \\ x_{i-1}^2 & x_{i-1} & 1 & f_{i-1} \\ x_i^2 & x_i & 1 & f_i \end{vmatrix} = 0$$

This can be written as

$$f = \frac{(x-x_{i-1})(x-x_i)}{(x_{i-2}-x_{i-1})} f_{i-2} + \frac{(x-x_{i-2})(x-x_i)}{(x_{i-1}-x_{i-2})(x_{i-1}-x_i)} f_{i-1} + \frac{(x-x_{i-2})(x-x_{i-1})}{(x_i-x_{i-2})(x_i-x_{i-1})} f_i$$

We further define

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$$\lambda = \frac{h}{h_i} = \frac{x - x_i}{x_i - x_{i-1}}$$

$$\lambda_i = \frac{h_i}{h_{i-1}}$$

$$\delta_i = 1 + \lambda_i$$

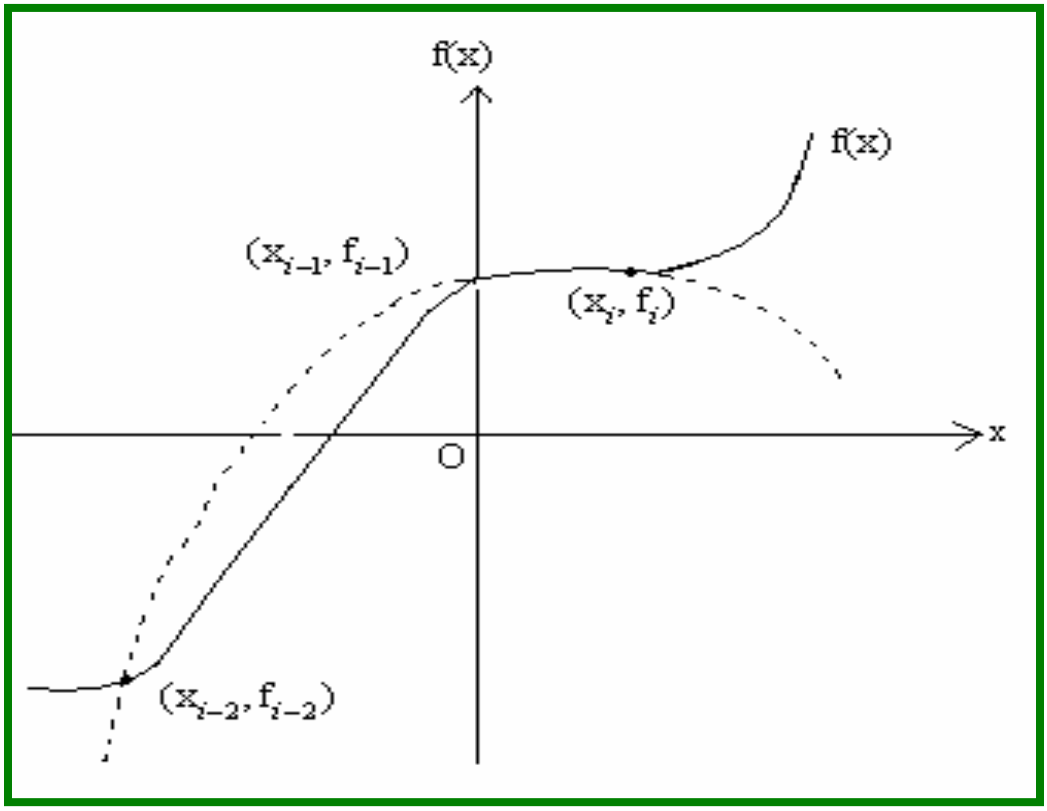


Fig: Quadratic polynomial

With these substitutions we get a simplified Equation as

$$f = \frac{1}{\delta_i}[\lambda(\lambda+1)\lambda_i^2 f_{i-2}$$

$$-\lambda(\lambda+1+\lambda_i^{-1})\lambda_i\delta_i f_{i-1}$$

$$+(\lambda+1+\lambda_i^{-1})\lambda_i f_i]$$

Or

$$f = \lambda^2(f_{i-2}\lambda_i^2 - f_{i-1}\lambda_i\delta_i + f_i\lambda_i)\delta_i^{-1}$$
$$+ \lambda[f_{i-2}\lambda_i^2 - f_{i-1}\delta_i^2$$
$$+ f_i(\lambda_i + \delta_i)]\delta_i^{-1} + f_i$$

To compute λ, set f = 0, we obtain

$$\lambda_i(f_{i-2}\lambda_i - f_{i-1}\delta_i + f_i)\lambda^2 + g_i\lambda + \delta_i f_i = 0$$

Where

$$g_i = f_{i-2}\lambda_i^2 - f_{i-1}\delta_i^2 + f_i(\lambda_i + \delta_i)$$

A direct solution will lead to loss of accuracy and therefore to obtain max accuracy we rewrite as:

$$\frac{f_i\delta_i}{\lambda^2} + \frac{g_i}{\lambda} + \lambda_i(f_{i-2}\lambda_i - f_{i-1}\delta_i + f_i) = 0$$

So that,

$$\frac{1}{\lambda} = \frac{-g_i \pm [g_i^2 - 4f_i\delta_i\lambda_i(f_{i-2}\lambda_i - f_{i-1}\delta_i + f_i)]^{1/2}}{2f_i\ \delta_i}$$

Or

$$\lambda = \frac{-2f_i\delta_i}{g_i \pm [g_i^2 - 4f_i\delta_i\lambda_i(f_{i-2}\lambda_i - f_{i-1}\delta_i + f_i)]^{1/2}}$$

Here, the positive sign must be so chosen that the denominator becomes largest in magnitude.

We can get a better approximation to the root, by using

$$x_{i+1} = x_i + h_i\lambda$$

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Muller's Method

Muller's method is a generalization of the secant method, in the sense that it does not require the derivative of the function. It is an iterative method that requires three starting points $(p_0, f(p_0))$, $(p_1, f(p_1))$, and $(p_2, f(p_2))$. A parabola is constructed that passes through the three points; then the quadratic formula is used to find a root of the quadratic for the next approximation. It has been proved that near a simple root Muller's method converges faster than the secant method and almost as fast as Newton's method. The method can be used to find real or complex zeros of a function and can be programmed to use complex arithmetic.

Example

Do two iterations of Muller's method to solve $x^3 - 3x + 1 = 0$ starting with $x_0 = 0.5,\ x_2 = 0,\ x_1 = 1$

Solution

$$f(x_0) = f_0 = (0.5)^3 - 3(0.5) + 1 = -0.375$$

$$f(x_1) = f_1 = (1)^3 - 3(1) + 1 = -1$$

$$f(x_2) = f_2 = 0 - 3 \times 0 + 1 = +1$$

$$c = f_0 = -0.375$$

$$h_1 = x_1 - x_0 = 0.5$$

$$h_2 = x_0 - x_2 = 0.5$$

$$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$$

$$= \frac{(0.5)(-1) - (-0.375) + (0.5)}{0.25} = 1.5$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1} = -2$$

$$\therefore x = x_0 - \frac{2c}{+b - \sqrt{b^2 - 4ac}}$$

$$= 0.5 - \frac{2(-0.375)}{-2 - \sqrt{4 + 4(1.5)(0.375)}}$$

$$= 0.5 - \frac{-0.75}{-2 - \sqrt{4 + 2.25}} = 0.33333 \prec 0.5$$

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Take

$x_2 = 0, x_0 = 0.33333, x_1 = 0.5$

$h_1 = x_1 - x_0 = 0.16667, h_2 = x_0 - x_2 = 0.33333$

$c = f_0 = f(0.33333)$

$= x_0^3 - 3x_0 + 1 = 0.037046$

$f_1 = x_1^3 - 3x_1 + 1 = -0.375$

$f_2 = x_2^3 - 3x_2 + 1 = 1$

$$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)} = \frac{0.023148}{0.027778}$$

$= 0.8333$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1} = -2.6$$

$$x = x_0 - \frac{2c}{b - \sqrt{b^2 - 4ac}}$$

$$= 0.333333 - \frac{0.074092}{-5.2236}$$

$= 0.3475 \succ 0.33333 = x_0$

For third iteration take,

$x_2 = 0.333333, \quad x_0 = 0.3475, \quad x_1 = 0.5$

Graeffe's Root Squaring Method

GRAEFFE'S ROOT SQUARING METHOD is particularly attractive for finding all the roots of a polynomial equation. Consider a polynomial of third degree $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$f(-x) = a_0 - a_1 x + a_2 x^2 - a_3 x^3$

$f(x) f(-x) = a_3^2 x^6 - (a_2^2 - 2a_1 a_3) x^4$

$+(a_1^2 - 2a_0 a_2) x^2 - a_0^2$

$f(x) f(-x) = a_3^2 t^3 - (a_2^2 - 2a_1 a_3) t^2$

$+(a_1^2 - 2a_0 a_2) t - a_0^2$

The roots of this equation are squares or 2*i* (*i* = 1), powers of the original roots. Here *i* = 1 indicates that squaring is done once.
The same equation can again be squared and this squaring process is repeated as many times as required. After each squaring, the coefficients become large and overflow is possible as *i* increase.

Suppose, we have squared the given polynomial '*i*' times, then we can estimate the value of the roots by evaluating *2i* root of

$$\left| \frac{a_i}{a_{i-1}} \right|, \quad i = 1, 2, \ldots, n$$

Where *n* is the degree of the given polynomial.

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The proper sign of each root can be determined by recalling the original equation. This method fails, when the roots of the given polynomial are repeated.

This method has a great advantage over other methods in that it does not require prior information about approximate values etc. of the roots. But it is applicable to polynomials only and it is capable of giving all the roots. Let us see the case of the polynomial equation having real and distinct roots. Consider the following polynomial equation

$$f(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n \qquad (1)$$

separating the even and odd powers of x *and squaring we get*

$$(x^n + a_2 x^{n-2} + a_4 x^{n-4} + \ldots)^2 = (a_1 x^{n-1} + a_3 x^{n-3} + a_5 x^{n-5} + \ldots)^2$$

puttig $x^2 = y$ *and simplifying we get the new equation*

$$y^n + b_1 y^{n-1} + b_1 y^{n-1} + \ldots + b_1 y^{n-1} + b_n = 0 \qquad (2)$$

$$b_1 = -a_1^2 + 2a^2$$

$$b_2 = a_2^2 - 2a_1 a_3 + 2a_4$$

….. …. …

$$b_n = (-1)^n a_n^2$$

if $p_1, p_2, \ldots p_n$ *be the roots of eq* 1 *then the roots of the equation* 2 *are* $p_1^2, p_2^2, \ldots p_n^2$

Example

Using Graeffe root squaring method, find all the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$

Solution

Using Graeffe root squaring method, the first three squared polynomials are as under:
For $i = 1$, the polynomial is

$$x^3 - (36 - 22)x^2 + (121 - 72)x - 36$$

$$= x^3 - 14x^2 + 49x - 36$$

For $i = 2$, the polynomial is

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$x^3 - (196-98)x^2 + (2401-1008)x - 1296$

$= x^3 - 98x^2 + 1393x - 1296$

For i = 3, the polynomial is

$x^3 - (9604-2786)x^2 + (1940449-254016)x - 1679616$

$= x^3 - 6818x^2 + 16864333x - 1679616$

The roots of polynomial are

$\sqrt{\frac{36}{49}} = 0.85714, \qquad \sqrt{\frac{49}{14}} = 1.8708, \qquad \sqrt{\frac{14}{1}} = 3.7417$

Similarly the roots of the p;oynomial 2 are

$\sqrt[4]{\frac{1296}{1393}} = 0.9821, \qquad \sqrt[4]{\frac{1393}{98}} = 1.9417, \qquad \sqrt[4]{\frac{98}{1}} = 3.1464$

Still better estimates of the roots obtained from polynomial (3) are

$\sqrt[8]{\frac{1679616}{1686433}} = 0.99949, \qquad \sqrt[8]{\frac{1686433}{6818}} = 1.99143, \qquad \sqrt[8]{\frac{6818}{1}} = 3.0144$

The exact values of the roots of the given polynomial are 1, 2 and 3.

Example

Apply Graeffe,s root squaring method to solve the equation $x^3 - 8x^2 + 17x - 10 = 0$

Solution

$f(x) = x^3 - 8x^2 + 17x - 10$

here three chnges are observed form $+ve$ *to* $-ve$, $-ve$ *to* $+ve$, $+ve$ *to* $-ve$

hence according to deccarte rule of signs $f(x)$ *may have three positive roots*

rewriting eq as $x(x^2+17) = (8x^2+10)$

and squaring on both sides and putting $x^2 = y$

$y(y+17)^2 = (8y+10)^2$

$y^3 + 34y^2 + 289 = 64y^2 + 160y + 100$

$y(y^2+129) = 30y^2 + 100$

putting $y^2 = z$ *we get*

$z(z+129)^2 = (30z+100)^2$

$z^3 + 258z^2 + 16641z = 900z^2 + 6000z + 10000$

$z(z^2+16641) = (642z^2+10000)$

squaring again and putting $z^2 = u$ *we get*

$u(u+16641)^2 = (642u+10000)^2$

$u^3 + 33282u^2 + 276922881u = 412164u^2 + 12840000u + 10^8$

$u^3 - 378882u^2 + 264082u - 10^8 = 0$

*if the roots of the eq*1 *are* p_1, p_2, p_3, *and those of eqiuation* 5 *are* q_1, q_2, q_3

$p_1 = (q_1)^{1/8} = (-\lambda_1)^{1/8} = (378882)^{1/8} = 4.9809593 \cong 5$

$p_2 = (q_2)^{1/8} = (-\lambda_2 / \lambda_1)^{1/8} = (264082/378882)^{1/8} = 0.9558821 \cong 1$

$p_3 = (q_3)^{1/8} = (-\lambda_3 / \lambda_2)^{1/8} = (10^8 / 264082)^{1/8} = 2.1003064 \cong 2$

here $f(5) = f(2) = f(1) = 0$

hence all are the roots of the eqiuation

Example

Find all the root of the equation $x^4 - 3x + 1 = 0$

Solution

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$f(x) = x^4 - 3x + 1$ (1)

here there are two changes in the sign so there are two positive roots of the equation and there are no chnge in the roots of the equation $f(-x)$ *so two positive real roots and two are complex roots*

rewriting the above equation as

$x^4 + 1 = 3x$

squaring the equation and putting $x^2 = y$

we have

$(y^2 + 1)^2 = 9y$

squaring again and putting $y^2 = z$

$(z+1)^4 = 81z$

$z^4 + 4z^3 + 6z^2 - 77z + 1 = 0$

$z^4 + 6z^2 + 1 = -z(4z^2 - 77)$ (2)

squaring once again and putting $z^2 = u$ *, we get*

$(u^2 + 6u + 1) = u(4u - 77)^2$

$u^4 - 4u^3 + 645u^2 - 5917u + 1 = 0$ (3)

if p_1, p_2, p_3, p_4 *are the roots of the equation* 1 *and* q_1, q_2, q_3, q_4 *are roots of equation* 3 *then*

$p_1 = (q_1)^{1/8} = (-\lambda_1)^{1/8} = (4)^{1/8} = 1.189207$

$p_2 = (q_2)^{1/8} = [\frac{-\lambda_2}{\lambda_1}]^{1/8} = [\frac{654}{4}]^{1/8} = 1.8909921$

$p_3 = (q_3)^{1/8} = [\frac{-\lambda_3}{\lambda_2}]^{1/8} = [\frac{5917}{654}]^{1/8} = 1.3169384$

$p_4 = (q_4)^{1/8} = [\frac{-\lambda_4}{\lambda_3}]^{1/8} = [\frac{1}{5971}]^{1/8} = 0.3376659$

from equation (2) *and* (3)

from equation 2 *and* 3 *, we observe the magnitude of the cofficients* λ_1 *and* λ_2 *have become cons* tan *t which implies* p_1 *and* p_4 *are the real roots ,then* p_2 *and* p_3 *are real roots,*

let the complex roots be $\rho_2 e^{\pm i\varphi_2} = \xi_2 + i\eta_2$

from equation (3) *it's magnitude is given by*

$\rho^{2(2^3)}{}_2 = \frac{\lambda_3}{\lambda_1} = \frac{5917}{4} \quad \therefore \rho_2 = 1.5780749$

also from equation 1 *sum of roots is zero i.e* $\rho_1 + 2\xi_2 + p_4 = 0$

$\xi_2 = -1/2(p_1 + p_4) = -0.7634365$

$\eta_2 = \sqrt{\rho_2{}^2 - \xi_2{}^2} = \sqrt{1.9074851} = 1.3811173$

hence, the four roots are $1.1892071, 0.3376659, -0.734365$ *and* $\pm 1.3811173i$

Revision Example

Obtain the Newton-Raphson extended formula

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} - \frac{1}{2}\frac{[f(x_0)]^2}{[f(x_0)]^3} f''(x_0)$$

For finding the root of the equation f(x)=0

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Solution

Expanding f (*x*) by Taylor's series and retaining up to second order term,

$$0 = f(x) = f(x_0) + (x - x_0) f'(x_0)$$

$$+ \frac{(x - x_0)^2}{2} f''(x_0)$$

Therefore,

$$f(x_1) = f(x_0) + (x_1 - x_0) f'(x_0)$$

$$+ \frac{(x_1 - x_0)^2}{2} f''(x_0) = 0$$

This can also be written as

$$f(x_0) + (x_1 - x_0) f'(x_0) + \frac{1}{2} \frac{[f(x_0)]^2}{[f'(x_0)]^2} f''(x_0) = 0$$

Thus, the Newton-Raphson extended formula is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} - \frac{1}{2} \frac{[f(x_0)]^2}{[f'(x_0)]^3} f''(x_0)$$

This is also known as Chebyshev's formula of third order

Solution of Linear System of Equations and Matrix Inversion

The solution of the system of equations gives n unknown values $x1, x2, \ldots, x$n, which satisfy the system simultaneously. If $m > n$, the system usually will have an infinite number of solutions.

If $|A| \neq 0$ then the system will have a unique solution.

If $|A| = 0,$

Then there exists no solution.

Numerical methods for finding the solution of the system of equations are classified as direct and iterative methods. In direct methods, we get the solution of the system after performing all the steps involved in the procedure. The direct methods consist of elimination methods and decomposition methods.

Under elimination methods, we consider, Gaussian elimination and Gauss-Jordan elimination methods

Crout's reduction also known as Cholesky's reduction is considered under decomposition methods.

Under iterative methods, the initial approximate solution is assumed to be known and is improved towards the exact solution in an iterative way. We consider Jacobi, Gauss-Seidel and relaxation methods under iterative methods.

Gaussian Elimination Method

In this method, the solution to the system of equations is obtained in two stages.

i) the given system of equations is reduced to an equivalent upper triangular form using elementary transformations

ii) the upper triangular system is solved using back substitution procedure

This method is explained by considering a system of *n* equations in *n* unknowns in the form as follows

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}\right\}$$

Stage I: Substituting the value of *x1* from first equation into the rest

$$\left.\begin{aligned} x_1 + a'_{12}x_2 + \cdots + a'_{1n}x_n &= b'_1 \\ a'_{22}x_2 + \cdots + a'_{2n}x_n &= b'_2 \\ &\vdots \\ a'_{n2}x_2 + \cdots + a'_{nn}x_n &= b'_n \end{aligned}\right\}$$

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Now, the last $(n-1)$ equations are independent of $x1$, that is, $x1$ is eliminated from the last $(n-1)$ equations.
This procedure is repeated and $x2$ is eliminated from 3rd, 4th, …, n-th equations
The same procedure is repeated till the given system assumes the upper triangular form:

$$\left.\begin{aligned} c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n &= d_1 \\ c_{22}x_2 + \cdots + c_{2n}x_n &= d_2 \\ \vdots & \\ c_{nn}x_n &= d_n \end{aligned}\right\}$$

Stage II: The values of the unknowns are determined by backward substitution.
*First x*n is found from the last equation and then substitution this value of *x*n in the preceding equation will give the value of *x*n-1. Continuing this way, we can find the values of all other unknowns

Example

Solve the following system of equations using Gaussian elimination method

$$\begin{aligned} 2x + 3y - z &= 5 \\ 4x + 4y - 3z &= 3 \\ -2x + 3y - z &= 1 \end{aligned}$$

Solution

Stage I *(Reduction to upper-triangular form):*
Divide first equation by 2 and then subtract the resulting equation (multiplied by 4 and –2) from the 2nd and 3rd equations respectively. Thus, we eliminate *x* from the 2nd and 3rd equations.
The resulting new system is given by

$$\left.\begin{aligned} x + \frac{3}{2}y - \frac{z}{2} &= \frac{5}{2} \\ -2y - z &= -7 \\ 6y - 2z &= 6 \end{aligned}\right\}$$

Now, we divide the 2nd equation by –2 and eliminate *y* from the last equation and the modified system is given by

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$$\left.\begin{aligned} x+\frac{3}{2}y-\frac{z}{2}&=\frac{5}{2}\\ y+\frac{z}{2}&=\frac{7}{2}\\ -5z&=-15 \end{aligned}\right\}$$

Stage II (Backward substitution):
From the last equation, we get

$z = 3$
Using this value of z, the second equation gives
$y = \frac{7}{2} - \frac{3}{2} = 2$
Putting these values of *y* and *z* in the first equation, we get
$x = \frac{5}{2} + \frac{3}{2} - 3 = 1$
Thus, the solution of the given system is given by
x = 1, y = 2, z = 3

Partial and Full Pivoting
The Gaussian elimination method fails if any one of the pivot elements becomes zero. In such a situation, we rewrite the equations in a different order to avoid zero pivots. Changing the order of equations is called pivoting.

Partial pivoting

If the pivot happens to be zero, then the i-*th* column elements are searched for the numerically largest element. Let the *j-th* row (*j* > *i*) contains this element, then we interchange the *i-th* equation with the *j-th* equation and proceed for elimination. This process is continued whenever pivots become zero during elimination.
For example, let us examine the solution of the following simple system
$10^{-5} x_1 + x_2 = 1$
$x_1 + x_2 = 2$
Using Gaussian elimination method with and without partial pivoting, assuming that we require the solution accurate to only four decimal places. The solution by Gaussian elimination gives $x1 = 0$, $x2 = 1$.

If we use partial pivoting, the system takes the form

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$$x_1 + x_2 = 2$$
$$10^{-5} x_1 + x_2 = 1$$

Using Gaussian elimination method, the solution is found to be $x1 = 1$, $x2 = 1$, which is a meaningful and perfect result.

In full pivoting (or complete pivoting), we interchange rows ***and*** columns, such that the largest element in the matrix of the variables also get changed. Full pivoting, in fact, is more complicated than the partial pivoting. Partial pivoting is preferred for hand computation.

The general form of a system of m linear equations in n unknowns $x1, x2, x3, \ldots, x$n can be represented in matrix form as under:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Using matrix notation, the above system can be written in compact form as

$$[A](X) = (B)$$

Note:

1. This method fails if any of he pivots become zero in such cases, by interchanging the rows we can get the non-zero pivots.

Example

Solve the system of equations by Gaussian elimination method with partial pivoting**.**

$$x + y + z = 7$$
$$3x + 3y + 4z = 24$$
$$2x + y + 3z = 16$$

Solution

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 24 \\ 16 \end{pmatrix}$$

To start with, we observe that the pivot element

$a_{11} = 1 (\neq 0)$.

However, a glance at the first column reveals that the numerically largest element is 3 which is in second row. Hence *R12*

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Thus the given equation takes the form after partial pivoting

$$\begin{bmatrix} 3 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 24 \\ 7 \\ 16 \end{pmatrix}$$

Stage I (Reduction to upper triangular form):

$$\begin{bmatrix} 1 & 1 & \frac{4}{3} \\ 0 & -1 & \frac{1}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ -1 \end{bmatrix}$$

Stage II (Back substitution):

$z = 3$

$-y + 1 = 0$ or $y = 1$

$x + 1 + 4 = 8$ or $x = 3$

Example

Solve the following system of equations by Gaussian elimination method with partial pivoting

$$0x_1 + 4x_2 + 2x_3 + 8x_4 = 24$$
$$4x_1 + 10x_2 + 5x_3 + 4x_4 = 32$$
$$4x_1 + 5x_2 + 6.5x_3 + 2x_4 = 26$$
$$9x_1 + 4x_2 + 4x_3 + 0x_4 = 21$$

Solution

In matrix notation, the given system can be written as

$$\begin{bmatrix} 0 & 4 & 2 & 8 \\ 4 & 10 & 5 & 4 \\ 4 & 5 & 6.5 & 2 \\ 9 & 4 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 24 \\ 32 \\ 26 \\ 21 \end{bmatrix}$$

To start with, we observe that the pivot row, that is, the first row has a zero pivot element ($a11 = 0$). This row should be interchanged with any row following it, which on becoming a pivot row should not have a zero pivot element.

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While interchanging rows it is better to interchange the first and fourth rows, which is called partial pivoting and get,

$$\begin{bmatrix} 9 & 4 & 4 & 0 \\ 4 & 10 & 5 & 4 \\ 4 & 5 & 6.5 & 2 \\ 0 & 4 & 2 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 21 \\ 32 \\ 26 \\ 24 \end{bmatrix}$$

Stage I (Reduction to upper-triangular form):

$$\begin{bmatrix} 1 & \frac{4}{9} & \frac{4}{9} & 0 \\ 0 & 8.2222 & 3.2222 & 4 \\ 0 & 3.2222 & 4.7222 & 2 \\ 0 & 4 & 2 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2.333 \\ 22.6666 \\ 16.6666 \\ 24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{4}{9} & \frac{4}{9} & 0 \\ 0 & 1 & 0.3919 & 0.4865 \\ 0 & 0 & 3.4594 & 0.4324 \\ 0 & 0 & 0.4324 & 6.0540 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2.333 \\ 2.7568 \\ 7.78836 \\ 12.9728 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{4}{9} & \frac{4}{9} & 0 \\ 0 & 1 & 0.3919 & 0.4865 \\ 0 & 0 & 1 & 0.1250 \\ 0 & 0 & 0 & 5.999 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2.3333 \\ 2.7568 \\ 2.2500 \\ 11.9999 \end{bmatrix}$$

Stage II
Back substitution

$$x_1 = 1.0, \qquad x_2 = 1.0, \qquad x_3 = 2.0, \qquad x_4 = 2.0$$

Example

Solve the system of equations

$$\begin{aligned} 3x + y - z &= 3 \\ 2x - 8y + z &= -5 \\ x - 2y + 9z &= 8 \end{aligned}$$

using Gauss elimination method.

Solution
The given system is equivalent to

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$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & -8 & 1 \\ 1 & -2 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$$

$$\text{A} \qquad X = B$$

the arg *umented matrix is*

$$[A \mid B] = \left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array}\right]$$

now making A *as an upper triangular matrix*

$$\sim \left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 0 & \frac{-26}{3} & \frac{5}{3} & -7 \\ 0 & \frac{-7}{3} & \frac{28}{3} & 7 \end{array}\right] R_2 - \frac{2}{3}R_1, R_3 - \frac{1}{3}R_1$$

now choo sin *g* $\frac{-26}{3}$ *as the pivot from the* sec *ond column*,

$$\sim \left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 0 & \frac{-26}{3} & \frac{5}{3} & -7 \\ 0 & 0 & \frac{693}{78} & \frac{231}{26} \end{array}\right] R_2 - \frac{2}{3}R_1, R_3 - \frac{1}{3}R_1$$

from this we get

$$3x + y - z = 3$$

$$\frac{-26}{3}y + \frac{5}{3}z = -7$$

$$\frac{693}{78}z = \frac{231}{26}$$

now by back substitution $z = 1$

$$\frac{-26}{3}y = -7 - \frac{5}{3}(z) = -7 - \frac{5}{3}(1) = \frac{-26}{3} \Rightarrow y = 1$$

now $x = \frac{1}{3}[3 - y + z] = \frac{1}{3}[3 - 1 + 1] = 1$

so the solution is $x = y = z = 1$

Example

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Using Gauss elimination method, solve the system.

$3.15x - 1.96y + 3.85z = 12.95$

$2.13x + 5.12y - 2.89z = -8.61$

$5.92x + 3.05y + 2.15z = 6.88$

Solution

The given system is equivalent to

$$\begin{bmatrix} 3.15 & -1.96 & 3.85 \\ 2.13 & 5.12 & -2.89 \\ 5.92 & 3.05 & 2.15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12.95 \\ -8.61 \\ 6.88 \end{bmatrix}$$

$$A \qquad X = \quad \text{B}$$

$$[A \mid B] = \begin{bmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 2.13 & 5.12 & -2.89 & \mid -8.61 \\ 5.92 & 3.05 & 2.15 & 6.88 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 0 & 6.4453 & -5.4933 & \mid -17.3667 \\ 0 & 6.7335 & -5.0855 & -17.4578 \end{bmatrix} \quad R_2 - \frac{2.13}{3.15}R_1, R_3 - \frac{5.92}{3.15}R_1$$

choosing 6.4453 *as pivot*

$$\sim \begin{bmatrix} 3.15 & -1.96 & 3.85 & 12.95 \\ 0 & 6.4453 & -5.4933 & \mid -17.3667 \\ 0 & 0 & 0.6534 & 0.6853 \end{bmatrix} \quad R_3 - \frac{6.7335}{6.4453}R_1$$

form this, we get

$3.15x - 1.96y + 3.85z = 12.95$

$6.4453y - 5.4933z = -17.3667$

$0.6534z = 0.6853$

by backward substitution

$$z = \frac{0.6853}{0.6534} = 1.0488215$$

$$y = \frac{5.4933 - 17.3667}{6.4453} = -1.8005692$$

$$x = \frac{1.96y - 3.85z + 12.95}{3.15} = 1.708864$$

Example

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Solve the system of equation

$$x_1 + x_2 + x_3 + x_4 = 2$$
$$x_1 + x_2 + 3x_3 - 2x_4 = -6$$
$$2x_1 + 3x_2 - x_3 + 2x_4 = 7$$
$$x_1 + 2x_2 + x_3 - x_4 = -2$$

By using Gauss elimination method.

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$$x_1 + x_2 + x_3 + x_4 = 2$$
$$x_1 + x_2 + 3x_3 - 2x_4 = -6$$
$$2x_1 + 3x_2 - x_3 + 2x_4 = 7$$
$$x_1 + 2x_2 + x_3 - x_4 = -2$$

the given system in matrix form is

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & -2 \\ 2 & 3 & -1 & 2 \\ 1 & 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 7 \\ -2 \end{bmatrix}$$

$$A \qquad\qquad X = B$$

$$[A \mid B] = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 3 & -2 & -6 \\ 2 & 3 & -1 & 2 & 7 \\ 1 & 2 & 1 & -1 & -2 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & -3 & -8 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 1 & 0 & -2 & -4 \end{array}\right] R_2 - R_1, R_3 - 2R_1, R_4 - R_1$$

sin*ce the element in the* sec*ond column is zero so* int*erchanging the rows*

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 0 & 2 & -3 & -8 \\ 0 & 1 & 0 & -2 & -4 \end{array}\right] R_{23}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 0 & 2 & -3 & -8 \\ 0 & 1 & 3 & -2 & -4 \end{array}\right] R_4 - R_2$$

now the pivot is 2*, therefore*

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 0 & 2 & -3 & -8 \\ 0 & 0 & 0 & \frac{5}{2} & 5 \end{array}\right] R_4 - \frac{3}{2} R_3$$

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from this we get

$x_1 + x_2 + x_3 + x_4 = 2$

$x_2 - 3x_3 = 3$

$2x_3 - 3x_4 = -8$

$(\frac{5}{2})x_4 = 5$

now $x_4 = 2$

$x_3 = \frac{1}{2}(-8 + 3x_4) = \frac{1}{2}(-8 + 6) = -1$

now

$x_2 = 3 + 3x_3 = 3 - 3 = 0$

now from equation 1

$x_1 = 2 - x_2 - x_3 - x_4 = 2 - 0 + 1 - 2 = 1$

so $x_1 = 1, x_2 = 0, x_3 = -1, x_4 = 2$

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Solution of Linear System of Equations and Matrix Inversion

Gauss–Jordon Elimination Method

This method is a variation of Gaussian elimination method. In this method, the elements above and below the diagonal are simultaneously made zero. That is a given system is reduced to an equivalent diagonal form using elementary transformations. Then the solution of the resulting diagonal system is obtained. Sometimes, we normalize the pivot row with respect to the pivot element, before elimination. Partial pivoting is also used whenever the pivot element becomes zero.

Example

Solve the system of equations using Gauss-Jordan elimination method:

$$\left.\begin{aligned} x+2y+z&=8\\ 2x+3y+4z&=20\\ 4x+3y+2z&=16 \end{aligned}\right\}$$

Solution

In matrix notation, the given system can be written as

$$\begin{bmatrix}1&2&1\\2&3&4\\4&3&2\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}8\\20\\16\end{pmatrix}$$

$$\begin{bmatrix}1&2&1\\0&-1&2\\0&-5&-2\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}8\\4\\-16\end{pmatrix}$$ (-2) R1 +R2 and (-4) R1+R3

Now, we eliminate y from the first and third rows using the second row. Thus, we get

$$\begin{bmatrix}1&0&5\\0&-1&2\\0&0&-12\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}16\\4\\-36\end{pmatrix}$$

Before, eliminating z from the first and second row, normalize the third row with respect to the pivot element, we get

$$\begin{bmatrix}1&0&5\\0&-1&2\\0&0&1\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}16\\4\\3\end{pmatrix}$$

Using the third row of above Equation, and eliminating z from the first and second rows, we obtain

$$\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-2\\3\end{pmatrix}$$

The solution is

x =1, y =2, z =3.

Example

Solve the system of equation by using the Gauss Jordan elimination method

$10x+y+z=12$

$2x+10y+z=13$

$x+y+5z=7$

Solution

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$10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 5z = 7$

the given system

$$\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

the arg*umented matrix may be written as*

$$[A/B] = \begin{bmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -8 & -44 & -51 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix} R_1 - 9R_3$$

$$\sim \begin{bmatrix} 1 & -8 & -44 & -51 \\ 0 & 26 & 89 & 115 \\ 0 & 9 & 49 & 58 \end{bmatrix} R_2 - 2R_1, R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & -8 & -44 & -51 \\ 0 & 1 & 89 & 59 \\ 0 & 9 & 49 & 58 \end{bmatrix} R_2 - 3R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 420 & 421 \\ 0 & 1 & 58 & 59 \\ 0 & 0 & -473 & -473 \end{bmatrix} R_1 + 8R_2, R_3 - 9R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \frac{1}{473} R_3, R_1 - 420R_3, R_2 - 58R_2$$

thus the system reduces to reduced echlon form

so $x = y = z = 1$

Example

Solve the system of equations by Gauss Jordan method

$10x_1 + x_2 + x_3 = 12$

$x_1 + 10x_2 - x_3 = 10$

$x_1 - 2x_2 + 10x_3 = 9$

Solution

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$10x_1 + x_2 + x_3 = 12$

$x_1 + 10x_2 - x_3 = 10$

$x_1 - 2x_2 + 10x_3 = 9$

the matrix form of the given system may be written as

$$\begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & -1 \\ 1 & -2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \\ 9 \end{bmatrix}$$

the arg*umented mairix may be written as*

$$[A/B] = \begin{bmatrix} 10 & 1 & 1 & 12 \\ 1 & 10 & -1 & 10 \\ 1 & -2 & 10 & 9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -89 & 10 & -78 \\ 1 & 10 & -1 & 88 \\ 1 & -2 & 10 & 87 \end{bmatrix} R_1 - 9R_2$$

$$\sim \begin{bmatrix} 1 & -89 & 10 & -78 \\ 0 & 9 & -1 & 8 \\ 0 & 1 & 0 & 1 \end{bmatrix} R_2 - R_1, R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & -89 & 10 & -78 \\ 0 & 9 & -1 & 8 \\ 0 & 1 & 0 & 1 \end{bmatrix} \frac{R_2}{99}, \frac{R_3}{87}$$

$$\sim \begin{bmatrix} 1 & -89 & 10 & -78 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} R_2 - 8R_3$$

$$\sim \begin{bmatrix} 1 & 0 & -79 & -78 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_1 - 89R_2, R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} R_1 + 79R_3, R_2 + R_3$$

so the system gives the values of all the three var *iables*

$x_1 = x_2 = x_3 = 1$

Example

Solve the system of equations by using Gauss Jordan method.

$x + 2y + z - w = -2$

$2x + 3y - z + 2w = 7$

$x + y + 3z - 2w = -6$

$x + y + z + w = 2$

Solution

$x + 2y + z - w = -2$

$2x + 3y - z + 2w = 7$

$x + y + 3z - 2w = -6$

$x + y + z + w = 2$

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the system may be written as

$$\begin{bmatrix} 1 & 2 & 1 & -1 \\ 2 & 3 & -1 & 2 \\ 1 & 1 & 3 & -2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ -6 \\ 2 \end{bmatrix}$$

the arg*umented matrix may be written as*

$$[A/B] = \left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & -2 \\ 2 & 3 & -1 & 2 & 7 \\ 1 & 1 & 3 & -2 & -6 \\ 1 & 1 & 1 & 1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & -2 \\ 0 & 1 & 3 & -4 & 11 \\ 0 & -1 & 2 & -1 & -4 \\ 0 & -1 & 0 & 2 & 4 \end{array}\right] \quad R_2 - 2R_1, R_3 - R_1, R_4 - R_1$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -5 & 7 & 20 \\ 0 & 1 & 3 & -4 & -11 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 3 & -2 & -7 \end{array}\right] \quad R_1 - 2R_2, \frac{1}{5}(R_3 + R_2), R_4 + R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 + 5R_3, R_2 - 3R_3, R_4 - 3R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 - 2R_4, R_2 + R_4, R_3 + R_4$$

the system maybe written as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$$

the values of all the ariables are

$x = 1, y = 0, z = -1, w = 2$

Crout's ReductionMethod

Here the coefficient matrix [A] of the system of equations is decomposed into the product of two matrices [L] and [U], where [L] is a lower-triangular matrix and [U] is an upper-triangular matrix with 1's on its main diagonal.

For the purpose of illustration, consider a general matrix in the form

$[L][U] = [A]$

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$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The sequence of steps for getting [L] and [U] are given below:
Step I: Multiplying all the rows of [L] by the first column of [U], we get

$l_{11} = a_{11}, \qquad l_{21} = a_{21}, \qquad l_{31} = a_{31}$
Thus, we observe that the first column of [L] is same as the first column of [A].

Step II: Multiplying the first row of [L] by the second and third columns of [U], we obtain

$l_{11}u_{12} = a_{12}, \qquad l_{11}u_{13} = a_{13}$
Or

$$u_{12} = \frac{a_{12}}{l_{11}}, \qquad u_{13} = \frac{a_{13}}{l_{11}}$$

Thus, the first row of [U] is obtained. We continue this process, getting alternately the column of [L] and a row of [U].
Step III: Multiply the second and third rows of [L] by the second column of [U] to get

$$l_{21}u_{12} + l_{22} = a_{22}, \qquad l_{31}u_{12} + l_{32} = a_{32}$$

This gives

$l_{22} = a_{22} - l_{21}u_{12}, \qquad l_{32} = a_{32} - l_{31}u_{12}$
Step IV: Now, multiply the second row of [L] by the third column of [U] which yields

$l_{21}u_{13} + l_{22}u_{23} = a_{23}$

$$u_{23} = \frac{a_{23} - l_{21}u_{13}}{l_{22}}$$

Step V: Lastly, we multiply the third row of [L] by the third column of [U] and get

$l_{31}u_{13} + l_{32}u_{23} + l_{33} = a_{33}$
This gives

$l_{33} = a_{33} - l_{33}u_{13} - l_{32}u_{23}$
Thus, the above five steps determine [L] and [U].
This algorithm can be generalized to any linear system of order n.
Consider a system of equations

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}\right\}$$

In matrix notation as [A](X) = (B). Let [A] = [L] [U], then we get,
$[L][U](X) = (B)$
Substituting [U] (X) = (Z) in Eq. we obtain [L] (Z) = (B)

$$\left.\begin{aligned} l_{11}z_1 &= b_1 \\ l_{21}z_1 + l_{22}z_2 &= b_2 \\ l_{31}z_1 + l_{32}z_2 + l_{33}z_3 &= b_3 \end{aligned}\right\}$$

Having computed z1, z2 and z3, we can compute x1, x2, and x3 from equation [U] (X) = (Z) or from

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

This method is also known as Cholesky reduction method. This technique is widely used in the numerical solutions of partial differential equations.

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This method is very popular from computer programming point of view, since the storages space reserved for matrix [A] can be used to store the elements of [L] and [U] at the end of computation.
This method fails if any

$a_{ii} = 0$

Example

Solve the following system of equations by Crout's reduction method

$5x_1 - 2x_2 + x_3 = 4$

$7x_1 + x_2 - 5x_3 = 8$

$3x_1 + 7x_2 + 4x_3 = 10$

Solution

Let the coefficient matrix [A] be written as [L] [U]. Thus,

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

Step I: Multiply all the rows of [L] by the first column of [U], we get

$l_{11} = 5, \quad l_{21} = 7, \quad l_{31} = 3$

Step II: Multiply the first row of [*L*] by the second and third columns of [*U*], we have

$l_{11}u_{12} = -2, \quad l_{11}u_{13} = 1$

$u_{12} = -\frac{2}{5}, u_{13} = \frac{1}{5}$

STEP III: Multiply the 2nd and 3rd rows of [*L*] by the 2nd column of [*U*], we get

$$\left.\begin{aligned} l_{21}u_{12} + l_{22} = 1 \quad or \quad l_{22} = 1 + \frac{14}{5} = \frac{19}{5} \\ l_{31}u_{12} + l_{32} = 7 \quad or \quad l_{32} = 7 + \frac{6}{5} = \frac{41}{5} \end{aligned}\right\}$$

STEP IV: Multiply the 2nd row of [L] by the 3rd column of [U]

$l_{21}u_{13} + l_{22}u_{23} = -5$

$\frac{19}{5}u_{23} = -5 - \frac{7}{5}$

$u_{23} = -\frac{32}{19}$

STEP V:Finally, multiply the 3rd row of [L] with the 3rd column of [U], we obtain

$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 4$

$l_{33} = \frac{327}{19}$

Thus, the given system of equations takes the form [*L*][*U*][*X*] = (*B*).

$$\begin{bmatrix} 5 & 0 & 0 \\ 7 & \frac{19}{5} & 0 \\ 3 & \frac{41}{5} & \frac{327}{19} \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{5} & \frac{1}{5} \\ 0 & 1 & -\frac{32}{19} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

That is,

$$\begin{bmatrix} 5 & 0 & 0 \\ 7 & \frac{19}{5} & 0 \\ 3 & \frac{41}{5} & \frac{327}{19} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

Let [U](X) = (Z), then

$[L](Z) = (4 \quad 8 \quad 10)^T$

Or

$$\begin{bmatrix} 5 & 0 & 0 \\ 7 & \frac{19}{5} & 0 \\ 3 & \frac{41}{5} & \frac{327}{19} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

Which gives utilizing these values of z, the Eq becomes

By back substitution method, we obtain

$$x_3 = \frac{46}{327}, \qquad x_2 = \frac{284}{327}, \qquad x_1 = \frac{366}{327}$$

This is the required solution.

Example

Solve the following system

$$2x - 3y + 10z = 3$$
$$-x + 4y + 2x = 20$$
$$5x + 2y + z = -12$$

Solution

$$2x - 3y + 10z = 3$$
$$-x + 4y + 2x = 20$$
$$5x + 2y + z = -12$$

the given system is $AX = B$

$$A = \begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \mathrm{B} = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

let $LU = A$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix} = \begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix}$$

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here

$u_{11} = 2,\ u_{12} = -3\ \ u_{13} = 10$

$l_{21}u_{11} = -1 \ \Rightarrow l_{21} = \frac{-1}{2}$

$l_{31}u_{11} = 5 \quad \Rightarrow l_{31} = \frac{5}{2}$

$l_{21}u_{12} + u_{22} = 4 \quad \Rightarrow u_{22} = 4 - l_{21}u_{12} = 4 - (\frac{-1}{2})(-3) = \frac{5}{2}$

$l_{21}u_{13} + u_{23} = 2 \quad \Rightarrow u_{23} = 2 - l_{21}u_{13} = 2 - (\frac{-1}{2})10 = 7$

$l_{31}u_{12} + l_{32}u_{22} = 5 \Rightarrow l_{32} = \frac{1}{u_{22}}[5 - l_{31}u_{12}] = \frac{19}{5}$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} \Rightarrow u_{33} = 1 - l_{31}u_{13} - l_{32}u_{23} = \frac{253}{5}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-1}{2} & 1 & 0 \\ \frac{5}{2} & \frac{19}{5} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & -3 & 10 \\ 0 & \frac{5}{2} & 7 \\ 0 & 0 & \frac{-253}{5} \end{bmatrix}$$

let $UX = Y$ *where* $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$, *then* $LY = B$,

$$i.e \begin{bmatrix} 1 & 0 & 0 \\ \frac{-1}{2} & 1 & 0 \\ \frac{5}{2} & \frac{19}{5} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{-1}{2} & 1 & 0 \\ \frac{5}{2} & \frac{19}{5} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

now eqn (1) *implies*

$y_1 = 3$

$\frac{-1}{2}y_1 + y_2 = 20 \Rightarrow y_2 = \frac{43}{2}$

$\frac{5}{2}y_1 + \frac{19}{5}y_2 + y_3 = -12 \ \Rightarrow y_3 = \frac{-506}{5}$

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$2x-3y+10z=3\ ,\frac{5}{2}y+7z=\frac{43}{2},-\frac{253}{5}z=\frac{-506}{3}$

by back substitution $x=-4, y=3$ *and* $z=2$

Example

Solve the following system

$x+3y+4z=4$

$x+4y+3z=-2$

$x+3y+4z=1$

Solution

$x+3y+4z=4$

$x+4y+3z=-2$

$x+3y+4z=1$

the given system is $AX=B$

$$A=\begin{bmatrix}1 & 3 & 8\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\quad \mathrm{B}=\begin{bmatrix}4\\ -2\\ 1\end{bmatrix}$$

let $LU=A$

$$L=\begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix}\quad U=\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

$$\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23}\\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33}\end{bmatrix}=\begin{bmatrix}1 & 3 & 8\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$$

here

$u_{11}=1,\ u_{12}=3\ \ u_{13}=8$

$l_{21}u_{11}=-1\ \Rightarrow l_{21}=1$

$l_{31}u_{11}=1\ \ \Rightarrow l_{31}=1$

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$l_{21}u_{12}+u_{22}=4 \quad \Rightarrow u_{22}=4-l_{21}u_{12}=4-(\frac{-1}{2})(-3)=\frac{5}{2}$

$l_{21}u_{13}+u_{23}=2 \quad \Rightarrow u_{23}=4-l_{21}u_{13}=4-(1)3=1$

$l_{31}u_{12}+l_{32}u_{22}=3 \Rightarrow l_{32}=\frac{1}{u_{22}}[3-l_{31}u_{12}]=0$

$l_{31}u_{13}+l_{32}u_{23}+u_{33} \Rightarrow u_{33}=4-l_{31}u_{13}-l_{32}u_{23}=-4$

$$L=\begin{bmatrix}1&0&0\\1&1&0\\1&0&1\end{bmatrix} \qquad U=\begin{bmatrix}1&3&8\\0&1&-5\\0&0&-4\end{bmatrix}$$

let $UX=Y$ *where* $y=\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}$, *then* $LY=B$,

$$i.e\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}4\\-2\\1\end{bmatrix}$$

and

$$\begin{bmatrix}1&0&0\\\frac{-1}{2}&1&0\\\frac{5}{2}&\frac{19}{5}&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}$$

now eqn (1) *implies*

$y_1=4$

$y_2=-2$

$y_1+y_3=1 \Rightarrow y_3=1-y_1=-3$

we also have

$x+3y+8z=4$

$y-5z=-2$

$-4z=-3$

by back substitutiton

$z=\frac{3}{4}$

$y=-2+5z=-2+5(\frac{3}{4})=\frac{7}{4}$

$x=4-3y-8z=4-3(\frac{7}{4})-8(\frac{3}{4})=\frac{29}{4}$

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Solution of Linear System of Equationsand Matrix Inversion

Jacobi's Method

This is an iterative method, where initial approximate solution to a given system of equations is assumed and is improved towards the exact solution in an iterative way.

In general, when the coefficient matrix of the system of equations is a sparse matrix (many elements are zero), iterative methods have definite advantage over direct methods in respect of economy of computer memory

Such sparse matrices arise in computing the numerical solution of partial differential equations

Let us consider

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \quad \vdots \qquad\qquad \vdots \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_1 \end{array}\right\}$$

In this method, we assume that the coefficient matrix [A] is strictly diagonally dominant, that is, in each row of [A] the modulus of the diagonal element exceeds the sum of the off-diagonal elements.

We also assume that the diagonal element do not vanish. If any diagonal element vanishes, the equations can always be rearranged to satisfy this condition.

Now the above system of equations can be written as

$$\left.\begin{array}{l} x_1 = \dfrac{b_1}{a_{11}} - \dfrac{a_{12}}{a_{11}}x_2 - \cdots - \dfrac{a_{1n}}{a_{11}}x_n \\ x_2 = \dfrac{b_2}{a_{22}} - \dfrac{a_{21}}{a_{22}}x_1 - \cdots - \dfrac{a_{2n}}{a_{22}}x_n \\ \vdots \quad \vdots \quad \vdots \qquad\qquad \vdots \\ x_n = \dfrac{b_n}{a_{nn}} - \dfrac{a_{n1}}{a_{nn}}x_1 - \cdots - \dfrac{a_{n(n-1)}}{a_{nn}}x_{n-1} \end{array}\right\}$$

We shall take this solution vector $(x_1, x_2, ..., x_n)^T$ as a first approximation to the exact solution of system. For convenience, let us denote the first approximation vector by $(x_1^{(1)}, x_2^{(1)}, ..., x_n^{(1)})$ got after taking as an initial starting vector.

Substituting this first approximation in the right-hand side of system, we obtain the second approximation to the given system in the form

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$$\left.\begin{aligned} x_1^{(2)} &= \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2^{(1)} - \cdots - \frac{a_{1n}}{a_{11}}x_n^{(1)} \\ x_2^{(2)} &= \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1^{(1)} - \cdots - \frac{a_{2n}}{a_{22}}x_n^{(1)} \\ \vdots \quad &\vdots \qquad \vdots \qquad\qquad \vdots \\ x_n^{(2)} &= \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1^{(1)} - \cdots - \frac{a_{n(n-1)}}{a_{nn}}x_{n-1}^{(1)} \end{aligned}\right\}$$

This second approximation is substituted into the right-hand side of Equations and obtain the third approximation and so on.

This process is repeated and *(r+1)th* approximation is calculated

$$\left.\begin{aligned} x_1^{(r+1)} &= \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2^{(r)} - \cdots - \frac{a_{1n}}{a_{11}}x_n^{(r)} \\ x_2^{(r+1)} &= \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1^{(r)} - \cdots - \frac{a_{2n}}{a_{22}}x_n^{(r)} \\ \vdots \quad &\vdots \qquad \vdots \qquad\qquad \vdots \\ x_n^{(r+1)} &= \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1^{(r)} - \cdots - \frac{a_{n(n-1)}}{a_{nn}}x_{n-1}^{(r)} \end{aligned}\right\}$$

Briefly, we can rewrite these Equations as

$$x_i^{(r+1)} = \frac{b_i}{a_{ii}} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{a_{ij}}{a_{ii}}x_j^{(r)},$$

$$r = 1, 2, ..., \qquad i = 1, 2, ..., n$$

It is also known as method of simultaneous displacements,

since no element of $x_i^{(r+1)}$ is used in this iteration until every element is computed.

A sufficient condition for convergence of the iterative solution to the exact solution is

$|a_{ii}| > \sum_{\substack{j=1 \\ j\neq 1}}^{n} |a_{ij}|, \qquad i = 1, 2, ..., n$ When this condition (diagonal dominance) is true, Jacobi's method converges

Example

Find the solution to the following system of equations using Jacobi's iterative method for the first five iterations:

$83x + 11y - 4z = 95$

$7x + 52y + 13z = 104$

$3x + 8y + 29z = 71$

Solution

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$$\left.\begin{aligned} x &= \frac{95}{83} - \frac{11}{83}y + \frac{4}{83}z \\ y &= \frac{104}{52} - \frac{7}{52}x - \frac{13}{52}z \\ z &= \frac{71}{29} - \frac{3}{29}x - \frac{8}{29}y \end{aligned}\right\}$$

Taking the initial starting of solution vector as $(0,0,0)^T$, from Eq. ,we have the first approximation as

$$\begin{pmatrix} x^{(1)} \\ y^{(1)} \\ z^{(1)} \end{pmatrix} = \begin{pmatrix} 1.1446 \\ 2.0000 \\ 2.4483 \end{pmatrix}$$

Now, using Eq. ,the second approximation is computed from the equations

$$\left.\begin{aligned} x^{(2)} &= 1.1446 - 0.1325y^{(1)} + 0.0482z^{(1)} \\ y^{(2)} &= 2.0 - 0.1346x^{(1)} - 0.25z^{(1)} \\ z^{(2)} &= 2.4483 - 0.1035x^{(1)} - 0.2759y^{(1)} \end{aligned}\right\}$$

Making use of the last two equations we get the second approximation as

$$\begin{pmatrix} x^{(2)} \\ y^{(2)} \\ z^{(2)} \end{pmatrix} = \begin{pmatrix} 0.9976 \\ 1.2339 \\ 1.7424 \end{pmatrix}$$

Similar procedure yields the third, fourth and fifth approximations to the required solution and they are tabulated as below;

Variables			
Iteration number *r*	*x*	*y*	*z*
1	1.1446	2.0000	2.4483
2	0.9976	1.2339	1.7424
3	1.0651	1.4301	2.0046

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4	1.0517	1.3555	1.9435
5	1.0587	1.3726	1.9655

Example

Solve the system by jacobi's iterative method

$8x-3y+2z=20$

$4x+11y-z=33$

$6x+3y+12z=35$

(Perform only four iterations)

Solution

Consider the given system as

$8x-3y+2z=20$

$4x+11y-z=33$

$6x+3y+12z=35$

the system is diagonally do min *ant*

$$x=\frac{1}{8}\left[20+3y-2z\right]$$

$$y=\frac{1}{11}\left[33-4x+z\right]$$

$$z=\frac{1}{12}\left[35-6x-3y\right]$$

we start with an initial aproximation $x_0=y_0=z_0=0$

substituting these

first iteration

$$x_1=\frac{1}{8}\left[20+3(0)-2(0)\right]=2.5$$

$$y_1=\frac{1}{11}\left[33-4(0)+0\right]=3$$

$$z_1=\frac{1}{12}\left[35-6(0)-3(0)\right]=2.916667$$

Second iteration

$$x_2=\frac{1}{8}\left[20+3(3)-2(2.9166667)\right]=2.895833$$

$$y_2=\frac{1}{11}\left[33-4(2.5)+2.9166667\right]=2.3560606$$

$$z_2=\frac{1}{12}\left[35-6(2.5)-3(3)\right]=0.9166666$$

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third iteration

$$x_3 = \frac{1}{8}\left[20 + 3(2.3560606) - 2(0.9166666)\right] = 3.1543561$$

$$y_3 = \frac{1}{11}\left[33 - 4(2.8958333) + 0.9166666\right] = 2.030303$$

$$z_3 = \frac{1}{12}\left[35 - 6(2.8958333) - 3(2.3560606)\right] = 0.8797348$$

fourth iteration

$$x_4 = \frac{1}{8}\left[20 + 3(2.030303) - 2(0.8797348)\right] = 3.0419299$$

$$y_4 = \frac{1}{11}\left[33 - 4(3.1543561) + 0.8797348\right] = 1.9329373$$

$$z_4 = \frac{1}{12}\left[35 - 6(3.1543561) - 3(2.030303)\right] = 0.8319128$$

Example

Solve the system by jacobi's iterative method

$$3x + 4y + 15z = 54.8$$

$$x + 12y + 3z = 39.66$$

$$10x + y - 2z = 7.74$$

(Perform only four iterations)

Solution

Consider the given system as

$$3x + 4y + 15z = 54.8$$

$$x + 12y + 3z = 39.66$$

$$10x + y - 2z = 7.74$$

the system is not diagonally do min *ant we rearrange the system*

$$10x + y - 2z = 7.74$$

$$x + 12y + 3z = 39.66$$

$$3x + 4y + 15z = 54.8$$

$$x = \frac{1}{10}\left[7.74 - y + 2z\right]$$

$$y = \frac{1}{12}\left[39.66 - x - 3z\right]$$

$$z = \frac{1}{15}\left[54.8 - 3x - 4y\right]$$

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we start with an initial aproximation $x_0 = y_0 = z_0 = 0$

substituting these

first iteration

$$x_1 = \frac{1}{10}\left[7.74 - (0) + 2(0)\right] = 0.774$$

$$y_1 = \frac{1}{12}\left[39.66 - (0) - 3(0)\right] = 1.1383333$$

$$z_1 = \frac{1}{15}\left[54.8 - 3(0) - 4(0)\right] = 3.6533333$$

Second iteration

$$x_2 = \frac{1}{10}\left[7.74 - 1.1383333 + 2(3.6533333)\right] = 1.3908333$$

$$y_2 = \frac{1}{12}\left[39.66 - 0.774 - 3(3.6533333)\right] = 2.3271667$$

$$z_2 = \frac{1}{15}\left[54.8 - 3(0.774) - 4(1.1383333)\right] = 3.1949778$$

third iteration

$$x_3 = \frac{1}{10}\left[7.74 - 2.3271667 + 2(3.1949778)\right] = 1.1802789$$

$$y_3 = \frac{1}{12}\left[39.66 - 1.3908333 - 3(3.1949778)\right] = 2.3903528$$

$$z_3 = \frac{1}{15}\left[54.8 - 3(1.3908333) - 4(2.3271667)\right] = 2.7545889$$

fourth iteration

$$x_4 = \frac{1}{10}\left[7.74 - 2.5179962 + 2(2.7798501)\right] = 1.0781704$$

$$y_4 = \frac{1}{12}\left[39.66 - 1.1802789 - 3(2.7545889)\right] = 2.51779962$$

$$z_4 = \frac{1}{15}\left[54.8 - 3(1.1802789) - 4(2.3903528)\right] = 2.7798501$$

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Solution of Linear System of Equations and Matrix Inversion
Gauss–Seidel Iteration Method

It is another well-known iterative method for solving a system of linear equations of the form

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \quad \vdots \quad & \quad \vdots \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}\right\}$$

In Jacobi's method, the $(r+1)th$ approximation to the above system is given by Equations

$$\left.\begin{aligned} x_1^{(r+1)} &= \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2^{(r)} - \cdots - \frac{a_{1n}}{a_{11}}x_n^{(r)} \\ x_2^{(r+1)} &= \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1^{(r)} - \cdots - \frac{a_{2n}}{a_{22}}x_n^{(r)} \\ \vdots \quad \vdots \quad & \quad \vdots \quad \vdots \\ x_n^{(r+1)} &= \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1^{(r)} - \cdots - \frac{a_{n(n-1)}}{a_{nn}}x_{n-1}^{(r)} \end{aligned}\right\}$$

Here we can observe that no element of $x_i^{(r+1)}$ replaces $x_i^{(r)}$ entirely for the next cycle of computation.

In Gauss-Seidel method, the corresponding elements of $x_i^{(r+1)}$ replaces those of $x_i^{(r)}$ as soon as they become available.

Hence, it is called the method of successive displacements. For illustration consider

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \quad \vdots \quad & \quad \vdots \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}\right\}$$

In Gauss-Seidel iteration, the (r + 1)th approximation or iteration is computed from:

$$\left.\begin{aligned} x_1^{(r+1)} &= \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2^{(r)} - \cdots - \frac{a_{1n}}{a_{11}}x_n^{(r)} \\ x_2^{(r+1)} &= \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1^{(r+1)} - \cdots - \frac{a_{2n}}{a_{22}}x_n^{(r)} \\ \vdots \quad \vdots \quad & \quad \vdots \quad \vdots \\ x_n^{(r+1)} &= \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1^{(r+1)} - \cdots - \frac{a_{n(n-1)}}{a_{nn}}x_{n-1}^{(r+1)} \end{aligned}\right\}$$

Thus, the general procedure can be written in the following compact form

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$$x_i^{(r+1)} = \frac{b_i}{a_{ii}} - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{(r+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{(r)} \qquad \text{for all } i = 1,2,...,n \text{ and } r = 1,2,...$$

To describe system in the first equation, we substitute the r-th approximation into the right-hand side and denote the result by $x_1^{(r+1)}$. In the second equation, we substitute $(x_1^{(r+1)}, x_3^{(r)}, ..., x_n^{(r)})$ and denote the result by $x_2^{(r+1)}$

In the third equation, we substitute $(x_1^{(r+1)}, x_2^{(r+1)}, x_4^{(r)}, ..., x_n^{(r)})$ and denote the result by $x_3^{(r+1)}$, and so on. This process is continued till we arrive at the desired result. For illustration, we consider the following example :

Note

The difference between jacobi's method and gauss Seidel method is that in jacobi's method the approximation calculated are used in the next iteration for next approximation but in Gauss-seidel method the new approximation calculated is instantly replaced by the previous one.

Example

Find the solution of the following system of equations using Gauss-Seidel method and perform the first five iterations:

$$\begin{aligned} 4x_1 - x_2 - x_3 &= 2 \\ -x_1 + 4x_2 - x_4 &= 2 \\ -x_1 + 4x_3 - x_4 &= 1 \\ -x_2 - x_3 + 4x_4 &= 1 \end{aligned}$$

Solution

The given system of equations can be rewritten as

$$\left.\begin{aligned} x_1 &= 0.5 + 0.25x_2 + 0.25x_3 \\ x_2 &= 0.5 + 0.25x_1 + 0.25x_4 \\ x_3 &= 0.25 + 0.25x_1 + 0.25x_4 \\ x_4 &= 0.25 + 0.25x_2 + 0.25x_3 \end{aligned}\right\}$$

Taking $x_2 = x_3 = x_4 = 0$ on the right-hand side of the first equation of the system , we get $x_1^{(1)} = 0.5$. Taking $x_3 = x_4 = 0$ and the current value of x_1, we get from the 2nd equation of the system

$$x_2^{(1)} = 0.5 + (0.25)(0.5) + 0 = 0.625$$

Further, we take *x4* = 0 and the current value of *x1* we obtain from the third equation of the system

$$\begin{aligned} x_3^{(1)} &= 0.25 + (0.25)(0.5) + 0 \\ &= 0.375 \end{aligned}$$

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Now, using the current values of *x2* and *x3* the fourth equation of system gives

$$x_4^{(1)} = 0.25 + (0.25)(0.625)$$
$$+(0.25)(0.375) = 0.5$$

The Gauss-Seidel iterations for the given set of equations can be written as

$$x_1^{(r+1)} = 0.5 + 0.25x_2^{(r)} + 0.25x_3^{(r)}$$
$$x_2^{(r+1)} = 0.5 + 0.25x_1^{(r+1)} + 0.25x_4^{(r)}$$
$$x_3^{(r+1)} = 0.25 + 0.25x_1^{(r+1)} + 0.25x_4^{(r)}$$
$$x_4^{(r+1)} = 0.25 + 0.25x_2^{(r+1)} + 0.25x_3^{(r+1)}$$

Now, by Gauss-Seidel procedure, the 2nd and subsequent approximations can be obtained and the sequence of the first five approximations are tabulated as below:

	Variables			
Iteration number r	x1	x2	x3	x4
1	0.5	0.625	0.375	0.5
2	0.75	0.8125	0.5625	0.59375
3	0.84375	0.85938	0.60938	0.61719
4	0.86719	0.87110	0.62110	0.62305
5	0.87305	0.87402	0.62402	0.62451

Example

Solve the system by Gauss-Seidel iterative method

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35$$

(Perform only four iterations)

Solution

Consider the given system as

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$8x-3y+2z=20$

$4x+11y-z=33$

$6x+3y+12z=35$

the system is diagonally do min *ant*

$$x=\frac{1}{8}\left[20+3y-2z\right]$$

$$y=\frac{1}{11}\left[33-4x+z\right]$$

$$z=\frac{1}{12}\left[35-6x-3y\right]$$

we start with an initial aproximation $x_0=y_0=z_0=0$

substituting these

first iteration

$$x_1=\frac{1}{8}\left[20+3(0)-2(0)\right]=2.5$$

$$y_1=\frac{1}{11}\left[33-4(2.5)+0\right]=2.0909091$$

$$z_1=\frac{1}{12}\left[35-6(2.5)-3(2.0909091)\right]=1.1439394$$

Second iteration

$$x_2=\frac{1}{8}\left[20+3y_1-z_1\right]=\frac{1}{8}\left[20+3(2.0909091)-2(1.1439394)\right]=2.9981061$$

$$y_2=\frac{1}{11}\left[33-4x_2+z_1\right]=\frac{1}{11}\left[33-4(2.9981061)+1.1439394\right]=2.0137741$$

$$z_2=\frac{1}{12}\left[35-6x_2-3y_2\right]=\frac{1}{12}\left[35-6(2.9981061)-3(2.0137741)\right]=0.9141701$$

third iteration

$$x_3=\frac{1}{8}\left[20+3(2.0137741)-2(0.9141701)\right]=3.0266228$$

$$y_3=\frac{1}{11}\left[33-4(3.0266228)+0.9141701\right]=1.9825163$$

$$z_3=\frac{1}{12}\left[35-6(3.0266228)-3(1.9825163)\right]=0.9077262$$

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fourth iteration

$$x_4 = \frac{1}{8}\left[20 + 3(1.9825163) - 2(0.9077262)\right] = 3.0165121$$

$$y_4 = \frac{1}{11}\left[33 - 4(3.0165121) + 0.9077262\right] = 1.9856071$$

$$z_4 = \frac{1}{12}\left[35 - 6(3.0165121) - 3(1.9856071)\right] = 0.8319128$$

Example

Solve the system by suing Gauss-seidel iteration method

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

Solution

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

the given system is diagonally do min *ant so we will make it diagonaaly do* min *ant by iterchanaginhg the equations*

$$28x + 4y - z = 32$$
$$2x + 17y + 4z = 35$$
$$x + 3y + 10z = 24$$

hence we can apply Gauss – Seidel method
from the above equations

$$x = \frac{1}{28}[32 - 4y + z]$$

$$y = \frac{1}{17}[35 - 2x - 4z]$$

$$z = \frac{1}{10}[24 - x - 3y]$$

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First approximation

putting $y = z = 0$

$$x_1 = \frac{1}{28}[32] = 1.1428571$$

puting $x = 1.1428571 \quad , z = 0$

$$y_1 = \frac{1}{17}[35 - 2(1.1428571) - 4(0)] = 1.9243697$$

putting $x = 1.1428571$, $y = 1.9243697$

$$z_1 = \frac{1}{10}[24 - 1.1428571 - 3(1.9243697)] = 1.7084034$$

Second iteration

$$x_2 = \frac{1}{28}[32 - 4(1.9243697) + 1.7084034] = 0.9289615$$

$$y_2 = \frac{1}{17}[35 - 2(0.9289615) - 4(1.7084034)] = 1.5475567$$

$$z_2 = \frac{1}{10}[24 - 0.9289615 - 3(1.5475567)] = 1.8408368$$

third iteration

$$x_3 = \frac{1}{28}[32 - 4(1.5475567) + 1.8428368] = 0.9875932$$

$$y_3 = \frac{1}{17}[35 - 2(0.9875932) - 4(1.8428368)] = 1.5090274$$

$$z_3 = \frac{1}{10}[24 - 0.9875932 - 3(1.5090274)] = 1.8485325$$

fourth iteration

$$x_4 = \frac{1}{28}[32 - 4(1.5090274) + 1.8485325] = 0.9933008$$

$$y_4 = \frac{1}{17}[35 - 2(0.9933008) - 4(1.8428368)] = 1.5070158$$

$$z_4 = \frac{1}{10}[24 - 0.9933008 - 3(1.5070158)] = 1.8485652$$

Example

Using Gauss-Seidel iteration method, solve the system of the equation.

$$10x - 2y - z - w = 3$$

$$-2x + 10y - z - w = 15$$

$$-x - y + 10z - 2w = 27$$

$$-x - y - 2z + 10w = -9$$

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(Perform only four iterations)

Solution

$10x-2y-z-w=3$

$-2x+10y-z-w=15$

$-x-y+10z-2w=27$

$-x-y-2z+10w=-9$

it is diagonally do min *anat and we may write eqaution as*

$$x=\frac{1}{10}[3+2y+z+w]$$

$$y=\frac{1}{10}[15+2x+z+w]$$

$$z=\frac{1}{10}[27+x+y+2w]$$

$$w=\frac{1}{10}[-9+x+y+2z]$$

first approximation

putting $y=z=w=0$ *on RHS of* (1), *we get*

$x_1=0.3$

$$y_1=\frac{1}{10}[15+2(0.3)]=1.56$$

putting $x=0.3, y=1.56$ *and* $w=0$

$$z_1=\frac{1}{10}[27+0.3+1.56]=2.886$$

putting $x=0.3, y=1.56$ *and* $z=2.886$

$$w_1=\frac{1}{10}[-9+0.3+1.56+2(2.886)]=-0.1368$$

sec*ond iteration*

$$x_2=\frac{1}{10}[3+2(1.56)+2.886-0.1368]=0.88692$$

$$y_2=\frac{1}{10}[15+2(0.88692)+2.886-0.1368]=1.952304$$

$$z_2=\frac{1}{10}[27+0.88692+1.952304+2(-0.1368)]=2.9565624$$

$$w_2=\frac{1}{10}[-9+0.88692+1.952304+2(2.9565624)]=-0.0247651$$

third iteration

$$x_3=\frac{1}{10}[3+2(1.952304)+2.9565624-0.0.0247651]=0.9836405$$

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$$y_3 = \frac{1}{10}[15 + 2(0.9836405) + 2.9565624 - 0.0247651] = 1.9899087$$

$$z_3 = \frac{1}{10}[27 + 0.9836405 + 1.9899087 + 2(-0.0247651)] = 2.9924019$$

$$w_3 = \frac{1}{10}[-9 + 0.983405 + 1.9899087 + 2(2.9924019)] = -0.0041647$$

fourth iteration

$$x_4 = \frac{1}{10}[3 + 2(1.9899087) + 2.9924019 - 0.0041647] = 0.9968054$$

$$y_4 = \frac{1}{10}[15 + 2(0.9968054) + 2.9924019 - 0.0041647] = 1.9981848$$

$$z_4 = \frac{1}{10}[27 + 0.9968054 + 1.9981848 + 2(-0.0041647)] = 2.9986661$$

$$w_4 = \frac{1}{10}[-9 + 0.9968054 + 1.9981848 + 2(2.9986661)] = -0.0007677$$

Note

When to stop the iterative processes ,we stop the iterative process when we get the required accuracy means if your are asked that find the accurate up to four places of decimal then we will simply perform up to that iteration after which we will get the required accuracy. If we calculate the root of the equation and its consecutive values are 1.895326125, 1.916366125, 1.919356325, 1.919326355, 1.919327145, 1.919327128
Here the accuracy up to seven places of decimal is achieved so if you are asked to acquire the accuracy up to six places of decimal then we will stop here .
But in the solved examples only some iteration are carried out and accuracy is not considered here.

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Solution of Linear System of Equations and Matrix Inversion

Relaxation Method

This is also an iterative method and is due to Southwell.To explain the details, consider again the system of equations

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{array}\right\}$$

Let

$$X^{(p)} = (x_1^{(p)}, x_2^{(p)}, ..., x_n^{(p)})^T$$

be the solution vector obtained iteratively after p-th iteration. If $R_i^{(p)}$ denotes the residual of the i-th equation of system given above , that is of $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i$ defined by

$$R_i^{(p)} = b_i - a_{i1}x_1^{(p)} - a_{i2}x_2^{(p)} - \cdots - a_{in}x_n^{(p)}$$

we can improve the solution vector successively by reducing the largest residual to zero at that iteration. This is the basic idea of relaxation method.

To achieve the fast convergence of the procedure, we take all terms to one side and then reorder the equations so that the largest negative coefficients in the equations appear on the diagonal.

Now, if at any iteration, R_i is the largest residual in magnitude, then we give an increment to x_i; a_{ii} being the coefficient of *xi*

$$dx_i = \frac{R_i}{a_{ii}}$$

In other words, we change x_i. to $(x_i + dx_i)$ to relax R_i that is to reduce R_i to zero.

Example

Solve the system of equations

$$6x_1 - 3x_2 + x_3 = 11$$
$$2x_1 + x_2 - 8x_3 = -15$$
$$x_1 - 7x_2 + x_3 = 10$$

by the relaxation method, starting with the vector (0, 0, 0).

Solution

At first, we transfer all the terms to the right-hand side and reorder the equations, so that the largest coefficients in the equations appear on the diagonal.

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Thus, we get

$$\left.\begin{aligned} 0 &= 11 - 6x_1 + 3x_2 - x_3 \\ 0 &= 10 - x_1 + 7x_2 - x_3 \\ 0 &= -15 - 2x_1 - x_2 + 8x_3 \end{aligned}\right\}$$

after interchanging the 2nd and 3rd equations.

Starting with the initial solution vector (0, 0, 0), that is taking $x_1 = x_2 = x_3 = 0,$

we find the residuals $R_1 = 11,\ R_2 = 10,\ R_3 = -15$

of which the largest residual in magnitude is R3, i.e. the 3rd equation has more error and needs immediate attention for improvement.

Thus, we introduce a change, *dx3*in *x*3 which is obtained from the formula

$$dx_3 = -\frac{R_3}{a_{33}} = \frac{15}{8} = 1.875$$

Similarly, we find the new residuals of large magnitude and relax it to zero, and so on. We shall continue this process, until all the residuals are zero or very small.

Iteration	**Residuals**			**Maximum Difference**		**Variables**		
number	**R1**	**R2**	**R3**	R_i	dx_i	***x*1**	***x*2**	***x*3**
0	**11**	**10**	**-15**	**-15**	**1.875**	**0**	**0**	**0**
1	**9.125**	**8.125**	**0**	**9.125**	**1.5288**	**0**	**0**	**1.875**
2	**0.0478**	**6.5962**	**-3.0576**	**6.5962**	**-0.9423**	**1.5288**	**0**	**1.875**

Iteration **Residuals** **Maximum Difference** **Variables**

number	R1	R2	R3	□□□□□□□□□□Ü	□□□□□□□□□□Ü	*x*1	*x*2	*x*3
0	11	10	-15	-15	15/8 =1.875	0	0	0
1	9.125	8.125	0	9.125	-9.125/(-6) =1.5288	0	0	1.875
2	0.0478	6.5962	-3.0576	6.5962	-6.5962/7 =-0.9423	1.5288	0	1.875
3	-2.8747	0.0001	-2.1153	-2.8747	2.8747/(-6) =-0.4791	1.0497	-0.9423	1.875
4	-0.0031	0.4792	-1.1571	-1.1571	1.1571/8 =0.1446	1.0497	-0.9423	1.875

Iteration	Residuals			Maximum Difference	Variables			
number	R1	R2	R3	□□□□□□□□□□Ü	□□□□□□□□□□Ü	*x*1	*x*2	*x*3
5	-0.1447	0.3346	0.0003	0.3346	-.3346/7 =-0.0478	1.0497	-0.9423	2.0196
6	0.2881	0.0000	0.0475	0.2881	-.2881/(-6) =0.0480	1.0497	-0.9901	2.0196
7	-0.0001	0.048	0.1435	0.1435	=-0.0179	1.0017	-0.9901	2.0196
8	0.0178	0.0659	0.0003	-	-	1.0017	-0.9901	2.0017

At this stage, we observe that all the residuals *R1, R2* and *R3* are small enough and therefore we may take the corresponding values of *xi* at this iteration as the solution. Hence, the numerical solution is given by

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$x_1 = 1.0017, \quad x_2 = -0.9901, \quad x_3 = 2.0017,$
The exact solution is
$x_1 = 1.0, \quad x_2 = -1.0, \quad x_3 = 2.0$

Example

Solve by relaxation method, the equation
$10x - 2y - 2z = 6$
$-x - 10y - 2z = 7$
$-x - y + 10z = 8$

Solution

The residual r_1, r_2, r_3 are given by
$r_1 = 6 - 10x + 2y + 2z$
$r_2 = 7 + x - 10y + 2z$
$r_3 = 8 + x + y - 10z$
The operation table is as follows

x	y	z	r1	r2	r3	
1	0	0	-10	1	1	L1
0	1	0	2	-10	1	L2
0	0	1	2	2	-10	L3

The relaxation table is as follows

x	y	z	r1	r2	r3	
0	0	0	6	7	8	L4
0	0	1	8	9	-2	L5=L4+L3
0	1	0	10	-1	-1	L6=L5+L2
1	0	0	0	0	0	L7=L6+L1

Explanation

(1) In L4 ,the largest residual is 8.to reduce it, To reduce it ,we give an increment of
$\frac{8}{c_3} = \frac{8}{10} = 0.8 \cong 1$
the resulting residulas are obtained by
$L_4 + (1)L_3, i.e\ line\ L_5$

(2) In line L_5 the largest residual is 9

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Increment= $\frac{9}{b_2} = \frac{9}{10} = 0.9 \cong 1$

The resulting residuals $(= L_6) = L_5 + 1.L_2$

(3) In line L_6 ,the largest residual is 10

Increment = $\frac{10}{a_1} = \frac{10}{10} \cong 1$

The resulting residuals $(= L_6) = L_5 + 1.L_2$

Exact solution is arrived and it is x=1,y=1,z=1

Example

Solve the system by relaxation method, the equations

$9x - y + 2z = 7$

$x + 10y - 2z = 15$

$2x - 2y - 13z = -17$

Solution

The residuals r_1, r_2, r_3 are given by

$9x - y + 2z = 9$

$x + 10y - 2z = 15$

$2x - 2y - 13z = -17$

here

$r_1 = 9 - 9x + y - 2z$

$r_2 = 15 - x - 10y + 2z$

$r_3 = -17 - 2x + 2y + 13z$

Operation table

x	y	z	r1	r2	r3
1	0	0	-9	-1	-2
0	1	0	1	-10	2
0	0	1	-2	2	13

Relaxation table is

x	y	z	r1	r2	r3
0	0	0	9	15	-17
0	0	1	7	17	-4
0	1	0	8	7	-2
0.89	0	0	-0.01	6.11	-3.78
0	0.61	0	0.6	0.01	-2.56
0	0	0.19	0.22	0.39	-0.09
0	0.039	0	0.259	0	-0.012

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0.028	0	0	0.007	-0.028	-0.068
0	0	0.00523	-0.00346	-1.01754	-0.00001

Then x=0.89+0.028=0.918;y=1+0.61+0.039=1.694
And z=1+0.19+0.00523=1.19523
Now substituting the values of x,y,z in (1) ,we get
r1=9-9(0.918)+1.649-2(1.19523)=-0.00346
r2=15-0.918-10(1.649)+2(1.19523)=-0.1754
r3=-17-2(0.918) +2(1.649) +13(1.19523) =-0.00001
Which is agreement with the final residuals.

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Solution of Linear System of Equations and Matrix Inversion

Matrix Inversion

Consider a system of equations in the form
$[A](X) = (B)$
One way of writing its solution is in the form
$(X) = [A]^{-1}(B)$
Thus, the solution to the system can also be obtained if the inverse of the coefficient matrix [A] is known. That is the product of two square matrices is an identity matrix
$[A][B] = [I]$
then, $[B] = [A]^{-1}$
and $[A] = [B]^{-1}$
Every square non-singular matrix will have an inverse.
Gauss elimination and Gauss-Jordan methods are popular among many methods available for finding the inverse of a matrix.

Gaussian Elimination Method

In this method, if A is a given matrix, for which we have to find the inverse; at first, we place an identity matrix, whose order is same as that of A, adjacent to A which we call an augmented matrix.
Then the inverse of A is computed in two stages. In the first stage, A is converted into an upper triangular form, using Gaussian elimination method
In the second stage, the above upper triangular matrix is reduced to an identity matrix by row transformations. All these operations are also performed on the adjacently placed identity matrix.
Finally, when A is transformed into an identity matrix, the adjacent matrix gives the inverse of A.
In order to increase the accuracy of the result, it is essential to employ partial pivoting.

Example
Use the Gaussian elimination method to find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

Solution
At first, we place an identity matrix of the same order adjacent to the given matrix. Thus, the augmented matrix can be written as

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$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 3 & -1 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{bmatrix}$$

Stage I (Reduction to upper triangular form): Let $R1$, $R2$ and $R3$ denote the 1st , 2nd and 3rd rows of a matrix. In the 1st column, 4 is the largest element, thus interchanging R1 and R2 to bring the pivot element 4 to the place of $a11$, we have the augmented matrix in the form

$$\begin{bmatrix} 4 & 3 & -1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{bmatrix}$$

Divide R1 by 4 to get

$$\begin{bmatrix} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \\ \end{bmatrix}$$

Perform $R_2 - R_1 \rightarrow$, which gives

$$\begin{bmatrix} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{5}{4} & 1 & -\frac{1}{4} & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \\ \end{bmatrix}$$

Perform $R_3 - 3R_1 \rightarrow R_3$ in the above equation , which yields

$$\begin{bmatrix} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{11}{4} & \frac{15}{4} & 1 & -\frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & -\frac{3}{4} & 1 \end{bmatrix}$$

Now, looking at the second column for the pivot, the max (1/4. 11/4) is 11/4. Therefore, we interchange R2 and R3 in the last equation and get

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$$\begin{bmatrix} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{11}{4} & \frac{15}{4} & 0 & -\frac{3}{4} & 1 \\ 0 & \frac{1}{4} & \frac{5}{4} & 1 & -\frac{1}{4} & 0 \end{bmatrix}$$

Now, divide $R2$ by the pivot $a22 = 11/4$, and obtain

$$\begin{bmatrix} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 1 & \frac{15}{11} & 0 & -\frac{3}{11} & \frac{4}{11} \\ 0 & \frac{1}{4} & \frac{5}{4} & 1 & -\frac{1}{4} & 0 \end{bmatrix}$$

Performing $R_3 - (1/4)R_2 \rightarrow R_3$ yields

$$\begin{bmatrix} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 1 & \frac{15}{11} & 0 & -\frac{3}{11} & \frac{4}{11} \\ 0 & 0 & \frac{10}{11} & 1 & -\frac{2}{11} & -\frac{1}{11} \end{bmatrix}$$

Finally, we divide R3 by (10/11), thus getting an upper triangular form

$$\begin{bmatrix} 1 & \frac{3}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 1 & \frac{15}{11} & 0 & -\frac{3}{11} & \frac{4}{11} \\ 0 & 0 & 1 & \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

Stage II

Reduction to an identity matrix (1/4)R3 + R1 and (-15/11)R3 + R2

$$\begin{bmatrix} 1 & \frac{3}{4} & 0 & \frac{11}{40} & \frac{1}{5} & -\frac{1}{40} \\ 0 & 1 & 0 & -\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

Finally, performing $R_1 - (3/4)R_2 \rightarrow R_1$ we obtain

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$$\begin{bmatrix} 1 & 0 & 0 & \frac{7}{5} & \frac{1}{5} & -\frac{2}{5} \\ 0 & 1 & 0 & -\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

Thus, we have

$$A^{-1} = \begin{bmatrix} \frac{7}{5} & \frac{1}{5} & -\frac{2}{5} \\ -\frac{3}{2} & 0 & \frac{1}{2} \\ \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

Example

Find the inverse of the $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using gauss elimination method.

Solution

We have the argumented system

$$[A/I] = \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & \frac{1}{2} & \frac{3}{2} & \frac{-3}{2} & 1 & 0 \\ 1 & \frac{7}{2} & \frac{17}{2} & \frac{-1}{2} & 0 & 1 \end{bmatrix} R_2 - (\frac{3}{2})R_1, R_3 - (\frac{1}{2})R_1$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-3}{2} & 1 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{bmatrix} R_3 - 7R_2$$

$$now\ if \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

then the system 1 *is the inverse of the given matrix the system is eqivalent to three systems*

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$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{-3}{2} \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix}$$

$x_{11} = -3$ $x_{21} = 12$ $x_{31} = -5$

$x_{21} = \frac{5}{2}$ $x_{22} = \frac{-17}{2}$ $x_{23} = \frac{7}{2}$

$x_{31} = \frac{-1}{2}$ $x_{22} = \frac{3}{2}$ $x_{23} = -\frac{1}{2}$

and the inverse martix is

$$\frac{1}{2}\begin{bmatrix} -6 & 5 & -1 \\ 24 & -17 & 3 \\ -10 & 7 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

by back substitution ,the three systems of equation may be written as

Example

Fine the inverse of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$ using gauss elimination method.

Solution

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$$[A/I] = \left[\begin{array}{ccc|ccc} 4 & 1 & 2 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 1 & -2 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 4 & 1 & 2 & 1 & 0 & 0 \\ 0 & \frac{5}{2} & -2 & \frac{-1}{2} & 1 & 0 \\ 0 & \frac{-9}{4} & \frac{3}{2} & \frac{-1}{4} & 0 & 1 \end{array}\right] R_2 - \frac{1}{2}R_1, R_3 - \frac{1}{4}R_1$$

now if $\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$ *is the inverse of the given matrix , then the system*(1)*is equivalent to three systems*

$$\begin{bmatrix} 4 & 1 & 2 \\ 0 & \frac{5}{2} & -2 \\ 0 & \frac{-9}{4} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{-1}{2} \\ \frac{-14}{20} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & 2 \\ 0 & \frac{5}{2} & -2 \\ 0 & \frac{-9}{4} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \\ x_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \frac{9}{10} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & 2 \\ 0 & \frac{5}{2} & -2 \\ 0 & \frac{-9}{4} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x_{11} = \frac{-4}{3} \qquad x_{21} = \frac{5}{3} \qquad x_{31} = \frac{7}{3}$$

$$x_{21} = 2 \qquad x_{22} = -2 \qquad x_{23} = -3$$

$$x_{31} = \frac{7}{3} \qquad x_{22} = \frac{-8}{3} \qquad x_{23} = -\frac{10}{3}$$

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and the inverse martix is

$$\begin{bmatrix} -\frac{4}{3} & 2 & \frac{7}{3} \\ \frac{5}{3} & -2 & \frac{-8}{3} \\ \frac{7}{3} & -3 & \frac{-10}{3} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -4 & 6 & 7 \\ 5 & -6 & -8 \\ 7 & -9 & -10 \end{bmatrix}$$

Gauss - Jordan Method

This method is similar to Gaussian elimination method, with the essential difference that the stage I of reducing the given matrix to an upper triangular form is not needed. However, the given matrix can be directly reduced to an identity matrix using elementary row operations.

Example

Find the inverse of the given matrix by Gauss-Jordan method

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

Solution

Let R1, R2 and R3 denote the 1st, 2nd and 3rd rows of a matrix. We place the identity matrix adjacent to the given matrix. So the augmented matrix is given by

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 3 & -1 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{bmatrix}$$

Performing $R_2 - 4R_1 \to R_2$, we get

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -5 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{bmatrix}$$

Now, performing $R_3 - 3R_1 \to R_3$, we obtain

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -5 & -4 & 1 & 0 \\ 0 & 2 & 0 & -3 & 0 & 1 \end{bmatrix}$$

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Carrying out further operations $R_2 + R_1 \to R_1$ and $R_3 + 2R_2 \to R_3$,
we arrive at

$$\begin{bmatrix} 1 & 0 & -4 & -3 & 1 & 0 \\ 0 & -1 & -5 & -4 & 1 & 0 \\ 0 & 0 & -10 & -11 & 2 & 1 \end{bmatrix}$$

Now, dividing the third row by –10, we get

$$\begin{bmatrix} 1 & 0 & -4 & -3 & 1 & 0 \\ 0 & -1 & -5 & -4 & 1 & 0 \\ 0 & 0 & 1 & \frac{11}{10} & -1 & -\frac{1}{10} \end{bmatrix}$$

Further, we perform $R_1 + 4R_3 \to R_1$, and $R_2 + 5R_3 \to R_2$ to get

$$\begin{bmatrix} 1 & 0 & 0 & \frac{7}{5} & \frac{1}{5} & -\frac{2}{5} \\ 0 & -1 & 0 & \frac{3}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

Finally, multiplying *R*2 by –1, we obtain

$$\begin{bmatrix} 1 & 0 & 0 & \frac{7}{5} & \frac{1}{5} & -\frac{2}{5} \\ 0 & 1 & 0 & -\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

Hence, we have

$$A^{-1} = \begin{bmatrix} \frac{7}{5} & \frac{1}{5} & -\frac{2}{5} \\ -\frac{3}{2} & 0 & \frac{1}{2} \\ \frac{11}{10} & -\frac{1}{5} & -\frac{1}{10} \end{bmatrix}$$

Exercise

Solve all the above examples solved by Gauss elimination by using gauss Jordan method.

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Eigen Value Problems

Let [A] be an n x n square matrix. Suppose, there exists a scalar and a vector

$$X = (x_1 \quad x_2 \quad \dots \quad x_n)^T$$

such that

$$[A](X) = \lambda(X)$$

$$\frac{d}{dx}(e^{ax}) = a(e^{ax})$$

$$\frac{d^2}{dx^2}(\sin ax) = -a^2(\sin ax)$$

Then λ is the eigen value and X is the corresponding eigenvector of the matrix [A].

We can also write it as $[A - \lambda I](X) = (O)$

This represents a set of n homogeneous equations possessing non-trivial solution, provided

$$|A - \lambda I| = 0$$

This determinant, on expansion, gives an n-th degree polynomial which is called characteristic polynomial of [A], which has n roots. Corresponding to each root, we can solve these equations in principle, and determine a vector called eigenvector.

Finding the roots of the characteristic equation is laborious. Hence, we look for better methods suitable from the point of view of computation. Depending upon the type of matrix [A] and on what one is looking for, various numerical methods are available.

Power Method and Jacobi's Method

Note!

We shall consider only real and real-symmetric matrices and discuss power and Jacobi's methods

Power Method

To compute the largest eigen value and the corresponding eigenvector of the system

$$[A](X) = \lambda(X)$$

where [A] is a real, symmetric or un-symmetric matrix, the power method is widely used in practice.

Procedure

Step 1: Choose the initial vector such that the largest element is unity.

Step 2: The normalized vector $v^{(0)}$ is pre-multiplied by the matrix [A].

Step 3: The resultant vector is again normalized.

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Step 4: This process of iteration is continued and the new normalized vector is repeatedly pre-multiplied by the matrix $[A]$ until the required accuracy is obtained.
At this point, the result looks like
$u^{(k)} = [A]v^{(k-1)} = q_k v^{(k)}$
Here, q_k is the desired largest eigen value and $v^{(k)}$ is the corresponding eigenvector.

Example
Find the eigen value of largest modulus, and the associated eigenvector of the matrix by power method

$$[A] = \begin{bmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{bmatrix}$$

Solution
We choose an initial vector $\upsilon^{(0)}$
as $(1,1,1)^T$.
Then, compute first iteration

$$u^{(1)} = [A]v^{(0)} \begin{bmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 12 \\ 14 \end{pmatrix}$$

Now we normalize the resultant vector to get

$$u^{(1)} = 14 \begin{pmatrix} \frac{1}{2} \\ \frac{6}{7} \\ 1 \end{pmatrix} = q_1 v^{(1)}$$

The second iteration gives,

$$u^{(2)} = [A]v^{(1)} \begin{bmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{bmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{6}{7} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{39}{7} \\ \frac{67}{7} \\ \frac{171}{14} \end{pmatrix}$$

$$= 12.2143 \begin{pmatrix} 0.456140 \\ 0.783626 \\ 1.0 \end{pmatrix} = q_2 v^{(2)}$$

Continuing this procedure, the third and subsequent iterations are given in the following slides

$$u^{(3)} = [A]v^{(2)} = \begin{bmatrix} 2 & 3 & 2 \\ 4 & 3 & 5 \\ 3 & 2 & 9 \end{bmatrix} \begin{pmatrix} 0.456140 \\ 0.783626 \\ 1.0 \end{pmatrix}$$

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$$= \begin{pmatrix} 5.263158 \\ 9.175438 \\ 11.935672 \end{pmatrix} = 11.935672 \begin{pmatrix} 0.44096 \\ 0.776874 \\ 1.0 \end{pmatrix} = q_3 v^{(3)}$$

$$u^{(4)} = [A]v^{(3)} = \begin{pmatrix} 5.18814 \\ 9.07006 \\ 11.86036 \end{pmatrix}$$

$$= 11.8636 \begin{pmatrix} 0.437435 \\ 0.764737 \\ 1.0 \end{pmatrix} = q_4 v^{(4)}$$

$$u^{(5)} = [A]v^{(4)} = \begin{pmatrix} 5.16908 \\ 9.04395 \\ 11.84178 \end{pmatrix}$$

$$= 11.84178 \begin{pmatrix} 0.436512 \\ 0.763732 \\ 1.0 \end{pmatrix} = q_5 v^{(5)}$$

After rounding-off, the largest eigen value and the corresponding eigenvector as accurate to two decimals are

$$\lambda = 11.84 \quad (X) = \begin{pmatrix} 0.44 \\ 0.76 \\ 1.00 \end{pmatrix}$$

Example

Find the first three iterations of the power method of the given matrix

$$\begin{bmatrix} 7 & 6 & -3 \\ -12 & -20 & 24 \\ -6 & -12 & 16 \end{bmatrix}$$

Solution

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$$\begin{bmatrix} 7 & 6 & -3 \\ -12 & -20 & 24 \\ -6 & -12 & 16 \end{bmatrix}$$

we choose finitial vector as $v^{(0)} = (1,1,1)^t$

first iteration

$$u^{(1)} = [A]v^{(0)} = \begin{bmatrix} 7 & 6 & -3 \\ -12 & -20 & 24 \\ -6 & -12 & 16 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7+6-3 \\ -12-20+24 \\ -6-12+16 \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \\ -2 \end{bmatrix}$$

$$by\ diagonali\sin g \begin{bmatrix} 10 \\ -8 \\ -2 \end{bmatrix} \quad u^{(1)} = 10\begin{bmatrix} 1 \\ -0.8 \\ -0.2 \end{bmatrix} = q_1 v^{(1)}$$

sec*ond iteration*

$$u^{(2)} = [A]v^{(1)} = \begin{bmatrix} 7 & 6 & -3 \\ -12 & -20 & 24 \\ -6 & -12 & 16 \end{bmatrix}\begin{bmatrix} 1 \\ -0.8 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 7-4.8+0.6 \\ -12+16-4.8 \\ -6+9.6-3.2 \end{bmatrix} = \begin{bmatrix} 2.8 \\ -0.8 \\ 0.4 \end{bmatrix}$$

$$by\ diagonali\sin g \begin{bmatrix} 2.8 \\ -0.8 \\ 0.4 \end{bmatrix} \quad u^{(1)} = 2.8\begin{bmatrix} 1 \\ -0.2857 \\ 0.1428 \end{bmatrix} = q_2 v^{(2)}$$

third iteration

$$u^{(3)} = [A]v^{(2)} = \begin{bmatrix} 7 & 6 & -3 \\ -12 & -20 & 24 \\ -6 & -12 & 16 \end{bmatrix}\begin{bmatrix} 1 \\ -0.2857 \\ 0.1428 \end{bmatrix} = \begin{bmatrix} 7-1.7142-0.4284 \\ -12+5.714+3.4272 \\ -6+3.4284+2.2848 \end{bmatrix} = \begin{bmatrix} 4.8574 \\ -2.8588 \\ -0.2868 \end{bmatrix}$$

now daigonali sin *g*

$$\begin{bmatrix} 4.8574 \\ -2.8588 \\ -0.2868 \end{bmatrix} \quad now\ normali\sin g\ 4.8574\begin{bmatrix} 1 \\ -0.5885 \\ -0.0590 \end{bmatrix}$$

Example

Find the first three iteration of the power method applied on the following matrices

$$\begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix} \quad use\ x^0 = (-1,2,1)^t$$

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Solution

$$\begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix} \quad USE\ x^{(0)} = (-1,2,1)^t$$

1*st iterations*

$$u^{(1)} = [\mathrm{A}]x^{(0)} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1-2+0 \\ 2+8-2 \\ 0-2+2 \end{bmatrix} = \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}$$

now we normalize the resul tan *t vector to get*

$$u^{(1)} = \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix} = 8\begin{bmatrix} \frac{-3}{8} \\ 1 \\ 0 \end{bmatrix} = q_1 x^{(1)}$$

$$u^{(2)} = [\mathrm{A}]x^{(1)} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} \frac{-3}{8} \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-3}{8}-1+0 \\ \frac{6}{8}+4+0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1.375 \\ 4.75 \\ -1 \end{bmatrix}$$

$$u^{(2)} = \begin{bmatrix} -1.375 \\ 4.75 \\ -1 \end{bmatrix} = 4.75\begin{bmatrix} -0.28947 \\ 1 \\ -0.2152 \end{bmatrix}$$

$$u^{(3)} = [\mathrm{A}]x^{(2)} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} -0.28947 \\ 1 \\ -0.2152 \end{bmatrix} = \begin{bmatrix} -1.28947 \\ 4.99998 \\ -1.42104 \end{bmatrix} = 4.99998\begin{bmatrix} -0.25789 \\ 1 \\ -0.28420 \end{bmatrix}$$

Exercise

Find the largest eigen value and the corresponding eigen vector by power method after fourth iteration starting with the initial vector $\upsilon^{(0)} = (0,0,1)^T$

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$$[A] = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$$

Let
$\lambda_1, \lambda_2, \ldots, \lambda_n$
be the distinct eigen values of an n x n matrix [A], such that $|\lambda_1| > |\lambda_2| > \cdots > |\lambda_n|$ and suppose $v_1, v_2, \ldots, v_n$ are the corresponding eigen vectors
Power method is applicable if the above eigen values are real and distinct, and hence, the corresponding eigenvectors are linearly independent.
Then, any eigenvector v in the space spanned by the eigenvectors $v_1, v_2, \ldots, v_n$
can be written as their linear combination $v = c_1 v_1 + c_2 v_2 + \cdots + c_n v_n$

Pre-multiplying by A and substituting

$$Av_1 = \lambda_1 v_1, \qquad Av_2 = \lambda_2 v_2, \quad \ldots \qquad Av_n = \lambda_n v_n$$

We get

$$Av = \lambda_1 \left(c_1 v_1 + c_2 \frac{\lambda_2}{\lambda_1} v_2 + \cdots + c_n \frac{\lambda_n}{\lambda_1} v_n \right)$$

Again, pre-multiplying by A and simplifying, we obtain

$$A^2 v = \lambda_1^2 \left[c_1 v_1 + c_2 \left(\frac{\lambda_2}{\lambda_1} \right)^2 v_2 + \cdots + c_n \left(\frac{\lambda_n}{\lambda_1} \right)^2 v_n \right]$$

Similarly, we have

$$A^r v = \lambda_1^r \left[c_1 v_1 + c_2 \left(\frac{\lambda_2}{\lambda_1} \right)^r v_2 + \cdots + c_n \left(\frac{\lambda_n}{\lambda_1} \right)^r v_n \right]$$

and

$$A^{r+1} v = (\lambda_1)^{r+1} \left[c_1 v_1 + c_2 \left(\frac{\lambda_2}{\lambda_1} \right)^{r+1} v_2 + \cdots + c_n \left(\frac{\lambda_n}{\lambda_1} \right)^{r+1} v_n \right]$$

Now, the eigen value λ_1

can be computed as the limit of the ratio of the corresponding components of $A^r v$ and $A^{r+1} v$.
That is,

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$$\lambda_1 = \frac{\lambda_1^{r+1}}{\lambda_1^r} = \underset{r\to\infty}{Lt} \frac{(A^{r+1}v)_p}{(A^r v)_p}, \qquad p = 1, 2, \ldots, n$$

Here, the index *p* stands for the p-th component in the corresponding vector
Sometimes, we may be interested in finding the least eigen value and the corresponding eigenvector.
In that case, we proceed as follows.
We note that $[A](X) = \lambda(X)$.

Pre-multiplying by $[A^{-1}]$, we get

$[A^{-1}][A](X) = [A^{-1}]\lambda(X) = \lambda[A^{-1}](X)$

Which can be rewritten as

$$[A^{-1}](X) = \frac{1}{\lambda}(X)$$

which shows that the inverse matrix has a set of eigen values which are the reciprocals of the eigen values of [*A*].
Thus, for finding the eigen value of the least magnitude of the matrix [*A*], we have to apply power method to the inverse of [*A*].

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Jacobi's Method

Definition
An $n \times n$ matrix $[A]$ is said to be orthogonal if
$[A]^T[A] = [I]$,
i.e. $[A]^T = [A]^{-1}$
In order to compute all the eigen values and the corresponding eigenvectors of a real symmetric matrix, Jacobi's method is highly recommended. It is based on an important property from matrix theory, which states
that, if $[A]$ is an $n \times n$ real symmetric matrix, its eigen values are real, and there exists an orthogonal matrix $[S]$ such that the diagonal matrix D is
$[S^{-1}][A][S]$

This digitalization can be carried out by applying a series of orthogonal transformations
$S_1, S_2, ..., S_n$,

Let A be an $n \times n$ real symmetric matrix. Suppose $|a_{ij}|$ be numerically the largest element amongst the off-diagonal elements of A. We construct an orthogonal matrix $S1$ defined as

$s_{ij=} - \sin\theta, \quad s_{ji} = \sin\theta,$
$s_{ii} = \cos\theta, \quad s_{jj} = \cos\theta$
While each of the remaining off-diagonal elements are zero, the remaining diagonal elements are assumed to be unity. Thus, we construct $S1$ as under

$$S_1 = \begin{array}{c} \text{i-th column} \quad j\text{-th column} \\ \downarrow \qquad\qquad \downarrow \\ \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & \cos\theta & \cdots & -\sin\theta & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & \sin\theta & \cdots & \cos\theta & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \\ \leftarrow \text{i-th row} \\ \\ \leftarrow j\text{-th row} \\ \\ \end{array}$$

Where $\cos\theta, -\sin\theta, \sin\theta$ *and* $\cos\vartheta$ are inserted in $(i,i),(i,j),(j,i),(j,j) - \text{th}$ positions respectively, and elsewhere it is identical with a unit matrix.
Now, we compute
$D_1 = S_1^{-1} A S_1 = S_1^T A S_1$
Since S1 is an orthogonal matrix, such that .After the transformation, the elements at the position *(i , j), (j , i)* get annihilated, that is *dij and dji* reduce to zero, which is seen as follows:

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$$\begin{bmatrix} d_{ii} & d_{ij} \\ d_{ji} & d_{jj} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a_{ii} & a_{ij} \\ a_{ij} & a_{jj} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} a$$

$$\begin{bmatrix} a_{ii}\cos^2\theta \sin\theta\cos\theta + a_{jj}\sin^2\theta & (a_{jj} - a_{ii})\sin\theta\cos\theta + a_{ij}\cos 2\theta \\ (a_{ji} - a_{ii})\sin\theta\cos\theta + a_{ij}\cos 2\theta & a_{ii}\sin^2\theta + a_{jj}\cos^2\theta - 2a_{ij}\sin\theta\cos\theta \end{bmatrix}$$

Therefore, $d_{ij} = 0$ only if,

$$a_{ij}\cos 2\theta + \frac{a_{jj} - a_{ii}}{2}\sin 2\theta = 0$$

That is if

$$\tan 2\theta = \frac{2a_{ij}}{a_{ii} - a_{jj}}$$

Thus, we choose θ such that the above equation is satisfied, thereby, the pair of off-diagonal elements *dij* and *dji* reduces to zero.However, though it creates a new pair of zeros, it also introduces non-zero contributions at formerly zero positions.

Also, the above equation gives four values of , but to get the least possible rotation, we choose

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Example

Find all the eigen values and the corresponding eigen vectors of the matrix by Jacobi's method

$$A=\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

Solution

The given matrix is real and symmetric. The largest off-diagonal element is found to be $a_{13}=a_{31}=2.$

Now, we compute

$$\tan 2\theta=\frac{2a_{ij}}{a_{ii}-a_{jj}}=\frac{2a_{13}}{a_{11}-a_{33}}=\frac{4}{0}=\infty$$

This gives, $\theta=\pi/4$

Thus, we construct an orthogonal matrix *Si* as

$$S_1=\begin{bmatrix} \cos\frac{\pi}{4} & 0 & -\sin\frac{\pi}{4} \\ 0 & 1 & 0 \\ \sin\frac{\pi}{4} & 0 & \cos\frac{\pi}{4} \end{bmatrix}=\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

The first rotation gives,

$$D_1=S_1^{-1}AS_1$$

$$=\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$=\begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

We observe that the elements d13 and *d31* got annihilated. To make sure that calculations are correct up to this step, we see that the sum of the diagonal elements of *D1* is same as the sum of the diagonal elements of the original matrix *A*.

As a second step, we choose the largest off-diagonal element of *D1* and is found to be $d_{12}=d_{21}=2,$ and compute

$$\tan 2\theta=\frac{2d_{12}}{d_{11}-d_{22}}=\frac{4}{0}=\infty$$

This again gives $\theta=\pi/4$

Thus, we construct the second rotation matrix as

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$$S_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

At the end of the second rotation, we get

$$D_2 = S_2^{-1} D_1 S_2$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

This turned out to be a diagonal matrix, so we stop the computation. From here, we notice that the eigen values of the given matrix are 5,1 and –1. The eigenvectors are the column vectors of $S = S_1 S_2$

Therefore

$$S = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Example

Find all the eigen values of the matrix by Jacobi's method.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Solution

Here all the off-diagonal elements are of the same order of magnitude. Therefore, we can choose any one of them. Suppose, we choose *a12* as the largest element and compute

$$\tan 2\theta = \frac{-1}{0} = \infty$$

Which gives, $\theta = \pi/4.$

Then $\cos\theta = \sin\theta = 1/\sqrt{2}$

and we construct an orthogonal matrix *S1* such that

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$$S = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The first rotation gives

$$D_1 = S_1^{-1} A S_1$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 3 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 2 \end{bmatrix}$$

Now, we choose $d_{13} = -1/\sqrt{2}$

As the largest element of D1 and compute

$$\tan 2\theta = \frac{2d_{13}}{d_{11} - d_{33}} = \frac{-\sqrt{2}}{1-2}$$

$\theta = 27^o 22' 41''$.

Now we construct another orthogonal matrix S2, such that

$$S_2 = \begin{bmatrix} 0.888 & 0 & -0.459 \\ 0 & 1 & 0 \\ 0.459 & 0 & 0.888 \end{bmatrix}$$

At the end of second rotation, we obtain

$$D_2 = S_2^{-1} D_1 S_2 = \begin{bmatrix} 0.634 & -0.325 & 0 \\ 0.325 & 3 & -0.628 \\ 0 & -0.628 & 2.365 \end{bmatrix}$$

Now, the numerically largest off-diagonal element of $D2$ is found to be $d_{23} = -0.628$ and compute.

$$\tan 2\theta = \frac{-2 \times 0.628}{3 - 2.365}$$

$\theta = -31^o 35' 24''$.

Thus, the orthogonal matrix is

$$S_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.852 & 0.524 \\ 0 & -0.524 & 0.852 \end{bmatrix}$$

At the end of third rotation, we get

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$$D_3 = S_3^{-1} D_2 S_3 = \begin{bmatrix} 0.634 & -0.277 & 0 \\ 0.277 & 3.386 & 0 \\ 0 & 0 & 1.979 \end{bmatrix}$$

To reduce *D3* to a diagonal form, some more rotations are required. However, we may take 0.634, 3.386 and 1.979 as eigen values of the given matrix.

Example

Using Jacobi's method, find the eigenvalues and eigenvectors of the following matrix,

$$\begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$$

Solution:

The given matrix is real and symmetric. The largest off – diagonal element is found to be

$$a_{12} = a_{21} = \frac{1}{2}$$

Now we comute

$$\tan 2\theta = \frac{2a_{ij}}{a_{ii} - a_{jj}} = \frac{2a_{12}}{a_{11} - a_{22}} = \frac{2\left(\frac{1}{2}\right)}{1 - \frac{1}{3}} = \frac{3}{2}$$

$$\theta = \frac{\tan^{-1}\left(\frac{3}{2}\right)}{2} = 28.155$$

Thus we construct an orthogonal matrix S_1 *as*

$$S_1 = \begin{bmatrix} \cos 28.155 & -\sin 28.155 & 0 \\ \sin 28.155 & \cos 28.155 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.882 & -0.472 & 0 \\ 0.472 & 0.882 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The first rotation gives, $D_1 = S_1^{-1} A S_1$

$$= \begin{bmatrix} 0.882 & 0.472 & 0 \\ -0.472 & 0.882 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} \begin{bmatrix} 0.882 & -0.472 & 0 \\ 0.472 & 0.882 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.268 & 0.000 & 0.412 \\ 0.000 & 0.066 & 0.063 \\ 0.412 & 0.063 & 0.200 \end{bmatrix}$$

We see that sum of the diagonal elements of D_1 =1.53

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And the sum of the diagonal elements of A = 1.53
This means that our question is going right.

As a second step we choose the largest of off-diagonal element of D_1, which is $d_{13} = d_{31} = 0.412$, and comput

$$\tan 2\theta = \frac{2d_{13}}{d_{11} - d_{33}} = \frac{2(0.412)}{1.268 - 0.200} = 0.772$$

$$\theta = \frac{\tan^{-1}(0.772)}{2} = 18.834$$

Thus we construct an orthogonal matrix S_2 as

$$S_2 = \begin{bmatrix} \cos 18.834 & 0 & -\sin 18.834 \\ 0 & 1 & 0 \\ \sin 18.834 & 0 & \cos 18.834 \end{bmatrix} = \begin{bmatrix} 0.946 & 0 & -0.323 \\ 0 & 1 & 0 \\ 0.323 & 0 & 0.946 \end{bmatrix}$$

Thus the rotation gives,

$$D_2 = S_2^{-1} D_1 S_2$$

$$= \begin{bmatrix} 0.946 & 0 & 0.323 \\ 0 & 1 & 0 \\ -0.323 & 0 & 0.946 \end{bmatrix} \begin{bmatrix} 1.268 & 0.000 & 0.412 \\ 0.000 & 0.066 & 0.063 \\ 0.412 & 0.063 & 0.200 \end{bmatrix} \begin{bmatrix} 0.946 & 0 & -0.323 \\ 0 & 1 & 0 \\ 0.323 & 0 & 0.946 \end{bmatrix}$$

$$= \begin{bmatrix} 1.408 & 0.020 & -0.001 \\ 0.020 & 0.066 & 0.060 \\ -0.001 & 0.060 & 0.059 \end{bmatrix}$$

We again see that sum of the diagonal elements of D_2 =1.53

Also the sum of the diagonal elements of A = 1.53
This means that our question is going right.
Hence the eigenvalues are 1.408 , .066 and .059 and the corresponding eigenvectors are the columns of S.Where

$$S = S_1 S_2$$

$$= \begin{bmatrix} 0.882 & -0.472 & 0 \\ 0.472 & 0.882 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.946 & 0 & -0.323 \\ 0 & 1 & 0 \\ 0.323 & 0 & 0.946 \end{bmatrix}$$

$$= \begin{bmatrix} .8343 & -.472 & -.2848 \\ .446 & .88 & -.1524 \\ .323 & 0 & .946 \end{bmatrix}$$

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Interpolation

Introduction

Finite differences play an important role in numerical techniques, where tabulated values of the functions are available.

For instance, consider a function $y = f(x)$.

As x takes values $x_0, x_1, x_2, \ldots, x_n$,

Let the corresponding values of y be $y_0, y_1, y_2, \ldots, y_n$.

That is, for given a table of values, $(x_k, y_k), k = 0,1,2,\ldots,n;$

The process of estimating the value of y, for any intermediate value of x, is called interpolation. The method of computing the value of y, for a given value of x, lying outside the table of values of x is known as extrapolation. If the function $f(x)$ is known, the value of y corresponding to any x can be readily computed to the desired accuracy. For interpolation of a tabulated function, the concept of finite differences is important. The knowledge about various finite difference operators and their symbolic relations are very much needed to establish various interpolation formulae.

Finite Difference Operators

Forward Differences

For a given table of values $(x_k, y_k), k = 0,1,2,...,n$ with equally spaced abscissas of a function $y = f(x)$, we define the forward difference operator Δ as follows

$\Delta y_i = y_{i+1} - y_i, \qquad i = 0,1,...,(n-1)$

To be explicit, we write

$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$\Delta y_{n-1} = y_n - y_{n-1}$$

These differences are called *first differences of the function y* and are denoted by the symbol Δy_i Here, Δ is called the first difference operator

Similarly, the differences of the first differences are called second differences, defined by

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0, \qquad \Delta^2 y_1 = \Delta y_2 - \Delta y_1$$

Thus, in general

$$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i$$

Here Δ^2 is called the *second difference* operator. Thus, continuing, we can define, *r-th* difference of y, as

$$\Delta^r y_i = \Delta^{r-1} y_{i+1} - \Delta^{r-1} y_i$$

By defining a difference table as a convenient device for displaying various differences, the above defined differences can be written down systematically by constructing a difference table for values

$(x_k, y_k), k = 0,1,...,6$

Forward Difference Table

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x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
x_0	y_0	Δy_0					
x_1	y_1	Δy_1	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$		
x_2	y_2	Δy_2	$\Delta^2 y_1$	$\Delta^3 y_1$	$\Delta^4 y_1$	$\Delta^5 y_0$	
x_3	y_3	Δy_3	$\Delta^2 y_2$	$\Delta^3 y_2$	$\Delta^4 y_1$	$\Delta^5 y_1$	$\Delta^6 y_0$
x_4	y_4	Δy_4	$\Delta^2 y_3$	$\Delta^3 y_3$			
x_5	y_5	Δy_5	$\Delta^2 y_4$				
x_6	y_6						

This difference table is called *forward difference* table or *diagonal difference* table. Here, each difference is located in its appropriate column, midway between the elements of the previous column.

Please note that the subscript remains constant along each diagonal of the table. The first term in the table, that is $y0$ is called the leading term, while the differences $\Delta y_0, \Delta^2 y_0, \Delta^3 y_0, \ldots$ are called leading differences

Example

Construct a forward difference table for the following values of x and y:

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

Solution

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x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0.1	0.003					
0.3	0.067	0.064	0.017			
0.5	0.148	0.081	0.019	0.002		
0.7	0.248	0.1	0.022	0.003	0.001	
0.9	0.37	0.122	0.026	0.004	0.001	0
1.1	0.518	0.148	0.031	0.005	0.001	0
1.3	0.697	0.179				

Example

Express $\Delta^2 y_0$ and $\Delta^3 y_0$ in terms of the values of the function y.

Solution:

Noting that each higher order difference is defined in terms of the lower order difference, we have

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0 = (y_2 - y_1) - (y_1 - y_0)$$
$$= y_2 - 2y_1 + y_0$$

And

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0 = (\Delta y_2 - \Delta y_1) - (\Delta_{y1} - \Delta y_0)$$
$$= (y_3 - y_2) - (y_2 - y_1) - (y_2 - y_1) + (y_1 - y_0)$$
$$= y_3 - 3y_2 + 3y_1 - y_0$$

Hence, we observe that the coefficients of the values of y, in the expansion of $\Delta^2 y_0, \Delta^3 y_0$, are binomial coefficients.

Thus, in general, we arrive at the following result: -

$$\Delta^n y_0 = y_n - {}^n C_1 y_{n-1} + {}^n C_2 y_{n-2} - {}^n C_3 y_{n-3} + \cdots + (-1)^n y_0$$

Example

Show that the value of yn can be expressed in terms of the leading value y0 and the leading differences

$$\Delta y_0, \Delta^2 y_0, \ldots, \Delta^n y_0.$$

Solution

The forward difference table will be

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$$\left.\begin{aligned} y_1 - y_0 = \Delta y_0 \quad \text{or} \quad y_1 = y_0 + \Delta y_0 \\ y_2 - y_1 = \Delta y_1 \quad \text{or} \quad y_2 = y_1 + \Delta y_1 \\ y_3 - y_2 = \Delta y_2 \quad \text{or} \quad y_3 = y_2 + \Delta y_2 \end{aligned}\right\}$$

Similarly,

$$\left.\begin{aligned} \Delta y_1 - \Delta y_0 = \Delta^2 y_0 \quad \text{or} \quad \Delta y_1 = \Delta y_0 + \Delta^2 y_0 \\ \Delta y_2 - \Delta y_1 = \Delta^2 y_1 \quad \text{or} \quad \Delta y_2 = \Delta y_1 + \Delta^2 y_1 \end{aligned}\right\}$$

Similarly, we can also write

$$\left.\begin{aligned} \Delta^2 y_1 - \Delta^2 y_0 = \Delta^3 y_0 \quad \text{or} \quad \Delta^2 y_1 = \Delta^2 y_0 + \Delta^3 y_0 \\ \Delta^2 y_2 - \Delta^2 y_1 = \Delta^3 y_1 \quad \text{or} \quad \Delta^2 y_2 = \Delta^2 y_1 + \Delta^3 y_1 \end{aligned}\right\}$$

$$\Delta y_2 = (\Delta y_0 + \Delta^2 y_0) + (\Delta^2 y_0 + \Delta^3 y_0)$$

$$= \Delta y_0 + 2\Delta^2 y_0 + \Delta^3 y_0$$

$$\begin{aligned} y_3 = y_2 + \Delta y_2 &= (y_1 + \Delta y_1) + (\Delta y_1 + \Delta^2 y_1) \\ &= y_0 + 3\Delta y_0 + 3\Delta^2 y_0 + \Delta^3 y_0 \\ &= (1+\Delta)^3 y_0 \end{aligned}$$

Similarly, we can symbolically write

$$\begin{aligned} y_1 &= (1+\Delta) y_0, \\ y_2 &= (1+\Delta)^2 y_0, \\ y_3 &= (1+\Delta)^3 y_0 \\ &\cdots\cdots \\ y_n &= (1+\Delta)^n y_0 \end{aligned}$$

Hence, we obtain

$$y_n = y_0 + {}^n C_1 \Delta y_0 + {}^n C_2 \Delta^2 y_0 + {}^n C_3 \Delta^3 y_0 + \cdots + \Delta^n y_0$$

OR

$$y_n = \sum_{i=0}^{n} {}^n C_i \Delta^i y_0$$

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Backward Differences

For a given table of values $(x_k, y_k), k = 0,1,2,...,n$ of a function $y = f(x)$ with equally spaced abscissas, the first backward differences are usually expressed in terms of the backward difference operator ∇ as

$$\nabla y_i = y_i - y_{i-1} i = n, \ (n-1),\ldots,1$$

To be explicit, we write

OR

$$\begin{aligned} \nabla y_1 &= y_1 - y_0 \\ \nabla y_2 &= y_2 - y_1 \\ \vdots \quad &\quad \vdots \quad \vdots \\ \nabla y_n &= y_n - y_{n-1} \end{aligned}$$

The differences of these differences are called *second differences* and they are denoted by $\nabla^2 y_2, \nabla^2 y_3, \ldots, \nabla^2 y_n$.

That is

$$\begin{aligned} \nabla^2 y_1 &= \nabla y_2 - \nabla y_1 \\ \nabla^2 y_2 &= \nabla y_3 - \nabla y_2 \\ \vdots \quad &\quad \vdots \quad \vdots \\ \nabla^2 y_n &= \nabla y_n - \nabla y_{n-1} \end{aligned}$$

Thus, in general, the second backward differences are

$$\nabla^2 y_i = \nabla y_i - \nabla y_{i-1}, \ \ i = n,(n-1),...,2$$

While the *k-th* backward differences are given as

$$\nabla^k y_i = \nabla^{k-1} y_i - \nabla^{k-1} y_{i-1}, \ \ i = n,(n-1),...,k$$

These backward differences can be systematically arranged for a table of values $(x_k, y_k), k = 0,1,...,6$ shown below.

Backward Difference Table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
x_0	y_0						
x_1	y_1	∇y_1	$\nabla^2 y_2$	$\nabla^3 y_3$	$\nabla^4 y_4$		
x_2	y_2	∇y_2	$\nabla^2 y_3$	$\nabla^3 y_4$	$\nabla^4 y_5$	$\nabla^5 y_5$	
x_3	y_3	∇y_3	$\nabla^2 y_4$	$\nabla^3 y_5$	$\nabla^4 y_6$	$\nabla^5 y_6$	$\nabla^6 y_6$
x_4	y_4	∇y_4	$\nabla^2 y_5$	$\nabla^3 y_6$			
x_5	y_5	∇y_5	$\nabla^2 y_6$				
x_6	y_6	∇y_6					

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From this table, it can be observed that the subscript remains constant along every backward diagonal.

Example

Show that any value of y can be expressed in terms of y_n and its backward differences.

Solution:

From

$$\nabla y_i = y_i - y_{i-1} i = n, \quad (n-1), \ldots, 1$$

We get

$$y_{n-1} = y_n - \nabla y_n \qquad y_{n-2} = y_{n-1} - \nabla y_{n-1}$$

From $$\nabla^2 y_i = \nabla y_i - \nabla y_{i-1}, \quad i = n, (n-1), ..., 2$$

We get $$\nabla y_{n-1} = \nabla y_n - \nabla^2 y_n$$

From these equations, we obtain

$$y_{n-2} = y_n - 2\nabla y_n + \nabla^2 y_n$$

Similarly, we can show that

$$y_{n-3} = y_n - 3\nabla y_n + 3\nabla^2 y_n - \nabla^3 y_n$$

Symbolically, these results can be rewritten as follows:

$$y_{n-1} = (1-\nabla) y_n$$
$$y_{n-2} = (1-\nabla)^2 y_n$$
$$y_{n-3} = (1-\nabla)^3 y_n$$
$$.....$$
$$y_{n-r} = (1-\nabla)^r y_n$$

$$y_{n-r} = y_n - {}^nC_1 \nabla y_n + {}^nC_2 \nabla^2 y_n - \cdots + (-1)^r \nabla^r y_n$$

Central Differences

In some applications, central difference notation is found to be more convenient to represent the successive differences of a function. Here, we use the symbol δ to represent central difference operator and the subscript of δy bb for any difference as the average of the subscripts

$$\delta y_{1/2} = y_1 - y_0, \qquad \delta y_{3/2} = y_2 - y_1,$$

In General,

$$\delta y_i = y_{i+(1/2)} - y_{i-(1/2)}$$

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Higher order differences are defined as follows:

$$\delta^2 y_i = \delta y_{i+(1/2)} - \delta y_{i-(1/2)}$$

$$\delta^n y_i = \delta^{n-1} y_{i+(1/2)} - \delta^{n-1} y_{i-(1/2)}$$

These central differences can be systematically arranged as indicated in the Table

x	y	δy	$\delta^2 y$	$\delta^3 y$	$\delta^4 y$	$\delta^5 y$	$\delta^6 y$
x_0	y_0						
x_1	y_1	$\delta y_{1/2}$	$\delta^2 y_1$	$\delta^3 y_{3/2}$	$\delta^4 y_2$		
x_2	y_2	$\delta y_{3/2}$	$\delta^2 y_2$	$\delta^3 y_{5/2}$	$\delta^4 y_3$	$\delta^5 y_{5/2}$	
x_3	y_3	$\delta y_{5/2}$	$\delta^2 y_3$	$\delta^3 y_{7/2}$	$\delta^4 y_4$	$\delta^5 y_{7/2}$	$\delta^6 y_3$
x_4	y_4	$\delta y_{7/2}$	$\delta^2 y_4$	$\delta^3 y_{9/2}$			
x_5	y_5	$\delta y_{9/2}$	$\delta^2 y_5$				
x_6	y_6	$\delta y_{11/2}$					

Thus, we observe that all the odd differences have a fractional suffix and all the even differences with the same subscript lie horizontally.

The following alternative notation may also be adopted to introduce finite difference operators. Let $y = f(x)$ be a functional relation between x and y, which is also denoted by y_x.

Suppose, we are given consecutive values of x differing by h say x, $x + h$, $x + 2h$, $x + 3h$, etc. The corresponding values of y are $y_x, y_{x+h}, y_{x+2h}, y_{x+3h}$,

As before, we can form the differences of these values.

Thus

$$\Delta y_x = y_{x+h} - y_x = f(x+h) - f(x)$$

$$\Delta^2 y_x = \Delta y_{x+h} - \Delta y_x$$

Similarly,

$$\nabla y_x = y_x - y_{x-h} = f(x) - f(x-h)$$

$$\delta y_x = y_{x+(h/2)} - y_{x-(h/2)} = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$$

To be explicit, we write

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$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\vdots \quad \vdots \quad \vdots$$
$$\Delta y_{n-1} = y_n - y_{n-1}$$

$$\nabla y_i = y_i - y_{i-1} i = n, \ (n-1),\ldots,1$$

OR

$$\nabla y_1 = y_1 - y_0$$
$$\nabla y_2 = y_2 - y_1$$
$$\vdots \quad \vdots \quad \vdots$$
$$\nabla y_n = y_n - y_{n-1}$$

$$\delta y_{1/2} = y_1 - y_0, \qquad \delta y_{3/2} = y_2 - y_1,$$

In General,

$$\delta y_i = y_{i+(1/2)} - y_{i-(1/2)}$$

Higher order differences are defined as follows:

$$\delta^2 y_i = \delta y_{i+(1/2)} - \delta y_{i-(1/2)}$$

$$\delta^n y_i = \delta^{n-1} y_{i+(1/2)} - \delta^{n-1} y_{i-(1/2)}$$

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Shift operator, E

Let $y = f(x)$ be a function of x, and let x takes the consecutive values x, $x + h$, $x + 2h$, etc. We then define an operator having the property

$$E f(x) = f(x+h)$$

Thus, when E operates on $f(x)$, the result is the next value of the function. Here, E is called the shift operator. If we apply the operator E twice on $f(x)$, we get

$$E^2 f(x) = E[E f(x)]$$
$$= E[f(x+h)] = f(x+2h)$$

Thus, in general, if we apply the operator 'E' n times on $f(x)$, we get

$$E^n f(x) = f(x+nh)$$

OR

$$E^n y_x = y_{x+nh}$$

$$Ey_0 = y_1, \qquad E^2 y_0 = y_2, \qquad E^4 y_0 = y_4, \quad \ldots, \quad E^2 y_2 = y_4$$

The inverse operator E^{-1} is defined as

$$E^{-1} f(x) = f(x-h)$$

Similarly

$$E^{-n} f(x) = f(x-nh)$$

Average Operator, μ;

it is defined as

$$\mu f(x) = \frac{1}{2}\left[f\left(x + \frac{h}{2}\right) + f\left(x - \frac{h}{2}\right)\right]$$

$$= \frac{1}{2}\left[y_{x+(h/2)} + y_{x-(h/2)}\right]$$

Differential Operator, D

it is defined as

$$\left.\begin{aligned} Df(x) &= \frac{d}{dx} f(x) = f'(x) \\ D^2 f(x) &= \frac{d^2}{dx^2} f(x) = f''(x) \end{aligned}\right\}$$

Important Results Using $\{\Delta, \nabla, \delta, E, \mu\}$

$$\Delta y_x = y_{x+h} - y_x = Ey_x - y_x$$
$$= (E-1)y_x$$
$$\Rightarrow \Delta = E - 1$$

Also

$$\nabla y_x = y_x - y_{x-h} = y_x - E^{-1} y_x$$
$$= (1 - E^{-1}) y_x$$

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$$\Rightarrow \nabla = 1 - E^{-1} = \frac{E-1}{E}$$

And

$$\begin{aligned}\delta y_x &= y_{x+(h/2)} - y_{x-(h/2)} \\ &= E^{1/2} y_x - E^{-1/2} y_x \\ &= (E^{1/2} - E^{-1/2}) y_x \\ \delta &= E^{1/2} - E^{-1/2}\end{aligned}$$

The definition of μ and E similarly yields

$$\begin{aligned}\mu y_x &= \frac{1}{2}\left[y_{x+(h/2)} + y_{x-(h/2)} \right] \\ &= \frac{1}{2}(E^{1/2} + E^{-1/2}) y_x \\ \Rightarrow \mu &= \frac{1}{2}(E^{1/2} + E^{-1/2})\end{aligned}$$

We know that

$$\begin{aligned}Ey_x &= y_{x+h} = f(x+h) \\ Ey_x &= f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \cdots \\ &= f(x) + hDf(x) + \frac{h^2}{2!} D^2 f(x) + \cdots \\ &= \left(1 + \frac{hD}{1!} + \frac{h^2 D^2}{2!} + \cdots \right) f(x) = e^{hD} y_x\end{aligned}$$

Thus

$$hD = \log E$$

Example:

Prove that

$$\begin{aligned}hD &= \log(1+\Delta) \\ &= -\log(1-\nabla) \\ &= \sinh^{-1}(\mu\delta)\end{aligned}$$

Solution:

Using the standard relations we have

$$\begin{aligned}hD &= \log E \\ &= \log(1+\Delta) \\ &= -\log E^{-1} \\ &= -\log(1-\nabla)\end{aligned}$$

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$$\mu\delta = \frac{1}{2}(E^{1/2} + E^{-1/2})(E^{1/2} - E^{-1/2})$$
$$= \frac{1}{2}(E - E^{-1})$$
$$= \frac{1}{2}(e^{hD} - e^{-hD})$$
$$= \sinh(hD)$$
$$\Rightarrow hD = \sinh^{-1}\mu\delta$$

Example

Prove that

1) $1 + \delta^2\mu^2 = \left(1 + \frac{\delta^2}{2}\right)^2$

2) $E^{1/2} = \mu + \frac{\delta}{2}$

3) $\Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + (\delta^2/4)}$

4) $\mu\delta = \frac{\Delta E^{-1}}{2} + \frac{\Delta}{2}$

5) $\mu\delta = \frac{\Delta + \nabla}{2}$

Solution

From the definition, we have:

(1) $\mu\delta = \frac{1}{2}(E^{1/2} + E^{-1/2})(E^{1/2} - E^{-1/2}) = \frac{1}{2}(E - E^{-1})$

$\therefore \quad 1 + \mu^2\delta^2 = 1 + \frac{1}{4}(E^2 - 2 + E^{-2}) = \frac{1}{4}(E + E^{-1})^2$

$1 + \frac{\delta^2}{2} = 1 + \frac{1}{2}(E^{1/2} - E^{-1/2})^2 = \frac{1}{2}(E + E^{-1})^2$

(2)

$\mu + (\delta/2)$

$= \frac{1}{2}(E^{1/2} + E^{-1/2} + E^{1/2} - E^{-1/2}) = E^{1/2}$

(3)

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$$\frac{\delta^2}{2}+\delta\sqrt{1+(\delta^2/4)}=\frac{\left(E^{1/2}-E^{-1/2}\right)^2}{2}+\frac{\left(E^{1/2}-E^{-1/2}\right)\sqrt{1+\frac{1}{4}\left(E^{1/2}-E^{-1/2}\right)^2}}{1}$$

$$=\frac{E-2+E^{-1}}{2}+\frac{1}{2}(E^{1/2}-E^{-1/2})(E^{1/2}+E^{-1/2})$$

$$=\frac{E-2+E^{-1}}{2}+\frac{E-E^{-1}}{2}$$

$$=E-1=\Delta$$

(4)

$$\mu\delta=\frac{1}{2}(E^{1/2}+E^{-1/2})(E^{1/2}-E^{-1/2})=\frac{1}{2}(E-E^{-1})$$

$$=\frac{1}{2}(1+\Delta-E^{-1})=\frac{\Delta}{2}+\frac{1}{2}(1-E^{-1})$$

$$=\frac{\Delta}{2}+\frac{1}{2}\left(\frac{E-1}{E}\right)=\frac{\Delta}{2}+\frac{\Delta}{2E}$$

(5)

$$\mu\delta=\frac{1}{2}(E^{1/2}+E^{-1/2})(E^{1/2}-E^{-1/2})$$

$$=\frac{1}{2}(E-E^{-1})$$

$$=\frac{1}{2}(1+\Delta-1+\nabla)=\frac{1}{2}(\Delta+\nabla)$$

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Interpolation Newton's Forward Difference Formula

Let $y = f(x)$ be a function which takes values $f(x_0), f(x_0 + h), f(x_0 + 2h), \ldots$, corresponding to various equi-spaced values of x with spacing h, say $x_0, x_0 + h, x_0 + 2h, \ldots$.

Suppose, we wish to evaluate the function $f(x)$ for a value $x_0 + ph$, where p is any real number, then for any real number p, we have the operator E such that

$$E^p f(x) = f(x + ph).$$

$$f(x_0 + ph) = E^p f(x_0) = (1 + \Delta)^p f(x_0)$$

$$= \left[1 + p\Delta + \frac{p(p-1)}{2!}\Delta^2 + \frac{p(p-1)(p-2)}{3!}\Delta^3 + \cdots\right] f(x_0)$$

$$f(x_0 + ph) = f(x_0) + p\Delta f(x_0)$$

$$+ \frac{p(p-1)}{2!}\Delta^2 f(x_0) + \frac{p(p-1)(p-2)}{3!}\Delta^3 f(x_0)$$

$$+ \cdots + \frac{p(p-1)\cdots(p-n+1)}{n!}\Delta^n f(x_0) + \text{Error}$$

This is known as Newton's forward difference formula for interpolation, which gives the value of $f(x_0 + ph)$ in terms of $f(x_0)$ and its leading differences.

This formula is also known as Newton-Gregory forward difference interpolation formula. Here $p=(x-x_0)/h$.

An alternate expression is

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$$+ \frac{p(p-1)(p-n+1)}{n!}\Delta^n y_0 + \text{Error}$$

If we retain $(r + 1)$ terms, we obtain a polynomial of degree r agreeing with y_x at $x0, x1, \ldots, xr$.

This formula is mainly used for interpolating the values of y near the beginning of a set of tabular values and for extrapolating values of y, a short distance backward from y_0

Example:

Evaluate $f(15)$, given the following table of values:

x	10	20	30	40	50
$y = f(x)$	46	66	81	93	101

Solution:

The forward differences are tabulated as

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x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	46				
20	66	20			
30	81	15	–5		
40	93	12	–3	2	
50	101	8	–4	–1	–3

We have Newton's forward difference interpolation formula

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$
$$+ \frac{p(p-1)(p-n+1)}{n!}\Delta^n y_0 + \text{Error}$$

Here we have

$$x_0 = 10, \quad y_0 = 46, \quad \Delta y_0 = 20,$$
$$\Delta^2 y_0 = -5, \quad \Delta^3 y_0 = 2, \quad \Delta^4 y_0 = -3$$

Let y_{15} be the value of y when $x = 15$, then

$$p = \frac{x - x_0}{h} = \frac{15-10}{10} = 0.5$$
$$f(15) = y_{15} = 46 + (0.5)(20) + \frac{(0.5)(0.5-1)}{2}(-5)$$
$$+ \frac{(0.5)(0.5-1)(0.5-2)}{6}(2) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24}(-3)$$
$$= 46 + 10 + 0.625 + 0.125 + 0.1172$$

$f(15) = 56.8672$ correct to four decimal places.

Example

Find Newton's forward difference, interpolating polynomial for the following data:

x	0.1	0.2	0.3	0.4	0.5
$y = f(x)$	1.4	1.56	1.76	2	2.28

Solution;

The forward difference table to the given data is

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x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	0.14				
0.2	1.56	0.16			
0.3	1.76	0.2	0.04		
0.4	2	0.24	0.04	0	
0.5	2.28	0.28	0.04	0	0

Since, 3rd and 4th leading differences are zero, we have Newton's forward difference interpolating formula as

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0$$

In this problem,

$$x_0 = 0.1, y_0 = 1.40,$$

$$\Delta y_0 = 0.16, \Delta^2 y_0 = 0.04,$$

and

$$p = \frac{x - 0.1}{0.1} = 10x - 1$$

Substituting these values,

$$y = f(x) = 1.40 + (10x-1)(0.16) + \frac{(10x-1)(10x-2)}{2}(0.04)$$

This is the required Newton's interpolating polynomial.

Example

Estimate the missing figure in the following table:

x	1	2	3	4	5
$y = f(x)$	2	5	7	---	32

Solution

Since we are given four entries in the table, the function $y = f(x)$ can be represented by a polynomial of degree three.

$$\Delta^3 f(x) = \text{Constant}$$

$$\text{and } \Delta^4 f(x) = 0, \quad \forall x$$

In particular,

$$\Delta^4 f(x_0) = 0$$

Equivalently,

$$(E-1)^4 f(x_0) = 0$$

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Expanding, we have

$$(E^4 - 4E^3 + 6E^2 - 4E + 1) f(x_0) = 0$$

That is,

$$f(x_4) - 4f(x_3) + 6f(x_2) - 4f(x_1) + f(x_0) = 0$$

Using the values given in the table, we obtain

$$32 - 4f(x_3) + 6 \times 7 - 4 \times 5 + 2 = 0$$

which gives $f(x_3)$, the missing value equal to 14.

Example

Consider the following table of values

x	.2	.3	.4	.5	.6
F(x)	.2304	.2788	.3222	.3617	.3979

Find f (.36) using Newton's Forward Difference Formula.

Solution

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.2	0.2304	0.0484	-0.005	0.0011	-0.0005
0.3	0.2788	0.0434	-0.0039	0.0006	
0.4	0.3222	0.0395	-0.0033		
0.5	0.3617	0.0362			
0.6	0.3979				

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$
$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots + \frac{p(p-1)(p-2).....(p-n+1)}{n!}\Delta^n y_0$$

Where

$x_0 = 0.2, \quad y_0 = 0.2304, \quad \Delta y_0 = 0.0484,$

$\Delta^2 y_0 = -0.005, \quad \Delta^3 y_0 = 0.0011, \Delta^4 y_0 = -.0005$ $\quad p = \frac{x - x_0}{h} = \frac{0.36 - 0.2}{0.1} = 1.6$

$$y_x = 0.2304 + 1.6(0.0484) + \frac{1.6(1.6-1)}{2!}(-0.005) + \frac{1.6(1.6-1)(1.6-2)}{3!}(0.0011) + \frac{1.6(1.6-1)(1.6-2)(1.6-3)}{4!}(-.0005)$$

$$= 0.2304 + .077441 - .0024 + \frac{1.6(.6)(-.4)}{6}(.0011) + \frac{1.6(.6)(-.4)(-1.4)}{24}(-.0005)$$

$$= 0.3078 - .0024 - .00007 - .00001$$

$$= .3053$$

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Example

Find a cubic polynomial in x which takes on the values
-3, 3, 11, 27, 57 and 107, when $x = 0, 1, 2, 3, 4$ and 5 respectively.

Solution

Here, the observations are given at equal intervals of unit width.
To determine the required polynomial, we first construct the difference table

Difference Table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	-3	6		
1	3	8	2	
2	11	16	8	6
3	27	30	14	6
4	57	30	20	6
5	107	50		

Since the 4th and higher order differences are zero, the required Newton's interpolation formula

$$f(x_0 + ph) = f(x_0) + p\Delta f(x_0) + \frac{p(p-1)}{2}\Delta^2 f(x_0) + \frac{p(p-1)(p-2)}{6}\Delta^3 f(x_0)$$

Here

$$p = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$$

$$\Delta f(x_0) = 6$$

$$\Delta^2 f(x_0) = 2$$

$$\Delta^3 f(x_0) = 6$$

Substituting these values into the formula, we have

$$f(x) = -3 + 6x + \frac{x(x-1)}{2}(2) + \frac{x(x-1)(x-2)}{6}(6)$$

$$f(x) = x^3 - 2x^2 + 7x - 3,$$

The required cubic polynomial.

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NEWTON'S BACKWARD DIFFERENCE INTERPOLATION FORMULA

For interpolating the value of the function $y = f(x)$ near the end of table of values, and to extrapolate value of the function a short distance forward from y_n, Newton's backward interpolation formula is used

Derivation

Let $y = f(x)$ be a function which takes on values $f(x_n), f(x_n\text{-}h), f(x_n\text{-}2h), \ldots, f(x_0)$ corresponding to equispaced values $x_n, x_n\text{-}h, x_n\text{-}2h, \ldots, x_0$. Suppose, we wish to evaluate the function $f(x)$ at $(x_n + ph)$, where p is any real number, then we have the shift operator E, such that

$$f(x_n + ph) = E^p f(x_n) = (E^{-1})^{-p} f(x_n) = (1-\nabla)^{-p} f(x_n)$$

Binomial expansion yields,

$$f(x_n + ph) = \left[1 + p\nabla + \frac{p(p+1)}{2!}\nabla^2 + \frac{p(p+1)(p+2)}{3!}\nabla^3 + \cdots + \frac{p(p+1)(p+2)\cdots(p+n-1)}{n!}\nabla^n + \text{Error}\right] f(x_n)$$

That is

$$f(x_n + ph) = f(x_n) + p\nabla f(x_n) + \frac{p(p+1)}{2!}\nabla^2 f(x_n) + \frac{p(p+1)(p+2)}{3!}\nabla^3 f(x_n) + \cdots + \frac{p(p+1)(p+2)\cdots(p+n-1)}{n!}\nabla^n f(x_n) + \text{Error}$$

This formula is known as Newton's backward interpolation formula. This formula is also known as Newton's-Gregory backward difference interpolation formula.

If we retain $(r + 1)$terms, we obtain a polynomial of degree r agreeing with $f(x)$ at x_n, $x_{n-1}, \ldots, x_{n-r}$. Alternatively, this formula can also be written as

$$y_x = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots + \frac{p(p+1)(p+2)\cdots(p+n-1)}{n!}\nabla^n y_n + \text{Error}$$

Here $p = \dfrac{x - x_n}{h}$

Example

For the following table of values, estimate $f(7.5)$.

x	1	2	3	4	5	6	7	8
$y = f(x)$	1	8	27	64	125	216	343	512

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Solution

The value to be interpolated is at the end of the table. Hence, it is appropriate to use Newton's backward interpolation formula. Let us first construct the backward difference table for the given data

Difference Table

x	$y = f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	1				
2	8	7			
3	27	19	12		
4	64	37	18	6	
5	125	61	24	6	0
6	216	91	30	6	0
7	343	127	36	6	0
8	512	169	42	6	0

Since the 4th and higher order differences are zero, the required Newton's backward interpolation formula is

$$y_x = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

In this problem,

$$p = \frac{x - x_n}{h} = \frac{7.5 - 8.0}{1} = -0.5$$

$$\nabla y_n = 169, \quad \nabla^2 y_n = 42, \quad \nabla^3 y_n = 6$$

$$y_{7.5} = 512 + (-0.5)(169) + \frac{(-0.5)(0.5)}{2}(42) + \frac{(-0.5)(0.5)(1.5)}{6}(6)$$

$$= 512 - 84.5 - 5.25 - 0.375$$

$$= 421.875$$

Example

The sales for the last five years is given in the table below. Estimate the sales for the year 1979

Year	1974	1976	1978	1980	1982
Sales (in lakhs)	40	43	48	52	57

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Solution

Newton's backward difference Table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1974	40				
1976	43	3			
1978	48	5	2		
1980	52	4	-1	3	
1982	57	5	1	2	5

In this example,

$$p = \frac{1979 - 1982}{2} = -1.5$$

and

$$\nabla y_n = 5, \quad \nabla^2 y_n = 1,$$
$$\nabla^3 y_n = 2, \quad \nabla^4 y_n = 5$$

Newton's interpolation formula gives

$$y_{1979} = 57 + (-1.5)5 + \frac{(-1.5)(-0.5)}{2}(1) + \frac{(-1.5)(-0.5)(0.5)}{6}(2)$$
$$+ \frac{(-1.5)(-0.5)(0.5)(1.5)}{24}(5)$$
$$= 57 - 7.5 + 0.375 + 0.125 + 0.1172$$

Therefore,

$$y_{1979} = 50.1172$$

Example

Consider the following table of values

x	1	1.1	1.2	1.3	1.4	1.5
F(x)	2	2.1	2.3	2.7	3.5	4.5

Use Newton's Backward Difference Formula to estimate the value of f(1.45) .

Solution

x	y=F(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1	2					
1.1	2.1	0.1				
1.2	2.3	0.2	0.1			
1.3	2.7	0.4	0.2	0.1		
1.4	3.5	0.8	0.4	0.2	0.1	
1.5	4.5	1	0.2	-0.2	-0.4	-0.5

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$p = \dfrac{x - x_n}{h} = \dfrac{1.45 - 1.5}{0.1} = -0.5$, $\nabla y_n =$ 1 , $\nabla^2 y_n = .2$, $\nabla^3 y_n =$ - .2 , $\nabla^4 y_n =$ -.4 ,

$\nabla^5 y_n =$ -.5

As we know that

$$y_x = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$
$$+ \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n$$

$$y_x = 4.5 + (-0.5)(1) + \frac{(-0.5)(-0.5+1)}{2!}(0.2) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(-0.2)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(-0.4) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{5!}(-0.5)$$

$= 4.5 - 0.5 - 0.025 + 0.0125 + 0.015625 + 0.068359$

$= 4.07148$

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LAGRANGE'S INTERPOLATION FORMULA

Newton's interpolation formulae developed earlier can be used only when the values of the independent variable x are equally spaced. Also the differences of y must ultimately become small.

If the values of the independent variable are not given at equidistant intervals, then we have the basic formula associated with the name of Lagrange which will be derived now.

Let $y = f(x)$ be a function which takes the values, $y_0, y_1, \ldots y_n$ corresponding to $x_0, x_1, \ldots x_n$. Since there are $(n + 1)$ values of y corresponding to $(n + 1)$ values of x, we can represent the function $f(x)$ by a polynomial of degree n.

Suppose we write this polynomial in the form .

$$f(x) = A_0 x^n + A_1 x^{n-1} + \cdots + A_n$$

or in the form,

$$\begin{aligned} y = f(x) &= a_0(x - x_1)(x - x_2)\cdots(x - x_n) \\ &+ a_1(x - x_0)(x - x_2)\cdots(x - x_n) \\ &+ a_2(x - x_0)(x - x_1)\cdots(x - x_n) + \cdots \\ &+ a_n(x - x_0)(x - x_1)\cdots(x - x_{n-1}) \end{aligned}$$

Here, the coefficients a_k are so chosen as to satisfy this equation by the $(n + 1)$ pairs (x_i, y_i). Thus we get

$$y_0 = f(x_0) = a_0(x_0 - x_1)(x_0 - x_1)(x_0 - x_2)\cdots(x_0 - x_n)$$

Therefore,

$$a_0 = \frac{y_0}{(x_0 - x_1)(x_0 - x_2)\cdots(x_0 - x_n)}$$

Similarily,we obtain

$$a_1 = \frac{y_1}{(x_1 - x_0)(x_1 - x_2)\cdots(x_1 - x_n)}$$

and

$$a_i = \frac{y_i}{(x_i - x_0)(x_i - x_1)\cdots(x_i - x_{i-1})(x_i - x_{i+1})\cdots(x_i - x_n)}$$

$$a_n = \frac{y_n}{(x_n - x_0)(x_n - x_1)\cdots(x_n - x_{n-1})}$$

Substituting the values of $a_0, a_1, \ldots, a_n$ we get

$$\begin{aligned} y = f(x) = &\frac{(x - x_1)(x - x_2)\cdots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\cdots(x_0 - x_n)} y_0 + \frac{(x - x_0)(x - x_2)\cdots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\cdots(x_1 - x_n)} y_1 + \\ &+ \frac{(x - x_0)(x - x_1)\cdots(x - x_{i-1})(x - x_{i+1})\cdots(x - x_n)}{(x_i - x_0)(x_i - x_1)\cdots(x_i - x_{i-1})(x_i - x_{i+1})\cdots(x_i - x_n)} y_i + \cdots \end{aligned}$$

The Lagrange's formula for interpolation

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This formula can be used whether the values $x_0, x_2, \ldots, x_n$ are equally spaced or not.

Alternatively, this can also be written in compact form as

$$y = f(x) = L_0(x)y_0 + L_1(x_1)y_1 + L_i(x_i)y_i + \cdots + L_n(x_n)y_n$$

$$= \sum_{k=0}^{n} L_k(x)y_k \qquad = \sum_{k=0}^{n} L_k(x)f(x_k)$$

Where

$$L_i(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$$

We can easily observe that,

$$L_i(x_i) = 1 \qquad \text{and} \qquad L_i(x_j) = 0, i \neq j.$$

Thus introducing *Kronecker* delta notation

$$L_i(x_j) = \delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

Further, if we introduce the notation

$$\prod(x) = \prod_{i=0}^{n}(x-x_i) = (x-x_0)(x-x_1)\cdots(x-x_n)$$

That is $\prod(x)$ is a product of $(n + 1)$ factors. Clearly, its derivative $\prod'(x)$ contains a sum of $(n + 1)$ terms in each of which one of the factors of will $\prod(x)$ be absent.

We also define,

$$P_k(x) = \prod_{i \neq k}(x-x_i)$$

Which is same as $\prod(x)$ except that the factor $(x–x_k)$ is absent. Then

$$\prod'(x) = P_0(x) + P_1(x) + \cdots + P_n(x)$$

But, when $x = x_k$, all terms in the above sum vanishes except $P_k(x_k)$

Hence

$$\prod'(x_k) = P_k(x_k) = (x_k - x_0)\cdots(x_k - x_{k-1})(_{xk} - x_{k+1})\cdots(x_k - x_n)$$

$$L_k(x) = \frac{P_k(x)}{P_k(x_k)} = \frac{P_k(x)}{\prod'(x_k)}$$

$$= \frac{\prod(x)}{(x-x_k)\prod'(x_k)}$$

Finally, the Lagrange's interpolation polynomial of degree n can be written as

$$y(x) = f(x) = \sum_{k=0}^{n} \frac{\prod(x)}{(x-x_k)\prod'(x_k)} f(x_k)$$

$$= \sum_{k=0}^{n} L_k(x)f(x_k) = \sum_{k=0}^{n} L_k(x)y_k$$

Example

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Find Lagrange's interpolation polynomial fitting the points
$y(1) = -3$, $y(3) = 0$,
$y(4) = 30$, $y(6) = 132$. Hence find $y(5)$.

Solution

The given data can be arranged as

x	1	3	4	6
$y = f(x)$	-3	0	30	132

Using Lagrange's interpolation formula, we have

$$y(x) = f(x) = \frac{(x-3)(x-4)x-6)}{(1-3)(1-4)(1-6)}(-3) + \frac{(x-1)(x-4)x-6)}{(3-1)(3-4)(3-6)}(0)$$
$$+\frac{(x-1)(x-3)(x-6)}{(4-1)(4-3)(4-6)}(30) + \frac{(x-1)(x-3)(x-4)}{(6-1)(6-3)(6-4)}(132)$$

On simplification, we get

$$y(x) = \frac{1}{10}\left(-5x^3 + 135x^2 - 460x + 300\right)$$
$$= \frac{1}{2}(-x^3 + 27x^2 - 92x + 60)$$

Which is required Lagrange's interpolation polynomial.
Now, $y(5) = 75$.

Example

Given the following data, evaluate $f(3)$ using Lagrange's interpolating polynomial.

x	1	2	5
$f(x)$	1	4	10

Solution

Using Lagrange's formula,

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2)$$

Therefore,

$$f(3) = \frac{(3-2)(3-5)}{(1-2)(1-5)}(1) + \frac{(3-1)(3-5)}{(2-1)(2-5)}(4) + \frac{(3-1)(3-2)}{(5-1)(5-2)}(10)$$
$$= 6.49999 = 6.5$$

Example

Find the interpolating polynomial for the data using Lagrange's formula

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x	1	2	-4
F(x)	3	-5	-4

Solution:

As Lagrange's formula for interpolating polynomial is given by,

$$y(x)=f(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0)+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1)+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2)$$

$$=\frac{(x-2)(x+4)}{(1-2)(1+4)}(3)+\frac{(x-1)(x+4)}{(2-1)(2+4)}(-5)+\frac{(x-1)(x-2)}{(-4-1)(-4-2)}(-4)$$

$$=-\frac{3}{5}(x^2+2x-8)-\frac{5}{6}(x^2+3x-4)-\frac{4}{30}(x^2-3x+2)$$

$$=-\frac{3}{5}x^2-\frac{6}{5}x+\frac{24}{5}-\frac{5}{6}x^2-\frac{15}{6}x+\frac{20}{6}-\frac{4}{30}x^2+\frac{12}{30}x-\frac{8}{30}$$

$$=\left(-\frac{3}{5}-\frac{5}{6}-\frac{4}{30}\right)x^2+\left(-\frac{6}{5}-\frac{15}{6}+\frac{12}{30}\right)x+\left(\frac{24}{5}+\frac{20}{6}-\frac{8}{30}\right)$$

$$=-\frac{47}{30}x^2-\frac{33}{10}x+\frac{118}{15}$$

Which is the required polynomial.

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DIVIDED DIFFERENCES

Let us assume that the function $y = f(x)$ is known for several values of x, (x_i, y_i), for $i=0,1,..n$. The divided differences of orders 0, 1, 2, …, n are now defined recursively as:

$$y[x_0] = y(x_0) = y_0$$

is the zero-th order divided difference

The first order divided difference is defined as

$$y[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0}$$

Similarly, the higher order divided differences are defined in terms of lower order divided differences by the relations of the form

$$y[x_0, x_1, x_2] = \frac{y[x_1, x_2] - y[x_0, x_1]}{x_2 - x_0}$$

Generally,

$$y[x_0, x_1, \ldots, x_n] = \frac{1}{x_n - x_0}\left[y[x_1, x_2, \ldots, x_n] - y[x_0, x_1, \ldots, x_{n-1}]\right]$$

Standard format of the Divided Differences

x	$y(x)$	1st order	2nd order	3rd order	4th order
x_0	y_0				
x_1	y_1	$y[x_1, x_0]$			
x_2	y_2	$y[x_2, x_1]$	$y[x_0, x_1, x_2]$	$y[x_0, x_1, x_2, x_3]$	
x_3	y_3	$y[x_3, x_2]$	$y[x_1, x_2, x_3]$	$y[x_1, x_2, x_3, x_4]$	$y[x_0, x_1, x_2, x_3, x_4]$
x_4	y_4	$y[x_4, x_3]$	$y[x_2, x_3, x_4]$		

We can easily verify that the divided difference is a symmetric function of its arguments. That is,

$$y[x_1, x_0] = y[x_0, x_1] = \frac{y_0}{x_0 - x_1} + \frac{y_1}{x_1 - x_0}$$

$$y[x_0, x_1, x_2] = \frac{y[x_1, x_2] - y[x_0, x_1]}{x_2 - x_0}$$

Now

$$= \frac{1}{x_2 - x_0}\left(\frac{y_2 - y_1}{x_2 - x_1} - \frac{y_1 - y_0}{x_1 - x_0}\right)$$

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Therefore

$$y[x_0, x_1, x_2] = \frac{y_0}{(x_0 - x_1)(x_0 - x_2)} + \frac{y_1}{(x_1 - x_0)(x_1 - x_2)} + \frac{y_2}{(x_2 - x_0)(x_2 - x_1)}$$

This is symmetric form, hence suggests the general result as

$$y[x_0, \ldots, x_k] = \frac{y_0}{(x_0 - x_1)\cdots(x_0 - x_k)} + \frac{y_1}{(x_1 - x_0)\cdots(x_1 - x_k)} + \cdots + \frac{y_k}{(x_k - x_0)\cdots(x_k - x_{k-1})}$$

OR

$$y[x_0, \ldots, x_k] = \sum_{i=0}^{k} \frac{y_i}{\prod_{\substack{i=0 \\ i \neq j}}^{k} (x_i - x_j)}$$

Example:

Construct the Newton's divided difference table for values of x=1,2,3,4,5,6 and f(x)= -3,0,15,48,105,192.

Solution:

x	**F(x)**	**1st difference**	**2nd difference**	**3rd difference**
1	**-3**		**15-3/3-1=6**	
2	**0**	**0-(-3)=3**	**33-15/4-2 =9**	**9-6/4-1=1**
3	**15**	**15-0=15**	**57-33/5-3 =12**	**12-9/5-2=1**
4	**48**	**48-15=33**	**87-57/6-4 =15**	**15-12/6-3 =1**
5	**105**	**105-48=57**		
6	**192**	**192-105=87**		

NEWTON'S DIVIDED DIFFERENCE INTERPOLATION FORMULA

Let $y = f(x)$ be a function which takes values $y_0, y_1, \ldots, y_n$ corresponding to $x = x_i$, $i = 0, 1, \ldots, n$. We choose an interpolating polynomial, interpolating at $x = x_i$, $i = 0, 1, \ldots, n$ in the following form

$$y = f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

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Here, the coefficients a_k are so chosen as to satisfy above equation by the $(n + 1)$ pairs (x_i, y_i).

Thus, we have

$$\left.\begin{aligned} &y(x_0) = f(x_0) = y_0 = a_0 \\ &y(x_1) = f(x_1) = y_1 = a_0 + a_1(x_1 - x_0) \\ &y(x_2) = f(x_2) = y_2 = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1) \\ &\cdots \\ &y_n = a_0 + a_1(x_n - x_0) + a_2(x_n - x_0)(x_n - x_1) + \cdots + a_n(x_n - x_0)\cdots(x_n - x_{n-1}) \end{aligned}\right\}$$

The coefficients a0, a_1, …, a_n can be easily obtained from the above system of equations, as they form a lower triangular matrix.

The first equation gives

$$a_0 = y(x_0) = y_0$$

The second equation gives

$$a_1 = \frac{y_1 - y_0}{x_1 - x_0} = y[x_0, x_1]$$

Third equation yields

$$a_2 = \frac{y_2 - y_0 - (x_2 - x_0)y[x_0, x_1]}{(x_2 - x_0)(x_2 - x_1)}$$

Which can be rewritten as

$$a_2 = \frac{\left[y_2 - y_1 + \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x_1 - x_0)\right] - (x_2 - x_0)y[x_0, x_1]}{(x_2 - x_0)(x_2 - x_1)}$$

That is

$$a_2 = \frac{y_2 - y_1 + y[x_0, x_1](x_1 - x_2)}{(x_2 - x_0)(x_2 - x_1)} = \frac{y[x_1, x_2] - y[x_0, x_1]}{x_2 - x_0}$$

Thus, in terms of second order divided differences, we have

$$a_2 = y[x_0, x_1, x_2]$$

Similarly, we can show that

Newton's divided difference interpolation formula can be written as

$$y = f(x) = y_0 + (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2]$$
$$+ \cdots + (x - x_0)(x - x_1)\ldots(x - x_{n-1})y[x_0, x_1, ..., x_n]$$

Newton's divided differences can also be expressed in terms of forward, backward and central differences. They can be easily derived.

Assuming equi-spaced values of abscissa, we have

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$$y[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y_0}{h}$$

$$y[x_0, x_1, x_2] = \frac{y[x_1, x_2] - y[x_0, x_1]}{x_2 - x_0} = \frac{\frac{\Delta y_1}{h} - \frac{\Delta y_0}{h}}{2h} = \frac{\Delta^2 y_0}{2!h^2}$$

By induction, we can in general arrive at the result

$$y[x_0, x_1, \ldots, x_n] = \frac{\Delta^n y_0}{n!h^n}$$

Similarly,

$$y[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\nabla y_1}{h}$$

$$y[x_0, x_1, x_2] = \frac{y[x_1, x_2] - y[x_0, x_1]}{x_2 - x_0} = \frac{\frac{\nabla y_2}{h} - \frac{\nabla y_1}{h}}{2h} = \frac{\nabla^2 y_2}{2!h^2}$$

In General,we have

$$y[x_0, x_1, \ldots, x_n] = \frac{\nabla^n y_n}{n!h^n}$$

Also, in terms of central differences, we have

$$y[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\delta y_{1/2}}{h}$$

$$y[x_0, x_1, x_2] = \frac{y[x_1, x_2] - y[x_0, x_1]}{x_2 - x_0} = \frac{\frac{\delta y_{3/2}}{h} - \frac{\delta y_{1/2}}{h}}{2h} = \frac{\delta^2 y_1}{2!h^2}$$

In general, we have the following pattern

$$\left.\begin{aligned} y[x_0, x_1, \ldots, x_{2m}] &= \frac{\delta^{2m} y_m}{(2m)!h^{2m}} \\ y[x_0, x_1, \ldots, x_{2m+1}] &= \frac{\delta^{2m+1} y_{m+(1/2)}}{(2m+1)!h^{2m+1}} \end{aligned}\right\}$$

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Example

Find the interpolating polynomial by (i) Lagrange's formula and
(ii) Newton's divided difference formula for the following data. Hence show that they

x	0	1	2	4
y	1	1	2	5

represent the same interpolating polynomial.

X	Y	1st D.D	2nd D.D	3rd D.D
0	1			
1	1	0		
2	2	1	1/2	-1/2
4	5	3/2	1/6	

Solution The divided difference table for the given data is constructed as follows:

i) Lagrange's interpolation formula gives

$$\begin{aligned} y = f(x) &= \frac{(x-1)(x-2)(x-4)}{(-1)(-2)(-4)}(1) + \frac{(x-0)(x-2)(x-4)}{(1-0)(1-2)(1-4)}(1) \\ &+ \frac{(x-0)(x-1)(x-4)}{(2)(2-1)(2-4)}(2) + \frac{(x-0)(x-1)(x-2)}{4(4-1)(4-2)}(5) \\ &= -\frac{(x^3-7x^2+14x-8)}{8} + \frac{x^3-6x^2+8x}{3} - \frac{x^3-5x^2+4x}{2} \\ &+ \frac{5(x^3-3x^2+2x)}{24} \\ &= -\frac{x^3}{12} + \frac{3x^2}{4} - \frac{2}{3}x + 1 \end{aligned}$$

(ii) Newton's divided difference formula gives

$$\begin{aligned} y = f(x) &= 1 + (x-0)(0) + (x-0)(x-1)\left(\frac{1}{2}\right) + (x-0)(x-1)(x-2)\left(-\frac{1}{12}\right) \\ &= -\frac{x^3}{12} + \frac{3x^2}{4} - \frac{2}{3}x + 1 \end{aligned}$$

We observe that the interpolating polynomial by both Lagrange's and Newton's divided difference formulae is one and the same.

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Note!

Newton's formula involves less number of arithmetic operations than that of Lagrange's.

Example

Using Newton's divided difference formula, find the quadratic equation for the following

X	0	1	4
Y	2	1	4

data.

Hence find y (2).

Solution:

The divided difference table for the given data is constructed as:

X	Y	1st D.D	2nd D.D
0	2		
1	1	-1	1/2
4	4	1	

Now, using Newton's divided difference formula, we have

$$y = 2 + (x-0)(-1) + (x-0)(x-1)\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}(x^2 - 3x + 4)$$

Hence, $y(2) = 1.$

Example

Find the equation of a cubic curve which passes through the points (4 , -43) , (7 , 83) , (9 , 327) and (12 , 1053) using Dividing Difference Formula.

Solution

The Newton's divided difference table is given by

X	Y	1st divided difference	2nd divided difference	3rd divided difference
4	-43			
7	83	42		
9	327	122	16	
12	1053	242	24	1

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Newton's Divided Difference formula is

$$y = f(x) = y_0 + (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2] + \cdots +$$
$$(x - x_0)(x - x_1)\ldots(x - x_{n-1})y[x_0, x_1, ..., x_n]$$
$$= -43 + (x-4)(42) + (x-4)(x-7)(16) + (x-4)(x-7)(x-9)(1)$$
$$= -43 + (x-4)\{42 + 16x - 112 + x^2 - 16x + 63\}$$
$$= -43 + (x-4)(x^2 - 7)$$
$$= -43 + x^3 - 7x - 4x^2 + 28$$
$$= x^3 - 4x^2 - 7x - 15$$

Which is the required polynomial.

Example

A function $y = f(x)$ is given at the sample points $x = x_0, x_1$ and x_2. Show that the Newton's divided difference interpolation formula and the corresponding Lagrange's interpolation formula are identical.

Solution

For the function $y = f(x)$, we have the data $(x_i, y_i), i = 0, 1, 2.$

The interpolation polynomial using Newton' divided difference formula is given as

$$y = f(x) = y_0 + (x - x_0)y[x_0, x_1]$$
$$+(x - x_0)(x - x_1)y[x_0, x_1, x_2]$$

Using the definition of divided differences, we can rewrite the equation in the form

$$y = y_0(x - x_0)\frac{(y_1 - y_0)}{(x_1 - x_0)} + (x - x_0)(x - x_1)\left[\frac{y_0}{(x_0 - x_1)(x_0 - x_2)}\right.$$
$$\left. + \frac{y_1}{(x_1 - x_0)(x_1 - x_2)} + \frac{y_2}{(x_2 - x_0)(x_2 - x_1)}\right]$$
$$= \left[1 - \frac{(x_0 - x)}{(x_0 - x_1)} + \frac{(x - x_0)(x - x_1)}{(x_0 - x_1)(x_0 - x_2)}\right]y_0$$
$$+\left[\frac{(x - x_0)}{(x_1 - x_0)} + \frac{(x - x_0)(x - x_1)}{(x_1 - x_0)(x_1 - x_2)}\right]y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}y_2$$

On simplification, it reduces to

$$y = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}y_2$$

which is the Lagrange's form of interpolation polynomial.

Hence two forms are identical.

Newton's Divided Difference Formula with Error Term

Following the basic definition of divided differences, we have for any x

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$$\left.\begin{aligned} y(x) &= y_0 + (x - x_0)y[x, x_0] \\ y[x, x_0] &= y[x_0, x_1] + (x - x_1)y[x, x_0, x_1] \\ y[x, x_0, x_1] &= y[x_0, x_1, x_2] + (x - x_2)y[x, x_0, x_1, x_2] \\ &\\ y[x, x_0, ..., x_{n-1}] &= y[x_0, x_1, ..., x_n] + (x - x_n)y[x, x_0, ..., x_n] \end{aligned}\right\}$$

Multiplying the second Equation by $(x - x_0)$, third by $(x - x_0)(x - x_1)$ and so on, and the last by

$(x - x_0)(x - x_1) \ldots (x - x_{n-1})$ and adding the resulting equations, we obtain

$$\begin{aligned} y(x) = y_0 &+ (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2] + \cdots \\ &+ (x - x_0)(x - x_1)\cdots(x - x_{n-1})y[x_0, x_1, ..., x_n] + \varepsilon(x) \end{aligned}$$

Where

$$\varepsilon(x) = (x - x_0)(x - x_1)\cdots(x - x_n)y[x, x_0, ..., x_n]$$

Please note that for $x = x_0, x_1, \ldots, x_n$, the error term $\varepsilon(x)$ vanishes

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Error Term in Interpolation Formulae

We know, if y (x) is approximated by a polynomial P_n (x) of degree n then the error is given by

$$\varepsilon(x) = y(x) - P_n(x),$$

Where

$$\varepsilon(x) = (x - x_0)(x - x_1)\cdots(x - x_n)y[x, x_0, ..., x_n]$$

Alternatively it is also expressed as

$$\varepsilon(x) = \prod(x)y[x, x_0, ..., x_n] = K\prod(x)$$

Now, consider a function $F(x)$, such that

$$F(x) = y(x) - P_n(x) - K\prod(x)$$

Determine the constant K in such a way that $F(x)$ vanishes for $x = x_0, x_1, \ldots, x_n$ and also for an arbitrarily chosen point $\bar{x}$, which is different from the given $(n + 1)$ points.

Let I denotes the closed interval spanned by the values $x_0, ..., x_n, \bar{x}$. Then $F(x)$ vanishes $(n + 2)$ times in the interval I.

By Rolle's theorem $F'(x)$ vanishes at least $(n + 1)$ times in the interval I, $F''(x)$ at least n times, and so on.

Eventually, we can show that $F^{(n+1)}(x)$ vanishes at least once in the interval I, say at $x = \xi$ Thus, we obtain

$$0 = y^{(n+1)}(\xi) - P_n^{(n+1)}(\xi) - K\prod^{(n+1)}(\xi)$$

Since $P_n(x)$ is a polynomial of degree n, its (n + 1)th derivative is zero. Also, from the definition of $\Pi(x)$

$$\prod^{(n+1)}(x) = (n+1)!.$$

Therefore we get

$$K = \frac{y^{(n+1)}(\xi)}{(n+1)!}$$

Hence

$$\varepsilon(x) = y(x) - P_n(x) = \frac{y^{(n+1)}(\xi)}{(n+1)!}\prod(x)$$

for some in the interval I.

Thus the error committed in replacing $y(x)$ by either Newton's divided difference formula or by an identical Lagrange's formula is

$$\varepsilon(x) = \prod(x)y[x, x_0, ..., x_n] = \prod(x)\frac{y^{(n+1)}(\xi)}{(n+1)!}$$

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INTERPOLATION IN TWO DIMENSIONS

Let *u* be a polynomial function in two variables, say *x* and *y*, in particular quadratic in *x* and cubic in *y*, which in general can be written as

$$u = f(x, y) = a_0 + a_1 x + a_2 y + a_3 x^2$$
$$+ a_4 xy + a_5 y^2 + a_6 y^3 + a_7 y^2 x + a_8 y x^2$$
$$+ a_9 y^3 x + a_{10} y^2 x^2 + a_{11} y^3 x^2$$

This relation involves many terms. If we have to write a relation involving three or more variables, even low degree polynomials give rise to long expressions. If necessary, we can certainly write, but such complications can be avoided by handling each variable separately.

If we let x, a constant, say $x = c$, the equation simplifies to the form

$$u = \Big|_{x=c} = b_0 + b_1 y + b_2 y^2 + b_3 y^3$$

Now we adopt the following procedure to interpolate at a point (1, *m*) in a table of two variables, by treating one variable a constant say $x = x_1$. The problem reduces to that of a single variable interpolation.

Any one of the methods discussed in preceding sections can then be applied to get $f(x_1, m)$. Then we repeat this procedure for various values of *x* say $x = x_2, x_3, \ldots, x_n$ keeping *y* constant. Thus, we get a new table with *y* constant at the value $y = m$ and with *x* varying. We can then interpolate at $x = 1$.

Example

Tabulate the values of the function $f(x) = x^2 + y^2 - y$ for $x = 0,1,2,3,4$ and $y = 0,1,2,3,4$.

Using the table of the values, compute f(2.5, 3.5) by numerical double interpolation

Solution

The values of the function for the given values of the given values of x and y are given in

X	……..	Y	……..		
	0	1	2	3	4
0	0	0	2	6	12

the following table

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1	1	1	3	7	13
2	4	4	6	10	16
3	9	9	11	15	21
4	16	16	18	22	28

Cont! Using quadratic interpolation in both x and y directions we need to consider three points in x and y directions. To start with we have to treat one variable as constant, say x. Keeping x = 2.5, y = 3.5 as near the center of the set, we choose the table of values corresponding to x = 1,2, 3 and y = 2, 3, 4.

Cont! The region of fit for the construction of our interpolation polynomial is shown in different color in the table

X	…….	Y	……		
	0	1	2	3	4
0	0	0	2	6	12
1	1	1	3	7	13
2	4	4	6	10	16
3	9	9	11	15	21
4	16	16	18	22	28

Cont !

Thus using Newton's forward difference formula, we have

	At x=1		
Y	f	Δf	$\Delta^2 f$
2	3		
3	7	4	2
4	13	6	

Cont !

Similarly

	At x=2		
Y	f	Δf	$\Delta^2 f$

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2	6		
3	10	4	2
4	16	6	

Cont !

Similarly

	At x=3		
Y	f	Δf	$\Delta^2 f$
2	11		
3	15	4	2
4	21	6	

$$with \quad p = \frac{y - y_0}{h} = \frac{3.5 - 2}{1} = 1.5$$

Cont !

$$with \quad p = \frac{y - y_0}{h} = \frac{3.5 - 2}{1} = 1.5$$

$$f(1, 3.5) = f_0 + p\Delta f_0 + \frac{p(p-1)}{2!}\Delta^2 f_0$$

$$= 3 + (1.5)(4) + \frac{(1.5)(0.5)}{2}(2) = 9.75$$

Cont !

$$f(2, 3.5) = 6 + (1.5)(4) + \frac{(1.5)(0.5)}{2}(2) = 12.75$$

$$f(3, 3.5) = 11 + (1.5)(4) + \frac{(1.5)(0.5)}{2}(2) = 17.75$$

Cont !

Therefore we arrive at the following result

	At x=3.5		
Y	f	Δf	$\Delta^2 f$
1	9.75		
2	12.75	3	2
3	17.75	5	

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Cont !

$$Now \quad defining \quad p = \frac{2.5-1}{1} = 1.5$$

$$f(2.5,3.5) = 9.75 + (1.5)(3) + \frac{(1.5)(0.5)}{2}(2) = 15$$

Cont !

From the functional relation, we also find that

$$f(2.5,3.5) = (2.5)^2 + (3.5)^2 - 3.5 = 15$$

And hence no error in interpolation!!!

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DIFFERENTIATION USING DIFFERENCE OPERATORS

Introduction

Consider a function of single variable y = f (x). If the function is known and simple, we can easily obtain its derivative (s) or can evaluate its definite integral

However, if we do not know the function as such or the function is complicated and is given in a tabular form at a set of points $x_0, x_1, \ldots, x_n$, we use only numerical methods for differentiation or integration of the given function.

We shall discuss numerical approximation to derivatives of functions of two or more variables in the lectures to follow when we shall talk about partial differential equations.

In what follows, we shall derive and illustrate various formulae for numerical differentiation of a function of a single variable based on finite difference operators and interpolation.

Subsequently, we shall develop Newton-Cotes formulae and related trapezoidal rule and Simpson's rule for numerical integration of a function.

We assume that the function $y = f(x)$ is given for the values of the independent variable $x = x_0 + ph$, for $p = 0, 1, 2, \ldots$ and so on. To find the derivatives of such a tabular function, we proceed as follows

Case I:

Using forward difference operator Δ and combining equations

$$E f(x) = f(x+h)$$

and

$$\Delta = E - 1$$

$$hD = \log E = \log(1+\Delta)$$

Remember the Differential operator, D is known to represents the property

$$\left.\begin{aligned} Df(x) &= \frac{d}{dx} f(x) = f'(x) \\ D^2 f(x) &= \frac{d^2}{dx^2} f(x) = f''(x) \end{aligned}\right\}$$

This would mean that in terms of Δ :

$$D = \frac{1}{h}\left(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - \cdots\right)$$

Therefore

$$Df(x_0) = f'(x_0) = \frac{1}{h}\left[\Delta f(x_0) - \frac{\Delta^2 f(x_0)}{2} + \frac{\Delta^3 f(x_0)}{3} - \frac{\Delta^4 f(x_0)}{4} + \frac{\Delta^5 f(x_0)}{5} - \cdots\right] = \frac{d}{dx} f(x_0)$$

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in other words

$$Dy_0 = y_0' = \frac{1}{h}\left(\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \cdots\right)$$

Also, we can easily verify

$$D^2 = \frac{1}{h^2}\left(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \cdots\right)^2$$

$$= \frac{1}{h^2}\left(\Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 - \frac{5}{6}\Delta^5 + \cdots\right)$$

$$D^2 y_0 = \frac{d^2 y_0}{dx^2} = y_0'' = \frac{1}{h^2}\left(\begin{array}{l}\Delta^2 y_0 - \Delta^3 y_0 \\ + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\end{array}\right)$$

Case II:

Using backward difference operator , we have ∇

$$hD = -\log(1-\nabla)$$

On expansion, we have

$$D = \frac{1}{h}\left(\nabla - \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \cdots\right)$$

we can also verify that

$$D^2 = \frac{1}{h^2}\left(\nabla - \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \cdots\right)^2$$

$$= \frac{1}{h^2}\left(\nabla^2 + \nabla^3 + \frac{11}{12}\nabla^4 + \frac{5}{6}\nabla^5 + \cdots\right)$$

Hence

$$\frac{d}{dx}y_n = Dy_n = y_n'$$

$$= \frac{1}{h}\left(\nabla y_n - \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \cdots\right)$$

and

$$y_n'' = D^2 y_n$$

$$= \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right)$$

The formulae for Dy_0 and $D^2 y_0$ are useful to calculate the first and second derivatives at the beginning of the table of values in terms of forward differences; while formulae for y'_n and y''_n

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The formulae for Dy_0 and D^2y_0 are useful to calculate the first and second derivatives at the beginning of the table of values in terms of forward differences; while formulae for y'_n and y''_n

Case III: Using central difference operator δ and following the definitions of differential operator D, central difference operator δ and the shift operator E, we have

$$\delta = E^{1/2} - E^{-1/2} = e^{hD/2} - e^{-hD/2}$$

$$= 2\sinh\frac{hD}{2}$$

Therefore, we have

$$\frac{hD}{2} = \sinh^{-1}\frac{\delta}{2}$$

But,

$$\sinh^{-1} x = x - \frac{1}{2}\frac{x^3}{3} + \frac{1\times 3}{2\times 4}\frac{x^5}{5} - \frac{1\times 3\times 5}{2\times 4\times 6}\frac{x^7}{7} + \cdots$$

Using the last two equations we get

$$\frac{hD}{2} = \left(\frac{\delta}{2} - \frac{\delta^3}{6\times 8} + \frac{3}{40\times 32}\delta^5 - \cdots\right)$$

That is,

$$D = \frac{1}{h}\left(\delta - \frac{1}{24}\delta^3 + \frac{3}{640}\delta^5 - \cdots\right)$$

Therefore

$$\frac{d}{dx}y = y' = Dy$$

$$= \frac{1}{h}\left(\delta y - \frac{1}{24}\delta^3 y + \frac{3}{640}\delta^5 y - \cdots\right)$$

Also

$$D^2 = \frac{1}{h^2}\left(\delta^2 - \frac{1}{12}\delta^4 + \frac{3}{90}\delta^6 - \cdots\right)$$

Hence

$$y'' = D^2 y = \frac{1}{h^2}\left(\delta^2 y - \frac{1}{12}\delta^4 y + \frac{1}{90}\delta^6 y - \cdots\right)$$

For calculating the second derivative at an interior tabular point, we use the equation

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$$D^2 = \frac{1}{h^2}\left(\delta^2 - \frac{1}{12}\delta^4 + \frac{3}{90}\delta^6 - \cdots\right)$$

while for computing the firstderivative at an interior tabular point, we in general use another convenient form for D, which is derived as follows. Multiply the right hand side of

$$\frac{d}{dx}y = y' = Dy = \frac{1}{h}\left(\delta y - \frac{1}{24}\delta^3 y + \frac{3}{640}\delta^5 y - \cdots\right)$$

by

$$\frac{\mu}{\sqrt{1+(\delta^2/4)}}$$

which is unity and noting the Binomial expansion

$$\left(1+\frac{1}{4}\delta^2\right)^{-1/2} = 1 - \frac{1}{8}\delta^2 +$$

$$\frac{3}{128}\delta^4 - \frac{15}{48\times 64}\delta^6 + \cdots$$

we get

$$D = \frac{\mu}{h}\left(1 - \frac{1}{8}\delta^2 + \frac{3}{128}\delta^4 - \cdots\right)$$

$$\left(\delta - \frac{1}{24}\delta^3 + \frac{3}{640}\delta^5 - \cdots\right)$$

On simplification, we obtain

$$D = \frac{\mu}{h}\left(\delta - \frac{1}{6}\delta^3 + \frac{4}{120}\delta^5 - \cdots\right)$$

Therefore the equation can also be written in another useful form as

$$y' = D = \frac{\mu}{h}\left(\delta y - \frac{1}{6}\delta^3 y + \frac{1}{30}\delta^5 y - \cdots\right)$$

The last two equations for y" and y' respectively are known as Stirling's formulae for computing the derivatives of a tabular function. Similar formulae can be derived for computing higher order derivatives of a tabular function**.**

The equation for y' can also be written as

$$y' = \frac{\mu}{h}\left[\delta y - \frac{1^2}{3!}\delta^3 y + \frac{(1)^2(2)^2}{5!}\delta^5 y - \frac{(1)^2(2)^2(3)^2}{7!}\delta^7 y + \cdots\right]$$

In order to illustrate the use of formulae derived so far, for computing the derivatives of a tabulated function, we shall consider some examples in the next lecture.

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DIFFERENTIATION USING DIFFERENCE OPREATORS:

Applications:

Remember Using forward difference operator Δ, the shift operator, the backward difference operator and the average difference operator, we obtained the following formulae:

$$Dy_0 = y_0' = \frac{1}{h}\left(\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \cdots\right)$$

$$D^2 y_0 = \frac{d^2 y_0}{dx^2} = y_0'' = \frac{1}{h^2}\left(\begin{array}{l}\Delta^2 y_0 - \Delta^3 y_0 \\ + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\end{array}\right)$$

$$\frac{d}{dx} y_n = Dy_n = y_n' = \frac{1}{h}\left(\nabla y_n - \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \cdots\right)$$

$$y_n'' = D^2 y_n = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right)$$

$$y'' = D^2 y = \frac{1}{h^2}\left(\delta^2 y - \frac{1}{12}\delta^4 y + \frac{1}{90}\delta^6 y - \cdots\right)$$

$$\frac{d}{dx} y = y' = Dy = \frac{1}{h}\left(\delta y - \frac{1}{24}\delta^3 y + \frac{3}{640}\delta^5 y - \cdots\right)$$

Recall from what we mentioned in the last lecture that for calculating the second derivative at an interior tabular point, we use the equation

$$D^2 = \frac{1}{h^2}\left(\delta^2 - \frac{1}{12}\delta^4 + \frac{3}{90}\delta^6 - \cdots\right)$$

While for computing the first derivative at an interior tabular point, we in general use another convenient form for D, which is derived as follows. Multiply the right hand side of

$$\frac{d}{dx} y = y' = Dy = \frac{1}{h}\left(\delta y - \frac{1}{24}\delta^3 y + \frac{3}{640}\delta^5 y - \cdots\right)$$

By $$\frac{\mu}{\sqrt{1 + (\delta^2/4)}}$$

Which is unity and noting the Binomial expansion

$$\left(1 + \frac{1}{4}\delta^2\right)^{-1/2} = 1 - \frac{1}{8}\delta^2 + \frac{3}{128}\delta^4 - \frac{15}{48 \times 64}\delta^6 + \cdots$$

We get

$$D = \frac{\mu}{h}\left(1 - \frac{1}{8}\delta^2 + \frac{3}{128}\delta^4 - \cdots\right)\left(\delta - \frac{1}{24}\delta^3 + \frac{3}{640}\delta^5 - \cdots\right)$$

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Simplification we get

$$D = \frac{\mu}{h}\left(\delta - \frac{1}{6}\delta^3 + \frac{4}{120}\delta^5 - \cdots\right)$$

Therefore the equation can also be written in another useful form as

$$y' = D = \frac{\mu}{h}\left(\delta y - \frac{1}{6}\delta^3 y + \frac{1}{30}\delta^5 y - \cdots\right)$$

The last two equations y" andy' respectively are known as Stirling's formulae for computing the derivatives of a tabular function. Similar formulae can be derived for computing higher order derivatives of a tabular function.
The equation for y' can also be written as

$$y' = \frac{\mu}{h}\left[\delta y - \frac{1^2}{3!}\delta^3 y + \frac{(1)^2(2)^2}{5!}\delta^5 y - \frac{(1)^2(2)^2(3)^2}{7!}\delta^7 y + \cdots\right]$$

In order to illustrate the use of formulae derived so far, for computing the derivatives of a tabulated function, we consider the following example :

Example:

Compute $f''(0)$ and $f'(0.2)$ from the following tabular data.

x	0.0	0.2	0.4	0.6	0.8	1.0
F(x)	1.00	1.16	3.56	13.96	41.96	101.00

Solution

Since x = 0 and 0.2 appear at and near beginning of the table, it is appropriate to use formulae based on forward differences to find the derivatives. The difference table for the given data is depicted below:

x	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
0.0	1.00					
0.2	1.16	0.16				
0.4	3.56	2.40	2.24	5.76		
0.6	13.96	10.40	8.00	9.60	3.84	0.0
0.8	41.96	28.00	17.60	13.44	3.84	
1.0	101.00	59.04	31.04			

Using forward difference formula for $D^2 f(x)$,

i.e

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$$D^2 f(x) = \frac{1}{h^2}\left[\Delta^2 f(x) - \Delta^3 f(x) + \frac{11}{12}\Delta^4 f(x) - \frac{5}{6}\Delta^5 f(x)\right]$$

we obtained

$$f''(0) = \frac{1}{(02)^2}\left[2.24 - 5.76 + \frac{11}{12}(3.84) - \frac{5}{6}(0)\right] = 0.0$$

Also, using the formula for $Df(x)$, we have

$$Df(x) = \frac{1}{h}\left[\Delta f(x) - \frac{\Delta^2 f(x)}{2} + \frac{\Delta^3 f(x)}{3} - \frac{\Delta^4 f(x)}{4}\right]$$

Hence,

$$f'(0.2) = \frac{1}{0.2}\left(2.40 - \frac{8.00}{2} + \frac{9.60}{3} - \frac{3.84}{4}\right) = 3.2$$

Example

Find $y'(2.2)$ and $y''(2.2)$ from the table.

x	1.4	1.6	1.8	2.0	2.2
Y(x)	4.0552	4.9530	6.0496	7.3891	9.0250

Solution:

Since x=2.2 occurs at the end of the table, it is appropriate to use backward difference formulae for derivatives. The backward difference table for the given data is shown below:

x	f(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.4	4.0552				
1.6	4.9530	0.8978			
1.8	6.0496	1.0966	0.1988	0.0441	
2.0	7.3891	1.3395	0.2429	0.0535	0.0094
2.2	9.0250	1.6359	0.2964		

Using backward difference formulae for $y'(x)$ and $y''(x)$, we have

$$y'_n = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4}\right)$$

Therefore,

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$$y'(2.2) = \frac{1}{0.2}\left(1.6359 + \frac{0.2964}{2} + \frac{0.0535}{3} + \frac{0.0094}{4}\right) = 5(1.8043) = 9.02$$

Also

$$y_n'' = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n\right)$$

Therefore

$$y''(2.2) = \frac{1}{(0.2)^2}\left(0.2964 + 0.0535 + \frac{11}{12}(0.0094)\right)$$
$$= 25(0.3585) = 8.9629$$

Example

Given the table of values, estimate, $y^{//}(1.3)$

x	1.3	1.5	1.7	1.9	2.1	2.3
y	2.9648	2.6599	2.3333	1.9922	1.6442	1.2969

Solution Since x = 1.3 appear at beginning of the table, it is appropriate to use formulae based on forward differences to find the derivatives. The difference table for the given data is depicted below:

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
1.3	2.9648					
1.5	2.6599	-0.3049				
1.7	2.3333	-0.3266	-0.0217			
1.9	1.9922	-0.3411	-0.0145	0.0072		
2.1	1.6442	-0.348	-0.0069	0.0076	0.0004	
2.3	1.2969	-0.3473	0.0007	0.0076	0	-0.0004

H=0.2

Using forward difference formula for $D^2 f(x)$,

$$D^2 f(x) = \frac{1}{h^2}\left[\Delta^2 f(x) - \Delta^3 f(x) + \frac{11}{12}\Delta^4 f(x) - \frac{5}{6}\Delta^5 f(x)\right]$$

We obtain

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$$f''(1.3) = \frac{1}{(0.2)^2}\left[-0.0217 - 0.0072 + \frac{11}{12}(0.0004) - \frac{5}{6}(-0.0004)\right]$$

$$f''(1.3) = \frac{1}{(0.2)^2}\left[-0.0217 - 0.0072 + (0.9167)(0.0004) - (0.8334)(-0.0004)\right]$$

$$f''(1.3) = \frac{1}{(0.2)^2}\left[-0.0217 - 0.0072 + (0.0003667) + (0.0003334)\right]$$

$$f''(1.3) = \frac{1}{(0.2)^2}\left[-0.0282\right] = -0.7050$$

Case IV: Derivation of Two and three point formulae:
Retaining only the first term in equation:

$$Dy_0 = y_0' = \frac{1}{h}\left(\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \cdots\right)$$

we can get another useful form for the first derivative as

$$_i = \frac{\Delta y_i}{h} = \frac{y_{i+1} - y_i}{h} = \frac{y(x_i + h) - y(x_i)}{h}$$

Similarly, by retaining only the first term in Eqn.

$$\frac{d}{dx}y_n = Dy_n = y_n' = \frac{1}{h}\left(\nabla y_n - \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \cdots\right)$$

$$y_i' = \frac{\nabla y_i}{h} = \frac{y_i - y_{i-1}}{h} = \frac{y(x_i) - y(x_i - h)}{h}$$

Adding the last two equations, we have

$$y_i' = \frac{y(x_i + h) - y(x_i - h)}{2h}$$

These Equations constitute two-point formulae for the first derivative. By retaining only the first term in Equation ,

$$D^2 y_0 = \frac{d^2 y_0}{dx^2} = y_0'' = \frac{1}{h^2}\left(\begin{array}{l}\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 \\ + \frac{5}{6}\Delta^5 y_0 + \cdots\end{array}\right)$$

we get,

$$y_i'' = \frac{\Delta^2 y_i}{h^2} = \frac{y_{i+2} - 2y_{i+1} + y_i}{h^2} = \frac{y(x_i + 2h) - 2y(x_i + h) + y(x_i)}{h^2}$$

Similarly we get

$$y_i'' = \frac{\Delta^2 y_i}{h^2} = \frac{y(x_i) - 2y(x_i - h) + y(x_i - 2h)}{h^2}$$

While retaining only the first term in the expression for y" in terms of δ we obtain

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$$y_i'' = \frac{\delta^2 y_i}{h^2} = \frac{\delta y_{i+(1/2)} - \delta y_{i-(1/2)}}{h^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$
$$= \frac{y(x_i - h) - 2y(x_i) + y(x_i + h)}{h^2}$$

The last three equations constitute three-point formulae for computing the second derivative. We shall see later that these two- and three-point formulae become handy for developing extrapolation methods to numerical differentiation and integration.

Example 1;

Write first derivative of f (x) at x = 0.1, 0.2, 0.3
where f (x) is given by

X	0.1	0.2	0.3	0.4	0.5	0.6
F(x)	0.425	0.475	0.400	0.452	0.525	0.575

solution;

Using the two point equation we have

$$f'(0.1) = \frac{f(0.2) - f(0.1)}{h} = \frac{0.475 - 0.425}{0.1} = \frac{0.050}{0.1} = 0.5.$$

$$f'(0.2) = \frac{f(0.3) - f(0.2)}{h} = \frac{0.400 - 0.475}{0.1} = \frac{-0.075}{0.1} = -0.75.$$

$$f'(0.3) = \frac{f(0.4) - f(0.3)}{h} = \frac{0.450 - 0.400}{0.1} = \frac{0.05}{0.1} = 0.5.$$

Example 2:

Find the 2nd derivative at 0.3, 0.4, 0.5 for the function given in the example above.

Solution;

$$f''(0.3) = \frac{f(0.4) - 2f(0.3) + f(0.2)}{h^2 = 0.01} = \frac{0.125}{0.01} = 12.5$$

$$f''(0.4) = \frac{f(0.5) - 2f(0.4) + f(0.3)}{h^2 = 0.01} = \frac{0.025}{0.01} = 2.5$$

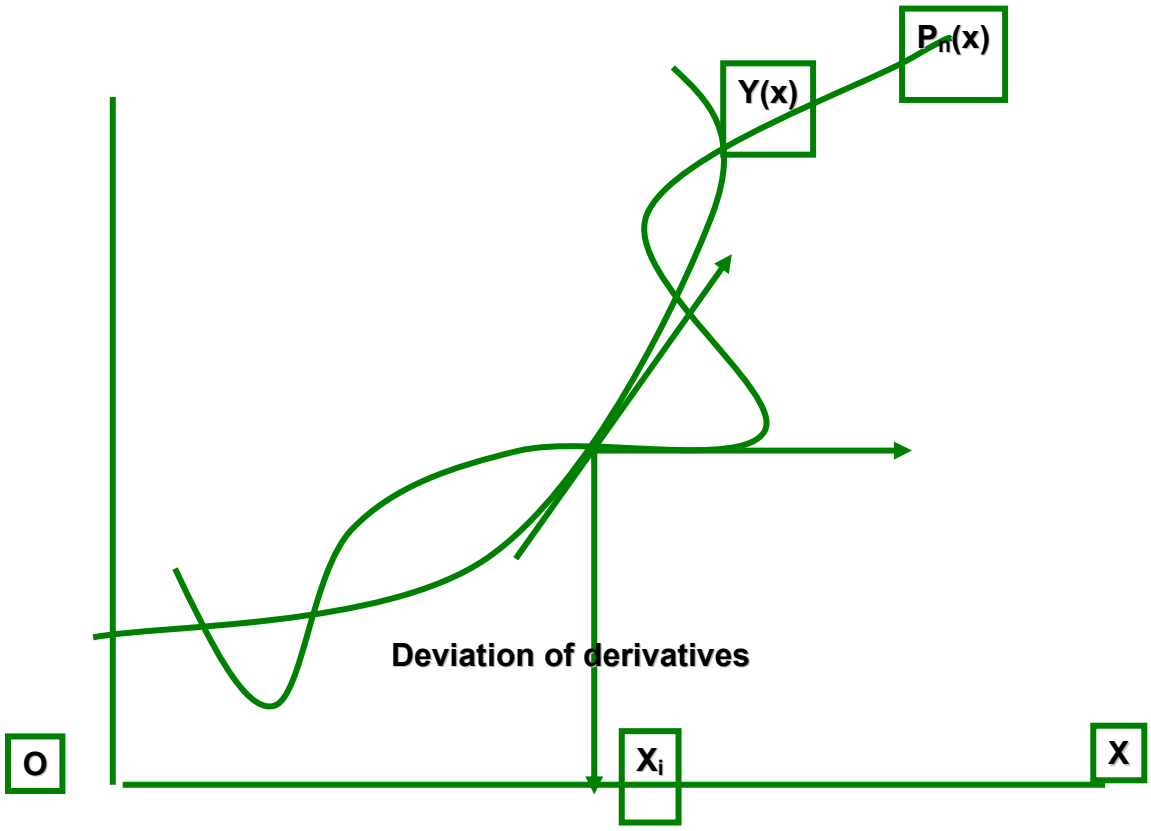
$$f''(0.5) = \frac{f(0.6) - 2f(0.5) + f(0.4)}{0.01} = \frac{0.075}{0.01} = 7.5$$

DIFFERENTIATION USING INTERPOLATION

If the given tabular function y(x) is reasonably well approximated by a polynomial $P_n(x)$ of degree n, it is hoped that the result of $P_n'(x)$ will also satisfactorily approximate the corresponding derivative of y(x).

However, even if $P_n(x)$ and $y(x)$ coincide at the tabular points, their derivatives or slopes may substantially differ at these points as is illustrated in the Figure below:

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For higher order derivatives, the deviations may be still worst. However, we can estimate the error involved in such an approximation.

For non-equidistant tabular pairs (x_i, y_i), i = 0, …, n we can fit the data by using either Lagrange's interpolating polynomial or by using Newton's divided difference interpolating polynomial. In view of economy of computation, we prefer the use of the latter polynomial.

Thus, recalling the Newton's divided difference interpolating polynomial for fitting this data as

$$P_n(x) = y[x_0] + (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2] + \cdots + \prod_{i=0}^{n-1}(x - x_i)y[x_0, x_1, ..., x_n]$$

Assuming that $P_n(x)$ is a good approximation to y(x), the polynomial approximation to can be $y'(x)$ obtained by differentiating $P_n(x)$. Using product rule of differentiation, the derivative of the products in $P_n(x)$ can be seen as follows:

$$\frac{d}{dx}\prod_{i=0}^{n-1}(x-x_i)=\sum_{i=0}^{n-1}\frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{x-x_i}$$

Thus, $y'(x)$ **is approximated by** $P_n'(x)$ **which is given by**

$$P_n'(x)=y[x_0,x_1]+[(x-x_1)+(x-x_0)]y[x_0,x_1,x_2]+\cdots+$$
$$\sum_{i=0}^{n=1}\frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{x-x_i}$$
$$y[x_0,x_1,\ldots,x_n]$$

The error estimate in this approximation can be seen from the following.

We have seen that if y(x) is approximated by $P_n(x)$, the error estimate is shown to be

$$E_n(x)=y(x)-P_n(x)=\frac{\prod(x)}{(n+1)!}y^{(n+1)}(\xi)$$

Its derivative with respect to x can be written as

$$E_n'(x)=y'(x)-P_n'(x)=\frac{\prod'(x)}{(n+1)!}y^{(n+1)}(\xi)+\frac{\prod(x)}{(n+1)!}\frac{d}{dx}y^{(n+1)}(\xi)$$

Its derivative with respect to x can be written as

$$E_n'(x)=y'(x)-P_n'(x)=\frac{\prod'(x)}{(n+1)!}y^{(n+1)}(\xi)+\frac{\prod(x)}{(n+1)!}\frac{d}{dx}y^{(n+1)}(\xi)$$

Since ξ(x) depends on x in an unknown way the derivative

$$\frac{d}{dx}y^{(n+1)}(\xi)$$

cannot be evaluated. However, for any of the tabular points $x = x_i$, $\prod(x)$ vanishes and the difficult term drops out. Thus, the error term in the last equation at the tabular point $x = x_i$ simplifies to

$$E_n'(x_i)=\text{Error}=\prod'(x_i)\frac{y^{(n+1)}(\xi)}{(n+1)!}$$

for some ξ in the interval I defined by the smallest and largest of $x, x_0, x_1, \ldots, x_n$ and

$$\prod'(x_i)=(x_i-x_0)\cdots(x_i-x_n)=\prod_{\substack{j=0\\ j\neq 1}}^{n}(x_i-x_j)$$

The error in the r-th derivative at the tabular points can indeed be expressed analogously.

To understand this method better, we consider the following example.

Example

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Find $f'(0.25)$ and $f'(0.22)$ from the following data using the method based on divided differences:

X	0.15	0.21	0.23
Y=f(x)	0.1761	0.3222	0.3617

X	0.27	0.32	0.35
Y=f(x)	0.4314	0.5051	0.5441

Solution We first construct divided difference table for the given data as shown below:

X	Y	1st D.D	2nd D.D	3rd D.D
0.15	0.1761			
0.21	0.3222	2.4350	-5.700	
0.23	0.3617	1.9750	-3.8750	15.6250
0.27	0.4314	1.7425	-2.9833	8.1064
0.32	0.5051	1.4740	-2.1750	6.7358
0.35	0.5441	1.3000		

Using divided difference formula

$$\sum_{i=0}^{n=1} \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{x-x_i}$$

$$y[x_0, x_1, \ldots, x_n]$$

from a quadratic polynomial, we have

$$y'(x) = P_3'(x) = y[x_0, x_1] + \{(x-x_1)+(x-x_0)\}y[x_0, x_1, x_2]$$
$$+\{(x-x_1)(x-x_2)+(x-x_0)(x-x_2)+(x-x_0)(x-x_1)\}y[x_0, x_1, x_2, x_3]$$

Thus, using first, second and third differences from the table, the above equation yields

$$y'(0.25) = 2.4350 + [(0.25-0.21)+(0.25-0.15)](-5.75)$$
$$+[(0.25-0.21)(0.25-0.23)+(0.25-0.15)(0.25-0.23)$$
$$+(0.25-0.15)(0.25-0.21)](15.625)$$

Therefore

$$f'(0.25) = 2.4350 - 0.805 + 0.10625$$
$$= 1.7363$$

Similarly we can show that

$$f'(0.22) = 1.9734$$

Example

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The following divided difference table is for y=1/x.use it to find $y'(0.75)$ from a quadratic polynomial
Fit.

x	Y=1/x	1st divided difference	2nd divided difference	3rd divided difference
.25	4.0000	-8.0000		
.50	2.0000	-2.6668	10.6664	
.75	1.3333	-1.3332	2.6672	-14.2219
1.00	1.0000	-.8000	1.0664	-2.1344
1.25	.8000	-.5332	0.5336	-0.7104
1.50	.6667			

Solution:
Using divided difference formula

$$\sum_{i=0}^{n=1} \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{x-x_i}$$

$$y[x_0, x_1, \ldots, x_n]$$

If we find the values for quadratic fit then we use the following formula,

$$y'(x) = P_3'(x)$$
$$= y[x_0, x_1] + \{(x-x_1)+(x-x_0)\}y[x_0, x_1, x_2]$$
$$+ \{(x-x_1)(x-x_2)+(x-x_0)(x-x_2)+(x-x_0)(x-x_1)\}y[x_0, x_1, x_2, x_3]$$

Thus, using first, second, third differences from the table, we get

$$y'(x) = P_3'(x) = \text{-}8.0000 + \{(x-.50)+(x-.25)\}10.6664$$
$$+ \{(x-.50)(x-.75)+(x-0.25)(x-0.75)+(x-0.25)(x-0.50)\}(\text{-}14.2219)$$

$Put\ x = 7.5$

$$y'(0.75) = P_3'(0.75) = \text{-}8.0000 + \{(0.75-.50)+(0.75-.25)\}10.6664$$
$$+ \{(0.75-.50)(0.75-.75)+(0.75-0.25)(0.75-0.75)+(0.75-0.25)(0.75-0.50)\}(\text{-}14.2219)$$
$$y'(0.75) = P_3'(0.75) = \text{-}8.0000 + \{(0.25)+(0.50)\}10.6664$$
$$+ \{(0.25)(0)+(0.50)(0)+(0.50)(0.25)\}(\text{-}14.2219)$$
$$y'(0.75) = P_3'(0.75) = \text{-}8.0000 + \{0.75\}10.6664 + \{0.125\}(\text{-}14.2219)$$
$$y'(0.75) = P_3'(0.75) = \text{-}8.0000 + 7.9998 - 1.7777375$$
$$y'(0.75) = P_3'(0.75) = -1.7779$$

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RICHARDSON'S EXTRAPOLATION METHOD

To improve the accuracy of the derivative of a function, which is computed by starting with an arbitrarily selected value of h, Richardson's extrapolation method is often employed in practice, in the following manner:

Suppose we use two-point formula to compute the derivative of a function, then we have

$$y'(x) = \frac{y(x+h) - y(x-h)}{2h} + E_T$$
$$= F(h) + E_T$$

Where E_T is the truncation error. Using Taylor's series expansion, we can see that

$$E_T = c_1 h^2 + c_2 h^4 + c_3 h^6 + \cdots$$

The idea of Richardson's extrapolation is to combine two computed values of $y'(x)$ using the same method but with two different step sizes usually h and h/2 to yield a higher order method. Thus, we have

$$y'(x) = F(h) + c_1 h^2 + c_2 h^4 + \cdots$$

And

$$y'(x) = F\left(\frac{h}{2}\right) + c_1 \frac{h^2}{4} + c_2 \frac{h^4}{16} + \cdots$$

Here, c_i are constants, independent of h, and F(h) and F(h/2) represent approximate values of derivatives. Eliminating c_1 from the above pair of equations, we get

$$y'(x) = \frac{4F\left(\frac{h}{2}\right) - F(h)}{3} + d_1 h^4 + O(h^6)$$

Now assuming that

$$F_1\left(\frac{h}{2}\right) = \frac{4F\left(\frac{h}{2}\right) - F(h)}{3}$$

Equation for y'(x) above reduces to

$$y'(x) = F_1\left(\frac{h}{2}\right) + d_1 h^4 + O(h^6)$$

Thus, we have obtained a fourth-order accurate differentiation formula by combining two results which are of second-order accurate. Now, repeating the above argument, we have

$$y'(x) = F_1\left(\frac{h}{2}\right) + d_1 h^4 + O(h^6)$$

$$y'(x) = F_1\left(\frac{h}{4}\right) + \frac{d_1 h^4}{16} + O(h^6)$$

Eliminating d_1 from the above pair of equations, we get a better approximation as

$$y'(x) = F_2\left(\frac{h}{4}\right) + O(h^6)$$

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Which is of sixth-order accurate, where

$$F_2\left(\frac{h}{4}\right)=\frac{4^2F_1\left(\frac{h}{2^2}\right)-F_1\left(\frac{h}{2}\right)}{4^2-1}$$

This extrapolation process can be repeated further until the required accuracy is achieved, which is called an extrapolation to the limit. Therefore the equation for F_2 above can be generalized as

$$F_m\left(\frac{h}{2^m}\right)=\frac{4^mF_{m-1}\left(\frac{h}{2^m}\right)-F_{m-1}\left(\frac{h}{2^{m-1}}\right)}{4^m-1},$$
$$m=1,2,3,\ldots$$

Where $F_0(h) = F(h)$.

To illustrate this procedure, we consider the following example.

Example: Using the Richardson's extrapolation limit, find y'(0.05) to the function y = -1/x, with h = 0.0128, 0.0064, 0.0032.

Solution

To start with, we take, h = 0.0128, then compute F (h) as

$$F(h)=\frac{y(x+h)-y(x-h)}{2h}=\frac{-\frac{1}{0.05+0.0128}+\frac{1}{0.05-0.0128}}{2(0.0128)}$$
$$=\frac{-15.923566+26.88172}{0.0256}$$
$$=428.05289$$

Similarly, F(h/2) = 406.66273. Therefore, using Eq. (7.30), we get

$$_1\left(\frac{h}{2}\right)=\frac{4F\left(\frac{h}{2}\right)-F\left(\frac{h}{2}\right)}{4-1}=399.5327$$

Which is accurate to $O(h^4)$. Halving the step size further, we compute

$$F\left(\frac{h}{2^2}\right)=\frac{-\frac{1}{0.05+0.0032}+\frac{1}{0.05-0.0032}}{2(0.0032)}$$
$$=401.64515$$

And

$$F_1\left(\frac{h}{2^2}\right) = \frac{4F\left(\frac{h}{2^2}\right) - F\left(\frac{h}{2}\right)}{4-1}$$

$$= 399.97263$$

Again, using Eq. , we obtain

$$F_2\left(\frac{h}{2^2}\right) = \frac{4^2 F_1\left(\frac{h}{2^2}\right) - F_1\left(\frac{h}{2}\right)}{4^2-1}$$

$$= 400.00195$$

The above computation can be summarized in the following table:

h	F	F1	F2
0.0128	428.0529		
0.00064	406.6627	399.5327	400.00195
0.0032	401.6452	399.9726	

Thus, after two steps, it is found that $y'(0.05) = 400.00915$ while the exact value is

$$y'(0.05) = \left(\frac{1}{x^2}\right)_{x=0.05} = \frac{1}{0.0025} = 400$$

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Numerical Differentiation and Integration

INTRODUCTION

DIFFERENTIATION USING DIFFERENCE OPREATORS

DIFFERENTIATION USING INTERPOLATION

RICHARDSON'S EXTRAPOLATION METHOD

NUMERICAL INTEGRATION

NEWTON-COTES INTEGRATION FORMULAE

THE TRAPEZOIDAL RULE (COMPOSITE FORM)

SIMPSON'S RULES (COMPOSITE FORM)

ROMBERG'S INTEGRATION

DOUBLE INTEGRATION

Basic Issues in Integration What does an integral represent?

$$AREA = \int_a^b f(x)\,dx$$

$$VOLUME = \int_c^d \int_a^b g(x,y)\;dx\;dy$$

Basic definition of an integral::

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

Sum of Height x Width

Objective:

Evaluate $I = \int_a^b f(x)\,dx$ without doing calculation analytically.

When would we want to do this?

1. Integrand is too complicated to integrate analytically.

$$\int_0^2 \frac{2+\cos(1+\sqrt{x})}{\sqrt{1+0.5x}} e^{0.5x}\,dx$$

2. Integrand is not precisely defined by an equation,i.e., we are given a set of data $(x_i, f(x_i))$, i=1,...,n.

All methods are applicable to integrands that are functions.

Some are applicable to tabulated values.

Key concepts:

1. Integration is a summing process. Thus virtually all numerical approximations can be represented by

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$$I = \int_a^b f(x)dx = \sum_{i=0}^{n} \Delta x f(x_i) + E_t$$

Where:

x = weights

x_i = sampling points

E_t = truncation error

2. Closed & Open forms:

Closed forms include the end points a & b in x_i. Open forms do not.

NUMERICAL INTEGRATION

Consider the definite integral

$$I = \int_{x=a}^{b} f(x)dx$$

Where f (x) is known either explicitly or is given as a table of values corresponding to some values of x, whether equispaced or not. Integration of such functions can be carried out using numerical techniques.

Of course, we assume that the function to be integrated is smooth and Riemann integrable in the interval of integration. In the following section, we shall develop Newton-Cotes formulae based on interpolation which form the basis for trapezoidal rule and Simpson's rule of numerical integration.

NEWTON-COTES INTERGRATION FORMULAE

In this method, as in the case of numerical differentiation, we shall approximate the given tabulated function, by a polynomial $P_n(x)$ and then integrate this polynomial.

Suppose, we are given the data (x_i, y_i), i = 0(1)n, at equispaced points with spacing h = $x_{i+1} - x_i$, we can represent the polynomial by any standard interpolation polynomial.

Suppose, we use Lagrangian approximation, then we have

$$f(x) = \sum L_k(x) y(x_k)$$

With associated error given by

$$E(x) = \frac{\prod(x)}{(n+1)!} y^{(n+1)}(\xi)$$

Where

$$L_k(x) = \frac{\prod(x)}{(x - x_k)\prod'(x_k)}$$

And

$$\prod(x) = (x - x_0)(x - x_1)\ldots(x - x_n)$$

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Then, we obtain an equivalent integration formula to the definite integral in the form

$$\int_a^b f(x)dx \approx \sum_{k=1}^{n} c_k y(x_k)$$

Where c_k are the weighting coefficients given by

$$c_k = \int_a^b L_k(x)dx$$

Which are also called Cotes numbers. Let the equispaced nodes are defined by

$$x_0 = a, x_n = b, h = \frac{b-a}{n}, \text{ and } x_k = x_0 + kh$$

So that $x_k - x_1 = (k-1)h$ etc. Now, we shall change the variable x to p such that, $x = x_0 + ph$, then we can rewrite equations.

$$L_k(x) = \frac{\prod(x)}{(x-x_k)\prod'(x_k)}$$

$$\prod(x) = (x-x_0)(x-x_1)\ldots(x-x_n)$$

As $$\prod(x) = h^{n+1} p(p-1)\ldots(p-n)$$

And $$L_k(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

$$= \frac{(ph)(p-1)h\cdots(p-k+1)h(p-k-1)h\cdots(p-n)h}{(kh)(k-1)h\cdots(1)(h)(-1)h\cdots(k-n)h}$$

Or

$$L_k(x) = (-1)^{(n-k)} \frac{p(p-1)\cdots(p-k+1)(p-k-1)\cdots(p-n)}{k!(n-k)!}$$

Also, noting that dx = h dp. The limits of the integral in Equation

$$c_k = \int_a^b L_k(x)dx$$

change from 0 to n and equation reduces to

$$c_k = \frac{(-1)^{n-k}(h)}{k!(n-k)!} \int_0^n \begin{matrix} p(p-1) \\ \cdots(p-k+1)(p-k-1) \\ \cdots(p-n)dp \end{matrix}$$

The error in approximating the integral can be obtained from

$$E_n = \frac{h^{n+2}}{(n+1)!} \int_0^n \begin{matrix} p(p-1) \\ \cdots(p-n)y^{(n+1)} \\ (\xi)dp \end{matrix}$$

Where $x_0 < \xi < x_n$. For illustration, consider the cases for n = 1, 2; For which we get

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$$c_0 = -h\int_0^1 (p-1)dp = \frac{h}{2},\ c_1 = h\int_0^1 pdp = \frac{h}{2}$$

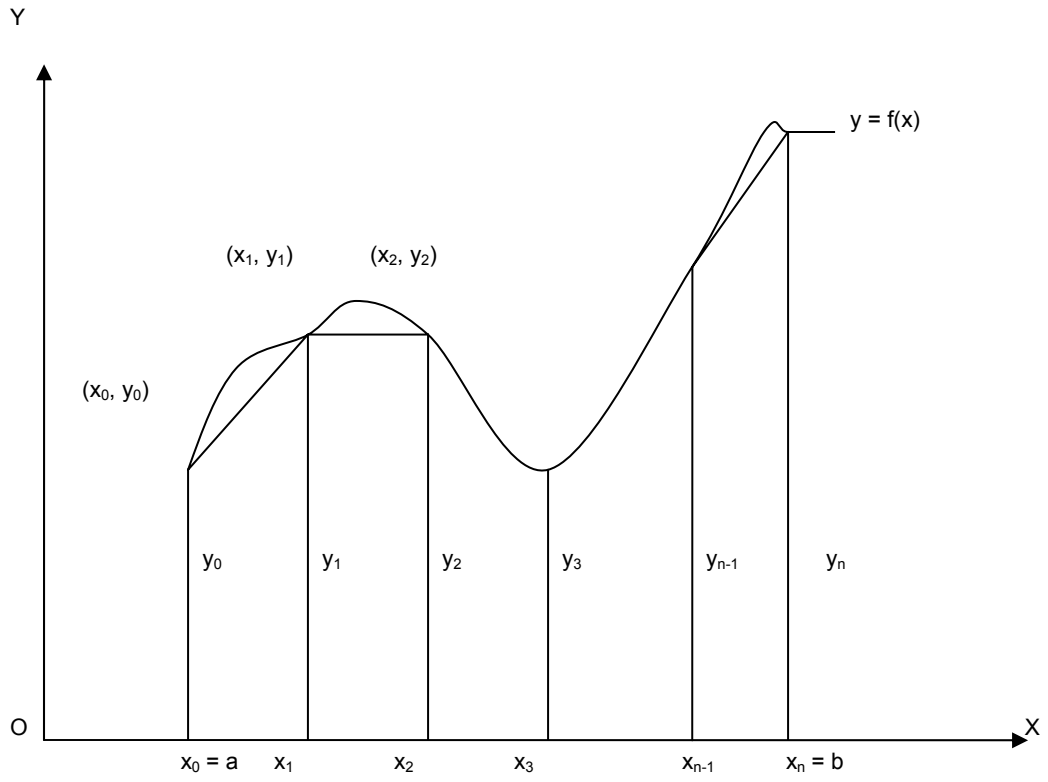
And

$$E_1 = \frac{h^3}{2} y''(\xi)\int_0^1 p(p-1)dp = -\frac{h^3}{12} y''(\xi)$$

Thus, the integration formula is found to be

$$\int_{x_0}^{x_1} f(x)dx = c_0 y_0 + c_1 y_1 + \text{Error} = \frac{h}{2}(y_0 + y_1) - \frac{h^3}{2} y''(\xi)$$

This equation represents the Trapezoidal rule in the interval $[x_0, x_1]$ with error term. Geometrically, it represents an area between the curve y = f (x), the x-axis and the ordinates erected at $x = x_0$ (= a) and $x = x_1$ as shown in the figure.



This area is approximated by the trapezium formed by replacing the curve with its secant line drawn between the end points (x_0, y_0) and (x_1, y_1).

For n =2, We have

$$c_0 = \frac{h}{2}\int_0^2 (p-1)(p-2)dp = \frac{h}{3}$$

$$c_1 = -h\int_0^2 p(p-2)dp = \frac{4}{3}h$$

$$c_2 = \frac{h}{2}\int_0^2 p(p-1)dp = \frac{h}{3}$$

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and the error term is given by

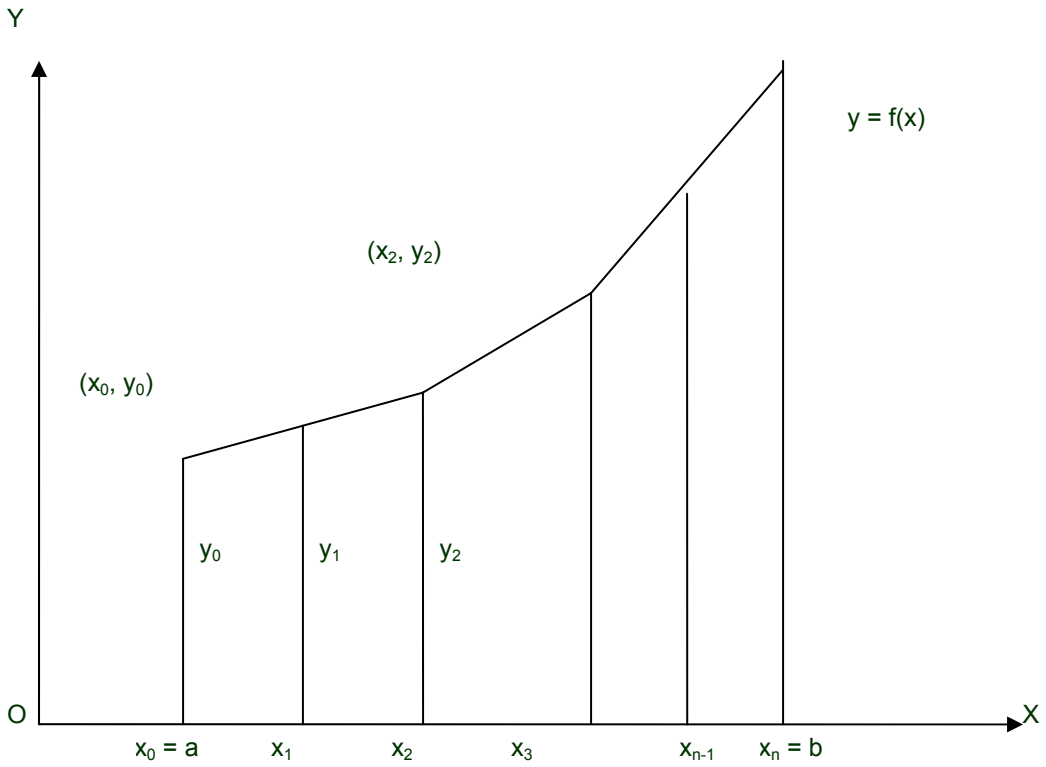
$$E_2 = \frac{h^5}{90} y^{(iv)}(\xi)$$

Thus, for n = 2, the integration takes the form

$$\int_{x_0}^{x_2} f(x)dx = x_0 y_0 + x_1 y_1 + x_2 y_2 + \text{Error}$$

$$= \frac{h}{3}(y_0 + 4y_1 + y_2) - \frac{h^5}{90} y^{(iv)}(\xi)$$

This is known as Simpson's 1/3 rule. Geometrically, this equation represents the area between the curve y = f (x), the x-axis and the ordinates at x = x_0 and x_2 after replacing the arc of the curve between (x_0, y_0) and (x_2, y_2) by an arc of a quadratic polynomial as in the figure



Thus Simpson's 1/3 rule is based on fitting three points with a quadratic.
Similarly, for n = 3, the integration is found to be

$$\int_{x_0}^{x_3} f(x)dx = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + 3y_3) - \frac{3}{80}h^5 y^{(iv)}(\xi)$$

This is known as Simpson's 3/8 rule, which is based on fitting four points by a cubic.
Still higher order Newton-Cotes integration formulae can be derived for large values of n.
But for all practical purposes, Simpson's 1/3 rule is found to be sufficiently accurate.

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The Trapezoidal Rule (Composite Form)

The Newton-Cotes formula is based on approximating y = f (x) between (x_0, y_0) and $(x_1, y_1)$by a straight line, thus forming a trapezium, is called trapezoidal rule. In order to evaluate the definite integral

$$I = \int_a^b f(x)dx$$

we divide the interval [a, b] into n sub-intervals, each of size $h = (b – a)/n$ and denote the sub-intervals by $[x_0, x_1]$, $[x_1, x_2]$, …, $[x_{n-1}, x_n]$, such that $x_0 = a$ and $x_n = b$ and $x_k = x_0 + k_h$, $k = 1, 2, \ldots, n – 1$.

Thus, we can write the above definite integral as a sum. Therefore,

$$I = \int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \cdots + \int_{x_{n-1}}^{x_n} f(x)dx$$

The area under the curve in each sub-interval is approximated by a trapezium. The integral I, which represents an area between the curve y = f (x), the x-axis and the ordinates at $x = x_0$ and $x = x_n$ is obtained by adding all the trapezoidal areas in each sub-interval.

Now, using the trapezoidal rule into equation:

$$\int_{x_0}^{x_1} f(x)dx = c_0 y_0 + c_1 y_1 + \text{Error} = \frac{h}{2}(y_0 + y_1) - \frac{h^3}{2} y''(\xi)$$

We get

$$I = \int_{x_0}^{x_n} f(x)dx = \frac{h}{2}(y_0 + y_1) - \frac{h^3}{2} y''(\xi_1) + \frac{h}{2}(y_1 + y_2) - \frac{h^3}{12} y''(\xi_2)$$

$$+ \cdots + \frac{h}{2}(y_{n-1} + y_n) - \frac{h^3}{12} y''(\xi_n)$$

Where $x_{k-1} < \xi < x_k$, for k = 1, 2, …, n – 1.

Thus, we arrive at the result

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n) + E_n$$

Where the error term E_n is given by

$$E_n = -\frac{h^3}{12}[y''(\xi_1) + y''(\xi_2) + \cdots + y''(\xi_n)]$$

Equation represents the trapezoidal rule over $[x_0, x_n]$, which is also called the composite form of the trapezoidal rule. The error term given by Equation:

$$E_n = -\frac{h^3}{12}[y''(\xi_1) + y''(\xi_2) + \cdots + y''(\xi_n)]$$

is called the global error.

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However, if we assume that $y''(x)$ is continuous over $[x_0, x_n]$ then there exists some ξ in $[x_0, x_n]$ such that $x_n = x_0 + nh$ and

$$E_n = -\frac{h^3}{12}[ny''(\xi)] = -\frac{x_n - x_0}{12}h^2 y''(\xi)$$

Then the global error can be conveniently written as $O(h^2)$.

Simpson's Rules (Composite Forms)

In deriving equation. ,

$$\int_{x_0}^{x_2} f(x)dx = x_0 y_0 + x_1 y_1 + x_2 y_2 + \text{Error} = \frac{h}{3}(y_0 + 4y_1 + y_2) - \frac{h^5}{90}y^{(iv)}(\xi)$$

The Simpson's 1/3 rule, we have used two sub-intervals of equal width. In order to get a composite formula, we shall divide the interval of integration [a, b] Into an even number of sub- intervals say 2N, each of width (b – a)/2N, thereby we have

$x_0 = a, x_1, \ldots, x_{2N} = b$ and $x_k = x_0 + kh$, k = 1,2, … (2N – 1).

Thus, the definite integral I can be written as

$$I = \int_a^b f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \cdots + \int_{x_{2N-2}}^{x_{2N}} f(x)dx$$

Applying Simpson's 1/3 rule as in equation

$$\int_{x_0}^{x_2} f(x)dx = x_0 y_0 + x_1 y_1 + x_2 y_2 + \text{Error} = \frac{h}{3}(y_0 + 4y_1 + y_2) - \frac{h^5}{90}y^{(iv)}(\xi)$$

to each of the integrals on the right-hand side of the above equation, we obtain

$$I = \frac{h}{3}[(y_0 + 4y_1 + y_2) + (y_2 + 4y_3 + y_4) + \cdots + (y_{2N-2} + 4y_{2N-1} + y_{2N})] - \frac{N}{90}h^5 y^{(iv)}(\xi)$$

That is

$$\int_{x_0}^{x_{2N}} f(x)dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots + y_{2N-1}) + 2(y_2 + y_4 + \cdots + y_{2N-2}) + y_{2N}] + \text{Error term}$$

This formula is called composite Simpson's 1/3 rule. The error term E, which is also called global error, is given by

$$E = -\frac{N}{90}h^5 y^{(iv)}(\xi) = -\frac{x_{2N} - x_0}{180}h^4 y^{(iv)}(\xi)$$

for some ξ in $[x_0, x_{2N}]$. Thus, in Simpson's 1/3 rule, the global error is of $O(h^4)$.

Similarly in deriving composite Simpson's 3/8 rule, we divide the interval of integration into n sub-intervals, where n is divisible by 3, and applying the integration formula

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$$\int_{x_0}^{x_3} f(x)dx = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + y_3) - \frac{3}{80}\mathrm{h}^5 y^{(iv)}(\xi)$$

to each of the integral given below

$$\int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_3} f(x)dx + \int_{x_3}^{x_6} f(x)dx + \cdots + \int_{x_{n-3}}^{x_n} f(x)dx$$

We obtain the composite form of Simpson's 3/8 rule as

$$\int_a^b f(x)dx = \frac{3}{8}h[y(a) + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + 2y_6 + \cdots$$
$$+ 2y_{n-3} + 3y_{n-2} + 3y_{n-1} + y(b)]$$

With the global error *E* given by

$$E = -\frac{x_n - x_0}{80}h^4 y^{(iv)}(\xi)$$

It may be noted that the global error in Simpson's 1/3 and 3/8 rules are of the same order. However, if we consider the magnitudes of the error terms, we notice that Simpson's 1/3 rule is superior to Simpson's 3/8 rule. For illustration, we consider few examples.

Example

Find the approximate value of $y = \int_0^{\pi} \sin x dx$ using

(i) Trapezoidal rule

(ii) Simpson's 1/3 rule by dividing the range of integration into six equal parts. Calculate the percentage error from its true value in both the cases.

Solution

We shall at first divide the range of integration $(0, \pi)$ into six equal parts so that each part is of width $\pi/6$ and write down the table of values:

X	0	π/6	π/3	π/2	2π/3	5π/6	π
Y=sinx	0.0	0.5	0.8660	1.0	0.8660	0.5	0.0

Applying trapezoidal rule, we have

$$\int_0^{\pi} \sin x dx = \frac{h}{2}[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

Here, h, the width of the interval is π /6. Therefore,

$$y = \int_0^{\pi} \sin x dx = \frac{\pi}{12}[0 + 0 + 2(3.732)] = \frac{3.1415}{6} \times 3.732 = 1.9540$$

Applying Simpson's 1/3 rule

$$\int_{x_0}^{x_2} f(x)dx = x_0 y_0 + x_1 y_1 + x_2 y_2 + \text{Error} = \frac{h}{3}(y_0 + 4y_1 + y_2) - \frac{h^5}{90}y^{(iv)}(\xi)$$

We have

$$\int_0^{\pi} \sin x dx = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{\pi}{18}[0 + 0 + (4 \times 2) + (2)(1.732)] = \frac{3.1415}{18} \times 11.464 = 2.0008$$

But the actual value of the integral is

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$$\int_0^{\pi} \sin x dx = [-\cos x]_0^{\pi} = 2$$

Hence, in the case of trapezoidal rule
The percentage of error

$$= \frac{2-1.954}{2} \times 100 = 23$$

While in the case of Simpson's rule the percentage error is

$$\frac{2-2.0008}{2} \times 100 = 0.04$$

(sign ignored)

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Example :

From the following data, estimate the value of $\int_1^5 \log x dx$ using Simpson's 1/3 rule.Also, obtain the value of h, so that the value of the integral will be accurate up to five decimal places.

X	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Y=logx	0.0000	0.4055	0.6931	0.9163	1.0986	1.2528	1.3863	1.5041	1.6094

Solution

We have from the data, n = 0, 1, …, 8, and h = 0.5. Now using Simpson's 1/3 rule,

$$\int_1^5 \log x dx = \frac{h}{3}[y_0 + y_8 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)]$$

$$= \frac{0.5}{3}[(0 + 1.6094) + 4(4.0787) + 2(3.178)]$$

$$= \frac{0.5}{3}(1.6094 + 16.3148 + 6.356) = 4.0467$$

The error in Simpson's rule is given by

$$E = -\frac{x_{2N} - x_0}{180} h^4 y^{(iv)}(\xi)$$

(ignoring the sign)

Since $$y = \log x, y' = \frac{1}{x}, y'' = -\frac{1}{x^2}, y''' = \frac{2}{x^3}, y^{(iv)} = -\frac{6}{x^4}$$

$$\underset{1 \le x \le 5}{Max} \quad y^{(iv)}(x) = 6,$$

$$\underset{1 \le x \le 5}{Max} \quad y^{(iv)}(x) = 0.0096$$

Therefore, the error bounds are given by

$$\frac{(0.0096)(4)h^4}{180} < E < \frac{(6)(4)h^4}{180}$$

If the result is to be accurate up to five decimal places, then

$$\frac{24h^4}{180} < 10^{-5}$$

That is, $h^4 < 0.000075$ or $h < 0.09$. It may be noted that the actual value of integrals is

$$\int_1^5 \log x dx = [x \log x - x]_1^5 = 5 \log 5 - 4$$

Example :

Evaluate the integral $I = \int_0^1 \frac{dx}{1 + x^2}$ by using

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(i) Trapezoidal rule
(ii) Simpson's 1/3 rule by taking h = ¼. Hence compute the approximate value of π.

Solution

At first, we shall tabulate the function as

X	0	¼	½	¾	1
$y = 1/ 1+x^2$	1	0.9412	0.8000	0.6400	0.5000

Using trapezoidal rule, and taking h = ¼

$$I = \frac{h}{2}\left[y_0 + y_4 + 4(y_1 + y_2 + y_3)\right] = \frac{1}{8}\left[1.5 + 2(2.312)\right] = 0.7828$$

using Simpson's 1/3 rule, and taking h = ¼, we have

$$I = \frac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2y_2\right] = \frac{1}{12}\left[1.5 + 4(1.512) + 1.6\right] = 0.7854$$

But the closed form solution to the given integral is

$$\int_0^1 \frac{dx}{1+x^2} + \left[\tan^{-1} x\right]_0^1 = \frac{\pi}{4}$$

Equating the last two equations, we get π = 3.1416.

Example: Compute the integral $I = \sqrt{\frac{2}{\pi}}\int_0^1 e^{-x^2/2}dx$ using Simpson's 1/3 rule,

Taking h = 0.125.

Solution At the outset, we shall construct the table of the function as required.

X	0	0.125	0.250	0.375	0.5	0.625	0.750	0.875	1.0
$y = \sqrt{\frac{2}{\pi}}\exp(-x^2/2)$	0.7979	0.7917	0.7733	0.7437	0.7041	0.6563	0.6023	0.5441	0.4839

Using Simpson's 1.3 rule, we have

$$= \frac{h}{3}[y_0 + y_8 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)]$$

$$= \frac{0.125}{3}[0.7979 + 0.4839 + 4(0.7917 + 0.7437 + 0.6563 + 0.5441) + 2(0.7733 + 0.7041 + 0.6023)]$$

$$= \frac{0.125}{3}(1.2812 + 10.9432 + 4.1594)$$

$$= 0.6827$$

Example :

A missile is launched from a ground station. The acceleration during first 80 seconds of flight, as recorded, is given in the following table:

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$t(s)$	0	10	20	30	40	50	60	70	80
$a(m/s^2)$	30.00	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

Compute the velocity of the missile when $t = 80\ s$, using Simpson's 1/3 rule.

Solution:

Since acceleration is defined as the rate of change of velocity, we have

$$v = \int_0^{80} a \quad dt \qquad \text{Or} \qquad \frac{dv}{dt} = a$$

Using Simpson's 1/3-rule, we have

$$v = \frac{h}{3}[(y_0 + y_8) + 4(y_1 + y_3 + y_7) + 2(y_2 + y_4 + y_6)]$$

$$= \frac{10}{3}[(30 + 50.67) + 4(31.63 + 35.47 + 40.33 + 46.69) + 2(33.34 + 37.75 + 43.25)]$$

$$= 3086.1 \quad m/s$$

Therefore, the required velocity is given by $y = 3.0861$ km/s.

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Simpson's 3/8 rule

Consider the definite integral $I = \int_{x=a}^{b} f(x)dx$

$$\int_{x_0}^{x_1} f(x)dx = c_0 y_0 + c_1 y_1 + \text{Error} = \frac{h}{2}(y_0 + y_1) - \frac{h^3}{12} y''(\xi)$$

Then, if n = 2, the integration takes the form

$$\int_{x_0}^{x_2} f(x)dx = x_0 y_0 + x_1 y_1 + x_2 y_2 + \text{Error}$$

$$= \frac{h}{3}(y_0 + 4y_1 + y_2) - \frac{h^5}{90} y^{(iv)}(\xi)$$

Thus Simpson's 1/3 rule is based on fitting three points with a quadratic.
Similarly, for n = 3, the integration is found to be

$$\int_{x_0}^{x_3} f(x)dx = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + y_3) - \frac{3}{80}\mathrm{h}^5 y^{(iv)}(\xi)$$

This is known as Simpson's 3/8 rule, which is based on fitting four points by a cubic. Still higher order Newton-Cotes integration formulae can be derived for large values of n.

TRAPEZOIDAL RULE

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n) + E_n$$

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n) + E_n$$

SIMPSON'S 1/3 RULE

$$I = \int_{x_0}^{x_2} f(x)dx = \frac{h}{3}(y_0 + 4y_1 + y_2) - \frac{h^5}{90} y^{(iv)}(\xi)$$

$$\int_{x_0}^{x_{2N}} f(x)dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots + y_{2N-1}) + 2(y_2 + y_4 + \cdots + y_{2N-2}) + y_{2N}] + \text{Error term}$$

$$E = -\frac{x_{2N} - x_0}{180} h^4 y^{(iv)}(\xi)$$

Simpson's 3/8 rule is

$$\int_a^b f(x)dx = \frac{3}{8}h[y(a) + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + 2y_6 + \cdots + 2y_{n-3} + 3y_{n-2} + 3y_{n-1} + y(b)]$$

With the global error *E* given by

$$E = -\frac{x_n - x_0}{80} h^4 y^{(iv)}(\xi)$$

ROMBERG'S INTEGRATION

We have observed that the trapezoidal rule of integration of a definite integral is of $O(\mathrm{h}^2)$, while that of Simpson's 1/3 and 3/8 rules are of fourth-order accurate.

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We can improve the accuracy of trapezoidal and Simpson's rules using Richardson's extrapolation procedure which is also called Romberg's integration method.
For example, the error in trapezoidal rule of a definite integral

$$I = \int_a^b f(x)dx$$

can be written in the form

$$I = I_T + c_1h^2 + c_2h^4 + c_3h^6 + \cdots$$

By applying Richardson's extrapolation procedure to trapezoidal rule, we obtain the following general formula

$$I_{Tm}\left(\frac{h}{2^m}\right) = \frac{4^m I_{T(m-1)}\left(\frac{h}{2^m}\right) - I_{T(m-1)}\left(\frac{h}{2^{m-1}}\right)}{4^{m-1}}$$

Where $m = 1, 2, \ldots,$ with
$I_{T0}(h) = I_T(h)$.

For illustration, we consider the following example.

Example:

Using Romberg's integration method, find the value of $\int_1^{1.8} y(x)dx$ starting with trapezoidal rule, for the tabular values

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
y = f(x)	1.543	1.669	1.811	1.971	2.151	2.352	2.577	2.828	3.107

Solution:

$$x_0 = 1, \qquad x_n = 1.8,$$

$$h = \frac{1.8 - 1.0}{N},\ x_i = x_0 + ih$$

Let I_T denote the integration by Trapezoidal rule, then for

$$N = 1, h = 0.8, I_T = \frac{h}{2}(y_0 + y_1)$$

$$= 0.4(1.543 + 3.107) = 1.8600$$

$$N = 2, h = 0.4, I_T = \frac{h}{2}(y_0 + 2y_1 + y_2) = 0.2[1.543 + 2(2.151) + 3.107]$$

$$= 1.7904$$

$$N = 4, h = 0.2, I_T = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3) + y_4] = 0.1[1.543 + 2(1.811 + 2.151 + 2.577) + 3.107]$$

$$= 1.7728$$

Similarly for

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$$N = 8, \qquad h = 0.1,$$

$$I_T = 1.7684$$

Now, using Romberg's formula , we have

$$I_{T1}\left(\frac{h}{2}\right) = \frac{4(1.7904) - 1.8600}{3}$$

$$= 1.7672$$

$$I_{T2}\left(\frac{h}{2^2}\right) = \frac{4^2(1.7728) - 1.7672}{4^2 - 1}$$

$$= 1.77317$$

$$I_{T3}\left(\frac{h}{2^3}\right) = \frac{4^3(1.7672) - 1.77317}{4^3 - 1}$$

$$= 1.7671$$

that the value of the tabulated integral is 1.7671**.**

DOUBLE INTEGRATION

To evaluate numerically a double integral of the form

$$I = \int \left[\int (x, y) dx \right] dy$$

over a rectangular region bounded by the lines $x = a$, $x = b$, $y = c$, $y = d$ we shall employ either trapezoidal rule or Simpson's rule, repeatedly With respect to one variable at a time. Noting that, both the integrations are just a linear combination of values of the given function at different values of the independent variable, we divide the interval [a, b] into N equal sub-intervals of size h, such that h = (b – a)/N; and the interval (c, d) into M equal sub-intervals of size k, so that k = (d – c)/M. Thus, we have

$$x_i = x_0 + ih, \quad x_0 = a,$$

$$x_N = b, \quad \text{for } i = 1, 2, ..., N - 1$$

$$y_i = y_0 + ik, \quad y_0 = c,$$

$$y_M = d, \quad \text{for } i = 1, 2, ..., M - 1$$

Thus, we can generate a table of values of the integrand, and the above procedure of integration is illustrated by considering a couple of examples.

Example Evaluate the double integral

$$I = \int_1^2 \int_1^2 \frac{dxdy}{x + y}$$

by using trapezoidal rule, with h = k = 0.25.

Solution

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Taking x = 1, 1.25, 1.50, 1.75, 2.0 and y = 1, 1.25, 1.50, 1.75, 2.0, the following table is generated using the integrand

$$f(x,y)=\frac{1}{x+y}$$

x	y				
	1.00	1.25	1.50	1.75	2.00
1.00	0.5	0.4444	0.4	0.3636	0.3333
1.25	0.4444	0.4	0.3636	0.3333	0.3077
1.50	0.4	0.3636	0.3333	0.3077	0.2857
1.75	0.3636	0.3333	0.307	0.2857	0.2667
2.00	0.3333	0.3077	0.2857	0.2667	0.25

Keeping one variable say *x* fixed and varying the variable y, the application of trapezoidal rule to each row in the above table gives

$$\int_1^2 f(1,y)dy=\frac{0.25}{2}[0.5+2(0.4444+0.4+0.3636)+0.3333]$$

$$=0.4062$$

$$\int_1^2 f(1.25,y)dy=\frac{0.25}{2}[0.4444+2(0.4+0.3636+0.3333)+0.3077]$$

$$=0.3682$$

$$\int_1^2 f(1.5,y)dy=\frac{0.25}{2}[0.4+2(0.3636+0.3333+0.3077)]+0.2857$$

$$=0.3369$$

$$\int_1^2 f(1.75,y)dy=\frac{0.25}{2}[0.3636+2(0.3333+0.3077+0.2857)+0.2667]$$

$$=0.3105$$

and

$$\int_1^2 f(2,y)dy=\frac{0.25}{2}[0.3333+2(0.3077+0.2857)+0.25]$$

$$=0.2879$$

Therefore

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$$I = \int_1^2 \int_1^2 \frac{dxdy}{x+y} = \frac{h}{2}\{f(1,y) + 2[f(1.25,y) + f(1.5,y) + f(1.75,y)] + f(2,y)\}$$

By use of the last equations we get the required result as

$$I = \frac{0.25}{2}\left[.04602 + 2(0.3682 + 0.3369 + 0.3105) + 0.2879\right] = 0.3407$$

Example :Evaluate

$$\int_0^{\pi/2} \int_0^{\pi/2} \sqrt{\sin(x+y)}dxdy$$

by numerical double integration.

Solution

Taking x = y = π/4, 3 π /8, π /2, we can generate the following table of the integrand

$$f(x,y) = \sqrt{\sin(x+y)}$$

x	y				
	0	π/8	π/4	3π/8	π/2
0	0.0	0.6186	0.8409	0.9612	1.0
π/8	0.6186	0.8409	0.9612	1.0	0.9612
π/4	0.8409	0.9612	1.0	0.9612	0.8409
3π/8	0.9612	1.0	0.9612	0.8409	0.6186
π/2	1.0	0.9612	0.8409	0.6186	0.0

Keeping one variable as say *x* fixed and *y* as variable, and applying trapezoidal rule to each row of the above table, we get

$$\int_0^{\pi/2} f(0,y)dx = \frac{\pi}{16}\left[0.0 + 2(0.6186 + 0.8409 + 0.9612) + 1.0\right] = 1.1469$$

$$\int_0^{\pi/2} \left(\frac{\pi}{8}, y\right)dx = \frac{\pi}{16}\left[0.6186 + 2(0.8409 + 0.9612 + 1.0) + 0.9612\right] = 1.4106$$

Similarly we get

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$$\int_0^{\pi/2} f\left(\frac{\pi}{4}, y\right)dx = 1.4778,$$

$$\int_0^{\pi/2} f\left(\frac{3\pi}{8}, y\right)dx = 1.4106$$

and

$$\int_0^{\pi/2} f\left(\frac{\pi}{2}, y\right)dx = 1.1469$$

Using these results, we finally obtain

$$\int_0^{\pi/2}\int_0^{\pi/2} \sqrt{\sin(x+y)}dxdy = \frac{\pi}{16}\left\{f(0,y) + 2\left(\frac{\pi}{8}, y\right) + f\left(\frac{\pi}{4}, y\right) + f\left(\frac{3\pi}{8}, y\right) + f\left(\frac{\pi}{2}, y\right)\right\}$$

$$= \frac{\pi}{16}\left[1.1469 + 2(1.4106 + 1.4778 + 1.4106) + 1.1469\right]$$

$$= 2.1386$$

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Ordinary Differential Equations

Introduction

Taylor Series

Euler Method

Runge-Kutta Method

Predictor Corrector Method

Introduction

Many problems in science and engineering when formulated mathematically are readily expressed in terms of ordinary differential equations (ODE) with initial and boundary condition.

Example

The trajectory of a ballistic missile, the motion of an artificial satellite in its orbit, are governed by ordinary differential equations.

Theories concerning electrical networks, bending of beams, stability of aircraft, etc., are modeled by differential equations.

To be more precise, the rate of change of any quantity with respect to another can be modeled by an ODE

Closed form solutions may not be possible to obtain, for every modeled problem, while numerical methods exist, to solve them using computers.

In general, a linear or non-linear ordinary differential equation can be written as

$$\frac{d^n y}{dt^n} = f\left(t, y, \frac{dy}{dt}, \cdots, \frac{d^{n-1} y}{dt^{n-1}}\right)$$

Here we shall focus on a system of first order differential equations of the form $\frac{dy}{dt} = f(t, y)$ with the initial condition $y(t_0) = y_0$, which is called an initial value problem (IVP).

It is justified, in view of the fact that any higher order ODE can be reduced to a system of first order differential equations by substitution.

For example, consider a second order differential equation of the form

$$y'' = f(t, y, y')$$

Introducing the substitution $p = y'$, the above equation reduces to a system of two first order differential equations, such as

$$y' = p, \qquad p' = f(t, y, p)$$

Theorem

Let $f(t, y)$ be real and continuous in the strip R, defined by $t \in [t_0, T], \qquad -\infty \le y \le \infty$

Then for any $t \in [t_0, T]$ and for any y_1, y_2, there exists a constant L, satisfying the inequality $|f(t, y_1) - f(t, y_2)| \le L|y_1 - y_2|$ so that $|f_y(t, y)| \le L,$ for every $t, y \in R$

Here, L is called *Lipschitz constant*.

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If the above conditions are satisfied, then for any y_0, the IVP has a unique solution $y(t)$, for $t \in [t_0, T]$

In fact, we assume the existence and uniqueness of the solution to the above IVP
The function may be linear or non-linear. We also assume that the function $f(t, y)$ is sufficiently differentiable with respect to either t or y.

TAYLOR'S SERIES METHOD

Consider an initial value problem described by

$$\frac{dy}{dt} = f(t, y), \qquad y(t_0) = y_0$$

Here, we assume that $f(t, y)$ is sufficiently differentiable with respect to x and y.
If $y(t)$ is the exact solution, we can expand $y(t)$ by Taylor's series about the point $t = t_0$ and obtain

$$y(t) = y(t_0) + (t - t_0)y'(t_0) + \frac{(t-t_0)^2}{2!}y''(t_0) + \frac{(t-t_0)^3}{3!}y'''(t_0) + \frac{(t-t_0)^4}{4!}y^{IV}(t_0) + \cdots$$

Since, the solution is not known, the derivatives in the above expansion are not known explicitly. However, f is assumed to be sufficiently differentiable and therefore, the derivatives can be obtained directly from the given differential equation.
Noting that f is an implicit function of y, we have

$$\left.\begin{aligned} y' &= f(t, y) \\ y'' &= \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx} = f_x + ff_y \end{aligned}\right\}$$

Similarly

$$\left.\begin{aligned} y''' &= f_{xx} + ff_{xy} + f(f_{xy} + ff_{yy}) + f_y(f_x + ff_y) \\ &= f_{xx} + 2ff_{xy} + f^2 f_{yy} + f_y(f_x + ff_y) \\ y^{IV} &= f_{xxx} + 3ff_{xxx} + 3f^2 f_{xyy} \\ &\quad + f_y(f_{xx} + 2ff_{xy} + f^2 f_{yy}) \\ &\quad + 3(f_x + ff_y)(ff_{xy} + ff_{yy}) \\ &\quad + f_y^2(f_x + ff_y) \end{aligned}\right\}$$

Continuing in this manner, we can express any derivative of y in terms of $f(t, y)$ and its partial derivatives.

Example

Using Taylor's series method, find the solution of the initial value problem

$$\frac{dy}{dt} = t + y, \qquad y(1) = 0$$

at $t = 1.2$, with $h = 0.1$ and compare the result with the closed form solution

Solution

Let us compute the first few derivatives from the given differential equation as follows:

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$$y' = t + y, \quad y'' = 1 + y', \quad y''' = y'',$$
$$y^{IV} = y''', \quad y^{V} = y^{IV}$$

Prescribing the initial condition, that is, at

$t_0 = 1, y_0 - y(t_0) = 0$, we have

$$y_0' = 1, y_0'' = 2,$$
$$y_0''' = y_0^{IV} = y_0^{V} = 2$$

Now, using Taylor's series method, we have

$$y(t) = y_0 + (t - t_0)y_0' + \frac{(t-t_0)^2}{2} y_0'' + \frac{(t-t_0)^3}{6} y_0'''$$
$$+ \frac{(t-t_0)^4}{24} y_0^{IV} + \frac{(t-t_0)^5}{120} y_0^{V} + \cdots$$

Substituting the above values of the derivatives, and the initial condition, we obtain

$$y(1.1) = 0 + (0.1)(1) + \frac{0.01}{2}(2) + \frac{0.001}{6}(2) + \frac{0.00001}{120}(2) + \cdots$$
$$= 0.1 + 0.01 + \frac{0.001}{3} + \frac{0.0001}{12} + \frac{0.00001}{60} + \cdots$$
$$= 0.1 + 0.01 + 0.000333 + 0.0000083 + 0.0000001 + \cdots$$
$$= 0.1103414$$

Therefore

$$y(1.1) = y_1 = 0.1103414 \cong 0.1103$$

Taking $y_1 = 0.1103$ at $t = 1.1$, the values of the derivatives are

$$y_1' = 1.1 + 0.1103 = 1.2103$$
$$y_1'' = 1 + 1.2103 = 2.2103$$
$$y_1''' = y_1^{IV} = y_1^{V} = 2.2103$$

Substituting the value of y_1 and its derivatives into Taylor's series expansion we get, after retaining terms up to fifth derivative only……

$$y(1.2) = y_1 + (t - t_1)y_1' + \frac{(t-t_1)^2}{2} y_1''$$
$$+ \frac{(t-t_1)^3}{6} y_1''' + \frac{(t-t_1)^4}{24} y_1^{IV} + \frac{(t-t_1)^5}{120} y_1^{V}$$
$$y(1.2) = 0.1103 + 0.12103$$
$$+0.0110515 + 0.0003683$$
$$+0.000184 + 0.0000003$$
$$= 0.2429341 \sim 0.2429 \approx 0.243$$

To obtain the closed form solution, we rewrite the given IVP as

$$\frac{dy}{dt} - y = t \qquad \text{or} \qquad d(ye^{-t}) = te^{-t}$$

On integration, we get

$$y = -e^t(te^{-t} + e^{-t}) + ce^t$$

$$= ce^t - t - 1$$

Using the initial condition, we get Therefore, the closed form solution is

$$y = -t - 1 + 2e^{t-1}$$

When $t = 1.2$, the closed form solution becomes

$$y(1.2) = -1.2 - 1 + 2(1.2214028)$$

$$-2.2 + 2.4428056$$

$$= 0.2428 \approx 0.243$$

Example

Using Taylor's Series method taking algorithm of order 3, solve the initial value problem

$$y' = 1 - y \quad ; \quad y(0) = 0 \ \ with \ \ h = 0.25 \quad \text{at} \quad x = .50$$

Solution:

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Let us compute the first three derivatives

$y' = 1 - y \qquad y'' = -y' \qquad y''' = -y''$

The initial condition is

$t_0 = 0 \quad y_0 = 0$, *we have*

$$y' = 1 - (0) = 1$$

$$y'' = -y' = -1$$

$$y''' = -y'' = -(-1) = 1$$

Now, Taylor's series methosd a lg *rithm is*

$$y(t_1) = y_0 + (t_1 - t_0)y_0' + \frac{(t_1 - t_0)^2}{2}y_0'' + \frac{(t_1 - t_0)^3}{3!}y_0'''$$

$$OR \quad y(t_1) = y_0 + hy_0' + h^2\frac{1}{2}y_0'' + h^3\frac{1}{3!}y_0''' \qquad h = 0.25$$

$$y(0.25) = 0 + (0.25)(1) + \frac{(0.25)^2}{2}(-1) + \frac{(0.25)^3}{3!}(1)$$

$$y(0.25) = 0.25 - \frac{0.0625}{2} + \frac{0.015625}{6}$$

$$y(0.25) = 0.25 - 0.03125 + 0.002604167$$

$$y(0.25) = 0.22135$$

Taking $y_1 = 0.22135$ *, Now*

$$y' = 1 - y = 1 - (0.22135) = 0.7786$$

$$y'' = -y' = -0.7786$$

$$y''' = -y'' = -(-0.7786) = 0.7786$$

$$y(t_2) = y_1 + (t_2 - t_1)y_1' + \frac{(t_2 - t_1)^2}{2}y_1'' + \frac{(t_2 - t_1)^3}{3!}y_1'''$$

$$OR \quad y(t_2) = y_1 + hy_1' + h^2\frac{1}{2}y_1'' + \frac{1}{6}h^3y_1''' \qquad h = .25$$

$$= 0.22135 + (0.25)(0.7786) + \frac{1}{2}(0.25)^2(-0.7786) + \frac{1}{6}(0.25)^3(0.7786)$$

$$= 0.22135 + 0.19465 - 0.02433 + 0.002027$$

$$= 0.39369$$

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EULER METHOD

Euler method is one of the oldest numerical methods used for integrating the ordinary differential equations. Though this method is not used in practice, its understanding will help us to gain insight into nature of predictor-corrector method

Consider the differential equation of first order with the initial condition $y(t_0) = y_0$.

$$\frac{dy}{dt} = f(t, y)$$

The integral of this equation is a curve in, *ty*-plane.

Here, we find successively $y_1, y_2, \ldots, y_m$; where y_m is the value of y at $t = t_m = t_0 + mh$, $m = 1, 2, \ldots$ and h being small.

Here we use a property that in a small interval, a curve is nearly a straight line. Thus at (t_0, y_0), we approximate the curve by a tangent at that point.

Therefore,

$$\left(\frac{dy}{dt}\right)_{(t_0, y_0)} = \frac{y - y_0}{t - t_0} = f(t_0, y_0)$$

That is $$y = y_0 + (t - t_0) f(t_0, y_0)$$

Hence, the value of y corresponding to $t = t_1$ is given by

$$y_1 = y_0 + (t_1 - t_0) f(t_0, y_0)$$

Similarly approximating the solution curve in the next interval (t_1, t_2) by a line through (t_1, y_1) having its slope *f(t 1, y 1), we obtain*

$$y_2 = y_1 + hf(t_1, y_1)$$

Thus, we obtain in general, the solution of the given differential equation in the form of a recurrence relation

$$y_{m+1} = y_m + hf(t_m, y_m)$$

Geometrically, this method has a very simple meaning. The desired function curve is approximated by a polygon train, where the direction of each part is determined by the value of the function *f* (*t, y*) at its starting point.

Example

Given $\frac{dy}{dt} = \frac{y - t}{y + t}$ with the initial condition $y = 1$ at $t = 0$. Using Euler method, find y approximately at x = 0.1, in five steps.

Solution

Since the number of steps are five, we shall proceed in steps of (0.1)/5 = 0.02.

Therefore, taking step size

$h = 0.02$, we shall compute the value of y at

$t = 0.02, 0.04, 0.06, 0.08$ and 0.1

Thus $y_1 = y_0 + hf(t_0, y_0)$, where $y_0 = 1, t_0 = 0$

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Therefore $y_1 = 1 + 0.02\dfrac{1-0}{1+0} = 1.02$

$$y_2 = y_1 + hf(t_1 + y_1) = 1.02 + 0.02\frac{1.02 - 0.02}{1.02 + 0.02} = 1.0392$$

$$y_3 = y_2 + hf(t_2, y_2) = 1.0392 + 0.02\frac{1.0392 - 0.04}{1.0392 + 0.04} = 1.0577$$

Similarly,

$$y_4 = y_3 + hf(t_3, y_3) = 1.0577 + 0.02\frac{1.0577 - 0.06}{1.0577 + 0.06} = 1.0738$$

$$y_5 = y_4 + hf(t_4, y_4) = 1.0738 + 0.02\frac{1.0738 - 0.08}{1.0738 + 0.08} = 1.0910$$

Hence the value of y corresponding to $t = 0.1$ is 1.091

Example

Solve the differential equation

$$y' = x + y \quad ; \quad y(0) = 1$$

in the interval [0,0.5] using Euler's method by taking h=0.1

Soution:

x_0	0
x_1	0.1
x_2	0.2
x_3	0.3
x_4	0.4
x_5	0.5

Since $h = 0.1$ *so we calculate values at* $x = 0, 0.1, 0.2, 0.3, 0.4, 0.5$

$y(0) = 1$ here $x_0 = 0$ $y_0 = 1$

$y_1 = y_0 + hf(x_0, y_0)$

$y_1 = 1 + (0.1)(0 + 1) = 1 + 0.1 = 1.1$

$y_1 = 1.1 \quad x_1 = 0.1$

$y_2 = y_1 + hf(x_1, y_1)$

$y_2 = 1.1 + (0.1)(0.1 + 1.1) = 1.1 + 0.12 = 1.22$

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now

$y_2 = 1.22 \quad x_2 = 0.2$

$y_3 = y_2 + hf(x_2, y_2)$

$y_3 = 1.22 + (0.1)(1.22 + 0.2) = 1.22 + 0.142 = 1.3620$

now

$y_3 = 1.3620 \quad x_3 = 0.3$

$y_4 = y_3 + hf(x_3, y_3)$

$y_4 = 1.3620 + (0.1)(1.3620 + 0.3) = 1.3620 + 0.1662 = 1.5282$

now

$y_4 = 1.3620 \quad x_4 = 0.4$

$y_5 = y_4 + hf(x_4, y_4)$

$y_5 = 1.5282 + (0.1)(1.5282 + 0.4) = 1.7210$

now

$y_5 = 1.7210 \quad x_5 = 0.5$

$y_6 = y_5 + hf(x_5, y_5)$

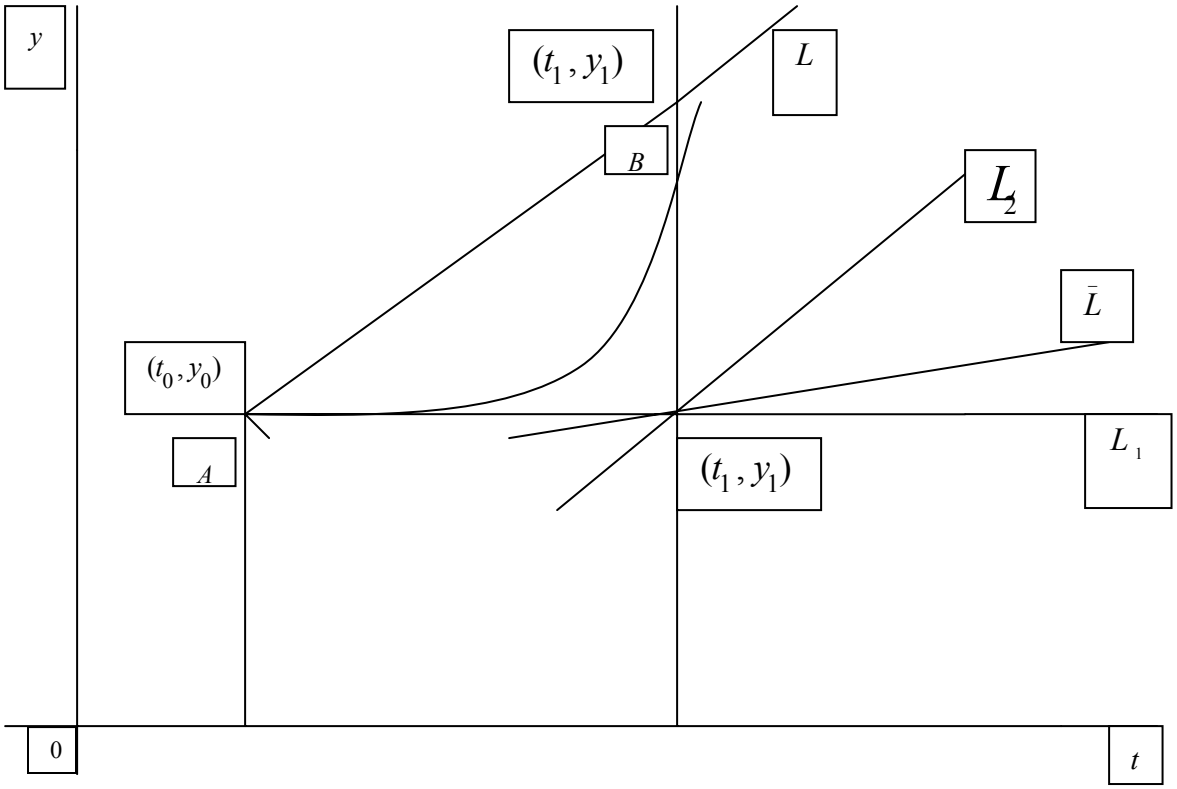
$y_6 = 1.7210 + (0.1)(1.7210 + 0.5) = 1.9431$

Hence the value of y *corresponding to* $t = 0.5$ *is* 1.9431

MODIFIED EULER'S METHOD

The modified Euler's method gives greater improvement in accuracy over the original Euler's method. Here, the core idea is that we use a line through (t_0, y_0) whose slope is the average of the slopes at (t_0, y_0) and $(t_1, y_1^{(1)})$ Where $y_1^{(1)} = y_0 + hf(t_0, y_0)$ is the value of y at $t = t_1$ as obtained in Euler's method, which approximates the curve in the interval (t_0, t_1)

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(Modified Euler's Method)

Geometrically, from Figure, if L_1 is the tangent at (t_0, y_0), L_2 is the line through $(t_1, y_1^{(1)})$ of slope $f(t_1, y_1^{(1)})$ and $\bar{L}$ is the line through $(t_1, y_1^{(1)})$ but with a slope equal to the average of $f(t_0, y_0)$ and $f(t_1, y_1^{(1)})$,.... the line L through (t_0, y_0) and parallel to L is used to approximate the curve in the interval (t_0, t_1).

Thus, the ordinate of the point B will give the value of y_1.

Now, the equation of the line AL is given by

$$y_1 = y_0 + \left[\frac{f(t_0, y_0) + f(t_1, y_1^{(1)})}{2}\right](t_1 - t_0)$$

$$= y_0 + h\left[\frac{f(t_0, y_0) + f(t_1, y_1^{(1)})}{2}\right]$$

Similarly proceeding, we arrive at the recurrence relation

$$y_{m+1} = y_m + h\left[\frac{f(t_m, y_m) + f(t_{m+1}, y_{m+1}^{(1)})}{2}\right]$$

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This is the *modified Euler's method*.

Example

Using modified Euler's method, obtain the solution of the differential equation

$$\frac{dy}{dt} = t + \sqrt{y} = f(t, y)$$

with the initial condition

$y_0 = 1$ at $t_0 = 0$ for the range $0 \le t \le 0.6$ in steps of 0.2

Solution

At first, we use Euler's method to get

$$y_1^{(1)} = y_0 + hf(t_0, y_0) = 1 + 0.2(0 + 1) = 1.2$$

Then, we use modified Euler's method to find

$$y(0.2) = y_1 = y_0 + h\frac{f(t_0, y_0) + f(t_1, y_1^{(1)})}{2}$$

$$= 1.0 + 0.2\frac{1 + \left(0.2 + \sqrt{1.2}\right)}{2} = 1.2295$$

Similarly proceeding, we have from Euler's method

$$y_2^{(1)} = y_1 + hf(t_1, y_1) = 1.2295 + 0.2(0.2 + \sqrt{1.2295})$$

$$= 1.4913$$

Using modified Euler's method, we get

$$y_2 = y_1 + h\frac{f(t_1, y_1) + f(t_2, y_2^{(1)})}{2}$$

$$= 1.2295 + 0.2\frac{\left(0.2 + \sqrt{1.2295}\right) + \left(0.4 + \sqrt{1.4913}\right)}{2}$$

$$= 1.5225$$

Finally,

$$y_3^{(1)} = y_2 + hf(t_2, y_2) = 1.5225 + 0.2\left(0.4 + \sqrt{1.5225}\right)$$

$$= 1.8493$$

Modified Euler's method gives

$$y(0.6) = y_3 = y_2 + h\frac{f(t_2, y_2) + f(t_3, y_3^{(1)})}{2}$$

$$= 1.5225 + 0.1\left[(0.4 + \sqrt{1.5225}) + (0.6 + \sqrt{1.8493})\right]$$

$$= 1.8819$$

Hence, the solution to the given problem is given by

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t	0.2	0.4	0.6
y	1.2295	1.5225	1.8819

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RUNGE – KUTTA METHOD

These are computationally, most efficient methods in terms of accuracy. They were developed by two German mathematicians, Runge and Kutta.

They are distinguished by their orders in the sense that they agree with Taylor's series solution up to terms of h^r, where r is the order of the method. These methods do not demand prior computation of higher derivatives of y(t) as in TSM.

Fourth-order Runge-Kutta methods are widely used for finding the numerical solutions of linear or non-linear ordinary differential equations, the development of which is complicated algebraically.

Therefore, we convey the basic idea of these methods by developing the second-order Runge-Kutta method which we shall refer hereafter as R-K method.

Please recall that the modified Euler's method: which can be viewed as

$$y_{n+1} = y_n + h \quad \text{(average of slopes)}$$

This, in fact, is the basic idea of R-K method.

Here, we find the slopes not only at t_n but also at several other interior points, and take the weighted average of these slopes and add to y_n to get y_{n+1}. Now, we shall derive the second order R-K method in the following slides.

Consider the IVP

$$\frac{dy}{dt} = f(t, y), \qquad y(t_n) = y_n$$

We also define

$$k_1 = hf(t_n, y_n), \qquad k_2 = hf(t_n + \alpha h, y_n + \beta k_1)$$

and take the weighted average of k_1 and k_2 and add to y_n to get y_{n+1}

We seek a formula of the form $y_{n+1} = y_n + W_1 k_1 + W_2 k_2$

Where $\alpha, \beta, W_1 \text{ and } W_2$ are constants to be determined so that the above equation agree with the Taylor's series expansion as high an order as possible

Thus, using Taylor's series expansion, we have

$$y(t_{n+1}) = y(t_n) + hy'(t_n) + \frac{h^2}{2} y''(t_n) + \frac{h^3}{6} y'''(t_n) + \cdots$$

Rewriting the derivatives of y in terms of f of the above equation, we get

$$y_{n+1} = y_n + hf(t_n, y_n) + \frac{h^2}{2}(f_t + ff_y)$$

$$+ \frac{h^3}{6}\left[f_{tt} + 2ff_{ty} + f^2 f_{yy} + f_y(f_t + ff_y) \right] + O(h^4)$$

Here, all derivatives are evaluated at (t_n, y_n).

Next, we shall rewrite the given equation after inserting the expressions or k_1 and k_2 as

$$y_{n+1} = y_n + W_1 hf(t_n, y_n) + W_2 hf(t_n + \alpha h, y_n + \beta k_1)$$

Now using Taylor's series expansion of two variables, we obtain

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$$y_{n+1} = y_n + W_1 hf(t_n, y_n) + W_2 h\Big[f(t_n, y_n) + (\alpha h f_t + \beta k_1 f_y)$$
$$\left(\frac{\alpha^2 h^2}{2} f_n + \alpha h \beta k_1 f_{ty} + \frac{\beta^2 k_1^2}{2} f_{yy} \right) + O(h^3) \Big]$$

Here again, all derivatives are computed at (t_n, y_n). On inserting the expression for k_1, the above equation becomes

$$y_{n+1} = y_n + (W_1 + W_2) hf + W_2 h\Big[(\alpha h f_t + \beta h f f_y)$$
$$+ \left(\frac{\alpha^2 h^2}{2} f_{tt} + \alpha\beta h^2 f f_{ty} + \frac{\beta^2 h^2}{2} f^2 f_{yy} \right) + O(h^3) \Big]$$

On rearranging in the increasing powers of *h*, we get

$$y_{n+1} = y_n + (W_1 + W_2) hf + W_2 h^2 (\alpha f_t + \beta f f_y)$$
$$+ W_2 h^3 \left(\frac{\alpha^2}{2} f_{tt} + \alpha\beta f f_{ty} + \frac{\beta^2 f^2 f f_y}{2} \right) + O(h^4) \Big]$$

Now, equating coefficients of *h* and h^2 in the two equations, we obtain

$$W_1 + W_2 = 1, \qquad W_2(\alpha f_t + \beta f f_y) = \frac{f_t + f f_y}{2}$$

Implying

$$W_1 + W_2 = 1, \qquad W_2 \alpha = W_2 \beta = \frac{1}{2}$$

Thus, we have three equations in four unknowns and so, we can chose one value arbitrarily. Solving we get

$$W_1 = 1 - W_2, \qquad \alpha = \frac{1}{2W_2}, \qquad \beta = \frac{1}{2W_2}$$

Where W_2 is arbitrary and various values can be assigned to it.

We now consider two cases, which are popular

Case I

If we choose $W_2 = 1/3$,

then $W_1 = 2/3$ and $\alpha = \beta = 3/2$

$$y_{n+1} = y_n + \frac{1}{3}(2k_1 + k_2)$$

$$k_1 = hf(t, y), \qquad k_2 = hf\left(t + \frac{3}{2}h, y + \frac{3}{2}k_1 \right)$$

Case II: If we consider

$W_2 = ½$, then $W_1 = ½$ and $\alpha = \beta = 1$.

Then $y_{n+1} = y_n + \dfrac{k_1 + k_2}{2}$

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$$k_1 = hf(t, y), \qquad k_2 = hf(t + h, y + k_1)$$

In fact, we can recognize that this equation is the modified Euler's method and is therefore a special case of a 2nd order Runge-Kutta method. These equations are known as 2nd order R –K Methods, since they agree with Taylor's series solution up to the term h^2.

Defining the local truncation error, TE, as the difference between the exact solution $y(t_{n+1})$ at $t = t_{n+1}$ and the numerical solution y_{n+1}, obtained using the second order R – K method, we have

$$TE = y(t_{n+1}) - y_{n+1}$$

Now, substituting

$$W_2 = \frac{1}{2\alpha}, \qquad W_1 = 1 - \frac{1}{2\alpha}, \quad \beta = \alpha,$$

into the above equation, we get

$$y_{n+1} = y_n + hf_{tt} + \frac{h^2}{2}(f_t + ff_y)_{t=t_n}$$

$$+ \frac{h^3\alpha}{4}(f_{tt} + 2ff_{ty} + f^2 f_{yy})_{t=t_n} + \cdots$$

Finally we obtain

$$TE = h^3\left[\left(\frac{1}{6} - \frac{\alpha}{4}\right)(f_{tt} + 2ff_{ty} + f^2 f_{yy}) + \frac{1}{6} f_y (f_t + ff_y)\right]$$

The expression can further be simplified to

$$TE = h^3\left[\left(\frac{1}{6} - \frac{\alpha}{4}\right)(y''' - f_y y') + \frac{1}{6} f_y y'\right]$$

Therefore, the expression for local truncation error is given by

$$TE = h^3\left[\left(\frac{1}{6} - \frac{\alpha}{4}\right) y''' + \frac{\alpha}{4} f_y y'\right]$$

Please verify that the magnitude of the TE in case I is less than that of case II

Following similar procedure, Runge-Kutta formulae of any order can be obtained. However, their derivations becomes exceedingly lengthy and complicated.

Amongst them, the most popular and commonly used in practice is the R-K method of fourth-order, which agrees with the Taylor series method up to terms of $O(h^4)$.

This well-known fourth-order R-K method is described in the following steps.

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

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where

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

Please note that the second-order Runge-Kutta method described above requires the evaluation of the function twice for each complete step of integration.
Similarly, fourth-order Runge-Kutta method requires the evaluation of the function four times. The discussion on optimal order R-K method is very interesting, but will be discussed some other time.

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Example

Use the following second order Runge-Kutta method described by

$$y_{n+1} = y_n + \frac{1}{3}(2k_1 + k_2)$$

where $k_1 = hf(x_n, y_n)$ and $k_2 = hf\left(x_n + \frac{3}{2}h, y_n + \frac{3}{2}k_1\right)$

and find the numerical solution of the initial value problem described as

$$\frac{dy}{dx} = \frac{y+x}{y-x}, \qquad y(0) = 1$$

at $x = 0.4$ and taking $h = 0.2$.

Solution

Here

$$f(x,y) = \frac{y+x}{y-x}, \qquad h = 0.2, \qquad x_0 = 0, \qquad y_0 = 1$$

We calculate $k_1 = hf(x_0, y_0) = 0.2\frac{1+0}{1-0} = 0.2$

$$k_2 = hf\left[x_0 + 0.3, y_0 + (1.5)(0.2)\right]$$

$$= hf(0.3, 1.3) = 0.2\frac{1.3+0.3}{1.3-0.3} = 0.32$$

Now, using the given R-K method, we get

$$y(0.2) = y_1 = 1 + \frac{1}{3}(0.4 + 0.32) = 1.24$$

Now, taking $x_1 = 0.2,\ y_1 = 1.24$, we calculate

$$k_1 = hf(x_1, y_1) = 0.2\frac{1.24+0.2}{1.24-0.2} = 0.2769$$

$$k_2 = hf\left(x_1 + \frac{3}{2}h, y_1 + \frac{3}{2}k_1\right) = hf(0.5, 1.6554)$$

$$= 0.2\frac{1.6554+0.5}{1.6554-0.5} = 0.3731$$

Again using the given R-K method, we obtain

$$y(0.4) = y_2 = 1.24 + \frac{1}{3}\left[2(0.2769) + 0.3731\right]$$

$$= 1.54897$$

Example

Solve the following differential equation

$$\frac{dy}{dt} = t + y$$

with the initial condition $y(0) = 1$, using fourth- order Runge-Kutta method from $t = 0$ to $t = 0.4$ taking $h = 0.1$

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Solution

The fourth-order Runge-Kutta method is described as

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad (1)$$

Where

$$k_1 = hf(t_n, y_n)$$
$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$
$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$
$$k_4 = hf(t_n + h, y_n + k_3)$$

In this problem

$$f(t, y) = t + y, \;\; h = 0.1, \;\; t_0 = 0, \;\; y_0 = 1.$$

As a first step, we calculate

$$k_1 = hf(t_0, y_0) = 0.1(1) = 0.1$$
$$k_2 = hf(t_0 + 0.05, y_0 + 0.05)$$
$$= hf(0.05, 1.05) = 0.1[0.05 + 1.05]$$
$$= 0.11$$
$$k_3 = hf(t_0 + 0.05, y_0 + 0.055)$$
$$= 0.1(0.05 + 1.055)$$
$$= 0.1105$$
$$k_4 = 0.1(0.1 + 1.1105) = 0.12105$$

Now, we compute from

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= 1 + \frac{1}{6}(0.1 + 0.22 + 0.2210 + 0.12105)$$
$$= 1.11034$$

Therefore $y(0.1) = y_1 = 1.1103$

In the second step, we have to find $y_2 = y(0.2)$

We compute

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$$k_1 = hf(t_1, y_1) = 0.1(0.1 + 1.11034) = 0.121034$$

$$k_2 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 0.1[0.15 + (1.11034 + 0.060517)] = 0.13208$$

$$k_3 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.1[0.15 + (1.11034 + 0.06604)] = 0.132638$$

$$k_4 = hf(t_1 + h, y_1 + k_3)$$

$$= 0.1[0.2 + (1.11034 + 0.132638)] = 0.1442978$$

From Equation (1), we see that

$$y_2 = 1.11034 + \frac{1}{6}[0.121034 + 2(0.13208)$$

$$+2(0.132638) + 0.1442978] = 1.2428$$

Similarly we calculate,

$$k_1 = hf(t_2, y_2) = 0.1[0.2 + 1.2428] = 0.14428$$

$$k_2 = hf\left(t_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right) = 0.1[0.25 + (1.2428 + 0.07214)] = 0.156494$$

$$k_3 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = 0.1[0.3 + (1.2428 + 0.078247)] = 0.1571047$$

$$k_4 = hf(t_2 + h, y_2 + k_3) = 0.1[0.3 + (1.2428 + 0.1571047)] = 0.16999047$$

Using equation (1), we compute

$$y(0.3) = y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1.399711$$

Finally, we calculate

$$k_1 = hf(t_3, y_3) = 0.1[0.3 + 1.3997] = 0.16997$$

$$k_2 = hf\left(t_3 + \frac{h}{2}, y_3 + \frac{k_1}{2}\right) = 0.1[0.35 + (1.3997 + 0.084985)] = 0.1834685$$

$$k_3 = hf\left(t_3 + \frac{h}{2}, y_3 + \frac{k_2}{2}\right) = 0.1[0.35 + (1.3997 + 0.091734)] = 0.1841434$$

$$k_4 = hf(t_3 + h, y_3 + k_3) = 0.1[0.4 + (1.3997 + 0.1841434)] = 0.19838434$$

Using them in equation (1), we get

$$y(0.4) = y_4 = y_3 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.58363$$

which is the required result.

Example

Apply Runge-Kutta Method of order four to solve the initial value problem at x=1.2

$$y' = xy^{1/3} \quad ; \quad y(1) = 1 \; ; \quad h = 0.1$$

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Solution:

The Fourth-Order Runge-Kutta Method is described as :-

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

where,

$$k_1 = hf\left(t_n, y_n\right)$$
$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$
$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$
$$k_4 = hf\left(t_n + h, y_n + k_3\right)$$

(Here, we are taking x as t)

First Iteration:-

$t_0 = 1 \quad , \quad y_0 = 1$

$$k_1 = hf(t_0, y_0) = 0.1\left[1 \times 1^{1/3}\right]$$
$$= 0.1$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$
$$= 0.1\left[\left(1+0.05\right) \times \left(1 + k_1/2\right)^{1/3}\right]$$
$$= 0.106721617$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$
$$= 0.1\left[\left(1+0.05\right) \times \left(1 + k_2/2\right)^{1/3}\right]$$
$$= 0.10683536$$

$$k_4 = hf\left(t_n + h, y_n + k_3\right)$$
$$= 0.1\left[\left(1+0.1\right) \times \left(1 + k_3\right)^{1/3}\right]$$
$$= 0.113785527$$

$$y_{n+1} = y_n + \left(k_1 + 2k_2 + 2k_3 + k_4\right)$$
$$y_1 = 1 + 0.10681658$$
$$y_1 = 1.10681658$$

Second Iteration:-

$t_1 = 1.1 \quad , \quad y_1 = 1.10681658$

$$k_1 = hf(t_1, y_1) = 0.1\left[1.1 \times 1.10681658^{1/3}\right]$$
$$= 0.113784884$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$
$$= 0.1\left[\left(1.1 + 0.05\right) \times \left(1 + 1.10681658/2\right)^{1/3}\right]$$
$$= 0.120961169$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$
$$= 0.1\left[\left(1.1 + 0.05\right) \times \left(1.10681658 + k_2/2\right)^{1/3}\right]$$
$$= 0.121085364$$

$$k_4 = hf\left(t_n + h, y_n + k_3\right)$$
$$= 0.1\left[\left(1.1 + 0.1\right) \times \left(1.10681658 + k_3\right)^{1/3}\right]$$
$$= 0.128499807$$

$$y_{n+1} = y_n + \left(k_1 + 2k_2 + 2k_3 + k_4\right)$$
$$y_2 = 1.10681658 + 0.12106296$$
$$y_2 = 1.22787954$$

So, at x = 1.2 , we get:

$y(1.2) = y_2 = 1.22787954$

RUNGE – KUTTA METHOD FOR SOLVING A SYSTEM OF EQUATIONS

The fourth-order Runge-Kutta method can be extended to numerically solve the higher-order ordinary differential equations- linear or non-linear

For illustration, let us consider a second order ordinary differential equation of the form

$$\frac{d^2y}{dt^2} = f\left(t, y, \frac{dy}{dt}\right)$$

Using the substitution $\frac{dy}{dt} = p$ this equation can be reduced to two first-order simultaneous differential equations, as given by

$$\frac{dy}{dt} = p = f_1(t, y, p), \qquad \frac{dp}{dt} = f_2(t, y, p)$$

Now, we can directly write down the Runge-Kutta fourth-order formulae for solving the system.

Let the initial conditions of the above system be given by

$$y(x_n) = y_n, \qquad y'(x_n) = p(x_n) = p_n$$

Then we define

$$\left.\begin{aligned} k_1 &= hf_1(t_n, y_n, p_n), & l_1 &= hf_2(t_n, y_n, p_n) \\ k_2 &= hf_1\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}, p_n + \frac{l_1}{2}\right), & l_2 &= hf_2\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}, p_n + \frac{l_1}{2}\right) \\ k_3 &= hf_1\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}, p_n + \frac{l_2}{2}\right), & l_3 &= hf_2\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}, p_n + \frac{l_2}{2}\right) \\ k_4 &= hf_1(t_n + h, y_n + k_3, p_n + l_3), & l_4 &= hf_2(t_n + h, y_n + k_3, p_n + l_3) \end{aligned}\right\}$$

Now, using the initial conditions y_n, p_n and 4th -order R-K formula, we compute

$$\left.\begin{aligned} y_{n+1} &= y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ p_{n+1} &= p_n + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \end{aligned}\right\}$$

This method can be extended on similar lines to solve system of n first order differential equations.

Example

Solve the following equation

$$y'' - (0.1)(1 - y^2)y' + y = 0$$

Using 4th order Runge-Kutta method for $x = 0.2$, with the initial values $y(0) = 1, y'(0)=0$

Solution:

Let $\frac{dy}{dx} = p = f_1(x, y, p)$

Then $\frac{dp}{dx} = (0.1)(1 - y_2)p - y = f_2(x, y, p)$

Thus, the given equation reduced to two first-order equations.

In the present problem, we are given

$$x_0 = 0, y_0 = 1, p_0 = y_o' = 0$$

Taking $h = 0.2$, we compute

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$$k_1 = hf_1(x_0, y_0, p_0) = 0.2(0.0) = 0.0$$

$$l_1 = hf_2(x_0, y_0, p_0) = 0.2(0.0-1) = -0.2$$

$$k_2 = hf_1\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, p_0 + \frac{l_1}{2}\right)$$

$$= hf_2(0.1, 1.0, -0.1) = -0.02$$

$$l_2 = hf_2\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, p_0 + \frac{l_1}{2}\right)$$

$$= hf_2(0.1, 1.0, -0.1) = 0.2(-0.1) = -0.02$$

$$k_3 = hf_1\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, p_0 + \frac{l_2}{2}\right)$$

$$= hf_1(0.1, 0.99, -0.1) = 0.2(-0.1) = -0.02$$

$$l_3 = hf_2\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, p_0 + \frac{l_2}{2}\right) = hf_2(0.1, 0.99, -0.1)$$

$$= 0.2[(0.1)(0.0199)(-0.1) - 0.99] = -0.1980$$

$$k_4 = hf_1(x_0 + h, y_0 + k_3, p_0 + l_3)$$

$$= hf_1(0.2, 0.98, -0.1980) = -0.0396$$

$$l_4 = hf_2(x_0 + h, y_0 + k_3, p_0 + l_3) = hf_2(0.2, 0.98, -0.1980)$$

$$= 0.2[(0.1)(1 - 0.9604)(-0.1980) - 0.98] = -0.19616$$

Now, $y(0.2) = y_1$ is given by

$$y(0.2) = y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$= 1 + \frac{1}{6}[0.0 + 2(-0.02) + 2(-0.02) + (-0.0396)]$$

$$= 1 - 0.019935 = 0.9801$$

$$y'(0.2) = p_1 = p_0 + \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$$

$$= 0 + \frac{1}{6}[-0.2 + 2(-0.2) + 2(-0.1980) + (-0.19616)]$$

$$= -0.19869 (= -0.1987)$$

Therefore, the required solution is

$$y(0.2) = 0.9801, \qquad y'(0.2) = -0.1987$$

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PREDICTOR – CORRECTOR METHOD

The methods presented so far are called single-step methods, where we have seen that the computation of y at t_{n+1} that is y_{n+1} requires the knowledge of y_n only.

In predictor-corrector methods which we will discuss now, is also known as multi-step methods.

To compute the value of y at t_{n+1}, we must know the solution y at t_n, t_{n-1}, t_{n-2}, etc.

Thus, a predictor formula is used to predict the value of y at t_{n+1} and then a corrector formula is used to improve the value of y_{n+1}.

Let us consider an IVP

$$\frac{dy}{dt} = f(t, y), \qquad y(t_n) = y_n$$

Using simple Euler's and modified Euler's method, we can write down a simple predictor-corrector pair (P – C) as

$$\left.\begin{aligned} P: y_{n+1}^{(0)} &= y_n + hf(t_n, y_n) \\ C: y_{n+1}^{(1)} &= y_n + \frac{h}{2}\left[f(t_n, y_n) + f(t_{n+1}, y_{n+1}^{(0)}) \right] \end{aligned}\right\}$$

Here, $y_{n+1}^{(1)}$ is the first corrected value of y_{n+1}. The corrector formula may be used iteratively as defined below:

$$y_{n+1}^{(r)} = y_n + \frac{h}{2}\left[f(t_n, y_n) + f(t_{n+1}, y_{n+1}^{(r-1)}) \right], \qquad r = 1, 2, \ldots$$

The iteration is terminated when two successive iterates agree to the desired accuracy

In this pair, to extrapolate the value of y_{n+1}, we have approximated the solution curve in the interval (t_n, t_{n+1}) by a straight line passing through (t_n, y_n) and (t_{n+1}, y_{n+1}).

The accuracy of the predictor formula can be improved by considering a quadratic curve through the equally spaced points $(t_{n-1}, y_{n-1}), (t_n, y_n), (t_{n+1}, y_{n+1})$

Suppose we fit a quadratic curve of the form

$$y = a + b(t - t_{n-1}) + c(t - t_n)(t - t_{n-1})$$

where *a, b, c* are constants to be determined As the curve passes through (t_{n-1}, y_{n-1}) and (t_n, y_n) and satisfies

$$\left(\frac{dy}{dt}\right)_{(t_n, y_n)} = f(t_n, y_n)$$

We obtain $y_{n-1} = a, \qquad y_n = a + bh = y_{n-1} + bh$

Therefore

$$b = \frac{y_n - y_{n-1}}{h}$$

and

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$$\left(\frac{dy}{dt}\right)_{(t_n,y_n)} = f(t_n,y_n) = \{b + c[(t-t_{n-1}) + (t-t_n)]\}_{(t_n,y_n)}$$

Which give

$$f(t_n,y_n) = b + c(t_n - t_{n-1}) = b + ch$$

or $$c = \frac{f(t_n,y_n)}{h} _ \frac{(y_n - y_{n-1})}{h^2}$$

Substituting these values of *a, b* and *c* into the quadratic equation, we get

$$y_{n+1} = y_{n-1} + 2(y_n - y_{n-1}) + 2[hf(t_n,y_n) - (y_n - y_{n-1})]$$

That is

$$y_{n+1} = y_{n-1} + 2hf(t_n,y_n)$$

Thus, instead of considering the P-C pair, we may consider the P-C pair given by

$$\left.\begin{aligned} P: y_{n+1} &= y_{n-1} + 2hf(t_n,y_n) \\ C: y_{n+1} &= y_n + \frac{h}{2}[f(t_n,y_n) + f(t_{n+1},y_{n+1})] \end{aligned}\right\}$$

The essential difference between them is, the one given above is more accurate However, this one can not be used to predict y_{n+1} for a given IVP, because its use require the knowledge of past two points. In such a situation, a R-K method is generally used to start the predictor method.

Milne's Method

It is also a multi-step method where we assume that the solution to the given IVP is known at the past four equally spaced point t_0, t_1, t_2 and t_3.

To derive Milne's predictor-corrector pair, let us consider a typical IVP

$$\frac{dy}{dt} = f(t,y), \qquad y(t_0) = y_0$$

On integration between the limits t_0 and t_4, we get

$$\int_{t_0}^{t_4} \frac{dy}{dt}dt = \int_{t_0}^{t_4} f(t,y)dt$$

$$y_4 - y_0 = \int_{t_0}^{t_4} f(t,y)dt$$

But we know from Newton's forward difference formula

$$f(t,y) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{6}\Delta^3 f_0 + \cdots$$

where $$s = \frac{t-t_0}{h}, \qquad t = t_0 + sh$$

$$y_4 = y_0 + \int_{t_0}^{t_4}\left[f_0 + s\Delta f_0 + \frac{s(s-1)}{2}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{6}\Delta^3 f_0 + \frac{s(s-1(s-2)(s-3)}{24}\Delta^4 f_0 + \cdots \right]dt$$

Now, by changing the variable of integration (from *t* to *s*), the limits of integration also changes (from 0 to 4), and thus the above expression becomes

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$$y_4 = y_0 + h\int_0^4 \left[4f_0 + s\Delta f_0 + \frac{s(s-1)}{2}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{6}\Delta^3 f_0 + \frac{s(s-1(s-2)(s-3)}{24}\Delta^4 f_0 + \cdots \right] ds$$

which simplifies to

$$y_4 = y_0 + h\left[4f_0 + 8\Delta f_0 + \frac{20}{3}\Delta^2 f_0 + \frac{8}{3}\Delta^3 f_0 + \frac{28}{90}\Delta^4 f_0 \right]$$

Substituting the differences

$$\Delta f_0 = f_1 - f_0, \Delta^2 f_0 = f_2 - 2f_1 + f_0,$$

It can be further simplified to

$$y_4 = y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3) + \frac{28}{90}h\Delta^4 f_0$$

Alternatively, it can also be written as

$$y_4 = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3') + \frac{28}{90}h\Delta^4 y_0'$$

This is known as *Milne's predictor formula*.

Similarly, integrating the original over the interval t_0 to t_2 or s = 0 to 2 and repeating the above steps, we get

$$y_2 = y_0 + \frac{h}{3}(y_0' + 4y_1' + y_2') - \frac{1}{90}h\Delta^4 y_0'$$

which is known as *Milne's corrector formula*.

In general, Milne's predictor-corrector pair can be written as

$$\left.\begin{aligned} P: y_{n+1} &= y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n) \\ C: y_{n+1} &= y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1}) \end{aligned}\right\}$$

From these equations, we observe that the magnitude of the truncation error in corrector formula is $1/90h\Delta^4 y_0'$ while the truncation error in predictor formula is $28/90h\Delta^4 y_0'$

Thus: TE in, c-formula is less than the TE in p-formula.

In order to apply this P – C method to solve numerically any initial value problem, we first predict the value of y_{n+1} by means of predictor formula, where derivatives are computed using the given differential equation itself.

Using the predicted value y_{n+1}, we calculate the derivative y'_{n+1} from the given differential equation and then we use the corrector formula of the pair to have the corrected value of y_{n+1} This in turn may be used to obtain improved value of y_{n+1} by using corrector again. This in turn may be used to obtain improved value of y_{n+1} by using the corrector again. This cycle is repeated until we achieve the required accuracy.

Example

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Find y (2.0) if y (t) is the solution of $\frac{dy}{dt} = \frac{1}{2}(t + y)$

y (0) = 2, y (0.5) = 2.636,
y (1.0) = 3.595 and y(1.5) = 4.968 Use Milne's P-C method.

Solution

Taking $t_0 = 0.0$, $t_1 = 0.5$, $t_2 = 1.0$, $t_3 = 1.5$ y_0, y_1, y_2 and y_3, are given, we have to compute y_4, the solution of the given differential equation corresponding to *t =2.0*

The Milne's P – C pair is given as

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

$$C: y_{n+1} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1})$$

From the given differential equation, $y' = (t + y)/2$

We have

$$y'_1 = \frac{t_1 + y_1}{2} = \frac{0.5 + 2.636}{2} = 1.5680$$

$$y'_2 = \frac{t_2 + y_2}{2} = \frac{1.0 + 3.595}{2} = 2.2975$$

$$y'_3 = \frac{t_3 + y_3}{2} = \frac{1.5 + 4.968}{2} = 3.2340$$

Now, using predictor formula, we compute

$$y_4 = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3)$$

$$= 2 + \frac{4(0.5)}{3}\left[2(1.5680) - 2.2975 + 2(3.2340)\right]$$

$$= 6.8710$$

Using this predicted value, we shall compute the improved value of y_4 from corrector formula

$$y_4 = y_2 + \frac{h}{3}(y'_2 + 4y'_3 + y'_4)$$

Using the available predicted value y_4 and the initial values, we compute

$$y'_4 = \frac{t_4 + y_4}{2} = \frac{2 + 6.68710}{2} = 4.4355$$

$$y'_3 = \frac{t_3 + y_3}{2} = \frac{1.5 + 4.968}{2} = 3.2340$$

and $y'_2 = 2.2975$

Thus, the first corrected value of y_4 is given by

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$$y_4^{(1)} = 3.595 + \frac{0.5}{3}[2.2975 + 4(3.234) + 4.4355]$$
$$= 6.8731667$$

Suppose, we apply the corrector formula again, then we have

$$y_4^{(2)} = y_2 + \frac{h}{3}\left[(y_2' + 4y_3' + (y_4^{(1)})'\right]$$
$$= 3.595 + \frac{0.5}{3}\left[2.2975 + 4(3.234) + \frac{2 + 6.8731667}{2}\right]$$
$$= 6.8733467$$

Finally, $y(2.0) = y_4 = 6.8734$.

Example

Tabulate the solution of $\frac{dy}{dt} = t + y, \qquad y(0) = 1$ in the interval [0, 0.4] with $h = 0.1$, using Milne's P-C method.

Solution

Milne's P-C method demand the solution at first four points t_0, t_1, t_2 and t_3. As it is not a self – starting method, we shall use R-K method of fourth order to get the required solution and then switch over to Milne's P – C method.

Thus, taking $t_0 = 0$, $t_1 = 0.1$, $t_2 = 0.2$, $t_3 = 0.3$ we get the corresponding y values using R–K method of 4th order; that is $y_0 = 1$, $y_1 = 1.1103$, $y_2 = 1.2428$ and $y_3 = 1.3997$

(Reference Lecture 38)

Now we compute

$$y_1' = t_1 + y_1 = 0.1 + 1.1103 = 1.2103$$
$$y_2' = t_2 + y_2 = 0.2 + 1.2428 = 1.4428$$
$$y_3' = t_3 + y_3 = 0.3 + 1.3997 = 1.6997$$

Using Milne's predictor formula

$$P: y_4 = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$
$$= 1 + \frac{4(0.5)}{3}\left[2(1.21103) - 1.4428 + 2(1.69971)\right]$$
$$= 1.58363$$

Before using corrector formula, we compute

$$y_4' = t_4 + y_4(predicted \quad value)$$
$$= 0.4 + 1.5836 = 1.9836$$

Finally, using Milne's corrector formula, we compute

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$$C: y_4 = y_2 + \frac{h}{3}(y_4' + 4y_3' + y_2')$$
$$= 1.2428 + \frac{0.1}{3}(1.9836 + 6.7988 + 1.4428)$$
$$= 1.5836$$

The required solution is:

t	0	0.1	0.2	0.3	0.4
y	1	1.1103	1.2428	1.3997	1.5836

Example

Using Milne's Predictor-Corrector Formula find f(0.4) from Ordinary Differential Equation

$$y' = x - y \quad ; \quad y(0) = 1 \; ; \; h = 0.1$$

with the help of following table.

X	0	0.1	0.2	0.3
Y	1	0.9097	0.8375	0.7816

Solution:

Here,

$x_0 = 0 \quad , \quad x_1 = 0.1 \quad , \quad x_2 = 0.2 \quad , \quad x_3 = 0.3 \quad , \quad x_4 = 0.4$

$y_1' = x_1 - y_1 = 0.1 - 0.9097 = -0.8097$
$y_2' = x_2 - y_2 = 0.2 - 0.8375 = -0.6375$
$y_3' = x_3 - y_3 = 0.3 - 0.7816 = -0.4816$

Now, using Predictor Formula

$$y_4 = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$
$$y_4 = 1 + \frac{4*0.1}{3}(-1.9451)$$
$$y_4 = 0.740653333$$

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Using the predicted value, we shall now compute the corrected value as;

$$y_4 = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right)$$

Now,

$$y_4' = x_4 - y_4 = 0.4 - 0.740653333 = -0.340653333$$

Putting the values, into *the Corrector Formula*;

$$y_4 = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right)$$

$$y_4 = 0.8375 + \frac{0.1}{3}\left(-0.6375 + \left(4^* - 0.4816\right) - 0.340653333\right)$$

$$y_4 = 0.8375 - 0.096818444$$

$$y_4 = 0.740681556 \qquad \text{Ans.}$$

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Adam-Moulton Method

It is another predictor-corrector method, where we use the fact that the solution to the given initial value problem is known at past four equally spaced points
$t_n, t_{n-1}, t_{n-2}, t_{n-3}$.
The task is to compute the value of y at t_{n+1}.
Let us consider the differential equation

$$\frac{dy}{dt} = f(t, y)$$

Integrating between the limits t_n to t_{n+1}, we have

$$\int_{t_n}^{t_{n+1}} \frac{dy}{dt} dt = \int_{t_n}^{t_{n+1}} f(t, y) dt$$

That is

$$y_{n+1} - y_n = \int_{t_n}^{t_{n+1}} f(t, y) dt$$

To carry out integration, we proceed as follows. We employ *Newton's backward interpolation formula*, so that

$$f(t, y) = f_n + s\nabla f_n + \frac{s(s+1)}{2}\nabla^2 f_n + \frac{s(s+1)(s+2)}{6}\nabla^3 f_n + \cdots$$

Where

$$s = \frac{t - t_n}{h}$$

After substitution, we obtain

$$y_{n+1} = y_n + \int_{t_n}^{t_{n+1}} \left[f_n + s\nabla f_n + \frac{s(s+1)}{2}\nabla^2 f_n \right.$$
$$\left. + \frac{s(s+1)(s+2)}{6}\nabla^3 f_n + \frac{s(s+1)(s+2)(s+3)}{24}\nabla^4 f_n + \cdots \right] dt$$

Now by changing the variable of integration (from t to s), the limits of integration also changes (from 0 to 1), and thus the above expression becomes

$$y_{n+1} = y_n + h\int_0^1 \left[f_n + s\nabla f_n + \frac{s(s+1)}{2}\nabla^2 f_n \right.$$
$$\left. + \frac{s(s+1)(s+2)}{6}\nabla^3 f_n + \frac{s(s+1)(s+2)(s+3)}{24}\nabla^4 f_n + \cdots \right] ds$$

Actual integration reduces the above expression to

$$y_{n+1} = y_n + h\left(f_n + \frac{1}{2}\nabla f_n + \frac{5}{12}\nabla^2 f_n + \frac{3}{8}\nabla^3 f_n + \frac{251}{720}\nabla^4 f_n \right)$$

Now substituting the differences such as

$$\nabla f_n = f_n - f_{n-1}$$

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$$\nabla^2 f_n = f_n - 2f_{n-1} + f_{n-2}$$
$$\nabla^3 f_n = f_n - 3f_{n-1} + 3f_{n-2} - f_{n-3}$$

Equation simplifies to

$$y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}) + \frac{251}{720}h\nabla^4 f_n$$

Alternatively, it can be written as

$$y_{n+1} = y_n + \frac{h}{24}[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}] + \frac{251}{720}h\nabla^4 y'$$

This is known as Adam's predictor formula.

The truncation error is $(251/720)h\nabla^4 y'_n$.

To obtain corrector formula, we use Newton's backward interpolation formula about f_{n+1} instead of f_n.

We obtain

$$y_{n+1} = y_n + h\int_{-1}^{0}\Big[f_{n+1} + s\nabla f_{n+1} + \frac{s(s+1)}{2}\nabla^2 f_{n+1}$$
$$+ \frac{s(s+1)(s+2)}{6}\nabla^3 f_{n+1} + \frac{s(s+1)(s+2)(s+3)}{24}\nabla^4 f_{n+1} + \cdots\Big]ds$$

Carrying out the integration and repeating the steps, we get the corrector formula as

$$y_{n+1} = y_n + \frac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right) + \left(\frac{-19}{720}\right)h\nabla^4 y'_{n+1}$$

Here, the truncation error is $(19/20)h\nabla^4 y'_{n+1}$.

The truncation error in Adam's predictor is approximately thirteen times more than that in the corrector, but with opposite sign.

In general, Adam-Moulton predictor-corrector pair can be written as

$$\left.\begin{aligned} P: y_{n+1} &= y_n + \frac{h}{24}\left(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right) \\ C: y_{n+1} &= y_n + \frac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right) \end{aligned}\right\}$$

Example

Using Adam-Moulton predictor-corrector method, find the solution of the initial value problem

$$\frac{dy}{dt} = y - t^2, \qquad y(0) = 1$$

at $t = 1.0$, taking $h = 0.2$. Compare it with the analytical solution.

Solution

In order to use Adam's P-C method, we require the solution of the given differential equation at the past four equally spaced points, for which we use R-K method of 4th order which is self starting.

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Thus taking $t_0 = 0, y_0 = 1, \quad h = 0.2$, we compute

$$k_1 = 0.2, k_2 = 0.218,$$
$$k_3 = 0.2198, k_4 = 0.23596,$$

and get

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1.21859$$

Taking $t_1 = 0.2$,

$y_1 = 1.21859, h = 0.2,$

we compute $k_1 = 0.23571,$

$k_2 = 0.2492,$

$k_3 = 0.25064, k_4 = 0.26184,$ and get

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1.46813$$

Now, we take $t_2 = 0.4,$

$y_2 = 1.46813, h = 0.2,$ and compute $k_1 = 0.2616,$

$k_2 = 0.2697,$

$k_3 = 0.2706, k_4 = 0.2757$

to get

$$y_3 = y(0.6) = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1.73779$$

Thus, we have at our disposal

$y_0 = y(0) = 1$

$y_1 = y(0,2) = 1.21859$

$y_2 = (0.4) = 1.46813$

$y_3 = y(0.6) = 1.73779$

Now, we use Adam's P-C pair to calculate *y (0.8)* and *y* (1.0) as follows:

$$P: y_{n+1} = y_n + \frac{h}{24}\left(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right)$$

Thus $$y_4^p = y_3 + \frac{h}{24}\left(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right) \qquad (1)$$

From the given differential equation, we have

$$y' = y - t^2.$$

Therefore

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$$y_0' = y_0 - t_0^2 = 1.0$$
$$y_1' = y_1 - t_1^2 = 1.17859$$
$$y_2' = y_2 - t_2^2 = 1.30813$$
$$y_3' = y_3 - t_3^2 = 1.37779$$

Hence, from Eq. (1), we get

$$y(0.8) = y_4^p = 1.73779 + \frac{0.2}{24}\left(75.77845 - 77.17967 + 43.60783 - 9\right) = 2.01451$$

Now to obtain the corrector value of y at $t = 0.8$, we use

$$y_4^c = y^c(0.8) = y_3 + \frac{h}{24}\left(9y_4' + 19y_3' - 5y_2' + y_1'\right) \qquad (2)$$

But $$9y_4' = 9(y_4^p - t_4^2) = 9[2.01451 - (0.8)^2] = 12.37059$$

Therefore

$$y_4 = y^c(0.8) = 1.73779 + \frac{0.2}{24}\left(12.37059 + 26.17801 - 6.54065 + 1.17859\right) = 2.01434 \qquad (3)$$

Proceeding similarly, we get

$$y_5^p = y^p(1.0) = y_4 + \frac{h}{24}\left(55y_4' - 59y_3' + 37y_2' - 9y_1'\right)$$

Noting that

$$y_4' = y_4 - t_4^2 = 1.3743,$$

We calculate

$$y_5^p = 2.01434 + \frac{0.2}{24}\left(75.5887 - 81.28961 + 48.40081 - 10.60731\right) = 2.28178$$

Now, the corrector formula for computing y_5 is given by

$$y_5^c = y^c(1.0) = y_4 + \frac{h}{24}\left(9y_5' + 19y_4' - 5y_3' + y_2'\right) \qquad (4)$$

But

$$9y_5' = 9\left(y_5^p - t_5^2\right) = 11.53602$$

Then finally we get

$$y_5 = y(1.0) = 2.01434 + \frac{0.2}{24}\left(11.53602 + 26.17801 - 6.54065 + 1.17859\right)$$
$$= 2.28393 \qquad (5)$$

The analytical solution can be seen in the following steps.

$$\frac{dy}{dt} - y = -t^2$$

After finding integrating factor and solving, we get

$$\frac{d}{dt} ye^{-1} = -e^{-t}t^2$$

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Integrating, we get

$$ye^{-t} = -\int e^{-t}t^2 dt = \int t^2 d(e^{-t}) = t^2e^{-t} + 2te^{-t} + c$$

That is

$$y = t^2 + 2t + 2 + \frac{c}{e^{-t}}$$

Now using the initial condition, $y(0) = 1$,
we get c = – 1.
Therefore, the analytical solution is given by

$$y = t^2 + 2t + 2 - e^t$$

From which we get

$$y(1.0) = 5 - e = 2.2817$$

Example

Using Adam-Moulton Predictor-Corrector Formula find f(0.4) from Ordinary Differential Equation

$$y' = 1 + 2xy \quad ; \quad y(0) = 0 \; ; \quad h = 0.1$$

with the help of following table.

X	0	0.1	0.2	0.3
Y	0	0.1007	0.2056	0.3199

Solution:

Here

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$h = 0.1$

$f(x, y) = 1 + 2xy$

$y_0' = 1 + 2x_0 y_0 = 1 + 2(0)(0) = 1$
$y_1' = 1 + 2x_1 y_1 = 1 + 2(0.1)(0.1007) = 1.02014$
$y_2' = 1 + 2x_2 y_2 = 1 + 2(0.2)(0.2056) = 1.08224$
$y_3' = 1 + 2x_3 y_3 = 1 + 2(0.3)(0.3199) = 1.19194$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24}\left(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24}\left(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right)$$
$$y_4 = 0.3199 + \frac{0.1}{24}\left(55(1.19194) - 59(1.08224) + 37(1.02014) - 9(1)\right)$$
$$y_4 = 0.446773833$$

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Computing y'_4 *for the Corrector Formula*;

$$y'_4 = 1 + 2x_4 y_4 = 1 + 2(0.4)(0.446773833)$$
$$y'_4 = 1.3574190664$$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right)$$
$$y_4 = y_3 + \frac{h}{24}\left(9y'_4 + 19y'_3 - 5y'_2 + y'_1\right)$$
$$y_4 = 0.3199 + \frac{0.1}{24}\left(9(1.3574190664) + 19(1.19194) - 5(1.08224) + 1.02014\right)$$

$$y_4 = 0.446869048$$

Convergence and Stability Considerations

The numerical solution of a differential equation can be shown to converge to its exact solution, if the step size *h* is very small.The numerical solution of a differential equation is said to be stable if the error do not grow exponentially as we compute from one step to another. Stability consideration are very important in finding the numerical solutions of the differential equations either by *single-step* methods or by using *multi-step* methods. However, theoretical analysis of stability and convergence of R -K methods and P –C methods are highly involved and obtain numerically stable solution using 4th order R – K method to the simple problem $y' = Ay$ gives us stability condition as $-2.78 \leq Ah$

In practice, to get numerically stable solutions to similar problems, we choose the value of h much smaller than the value given by the above condition and also check for consistency of the result.

Another topic of interest which is not considered, namely the stiff system of differential equations that arises in many chemical engineering systems, such as chemical reactors, where the rate constants for the reactions involved are widely different.

Most of the realistic stiff DE do not have analytical solutions and therefore only numerical solutions can be obtained. However, to get numerically stable solutions, a very small step size h is required, to use either R-K methods or P – C methods.

More computer time is required

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Lecture # 41

Examples of Differential Equations

Recall EULER METHOD

We considered the differential equation of first order with the initial condition y(t0) = y0.

$$\frac{dy}{dt} = f(t, y)$$

We obtained the solution of the given differential equation in the form of a recurrence relation

$$y_{m+1} = y_m + hf(t_m, y_m)$$

In fact Euler's method constructs wi ~ y(ti) for each i = 0, 1,…, N-1 by deleting the remainder term. Thus the Euler's Method is

$$w_0 = \alpha,$$

$$w_{i+1} = w_i + hf(t_i, w_i)$$

$$for\ \ each\ \ i = 0,1,...,N-1$$

Euler's algorithm

Let us try to approximate the solution of the given IVP at (N+1) equally spaced numbers in the interval [a ,b]

$$y' = f(t, y),$$

$$a \le t \le b, \qquad y(a) = \alpha$$

INPUT endpoints a, b; integer N, initial condition (alpha)

OUTPUT approximate w to y at the (N+1) values of t

Step 1

Set h=(b-a) / N

t = a

w = (alpha)

OUTPUT (t , w)

Step 2

For i = 0,1,…N do Step 3, 4.

Step 3

Set w = w + h f (t , w); (compute wi).

t = a + i h (compute ti)

Step 4 OUTPUT (t , w)

Step 5 STOP

Example

Use Euler's method to approximate the solution of IVP

y'= y – t^2 + 1, 0 < t < 2,

y (0) = 0.5 with N = 10.

Solution

Here, h = 0.2, ti = 0.2i, w0= 0.5 and wi+1 = wi + h (wi – ti^2 + 1)

= wi+0.2[wi - 0.04i2 +1]

=1.2 wi - 0.008i^2 + 0.2

for i = 0,1,…,9.

The exact solution is

y (t)= (t+1)^2 -0.5 e ^{t}

> alg051();

This is Euler's Method.

Input the function F(t,y) in terms of t and y

For example: y-t^2+1
> y-t^2+1
Input left and right endpoints separated by blank
> 0 2
Input the initial condition
> 0.5
Input a positive integer for the number of subintervals
> 10
Choice of output method:
1. Output to screen
2. Output to text file
Please enter 1 or 2
> 1
Output

t	w
0.000	0.5000000
0.200	0.8000000
0.400	1.1520000
0.600	1.5504000
0.800	1.9884800
1.000	2.4581760

> alg051();
This is Euler's Method.
Input the function F (t,y) in terms of t and y
For example: y-3*t^2+4
> y-3*t^2+4
Input left and right hand points separated by a blank
>0 1
Input the initial condition
> 0.5
Input a positive integer for the number of subintervals
> 10
Choice of output method:
1. Output to screen
2. Output to text file
Please enter 1 or 2
> 1
Output

t	w
0.000	0.5000000
0.100	0.9500000
0.200	1.4420000
0.300	1.9742000
0.400	2.5446200
0.500	3.1510820
0.600	3.7911902
0.700	4.4623092
0.800	5.1615401
0.900	5.8856942
1.000	6.6312636

Recall Runge-Kutta (Order Four) METHOD
The fourth-order R-K method was described as

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

Example
Solve the following differential equation

$\frac{dy}{dt} = t + y$ with the initial condition y(0) = 1, using fourth- order Runge-Kutta method from t = 0 to t = 0.4 taking h = 0.1

Solution
The fourth-order Runge-Kutta method is described as

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad .....(1)$$

where

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

In this problem,

$$f(t, y) = t + y,\ h = 0.1,\ t_0 = 0,\ y_0 = 1.$$

As a first step, we calculate

$$k_1 = hf(t_0, y_0) = 0.1(1) = 0.1$$

$$k_2 = hf(t_0 + 0.05, y_0 + 0.05)$$

$$= hf(0.05, 1.05) = 0.1[0.05 + 1.05]$$

$$= 0.11$$

$$k_3 = hf(t_0 + 0.05, y_0 + 0.055)$$

$$= 0.1(0.05 + 1.055)$$

$$= 0.1105$$

$$k_4 = 0.1(0.1 + 1.1105) = 0.12105$$

Now, we compute from

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + \frac{1}{6}(0.1 + 0.22 + 0.2210 + 0.12105)$$

$$= 1.11034$$

Therefore y(0.1) = y1=1.1103
In the second step, we have to find y2 = y(0.2)
We compute

$$k_1 = hf(t_1, y_1) = 0.1(0.1 + 1.11034) = 0.121034$$

$$k_2 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 0.1[0.15 + (1.11034 + 0.060517)] = 0.13208$$

$$k_3 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.1[0.15 + (1.11034 + 0.06604)] = 0.132638$$

$$k_4 = hf(t_1 + h, y_1 + k_3)$$

$$= 0.1[0.2 + (1.11034 + 0.132638)] = 0.1442978$$

From Equation (1), we see that

$$y_2 = 1.11034 + \frac{1}{6}[0.121034 + 2(0.13208)$$

$$+2(0.132638) + 0.1442978] = 1.2428$$

Similarly we calculate,

$$k_1 = hf(t_2, y_2) = 0.1[0.2 + 1.2428] = 0.14428$$

$$k_2 = hf\left(t_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right) = 0.1[0.25 + (1.2428 + 0.07214)] = 0.156494$$

$$k_3 = hf\left(t_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = 0.1[0.3 + (1.2428 + 0.078247)] = 0.1571047$$

$$k_4 = hf(t_2 + h, y_2 + k_3) = 0.1[0.3 + (1.2428 + 0.1571047)] = 0.16999047$$

Using equation (1), we compute

$$y(0.3) = y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1.399711$$

Finally, we calculate

$$k_1 = hf(t_3, y_3) = 0.1[0.3 + 1.3997] = 0.16997$$

$$k_2 = hf\left(t_3 + \frac{h}{2}, y_3 + \frac{k_1}{2}\right) = 0.1[0.35 + (1.3997 + 0.084985)] = 0.1834685$$

$$k_3 = hf\left(t_3 + \frac{h}{2}, y_3 + \frac{k_2}{2}\right) = 0.1[0.35 + (1.3997 + 0.091734)] = 0.1841434$$

$$k_4 = hf(t_3 + h, y_3 + k_3) = 0.1[0.4 + (1.3997 + 0.1841434)] = 0.19838434$$

Using them in equation (1), we get

$$y(0.4) = y_4$$

$$= y_3 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1.58363$$

which is the required result

Runge-Kutta Order Four

$$w_0 = \alpha$$

$$k_1 = hf(t_i, w_i)$$

$$k_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_i + h, w_i + k_3)$$

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad .....(1)$$

RK4 algorithm

Let us try to approximate the solution of the given IVP at (N+1) equally spaced numbers in the interval [a ,b]

$$y' = f(t, y),$$

$$a \le t \le b, \qquad y(a) = \alpha$$

INPUT endpoints a, b; integer N, initial condition (alpha)

OUTPUT approximate w to y at the (N+1) values of t

Step 1

Set h=(b-a) / N

t = a

w = (alpha)

OUTPUT (t , w)

Step 2

For i = 0,1,…N do Step 3 - 5.

Step 3

Set

$$K_1 = hf(t, w)$$

$$K_2 = hf\left(t + \frac{h}{2}, w + \frac{K_1}{2}\right)$$

$$K_3 = hf\left(t + \frac{h}{2}, w + \frac{K_2}{2}\right)$$

$$K_4 = hf(t + h, w + K_3)$$

$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Step 3

Set w = w + h f (t , w); (compute wi).

t = a + i h (compute ti)

Step 4 OUTPUT (t , w)

```
Step 5 STOP
> alg052();
This is the Runge-Kutta Order Four Method.
Input the function F(t,y) in terms of t and y
For example: y-t^2+1
> y-t^2+1
Input left and right endpoints separated by blank
> 0 2
Input the initial condition
> 0.5
Input a positive integer for the number of subintervals
> 10
Choice of output method:
1. Output to screen
2. Output to text file
Please enter 1 or 2
> 1
Output
   t          w
0.000  0.5000000
0.200   0.8292933
0.400   1.2140762
0.600   1.6489220
0.800   2.1272027
1.000   2.6408227
1.200   3.1798942
1.400   3.7323401
1.600   4.2834095
1.800   4.8150857
2.000   5.3053630
```

Examples of Numerical Differentiation

The simplest formula for differentiation is

$$f'(x_0) = \frac{1}{h}\left[f(x_0 + h) - f(x_0)\right] - \frac{h^2}{2} f^{(2)}(\xi),$$

Example

Let f(x)= ln x and x0 = 1.8. Then quotient

$$\frac{f(1.8+h) - f(1.8)}{h}, \qquad h > 0,$$

is used to approximate $f'(1.8)$ with error

$$\frac{|hf''(\xi)|}{2} = \frac{|h|}{2\xi^2} \le \frac{|h|}{2(1.8)^2}, \text{ where } 1.8 < \xi < 1.8 + h.$$

Let us see the results for h = 0.1, 0.01, and 0.001.

h	f(1.8 + h)	$\frac{f(1.8+h) - f(1.8)}{h}$	$\frac{\lvert h\rvert}{2(1.8)^2}$
0.1	0.64185389	0.5406722	0.0154321
0.01	0.59332685	0.5540180	0.0015432
0.001	0.58834207	0.5554013	0.0001543

Since $f'(x) = 1/x,$

The exact value of $f'(1.8)$ is $0.55\overline{5}$ and the error bounds are a appropriate.
The following two three point formulas become especially useful if the nodes are equally spaced, that is, when
x1 = x0 + h and
x2 = x0 + 2h,

$$f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)\right] + \frac{h^2}{3} f^{(3)}(\xi_0),$$

where ξ_0 lies between x0 and x0 + 2h, and

$$f'(x_0) = \frac{1}{2h}\left[f(x_0 + h) - f(x_0 - h)\right] - \frac{h^2}{6} f^{(3)}(\xi_1),$$

where ξ_1 lies between (x0 – h) and (x0 + h).
Given in Table below are values for f (x) = xe^x.

x	f (x)
1.8	10.889365
1.9	12.703199
2.0	14.778112
2.1	17.148957
2.2	19.855030

Since

$f'(x) = (x+1)e^x, f'(2.0) = 22.167168.$

Approximating $f'(2.0)$
using the various three-and five-point formulas produces the following results.

Three point formulas:

$$f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)\right] + \frac{h^2}{3} f^{(3)}(\xi_0),$$

$$f'(x_0) = \frac{1}{2h}\left[f(x_0 + h) - f(x_0 - h)\right] - \frac{h^2}{6} f^{(3)}(\xi_1),$$

Using three point formulas we get

$$h = 0.1 : \frac{1}{0.2}\left[-3f(2.0) + 4f(2.1) - f(2.2)\right]$$

$$= 22.032310,$$

$$h=-0.1:\frac{1}{-0.2}\left[-3f(2.0)+4f(1.9)-f(1.8)\right]$$
$$=22.0054525,$$
$$h=0.1:\frac{1}{0.2}\left[f(2.1)-f(1.9)\right]$$
$$=22.228790,$$
$$h=0.2:\frac{1}{0.4}\left[f(2.2)-f(1.8)\right]$$
$$=22.414163.$$

Five point formula

Using the five point formula with h = 0.1 (the only formula applicable):

$$f'(x_0)=\frac{1}{12h}\left[f(x_0-2h)-8f(x_0-h)+8f(x_0+h)-f(x_0+2h)\right]$$
$$=\frac{1}{0.2}\left[f(1.8)-8f(1.9)+8f(2.1)-f(2.2)\right]$$
$$=22.166999.$$

The errors in the formulas are approximately

$$1.35\times10^{-1},1.13\times10^{-1},\ -6.16\times10^{-2},-2.47\times10^{-1},$$

and 1.69×10^{-4},

respectively. Clearly, the five-point formula gives the superior result.

Consider approximating for f (x) = sin x, using the values in table [the true value is cos (0.900) = 0.62161.]

x	sin x	x	sin x
0.800	0.71736	0.901	0.78395
0.850	0.75128	0.902	0.78457
0.880	0.77074	0.905	0.78643
0.890	0.77707	0.910	0.78950
0.895	0.78021	0.920	0.79560
0.898	0.78208	0.950	0.81342
0.899	0.78270	1.000	0.84147

Using the formula

$$f'(0.900) \approx \frac{f(0.900+h) - f(0.900-h)}{2h}$$

with different values of h gives the approximations in table given below:

h	Approximation to $f'(0.900)$	Error
0.001	0.62500	0.00339
0.002	0.62250	0.00089
0.005	0.62200	0.00039
0.010	0.62150	-0.00011
0.020	0.62150	-0.00011
0.050	0.62140	-0.00021
0.100	0.62055	-0.00106

Examples of Numerical Integration

EXAMPLE

The Trapezoidal rule for a function f on the interval [0, 2] is

$$\int_0^2 f(x)dx = \frac{h}{2}[f(x_0) + f(x_1)]$$

$$\int_0^2 f(x)dx \approx f(0) + f(2),$$

while Simpson's rule for f on [0, 2] is

$$\int_0^2 f(x)dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)].$$

That is

$$\int_0^2 f(x)dx \approx \frac{1}{3}[f(0) + 4f(1) + f(2)].$$

f (x)	x2	x4	1/(x + 1)	$\sqrt{1+x^2}$	sin x	ex
Exact value	2.667	6.400	1.099	2.958	1.416	6.389
Trapezoidal	4.000	16.000	1.333	3.326	0.909	8.389
Simpson's	2.667	6.667	1.111	2.964	1.425	6.421

Use close and open formulas listed below to approximate

$$\int_0^{\pi/4} \sin x dx = 1 - \sqrt{2}/2$$

Some of the common closed Newton-Cotes formulas with their error terms are as follows:

n = 1: Trapezoidal rule

$$\int_{x_0}^{x_1} f(x)dx = \frac{h}{2}[f(x_0) + f(x_1)] - \frac{h^3}{12} f''(\xi),$$

Where $x_0 < \xi < x_1$.

n = 2: Simpson's rule

$$\int_{x_0}^{x_2} f(x)dx = \frac{h}{2}[f(x_0)+4f(x_1)+f(x_2)]-\frac{h^5}{90}f^{(4)}(\xi), \qquad x_0 < \xi < x_2.$$

n = 3: Simpson's rule

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}[f(x_0)+3f(x_1)+3f(x_2)+f(x_3)]-\frac{3h^5}{80}f^{(4)}(\xi), \qquad x_0 < \xi < x_3.$$

n = 4:

$$\int_{x_2}^{x_1} f(x)dx = \frac{2h}{45}[7f(x_0)+32f(x_1)+12f(x_2)+32f(x_3)+7f(x_4)]-\frac{8h^7}{945}f^{(6)}(\xi$$

Where $x_0 < \xi < x_4$.

n = 0: Midpoint rule

$$\int_{x_2}^{x_1} f(x)dx = 2hf(x_0)+\frac{h^3}{3}f''(\xi), \qquad \text{where } x_{-1} < \xi < x_1.$$

n	1	2	3	4
Closed formulas	0.27768018	0.29293264	0.29291070	0.29289318
Error	0.01521303	0.00003942	0.00001748	0.00000004
Open formulas	0.29798754	0.29285866	0.29286923	
Error	0.00509432	0.00003456	0.00002399	

Composite Numerical Integration

EXAMPLE 1

Consider approximating $\int_0^{\pi} \sin x dx$ with an absolute error less than 0.00002, using the Composite Simpson's rule. The Composite Simpson's rule gives

$$\int_0^{\pi} \sin x dx = \frac{h}{3}\left[2\sum_{j=1}^{(n/2)-1} \sin x_{2j} + 4\sum_{j=1}^{n/2} \sin x_{2j-1}\right] - \frac{\pi h^4}{180}\sin \mu.$$

Since the absolute error is required to be less than 0.00002, the inequality

$$\left|\frac{\pi h^4}{180}\sin \mu\right| \le \frac{\pi h^4}{180} = \frac{\pi^5}{180n^4} < 0.00002$$

is used to determine n and h. Computing these calculations gives n greater than or equal to 18. If n = 20, then the formula becomes

$$\int_0^{\pi} \sin x dx \approx \frac{\pi}{60}\left[2\sum_{j=1}^{9} \sin\left(\frac{j\pi}{10}\right) + 4\sum_{j=1}^{10} \sin\left(\frac{(2j-1)\pi}{20}\right)\right] = 2.000006.$$

To be assured of this degree of accuracy using the Composite Trapezoidal rule requires that

$$\left|\frac{\pi h^2}{12}\sin \mu\right| \le \frac{\pi h^2}{12} = \frac{\pi^3}{12n^2} < 0.00002$$

or that $n \ge 360$. Since this is many more calculations than are needed for the Composite Simpson's rule, it is clear that it would be undesirable to use the Composite Trapezoidal rule on this problem. For comparison purposes, the Composite Trapezoidal rule with n = 20 and gives

$$\int_0^{\pi} \sin x dx \approx \frac{\pi}{40}\left[2\sum_{j=1}^{19} \sin\left(\frac{j\pi}{20}\right) + \sin 0 + \sin \pi\right]$$

$$= \frac{\pi}{40}\left[2\sum_{j=1}^{19} \sin\left(\frac{j\pi}{20}\right)\right] = 1.9958860.$$

The exact answer is 2; so Simpson's rule with n = 20 gave an answer well within the required error bound, whereas the Trapezoidal rule with n = 20 clearly did not.

An Example of Industrial applications: A company advertises that every roll of toilet paper has at least 250 sheets. The probability that there are 250 or more sheets in the toilet paper is given by

$$P(y \ge 250) = \int_{250}^{\infty} 0.3515 \quad e^{-0.3881(y-252.2)^2} dy$$

Approximating the above integral as

$$P(y \ge 250) = \int_{250}^{270} 0.3515 \quad e^{-0.3881(y-252.2)^2} dy$$

a)use single segment Trapezoidal rule to find the probability that there are 250 or more sheets.
b)Find the true error, Et for part (a).
C)Find the absolute relative true error for part (a).

$$I \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$ where

$a = 250 \quad b = 270$

$f(y) = 0.3515e^{-0.3881(y-252.2)^2} \quad f(250) = 0.3515e^{-0.3881(250-252.2)^2}$

$= 0.053721$

$f(270) = 0.3515e^{-0.3881(270-252.2)^2} \quad = 1.3888\times10^{-54}$

$$I = (270-250)\left[\frac{0.053721+1.3888\times10^{-54}}{2}\right]$$

$= 0.53721$

b) The exact value of the above integral cannot be found. We assume the value obtained by adaptive numerical integration using Maple as the exact value for calculating the true error and relative true error.

$$P(y \geq 250) = \int_{250}^{270} 0.3515 \quad e^{-0.3881(y-252.2)^2} dy$$

$$= 0.97377$$

so the true error is $= 0.97377 - 0.53721 \ = 0.43656$

The absolute relative true error,

$|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100$$

$$= \left|\frac{0.97377-0.53721}{0.97377}\right| \times 100$$

$= 44.832\%$

Improper Integrals

EXAMPLE

To approximate the values of the improper integral

$$\int_0^1 \frac{e^x}{\sqrt{x}} dx,$$

we will use the Composite Simpson's rule with h = 0.25. Since the fourth Taylor polynomial for ex about x = 0 is

$$P_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}.$$

We have

$$\int_0^1 \frac{P_4(x)}{\sqrt{x}} dx = \lim_{M \to 0^+} \left[2x^{1/2} + \frac{2}{3}x^{3/2} + \frac{1}{5}x^{5/2} + \frac{1}{21}x^{7/2} + \frac{1}{180}x^{9/2} \right]_M^1$$

$$= 2 + \frac{2}{3} + \frac{1}{5} + \frac{1}{21} + \frac{1}{108} \approx 2.9235450.$$

Table below lists the approximate values of

$$G(x) = \begin{cases} \dfrac{e^x - P_4(x)}{\sqrt{4}} & \text{when} \quad 0 < x \le 1 \\ 0, & \text{when} \quad x = 0 \end{cases}$$

x	G(x)
0.00	0
0.25	0.0000170
0.50	0.0004013
0.75	0.0026026
1.00	0.0099485

Applying the Composite Simpson's rule to G using these data gives

$$\int_0^1 G(x)dx \approx$$

$$\frac{0.25}{3}[0 + 4(0.0000170) + 2(0.0004013)$$

$$+4(0.0026026) + 0.0099485]$$

$$= 0.0017691$$

Hence

$$\int_0^1 \frac{e^x}{\sqrt{x}}dx \approx 2.9235450 + 0.0017691 = 2.9253141.$$

This result is accurate within the accuracy of the Composite Simpson's rule approximation for the function G. Since $\left|G^{(4)}(x)\right| < 1$ on [0, 1], the error is bounded by

$$\frac{1-0}{180}(0.25)^4(1) = 0.0000217.$$

EXAMPLE

To approximate the value of the improper integral

$$I = \int_1^\infty x^{-3/2} \sin\frac{1}{x}dx.$$

we make the change of variable t = x^{-1} to obtain $I = \int_0^1 t^{1/2} \sin t \; dt.$

The fourth Taylor polynomial, P4(t), for sin t about 0 is

$$P_4(t) = t - \frac{1}{6}t^3,$$

So we have $I = \int_0^1 \frac{\sin t - t + \frac{1}{6}t^3}{t^{1/2}} dt + \int_0^1 t^{1/2} - \frac{1}{6}t^{5/2} dt$

$$= \int_0^1 \frac{\sin t - t + \frac{1}{6}t^3}{t^{1/2}} dt + \left[\frac{2}{3}t^{3/2} - \frac{1}{21}t^{7/2} \right]_0^1$$

$$= \int_0^1 \frac{\sin t - t + \frac{1}{6}t^3}{t^{1/2}} dt + 0.61904761.$$

Applying the Composite Simpson's rule with n = 8 to the remaining integral gives

$$I = 0.0014890097 + 0.61904761 = 0.62053661,$$

which is accurate to within 4.0×10^{-8}.

An Introduction to MAPLE

Maple is a comprehensive computer system for advanced mathematics.

It includes facilities for interactive algebra, calculus, discrete mathematics, graphics, numerical computation etc.

It provides a unique environment for rapid development of mathematical programs using its vast library of built-in functions and operations.
Syntax :As with any computer language, Maple has its own syntax.
We try to explain some of the symbols used in Maple

Symbol	Description	Examples	Sample Output
;	End-of-line. Tells Maple to process the line and show the output.	hello;	hello
:	End-of-line. Tells Maple to process the line and hide the output.	hello:	
:=	Assignment. Lets you assign values to variables.	a := 3; a;	a := 3 3
+, -	Addition, subtraction.	1 + 3; 1 - 3;	4 -2
*, /	Multiplication, division	3*412; 1236/3; 7/3;	1236 412 7/3
^, sqrt	Power, square root	2^3; sqrt(2); 2^(1/2);	
evalf, .	Floating-point (decimal) evaluation	evalf(7/3); 7.0/3;	2.333333333 2.333333333

I,Pi	Imaginary unit, Pi.	2 + 3*I; (2*I)^2; evalf(Pi);	2+3I -4 3.14159265
%, %%	Recall the last output, recall the second-to-last output, etc.	%; %%%;	3.14159265-4

Some syntactical Tips:
Maple is case sensitive. foo, Foo, and FOO are three different things.

x*y gives the product of x and y,
xy is one variable

To get the constant e use exp(1).

Using the % operator can give confusing results. It always returns the last output from the Kernel, which may have nothing to do with where the cursor is (or which worksheet is active). If Maple doesn't recognize something, it assumes it as a variable; e.g. typing i^2 will give you i2,while we may be wanted -1.

Spaces are optional.

Greek letters may be entered by spelling their name. For example, alpha is always displayed as α , and Gamma is displayed as Γ
(note upper-case).
Built-in Data Capabilities
Maple can handle arbitrary-precision floating point numbers. In other words, Maple can store as many digits for a number as you like, up to the physical limits of your computer's memory. To control this, use the Digits variable.

```
sqrt(2.0);
  1.414213562
Digits := 20:
sqrt(2.0);
  1.4142135623730950488
```

Maple sets Digits to be 10 by default. You can also temporarily get precision results by calling evalf with a second argument.

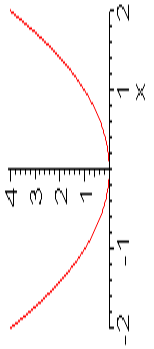
```
evalf(sqrt(2), 15);
  1.41421356237310
```

Large integers are handled automatically

Using symbolic computation
The main feature of Maple is symbolic computation.

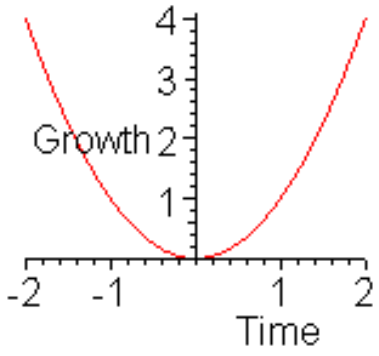
In other words, Maple does algebra.

Example	Output	Comments
(x + y)^2;	$(x + y)^2$	A basic expression.
k := x*y + y^2;	$k := xy + y^2$	k is now an alias for the expression. Note that k is simply another name for the expression - they are not equal in the mathematical sense.
p := k /(x+y);		You can now use k to refer to the expression. Maple immediately substitutes the value of k.
k := 'k';	k	You can unassign a variable by assigning it to its own name in single quotes.

simplify(p);	y	The simplify command does algebraic simplification.
p := x^2 - 8*x +15;	p := x2 - 8x + 15	Maple doesn't mind if you re-use names. The old value is lost.
solve(p=3,x);	2,6	Use the solve command to solve equations. Note the use of the = sign. Here, it is used in a mathematical sense. Maple will try different values for x until it finds all of them that make the mathematical statement x2 - 8x + 15 = 3 true.
dpdx := diff(p,x);	dpdx := 2x - 8	The diff command differentiates an expression with respect to a variable.
int(p,x);		The int command integrates an expression. Note that the constant of integration is left off.

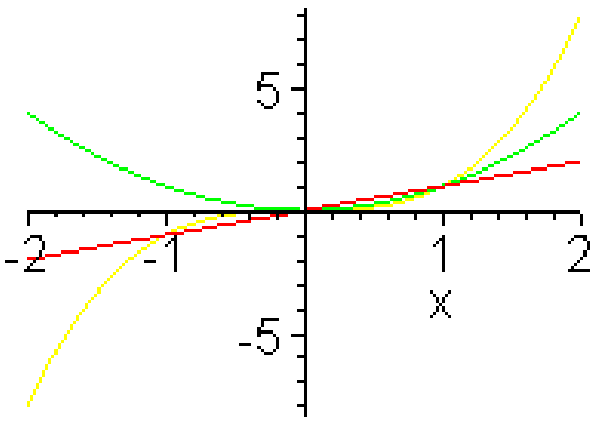
Basic Plotting

Maple can produce graphs very easily. Here are some examples, showcasing the basic capabilities.



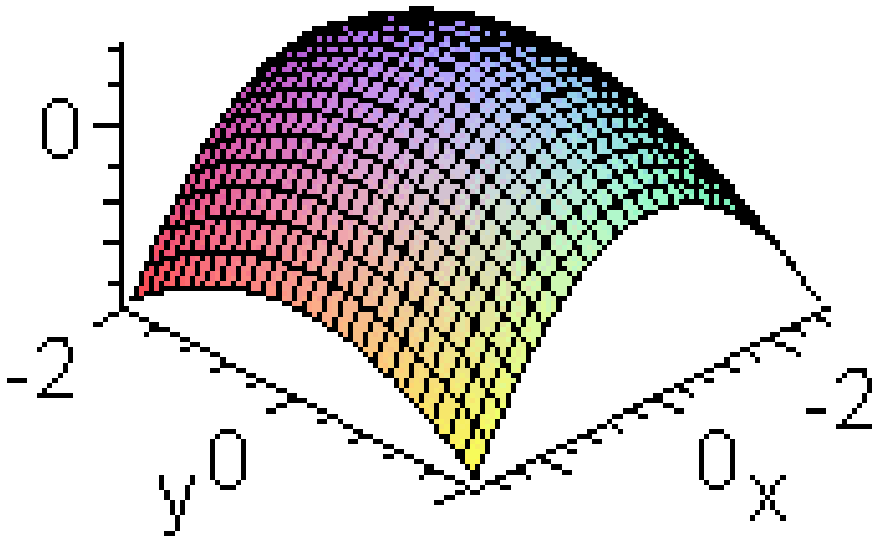
plot(x^2, x=-2..2);

A basic plot.



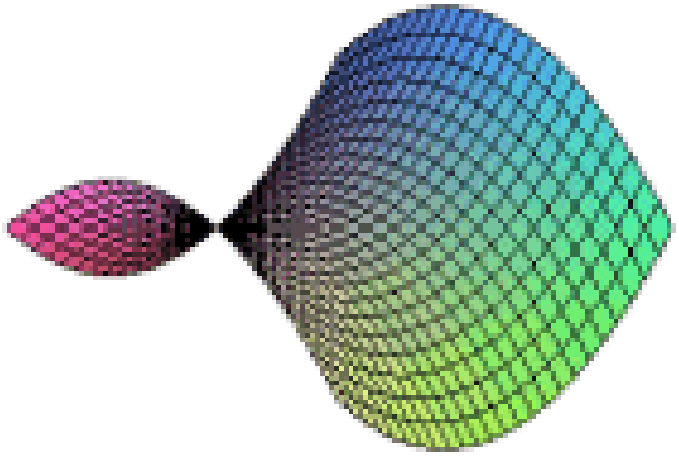
plot(x^2, x=-2..2, y=-10..10);

A plot with vertical axis control.



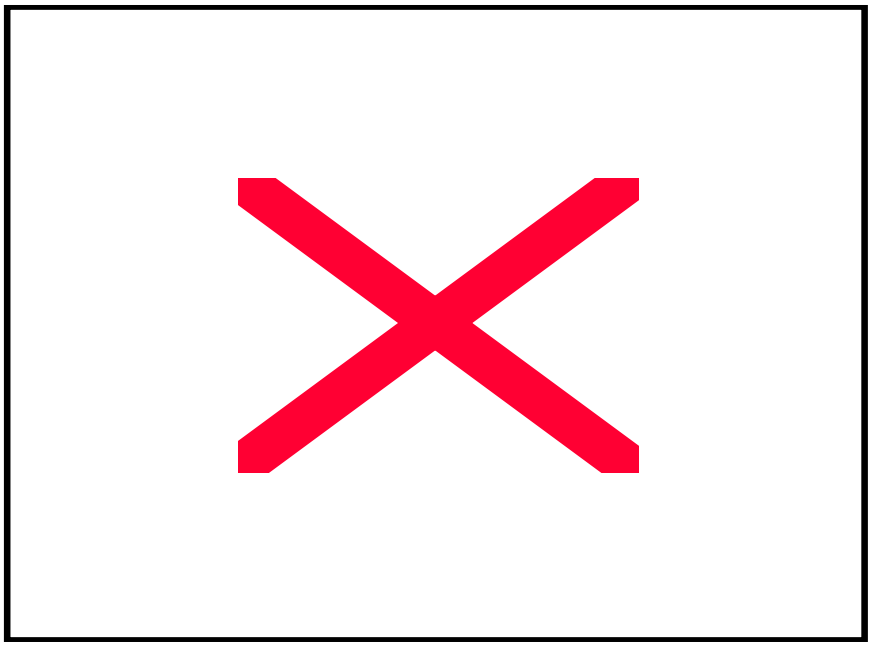
plot([x, x^2, x^3], x=-2..2);
Plot multiple expressions by enclosing them in brackets.

Live



plot3d(4-x^2-y^2, x=-3..3, y=-2..2);

A basic 3-d plot.



smartplot3d(x^2-y^2);

Using smartplot to let maple set it's own scaling.

Eigenvals and vectors of a numeric matrix :
Calling Sequence
  Eigenvals(A, vecs)
  Eigenvals(A, B, vecs)

Parameters
  A,B - square matrices of real or complex numbers
  vecs - (optional) name to be assigned the matrix of eigenvectors

Example

```
> A := array([[1,2,4],[3,7,2],[5,6,9]]);
```

$$A := \begin{bmatrix} 1 & 2 & 4 \\ 3 & 7 & 2 \\ 5 & 6 & 9 \end{bmatrix}$$

```
> evalf(Eigenvals(A));
[-.8946025434, 13.74788901, 4.146713483]
> lambda := evalf(Eigenvals(A,vecs));
λ := [-.8946025434, 13.74788901, 4.146713483]
> print(vecs);
```

$$\begin{bmatrix} 0.946575123 & 0.3592673648 & -.2987282688 \\ -.2818195168 & 0.4346670837 & 0.7371720079 \\ -.3074361518 & 0.9276415841 & -.6035890728 \end{bmatrix}$$

linalg[eigenvectors] - find the eigenvectors of a matrix

Calling Sequence
  eigenvectors(A)
  eigenvectors(A, 'radical')
  eigenvectors(A, 'implicit')

Parameters
A - square matrix
The command with(linalg,eigenvectors) allows the use of the abbreviated form of this command.
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
> A := matrix(3,3, [1,-3,3,3, -5,3,6,-6,4]);

$$A := \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

> e := eigenvalues(A);

$$e := 4, -2, -2$$

> v := [eigenvectors(A)];

$$v := [[4, 1, \{[1, 1, 2]\}], [-2, 2, \{[1, 1, 0], [-1, 0, 1]\}]]$$

> v[1][1]; # The first eigenvalue
4
> v[1][2]; # Its multiplicity
1
> v[1][3]; # Its eigenvectors

$$\{[1, 1, 2]\}$$

> v[2][1]; # The second eigenvalue
-2
> v[2][2]; # Its multiplicity
> v[2][2]; # Its multiplicity
2
Help
its worksheet interface

- Waiting for command;
- Restart; refresh memory;
- # #; comments so no action implied

Eval - Evaluate an expression
Calling Sequence
Eval(a, x=n))
Eval(a, {x1=n1,x2=n2,...})
Parameters
a - an expression
x, x1, x2,... - names
n, n1, n2,... - evaluation points
Description
The Eval function is a place holder for evaluation at a point.
The expression a is evaluated at
x = n (x1=n1, x2=n2, ... for the multivariate case).

The call Eval (a, x=n) mod p evaluates the polynomial a at x=n modulo p .
Note: The polynomial must be a multivariate polynomial over a finite field.
The call modp1(Eval(a,n),p) evaluates the polynomial a at x = n modulo p where a must be a univariate polynomial in the modp1 representation, with n an integer and p an integer > 1.
Examples

- Eval(x^2+1,x=3) mod 5;

0

- Eval(x^2+y,{x=3,y=2}) mod 5;

> Eval (int (f(x),x), x=y);

$$\int f(x)\,dx\Big|_{x=y}$$

Eigen values ?;

Solution of Problems

We can use Maple For:
Solution of non-linear equations
by Newton's Method
by Bisection Method
Solution of System of linear equations.
Numerical Integration.
Numerical Solution of ODE's.
Maple performs both numerical and symbolic itegration.
Please note that the Maple uses the int function for the both numerical and symbolic integration, but for numerical integration we have to use the additional evalf command
Some inbuilt functions in Maple being used for integration
Numerical Integration
Calling Sequences
evalf(Int(f, x=a..b))
evalf(Int(f, a..b))
evalf(Int(f, x=a..b, opts))
evalf(Int(f, a..b, opts))
evalf(int(f, x=a..b))
We Define Parameters
f - algebraic expression or procedure; integrand
x - name; variable of integration
a,b - endpoints of the interval of integration
opts - (optional) name or equation of the form
option=name; options
Description
In the case of a definite integral, which is returned unevaluated, numerical integration can be invoked by applying evalf to the unevaluated integral. To invoke numerical integration without
first invoking symbolic integration, use the inert function Int as in: evalf(Int(f, x=a..b)).

If the integrand f is specified as a procedure or a Maple operator, then the second argument must be the range a..b and not an equation. (i.e., a variable of integration must not be specified.)
>evalf(Int(exp(-x^3), x = 0..1));.8075111821
>evalf(Int(exp(-x^3), x = 0..1));.8075111821
>evalf(Int(exp(-x^3), x = 0..1));8075111821
>alg041(); This is Simpson's Method.
`Input the function F(x) in terms of x`
`For example:
> sin (x)
`Input lower limit of integration and upper limit of integration`
`separated by a blank`
> 0 3.14159265359
`Input an even positive integer N.`
> 20
The integral of F from 0.00000000
to
3.14159265

is 2.00000678
alg041(); This is Simpson's Method.
`Input the function F(x) in terms of x`
> x^2
`Input lower limit of integration and upper limit of integration separated by a blank'
>0 2
Input an even positive integer N
>20
The integral of F from
0.00000000
to
2.00000000

is 2.66666667
> alg041();
This is Simpson's Method.
Input the function F(x) in terms of x, for example: cos(x)
> exp(x-x^2/2)
Input lower limit of integration and upper limit of integration separated by a blank
> 0 3.14159265359
Input an even positive integer N.
> 20
The integral of F from
0.00000000
to
3.14159265

```
is 3.41046542
> alg044();
This is Simpson's Method for double integrals.

Input the functions F(X,Y), C(X), and D(X) in terms of x and y separated by a space.
For example: cos(x+y) x^3 x
> exp(y/x) x^3 x^2
Input lower limit of integration and upper limit of integration separated by a blank
> 0.1 0.5
Input two even positive integers N, M ; there will be N subintervals for outer integral and
M subintervals for inner integral - separate with blank
> 10 10
The double integral of F from
0.100 to    0.500
Is         .03330546
obtained with
N :=  10 and M :=  10
> alg045();
`This is Gaussian Quadrature for double integrals.`
`Input the function F(x,y) in terms of x and y`
`For example: sqrt(x^2+y^2)`
> exp (y/x)
Input the functions C(x), and D(x) in terms of x separated by a space
For example: cos (x) sin (x)
> x^3  x^2
Input lower limit of integration and upper limit of integration separated by a blank space
>0.1 0.5
Input two integers M > 1 and N > 1. This implementation of Gaussian quadrature
requires both to be less than or equal to 5.
M is used for the outer integral and N for the inner integral - separated by a space.
> 5 5
The double integral of F from
0.1000  to  0.5000
is   3.3305566120e-02
Or  0.03305566120
obtained with
M = 5 and N = 5
```

Lecture 44

Solution of
Non-Linear Equations

Bisection Method
Regula-Falsi Method
Method of iteration
Newton - Raphson Method
Muller's Method
Graeffe's Root Squaring Method

Newton -Raphson Method

An approximation to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Better and successive approximations *x*2, *x*3, …, *x*n to the root are obtained from

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

N-R Formula
Newton's algorithm

To find a solution to *f(x)=0* given an initial approximation *p0*
INPUT *initial approximation p0; tolerance TOL; maximum number of iterations N0*

OUTPUT *approximate solution p or message of failure*
Step 1
Set I = 1

Step 2
While i ≤ N0 do Steps 3-6
Step 3
Set p = p0 – f (p0) / f' (p0) *(compute pi).*
Step 4
If Abs (p – p0) < TOL OUTPUT *(p);*
(The procedure was successful.)
STOP
Step 5 *Set i = i + 1*
Step 6 *Set p0 = p (Update p0)*
Step 7 OUTPUT
(The method failed after N0 iterations, N0 = ',N0)
The procedure was unsuccessful
STOP

Example

Using Maple to solve a non-linear equation.

$$\cos(x) - x = 0$$

Solution

The Maple command will be as follows,
Fsolve (cos (x) -x);

- alg023();
- This is Newton's Method

Input the function F(x) in terms of x
For example:
> cos(x)-x
Input initial approximation
> 0.7853981635
Input tolerance
> 0.00005
Input maximum number of iterations - no decimal point
> 25
Select output destination
1. Screen
2. Text file
Enter 1 or 2
> 1
Select amount of output
1. Answer only
2. All intermediate approximations
Enter 1 or 2
> 2

Newton's Method

I	P	F(P)
1	0.739536134	-7.5487470e-04
2	0.739085178	-7.5100000e-08
3	0.739085133	0.0000000e-01

Approximate solution = 0.73908513
with F(P) = 0.0000000000
Number of iterations = 3
Tolerance = 5.0000000000e-05

Another Example

> alg023();
Input the function F(x) in terms of x ,
> sin(x)-1
Input initial approximation
> 0.17853
Input tolerance
> 0.00005
Input maximum number of iterations – no decimal point
> 25
Select output destination
1. Screen
2. Text file
Enter 1 or 2
>2
Select amount of output
1. Answer only
2. All intermediate approximations
Enter 1 or 2
> 2

Newton's Method

I	P	F(P)
1	1.01422964e+00	-1.5092616e-01
2	1.29992628e+00	-3.6461537e-02
3	1.43619550e+00	-9.0450225e-03
4	1.50359771e+00	-2.2569777e-03
5	1.53720967e+00	-5.6397880e-04
6	1.55400458e+00	-1.4097820e-04
7	1.56240065e+00	-3.5243500e-05

More…

8	1.56659852e+00	-8.8108000e-06
9	1.56869743e+00	-2.2027000e-06
10	1.56974688e+00	-5.5070000e-07
11	1.57027163e+00	-1.3770000e-07
12	1.57053407e+00	-3.4400000e-08
13	1.57066524e+00	-8.6000000e-09
14	1.57073085e+00	-2.1000000e-09
15	1.57076292e+00	-6.0000000e-10

Approximate solution
= 1.57076292
with
F(P) =6.0000000000e-10
Number of iterations = 15
Tolerance = 5.0000000000e-05

Bisection Method

```
> alg021();
This is the Bisection Method.
Input the function F(x) in terms of x
For example:
> x^3+4*x^2-10
Input endpoints A < B separated by blank
> 1 2
Input tolerance
> 0.0005
Input maximum number of iterations - no decimal point
> 25
Select output destination
      1.    Screen  ,
2. Text file
Enter 1 or 2
> 1
Select amount of output
1. Answer only
2. All intermediate approximations
Enter 1 or 2
> 2
```

Bisection Method

I	P	F(P)
1	1.50000000e+00	2.3750000e+00
2	1.25000000e+00	-1.7968750e+00
3	1.37500000e+00	1.6210938e-01
4	1.31250000e+00	-8.4838867e-01
5	1.34375000e+00	-3.5098267e-01
6	1.35937500e+00	-9.6408842e-02

```
 7   1.36718750e+00    3.2355780e-02
 8   1.36328125e+00   -3.2149969e-02
 9   1.36523438e+00    7.2030000e-05
10   1.36425781e+00   -1.6046697e-02
11   1.36474609e+00   -7.9892590e-03

Approximate solution P =  1.36474609
with F(P) =   -.00798926
Number of iterations =  11
Tolerance =  5.00000000e-04
alg021(); Another example of the Bisection Method.
Input the function F(x) in terms of x,
> cos(x)
Input endpoints A < B separated by blank
> 1 2
Input tolerance
> 0.0005
Input maximum number of iterations - no decimal point
> 25
Select output destination
1. Screen  ,      2. Text file
Enter 1 or 2
> 1
Select amount of output
1. Answer only
2. All intermediate approximations
Enter 1 or 2
> 2
```

Bisection Method

```
 1          P                    F(P)
 1   1.50000000e+00    7.0737202e-02
 2   1.75000000e+00   -1.7824606e-01
 3   1.62500000e+00   -5.4177135e-02
 4   1.56250000e+00    8.2962316e-03
 5   1.59375000e+00   -2.2951658e-02
6   1.57812500e+00   -7.3286076e-03
 7   1.57031250e+00    4.8382678e-04
 8   1.57421875e+00   -3.4224165e-03
 9   1.57226563e+00   -1.4692977e-03
10   1.57128906e+00   -4.9273519e-04
11   1.57080078e+00   -4.4542051e-06
Approximate solution P =  1.57080078
with F(P) =   -.00000445
Number of iterations =  11
Tolerance =  5.00000000e-04
```

- alg025(); This is the Method of False

```
Position
Input the function F(x) in terms of x
> cos(x)-x
Input endpoints P0 < P1 separated by a blank  space
```

- 0.5 0.7853981635

```
Input tolerance
>0.0005
Input maximum number of
iterations  - no decimal point
> 25
```

Select output destination
1. Screen
2. Text file
Enter 1 or 2
>1
Select amount of output
1. Answer only
2. All intermediate approximations
Enter 1 or 2
> 2

METHOD OF FALSE POSITION

I	P	F(P)
2	7.36384139e-01	4.51771860e-03
3	7.39058139e-01	4.51772000e-05
4	7.39084864e-01	4.50900000e-07

Approximate solution P = .73908486
with F(P) = .00000045
Number of iterations = 4
Tolerance = 5.00000000e-04

System of Linear Equations

Gaussian Elimination Gauss-Jordon Elimination

Crout's Reduction Jacobi's

Gauss- Seidal Iteration Relaxation

Matrix Inversion

> alg061();
This is Gaussian Elimination to solve a linear system.
The array will be input from a text file in the order:
A(1,1), A(1,2), ..., A(1,N+1), A(2,1), A(2,2), ..., A(2,N+1),..., A(N,1), A(N,2), ..., A(N,N+1)

Place as many entries as desired on each line, but separate entries with
at least one blank.
Has the input file been created? - enter Y or N.
> y
Input the file name in the form - drive:\name.ext
for example: A:\DATA.DTA
> d:\maple00\dta\alg061.dta
Input the number of equations - an integer.
> 4
Choice of output method:
1. Output to screen 2. Output to text file
Please enter 1 or 2.
> 1

GAUSSIAN ELIMINATION

The reduced system - output by rows:

1.00000000	-1.00000000	2.00000000	-1.00000000	-8.00000000
0.00000000	2.00000000	-1.00000000	1.00000000	6.00000000
0.00000000	0.00000000	-1.00000000	-1.00000000	-4.00000000
0.00000000	0.00000000	0.00000000	2.00000000	4.00000000

Has solution vector:
-7.00000000 3.00000000 2.00000000 2.00000000

with 1 row interchange (s)
> alg071();
This is the Jacobi Method for Linear Systems.
The array will be input from a text file in the order
A(1,1), A(1,2), ..., A(1,n+1), A(2,1), A(2,2), ...,
A(2,n+1),..., A(n,1), A(n,2), ..., A(n,n+1)
Place as many entries as desired on each line, but separate
entries with at least one blank.
The initial approximation should follow in the same format has the input file been created? - enter Y or N.
> y
Input the file name in the form - drive:\name.ext
for example: A:\DATA.DTA
> d:\maple00\alg071.dta
Input the number of equations - an integer.
> 4
Input the tolerance.
> 0.001
Input maximum number of iterations.
> 15
Choice of output method:
1. Output to screen
2. Output to text file
Please enter 1 or 2.
> 1

JACOBI ITERATIVE METHOD FOR LINEAR SYSTEMS

The solution vector is :
1.00011860 1.99976795
-.99982814 0.99978598
using 10 iterations
with Tolerance 1.0000000000e-03

An approximation to the root is given by

Better and successive approximations x_2, x_3, …, x_n to the root are obtained from

Example
Using Maple to solve a non-linear equation.

System of Linear Equations

Input the tolerance.
> 0.001
Input maximum number of iterations.
> 15
Choice of output method:
1. Output to screen
2. Output to text file
Please enter 1 or 2.
> 1

Summing up

Non-Linear Equations

Bisection Method (Bolzano)
Regula-Falsi Method
Method of iteration
Newton - Raphson Method
Muller's Method
Graeffe's Root Squaring Method

In the method of *False Position*, the first approximation to the root of $f(x) = 0$ is given by

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) \qquad \text{.....(2.2)}$$

Here f (xn-1) and f (xn+1) are of opposite sign. Successive approximations to the root of f (x) = 0 is given by Eq. (2.2).

METHOD OF ITERATION can be applied to find a real root of the equation f (x) = 0 by rewriting the same in the form, $x = \phi(x)$

$$\left.\begin{aligned} x_1 &= \phi(x_0) \\ x_2 &= \phi(x_1) \\ &\vdots \\ x_{n+1} &= \phi(x_n) \end{aligned}\right\}$$

In Newton – Raphson Method successive approximations
x2, x3, …, xn to the root are obtained from

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

N-R Formula

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}, n = 12,3\ldots$$

This sequence converges to the root 'b' of *f (x)* = 0 i.e. *f(b) = 0*.

The Secant method converges faster than linear and slower than Newton's quadratic.

In Muller's Method we can get a better approximation to the root, by using

$$x_{i+1} = x_i + h_i \lambda$$

$$\lambda = \frac{-2 f_i \delta_i}{g_i \pm [g_i^2 - 4 f_i \delta_i \lambda_i (f_{i-2} \lambda_i - f_{i-1} \delta_i + f_i)]^{1/2}}$$

Where we defined

$$\lambda = \frac{h}{h_i} = \frac{x - x_i}{x_i - x_{i-1}}$$

$$\lambda_i = \frac{h_i}{h_{i-1}}$$

$$\delta_i = 1 + \lambda_i$$

Systems of Linear Equations
Gaussian Elimination Gauss-Jordon Elimination
Crout's Reduction Jacobi's
Gauss- Seidal Iteration Relaxation
Matrix Inversion

In Gaussian Elimination method, the solution to the system of equations is obtained in two stages.

- the given system of equations is reduced to an equivalent upper triangular form using elementary transformations
- the upper triangular system is solved using back substitution procedure

Gauss-Jordon method is a variation of Gaussian method. In this method, the elements above and below the diagonal are simultaneously made zero

In Crout's Reduction Method the coefficient matrix [A] of the system of equations is decomposed into the product of two matrices [L] and [U], where [L] is a lower-

triangular matrix and [U] is an upper-triangular matrix with 1's on its main diagonal.

For the purpose of illustration, consider a general matrix in the form

$$[L][U]=[A]$$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Jacobi's Method is an iterative method, where initial approximate solution to a given system of equations is assumed and is improved towards the exact solution in an iterative way.
In Jacobi's method, the (r + 1)th approximation to the above system is given by Equations

$$\left.\begin{aligned} x_1^{(r+1)} &= \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2^{(r)} - \cdots - \frac{a_{1n}}{a_{11}}x_n^{(r)} \\ x_2^{(r+1)} &= \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1^{(r)} - \cdots - \frac{a_{2n}}{a_{22}}x_n^{(r)} \\ &\vdots \quad \vdots \quad \vdots \quad \vdots \\ x_n^{(r+1)} &= \frac{b_n}{a_{nn}} - \frac{a_{n1}}{a_{nn}}x_1^{(r)} - \cdots - \frac{a_{n(n-1)}}{a_{nn}}x_{n-1}^{(r)} \end{aligned}\right\}$$

Here we can observe that no element of $x_i^{(r+1)}$ replaces $x_i^{(r)}$ entirely for the next cycle of computation.

In Gauss-Seidel method, the corresponding elements of $x_i^{(r+1)}$ replaces those of $x_i^{(r)}$ as soon as they become available. It is also called method of Successive Displacement.

The Relaxation Method is also an iterative method and is due to Southwell.

$$dx_i = \frac{R_i}{a_{ii}}$$

Eigen Value Problems

Power Method
Jcobi's Method

In Power Method the result looks like

$$u^{(k)} = [A]v^{(k-1)} = q_k v^{(k)}$$

Here, q_k is the desired largest eigen value and $v^{(k)}$ is the corresponding eigenvector.

Interpolation

Finite Difference Operators
Newton's Forward Difference Interpolation Formula
Newton's Backward Difference Interpolation Formula
Lagrange's Interpolation Formula
Divided Differences
Interpolation in Two Dimensions
Cubic Spline Interpolation

Finite Difference Operators

Forward Differences

Backward Differences

Central Difference

$$\Delta^r y_i = \Delta^{r-1} y_{i+1} - \Delta^{r-1} y_i$$

$$\nabla^k y_i = \nabla^{k-1} y_i - \nabla^{k-1} y_{i-1},$$

$$i = n, (n-1), ..., k$$

$$\delta^n y_i = \delta^{n-1} y_{i+(1/2)} - \delta^{n-1} y_{i-(1/2)}$$

Thus

$$\Delta y_x = y_{x+h} - y_x = f(x+h) - f(x)$$

$$\Delta^2 y_x = \Delta y_{x+h} - \Delta y_x$$

Similarly

$$\nabla y_x = y_x - y_{x-h} = f(x) - f(x-h)$$

$$\delta y_x = y_{x+(h/2)} - y_{x-(h/2)} = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$$

Shift operator, E

$$E\,f(x) = f(x+h)$$

$$E^n f(x) = f(x+nh)$$

$$E^n y_x = y_{x+nh}$$

The inverse operator E-1 is defined as

$$E^{-1} f(x) = f(x-h)$$

Similarly,

$$E^{-n} f(x) = f(x-nh)$$

Average Operator, μ

$$\mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$$

$$= \frac{1}{2}\left[y_{x+(h/2)} + y_{x-(h/2)}\right]$$

Differential Operator, D

$$\left.\begin{aligned} Df(x) &= \frac{d}{dx} f(x) = f'(x) \\ D^2 f(x) &= \frac{d^2}{dx^2} f(x) = f^n(x) \end{aligned}\right\}$$

Important Results

$$\Delta = E - 1$$

$$\nabla = 1 - E^{-1} = \frac{E-1}{E}$$

$$\delta = E^{1/2} - E^{1/2}$$

$$hD = \log E$$

$$\mu = \frac{1}{2}(E^{1/2} + E^{-1/2})$$

The Newton's forward difference formula for interpolation, which gives the value of f (x0 + ph) in terms of f (x0) and its leading differences.

$$f(x_0 + ph) = f(x_0) + p\Delta f(x_0)$$

$$+ \frac{p(p-1)}{2!}\Delta^2 f(x_0) + \frac{p(p-1)(p-2)}{3!}\Delta^3 f(x_0)$$

$$+ \cdots + \frac{p(p-1)\cdots(p-n+1)}{n!}\Delta^n f(x_0) + \text{Error}$$

An alternate expression is

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$
$$+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$
$$+\frac{p(p-1)\cdots(p-n+1)}{n!}\Delta^n y_0 + \text{Error}$$

Newton's Backward difference formula is,

$$f(x_n + ph) = f(x_n) + p\nabla f(x_n)$$
$$+\frac{p(p+1)}{2!}\nabla^2 f(x_n)$$
$$+\frac{p(p+1)(p+2)}{3!}\nabla^3 f(x_n) + \cdots$$
$$+\frac{p(p+1)(p+2)\cdots(p+n-1)}{n!}\nabla^n f(x_n) + \text{Error}$$

Alternatively, this formula can also be written as

$$y_x = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n$$
$$+\frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$
$$+\frac{p(p+1)(p+2)\cdots(p+n-1)}{n!}\nabla^n y_n + \text{Error}$$

Here

$$p = \frac{x - x_n}{h}$$

The Lagrange's formula for interpolation

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}y_1 + \cdots$$
$$+\frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}y_i + \cdots + \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})}y_n$$

Newton's divided difference interpolation formula can be written as

$$y = f(x) = y_0 + (x-x_0)y[x_0, x_1]$$
$$+(x-x_0)(x-x_1)y[x_0, x_1, x_2]$$
$$+\cdots+$$
$$(x-x_0)(x-x_1)\ldots(x-x_{n-1})y[x_0, x_1, ..., x_n]$$

Where the first order divided difference is defined as

$$y[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0}$$

Numerical Differentiation and Integration

We expressed D in terms of Δ :

$$D = \frac{1}{h}\left(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - \cdots\right)$$

Using backward difference operator , we have

$hD = -\log(1 - \nabla).$

On expansion, we have

$$D = \frac{1}{h}\left(\nabla - \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \cdots\right)$$

Using Central difference Operator

$$D = \frac{1}{h}\left(\delta - \frac{1}{24}\delta^3 + \frac{3}{640}\delta^5 - \cdots\right)$$

Differentiation Using Interpolation

Richardson's Extrapolation

$$\frac{d}{dx}\prod_{i=0}^{n-1}(x - x_i)$$

$$= \sum_{i=0}^{n-1}\frac{(x - x_0)(x - x_1)\cdots(x - x_n)}{x - x_i}$$

Thus, $y'(x)$ is approximated by $P_n'(x)$ which is given by

$$P_n'(x) = y[x_0, x_1] + [(x - x_1) + (x - x_0)]y[x_0, x_1, x_2] + \cdots$$

$$+ \sum_{i=0}^{n=1}\frac{(x - x_0)(x - x_1)\cdots(x - x_{n-1})}{x - x_i}y[x_0, x_1, \ldots, x_n]$$

$$F_m\left(\frac{h}{2^m}\right)$$

$$= \frac{4^m F_{m-1}\left(\frac{h}{2^m}\right) - F_{m-1}\left(\frac{h}{2^{m-1}}\right)}{4^m - 1},$$

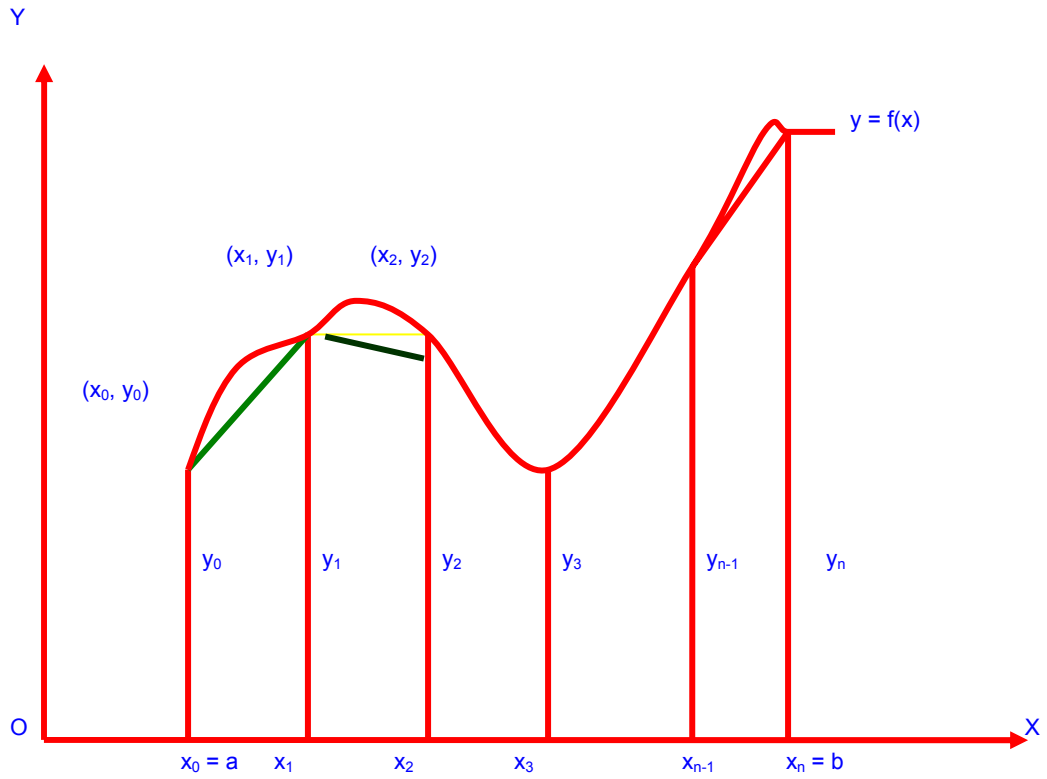
$$m = 1, 2, 3,$$

Basic Issues in Integration

What does an integral represent?

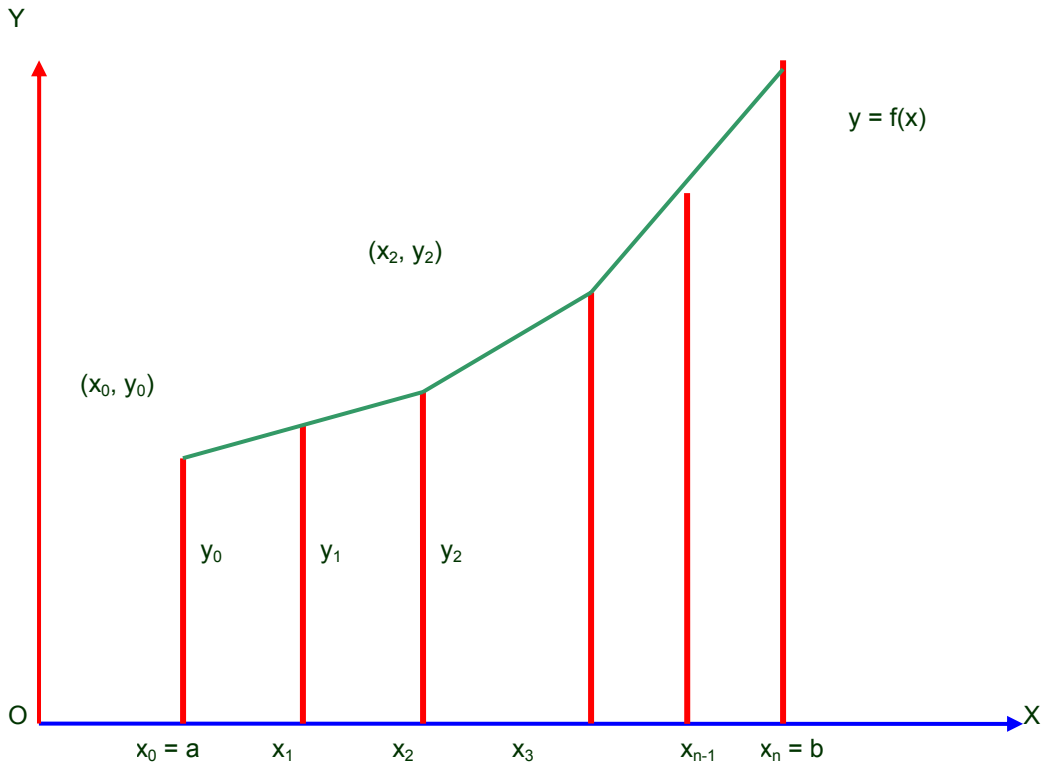
$\int_a^b f(x)\,dx$ = AREA

$$\int_c^d \int_a^b g(x,y)\ dx\ dy \quad = \text{VOLUME}$$



$$\int_{x_0}^{x_3} f(x)dx$$

$$= \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + y_3) - \frac{3}{80}\mathrm{h}^5 y^{(iv)}(\xi)$$



TRAPEZOIDAL RULE

$$\int_{x_0}^{x_n} f(x)dx$$

$$= \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n) + E_n$$

DOUBLE INTEGRATION

We described procedure to evaluate numerically a double integral of the form

$$I = \int \left[\int (x, y)dx \right] dy$$

Differential Equations

Taylor Series
Euler Method
Runge-Kutta Method
Predictor Corrector Method

In Taylor's series we expanded y (t) by Taylor's series about the point t = t0 and obtain

$$y(t) = y(t_0) + (t-t_0)y'(t_0) + \frac{(t-t_0)^2}{2!}y''(t_0) + \frac{(t-t_0)^3}{3!}y'''(t_0) + \frac{(t-t_0)^4}{4!}y^{IV}(t_0) + \cdots$$

In Euler Method we obtained the solution of the differential equation in the form of a recurrence relation

$$y_{m+1} = y_m + hf(t_m, y_m)$$

We derived the recurrence relation

$$y_{m+1} = y_m + h\left[\frac{f(t_m, y_m) + f(t_{m+1}, y_{m+1}^{(1)})}{2}\right]$$

Which is the modified Euler's method.

The fourth-order R-K method was described as

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

In general, Milne's predictor-corrector pair can be written as

$$\left.\begin{aligned} P: y_{n+1} &= y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n) \\ C: y_{n+1} &= y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1}) \end{aligned}\right\}$$

This is known as Adam's predictor formula.

$$y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}) + \frac{251}{720}h\nabla^4 f_n$$

Alternatively, it can be written as

$$y_{n+1} = y_n + \frac{h}{24}\left[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right] + \frac{251}{720}h\nabla^4 y'_n$$

MTH603 FINALTERM SUBJECTIVE

Question No: 50 (Marks: 5)

Solve $\sin x = 1 + x^3$ by using the Newton Raphson method up to one iteration (Note: Accuracy up to four decimal places is required)

Answer:

$f(x) = \sin x = 1 + x^3$

$f(x) = \sin x - 1 - x^3$

$f(1) = \sin 1 - 1 - 1^3 = 0.0174 - 1 - 1 = -1.9826$

$f(2) = \sin 2 - 1 - 2^3 = 0.0348 - 1 - 8 = -8.9652$

$f'(1) = -\cos x - 3x^2 = -\cos 1 - 3(1)^2 = -0.9998 - 3 = -3.9998$

$f'(2) = -\cos x - 3x^2 = -\cos 2 - 3(2)^2 = -0.9993 - 12 = -12.9993$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = -3.9998 - \frac{-8.9652}{-12.9993} = -3.9998 - 0.6896 = -4.6894$$

Q: Evaluate the integral $\int_0^1 (x+1)\,dx$ By Trapezoidal Rule, h=0.2

Answer:

X	0	0.2	0.4	0.6	0.8	1
Y= (x+1)	1.0	1.2	1.4	1.6	1.8	2

Using trapezoidal rule, taking h=0.2

$$I=\frac{h}{2}\left(y_0+y_5+2\left(y_1+y_2+y_3+y_4\right)\right)$$

$$I=\frac{0.2}{2}\left(1.0+2+2(1.2+1.4+1.6+1.8)\right)$$

$$I=\frac{0.2}{2}\left(3+12\right)$$

$$I=\frac{3}{2}=1.5$$

Q: Evaluate the integral $\int_3^5(\sin x+\cos x)dx$ By Simpson`s 3/8 rule. Take the interval h=1.

Answer:

X	3	4	5
Y=(sin x+ cos x)	1.0509	1.0673	1.0833

$$\int_3^5(\sin x+\cos x)dx=\frac{3h}{8}\left(y_0+y_2+3\left(y_1\right)\right)$$

$$\int_3^5(\sin x+\cos x)dx=\frac{3(1)}{8}\left(1.0509+1.0833+3(1.0673)\right)$$

$$\int_3^5(\sin x+\cos x)dx=\frac{3}{8}\left(2.1342+3.2019\right)$$

$$\int_3^5(\sin x+\cos x)dx=\frac{3}{8}\left(5.3361\right)=\frac{16.0083}{8}=2.0010375$$

Q: Using Milne's method find y(0.1) dy/dx=x-y/x+y, Y(0)=1.

Answer:

$$\frac{dy}{dx}=\frac{x-y}{x+y},\ \ y(0)=1,$$

$$find\ y(0.1)$$

$$y_0'=\frac{x-y}{x+y}=\frac{0-1}{0+1}=-\frac{1}{1}=-1$$

$$now, using\ Predictor\ formula$$

$$y_1=y_0+\frac{4h}{3}\left(2y_0'\right)$$

Q: y'=2t+3y then find next two derivatives w.r.t t or y.

Answer:

w.r.t t 2nd dravative Y'' = 2 3rd dravative Y''' = 0
w.r.t y. 2nd dravative Y'' = 3 3rd dravative Y''' = 0

Q: Make backward difference table from;

X	**0**	**3**	**4**
Y	**4**	**7**	**9**

Q:Solve by Gauss-elimination method:
1 0 8/3 x 8

0 -1 1/3 y = 0

0 0 -1/3 z 1

Q: Find y by Euler's method when x=1, dy/dx=1/x+y, y(0)=1, h=1
Answer:

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 1 + 1\left(\frac{1}{x_0 + y_0}\right) = 1 + 1\left(\frac{1}{1+1}\right) = 1 + 0.5 = 1.5$$

Q: Evaluate the integral by Simpson's 1/3 rule , $\int_0^{\frac{\pi}{4}} (\sin x + 1)\,dx$ **taking interval of** $\frac{\pi}{4}$**.Answer:**

X	0	$\frac{\pi}{4}$
Y=(sin x+ 1)	1	1.7071

$$\int_3^5 (\sin x + 1)dx = \frac{h}{3}(y_0 + y_1)$$

$$\int_3^5 (\sin x + 1)dx = \frac{\frac{\pi}{4}}{3}(1 + 1.7071)$$

$$\int_3^5 (\sin x + 1)dx = \frac{3.14}{12}(2.7071) = \frac{8.500294}{12} = 0.7083$$

2012 paper1:

Q.1. State the sufficient condition of convergence of the iterative solution to the exact solution? 2m

Answer: page 72

A sufficient condition for convergence of the iterative solution to the exact solution is $|a_{ii}| > \sum_{\substack{j=1 \\ j\neq 1}}^{n} |a_{ii}|$, i=1, 2...n When this condition (diagonal dominance) is true, Jacobi's method converges

Q.2. Evaluate the integral $\int_3^5 (\log x + 1)dx$ Using trapezoidal rule

Take h=1…..2m

Answer:

X	3	4	5
Y= logx+1	0.6020	0.6989	0.7781

Using trapezoidal rule, taking h=1

$$I = \frac{h}{2}(y_0 + y_3 + 2y_1)$$

$$I = \frac{1}{2}(0.6020 + 0.7781 + 2(0.6989))$$

$$I = \frac{1}{2}(1.3801 + 1.3978)$$

$$I = \frac{2.7779}{2} = 1.38895$$

Q3. Write the formula for finding the value of k_1 in the fourth order R-K Method. 2m

Answer: page 205

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

Q4. Write Adam-Moulton's Predictor formula for finding the solution of a differential equation. 2m

Answer: page 223

$$P: y_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Q5. Obtain numerically the solution of y` = x^2 +2x+y^2, y (0) =1 Using Euler's method to find y at x=1, h=1 …. 3m

Answer:

$Y_1 = y_0 + hf(x_0, y_0)$

$Y_1 = 1 + (1)(1+1)$

$Y_1 = 1+2=3$

Q6. If f(2)=-2.6146 and f(3)=4.7610, then find the first approximation using the Regula-Falsi method? 3m

Answer:

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$x_3 = 3 - \frac{3-2}{4.7610 - (-2.6146)}(4.7610)$$

$$x_3 = 3 - \frac{4.7610}{7.3756}$$

$$x_3 = 3 - \frac{4.7610}{7.3756} = 3 - 0.6455 = 2.3545$$

Q7. If in a solving a giving differential equations

$y_0 = 1, y_1' = 1.1, y_2' = 1.2, y_3' = 1.3, h = 1$ ……3m

then find y_4 *by Mi*ln*e*Pr*edictor formula*

Answer:

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

$$P: y_4 = y_0 + \frac{4h}{3}\left(2y'_1 - y'_2 + 2y'_3\right)$$

$$P: y_4 = 1 + \frac{4(1)}{3}\left(2(1.1) - 1.2 + 2(1.3)\right)$$

$$P: y_4 = 1 + \frac{4}{3}\left(2.2 - 1.2 + 2.6\right)$$

$$P: y_4 = 1 + \frac{4}{3}\left(3.6\right)$$

$$P: y_4 = 1 + \frac{14.4}{3}$$

$$P: y_4 = \frac{3 + 14.4}{3} = \frac{17.4}{3} = 5.8$$

Q8. Find F(h), using Richardson's extrapolation limit, while finding

$y'(0.01)$ *to the function* $y = \frac{1}{x}$ *with* $h = 0.005$ ……..3m

Answer:

$$F(h) = \frac{y(x+h) - y(x-h)}{2h}$$

$$F(h) = \frac{\frac{1}{0.01 + 0.005} - \frac{1}{0.01 - 0.005}}{2(0.005)}$$

$$F(h) = \frac{\frac{1}{0.015} - \frac{1}{0.005}}{0.01}$$

$$F(h) = \frac{66.66 - 200}{0.01}$$

$$F(h) = \frac{-133.34}{0.01} = -13334$$

Q.9

Evaluate the int *egral*

$$\int_0^3 (x^2 + x)dx$$

u sin g *Simpson's* $\frac{3}{8}$ *rule* take h=1……5m

Answer:

X	0	1	2	3
Y=(x^2+x)	0	2	6	12

$$\int_0^3 (x^2+x)dx = \frac{3}{8}h\left[y_0+y_3+3(y_1+y_2)\right]$$

$$\int_0^3 (x^2+x)dx = \frac{3(1)}{8}\left[0+12+3(2+6)\right]$$

$$\int_0^3 (x^2+x)dx = \frac{3}{8}\left[12+24\right] = \frac{3}{8}(36) = \frac{108}{8} = 13.5$$

Q10. Construct a backward difference table from the following values of x and y…..5m

x	-1	0	1	2	3
Y=f(x)	10	2	10	62	80

Answer:

x	Y=f(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
-1	10				
0	2	-8			
1	10	8	16		
2	62	52	44	28	
3	80	18	-34	-78	-106

Q11. From the following table of values, construct forward difference table…..5m

X	1.00	1.05	1.10	1.15	1.20
y	1.000	1.0247	1.0488	1.0724	1.0954

Answer:

x	Y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.00	1.000				
1.05	1.0247	0.0247			
1.10	1.0488	0.0241	0.0006		
1.15	1.0724	0.0236	-0.0005	-0.0011	
1.20	1.0954	0.023	-0.0006	-0.0001	0.0000

Q12. Use Runge-Kutta Method of order four to find the values of $k_1k_2k_3k_4$ for the initial value problem
$y' = \frac{1}{2}(2x^3+y)$, $y(1)=2$ taking $h=0.1$…….5m
Answer:

$$here\ f(x,y)=\frac{1}{2}\left(2x^3+y\right),\quad h=0.1,\quad x_1=1,\quad y_1=2$$

$$K_1=hf(x_1,y_1)=0.1\left(\frac{1}{2}\left(2\left(1^3+2\right)\right)\right)$$

$$K_1=hf(x_1,y_1)=0.1\left(\frac{6}{2}\right)=\frac{0.6}{2}=0.3$$

$$K_2=hf\left(x_1+\frac{h}{2},y_1+\frac{k_1}{2}\right)=0.1\left(1+\frac{0.1}{2},2+\frac{0.3}{2}\right)$$

$$K_2=hf\left(x_1+\frac{h}{2},y_1+\frac{k_1}{2}\right)=0.1(1.05,2+0.15)$$

$$K_2=hf\left(x_1+\frac{h}{2},y_1+\frac{k_1}{2}\right)=0.1(1.05,2.15)=0.1(1.05+2.15)=0.32$$

$$K_3=hf\left(x_1+\frac{h}{2},y_1+\frac{k_2}{2}\right)=0.1\left(1+\frac{0.1}{2},2+\frac{0.32}{2}\right)$$

$$K_3=hf\left(x_1+\frac{h}{2},y_1+\frac{k_2}{2}\right)=0.1(1.05,2+0.16)$$

$$K_3=hf\left(x_1+\frac{h}{2},y_1+\frac{k_2}{2}\right)=0.1(1.05,2.16)=0.1(1.05+2.16)=0.321$$

$$K_4=hf(x_1+h,y_1+k_3)=0.1(1+0.1,2+0.321)$$

$$K_4=hf(x_1+h,y_1+k_3)=0.1(1.1,2.321)=0.1(0.1+2.21575)=0.3421$$

Now,

$$y_2=y_1+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

$$y_2=2+\frac{1}{6}(0.3+2(0.32)+2(0.321)+0.3421)$$

$$y_2=2+\frac{1}{6}(0.3+0.64+0.642+0.3421)=2+\frac{1.9241}{6}=2+0.320683=2.320683$$

2012 Paper2

Q1. Obtain numerically the solution of $y^{/}=x+y^2, y(0)=2$ Using Euler`s method to find y at x=1, h=1….2m

Answer:

$$h=1, x=1 and\ y(0)=2$$

$$y=y_0+hf(x_0,y_0)$$

$$y=2+1(1+2)$$

$$y=2+3=5$$

Q2. If f(0)=3 and f(1)=9, then the next approximate value of the function using secant method….2m

Answer:

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(0)9 - (1)3}{9 - 3}$$

$$x_2 = \frac{0 - 3}{9 - 3} = \frac{-3}{6} = -0.5$$

Q3. Write a backward difference formula of $D^2f(x)$….2m

Answer:

$$y_n'' = D^2 y_n = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right)$$

Q4. Write a formula for Simpson's 1/3 rule……2m

Answer:

$$\int_{x_0}^{x_{2N}} f(x)dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots + y_{2N-1}) + 2(y_2 + y_4 + \cdots + y_{2N-2}) + y_{2N}] + \text{Error term}$$

Q5.

$Evaluate\ the\ \text{integral}$

$$\int_3^5 (\log x + x)dx$$

…..3m

$$using\ Simpson's\ \frac{3}{8}\ rule$$

$$take\ h = 1$$

Answer:

X	3	4	5
Y=(log x+ x)	3.4771	4.6020	5.6989

$$\int_3^5 (\log x + x)dx = \frac{3}{8}h\left(y_0 + y_2 + 3y_1\right)$$

$$\int_3^5 (\log x + x)dx = \frac{3}{8}(1)\left(3.4771 + 5.6989 + 3(4.6020)\right)$$

$$\int_3^5 (\log x + x)dx = \frac{3}{8}\left(9.176 + 13.806\right)$$

$$\int_3^5 (\log x + x)dx = \frac{3}{8}(22.982) = \frac{68.946}{8} = 8.61825$$

Q6. Write the two steps of solving the linear equations using Gaussian Elimination method….3m

Answer:

In this method, the solution to the system of equations is obtained in two stages.

I) the given system of equations is reduced to an equivalent upper triangular form using elementary transformations

ii) The upper triangular system is solved using back substitution procedure

Q.7

Use Runge – Kutta Method of order four to find the values of

k_1 *and* k_2 *for the* int *ial value problem* ….3m

$y' = yx^2, y(0) = 1, h = 0.1$

Answer:

$f(x,y) = yx^2, y(0) = 1 \text{ and } h = 0.1, \quad x_0 = 0$

$k_1 = hf(x_0, y_0) = 0.1\left(1\left(0^2\right)\right) = 0$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.1\left(0 + \frac{0.1}{2}, 1 + \frac{0}{2}\right) = 0.1(0.05, 1) = 0.1(0.05 + 1) = 0.105$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2)$

$y_1 = 1 + \frac{1}{6}(0 + 2(0.105)) = 1 + \frac{0.21}{6} = 1.035$

Q.8

Evaluate the int *egral*

$\int_0^3 (x^2 + x)dx$ …..3m

$U \sin g\ T\ (apezoidal)\ rule$

$take\ h = 1$

Asnwer:

x	0	1	2	3
$Y=(x^2 + x)$	0	2	6	12

$$\int_0^3 (x^2+x)dx = \frac{h}{2}\left(y_0 + y_3 + 2(y_1+y_2)\right)$$

$$\int_0^3 (x^2+x)dx = \frac{1}{2}\left(0+12+2(2+6)\right)$$

$$\int_0^3 (x^2+x)dx = \frac{1}{2}(28)$$

$$\int_0^3 (x^2+x)dx = \frac{28}{2} = 14$$

Q.9

Evaluate the integral

$$\int_0^3 (x+1)dx$$ 5m

Using Simpson's $\frac{3}{8}$ *rule*

Answer:

x	0	1	2	3
Y=(x + 1)	1	2	3	4

$$\int_0^3 (x+1)dx = \frac{3}{8}h\left(y_0 + y_3 + 3(y_1+y_2)\right)$$

$$h = \frac{b-a}{3n} = \frac{3-0}{3(3)} = \frac{3}{9} = 0.3$$

$$\int_0^3 (x^2+x)dx = \frac{3}{8}(0.3)\left(1+4+3(2+3)\right)$$

$$\int_0^3 (x^2+x)dx = \frac{0.9}{8}(5+15)$$

$$\int_0^3 (x^2+x)dx = \frac{0.9}{8}(20) = \frac{18}{8} = 2.25$$

Q10. Find the dominant Eigen-value and the corresponding eigenvector of the matrix

$$\begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix}$$ *by Power Method with vector with vector* $v^0 = (1,1,1)^T$ *as the initial vector*

....5 m

Answer:

vector $v^0 = (1,1,1)^T$ *as the initial vector. Then, compute first iteration*

$$u^1 = [A]v^0 \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 8+1+2 \\ 0+10-1 \\ 6+2+15 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ 23 \end{pmatrix}$$

$$u^1 = 23 \begin{pmatrix} \frac{11}{23} \\ \frac{9}{23} \\ 1 \end{pmatrix} = q_1 v^1$$

sec*ond iteration*

$$u^2 = [A]v^1 \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{pmatrix} \frac{11}{23} \\ \frac{9}{23} \\ 1 \end{pmatrix} = \begin{bmatrix} \frac{88}{23} + \frac{9}{23} + 2 = \frac{143}{23} \\ 0 + \frac{90}{23} - 1 = \frac{67}{23} \\ \frac{66}{23} + \frac{18}{23} + 15 = \frac{429}{23} \end{bmatrix}$$

$$u^2 = 23 \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = q_2 v^2$$

third iteration

$$u^3 = [A]v^2 \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = \begin{bmatrix} 8+1+2 \\ 0+10-1 \\ 6+2+15 \end{bmatrix} \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = \begin{bmatrix} 1573 \\ 603 \\ 9867 \end{bmatrix}$$

$$u^3 = 9867 \begin{bmatrix} \frac{1573}{9867} = 0.0159 \\ \frac{603}{9867} = 0.0611 \\ 1.0 \end{bmatrix} = 9867 \begin{bmatrix} 0.0159 \\ 0.0611 \\ 1.0 \end{bmatrix} q_3 v^2$$

The l arg *est eign – value and the corresponding eignvector as accurate to two decimals are*

$$\lambda = 9867 \; and \; (X) = \begin{bmatrix} 0.01 \\ 0.06 \\ 1.0 \end{bmatrix}$$

Q11. From the following table of values, construct backward difference table….5m

X	**0.1**	**0.2**	**0.3**	**0.4**
F(x)	**1.10517**	**1.22140**	**1.34986**	**1.49182**

Answer:

x	F(x)	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
0.1	1.10517			
0.2	1.22140	0.11623		
0.3	1.34986	0.12846	0.01223	
0.4	1.49182	0.14196	0.0135	0.0012 7

Q12. Evaluate the integral

$\int_0^4 (x^2+1)dx \; U\,\text{si}\,ng\; Simpson's \frac{1}{3} rule$

$take\ h = 1$

….5m

Answer:

x	0	1	2	3	4
$Y=(x^2+1)$	1	2	5	10	17

$$\int_0^4 (x^2+1)dx = \frac{h}{3}\left(y_0 + y_4 + 4(y_1 + y_3) + 2\left(y_2\right)\right)$$

$$\int_0^4 (x^2+1)dx = \frac{1}{3}\left(1 + 17 + 4(2+10) + 2\left(5\right)\right)$$

$$\int_0^4 (x^2+1)dx = \frac{1}{3}\left(18 + 48 + 10\right)$$

$$\int_0^4 (x^2+1)dx = \frac{1}{3}\left(76\right) = \frac{76}{3} = 25.33$$

2 Marks Questions:

Q. Evaluate the integral

$\int_0^{\pi/2} \cos 2x dx$ **Using Simpson's 1/3 rule. Where** $h=\pi/4$

Answer:

x	0	1	2	3	4
Y= cos 2x	0	0.999 3	0.9975	0.9945	0.9902

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{h}{3}\left(y_0 + y_4 + 4(y_1 + y_3) + 2\left(y_2\right)\right)$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{\frac{\pi}{4}}{3}\left(0 + 0.9902 + 4(0.9993 + 0.9945) + 2\left(0.9975\right)\right)$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{3.14}{12}\left(0.9902 + 7.9752 + 1.995\right)$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{3.14}{12}\left(10.9604\right) = \frac{34.415656}{12} = 2.8679$$

Q. Difference between Jacobi's Method and Gauss Seidel Method.

Answer:

The **Jacobi method** is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal. Each diagonal element is solved for, and an approximate value plugged in. The process is then iterated until it converges. This algorithm is a stripped-down version of the Jacobi transformation method of matrix diagonalization.

The Gauss-Seidel Method

With the Jacobi method, the values of obtained in the th iteration remain unchanged until the entire th iteration has been calculated. With the Gauss-Seidel method, we use the new values as soon as they are known. For example, once we have computed from the first equation, its value is then used in the second equation to obtain the new and so on.

Q. if $y=t+2y$, then find next two derivation in term of t and/or y.

Answer:

w.r.t t 2nd dravative t" = 1 3rd dravative t"' = 0
w.r.t y. 2nd dravative Y" = 2 3rd dravative Y"' = 0

3 Marks Questions:

Q. Evaluate the integral

$\int_{\pi/2}^{\pi} \sin x dx$ **Using Simpson's 3/8 rule. Where** $h = \pi / 4$

Answer:

X	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Y=sinx	1.0	0.7071	0.0

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3h}{8}\left(y_0 + y_2 + 3\left(y_1\right)\right)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3\left(\frac{\pi}{4}\right)}{8}\left(1.0 + 0.0 + 3(0.7071)\right)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{\frac{3\pi}{4}}{8}\left(1.0 + 2.1213\right)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{2.355}{8}\left(3.1213\right) = \frac{7.3506615}{8} = 0.9188$$

Q. Use Ruge-Kutta method of order four to find value of k_1 and k_2 for initial value problem. $y' = 1 + xy,\ y(0) = 2,\quad h = 0.2$

Answer:

$here\ f\left(x, y\right) = 1 + xy,\quad h = 0.2,\quad x_0 = 0,\quad y_0 = 2$

$$K_1 = hf\left(x_0, y_0\right) = 0.2\left(1 + \left(0\right)\left(2\right)\right)$$

$$K_1 = hf\left(x_0, y_0\right) = 0.2\left(1 + 0\right) = 0.2\left(1\right) = 0.2$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\left(0 + \frac{0.2}{2}, 2 + \frac{0.2}{2}\right)$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\left(0.1, 2 + 0.1\right)$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\left(0.1, 2.1\right) = 0.2\left(0.1 + 2.1\right) = 0.44$$

Now,

$$y_1 = y_0 + \frac{1}{6}\left(k_1 + 2k_2\right)$$

$$y_1 = 2 + \frac{1}{6}\left(0.2 + 2\left(0.44\right)\right)$$

$$y_1 = 2 + \frac{1}{6}\left(0.2 + 0.88\right) = 2 + \frac{1.08}{6} = 2 + 0.18 = 2.18$$

Q. Find the residuals by relation method

$$6x_1 - 3x_2 + x_3 = 11$$
$$2x_1 + x_2 - 8x_3 = -15$$
$$x_1 - 7x_2 + x_3 = 10$$

With starting vector (0,0,0)

Answer:

Re-arrange the equation:

$6x_1-3x_2+x_3=11$

$X_1-7x_2+x_3=10$

$2x_1+x_2-8x_3=-15$

Thus, we get

$R_1=11-6x_1+3x_2-x_3=11-6(0)+3(0)-(0)=11$

$R_2=10-x_1+7x_2-x_3=10-(0)+7(0)-(0)=10$

$R_3=-15-2x_1-3x_2+8x_3=-15-2(0)-3(0)+8(0)=-15$

The largest residual in magnitude is R_3

$Dx_3=\frac{R_3}{a_{33}}=\frac{-15}{8}=1.87$

Similarly, we find the new residuals of large magnitude and relax it to zero, and so on.

5 Marks Questions:

Q. Evaluate the integral

$$\int_0^3 (x^2+1)dx$$

Using Simpson's 3/8 rule. Where $h=1$

Answer:

x	0	1	2	3
$Y=(x^2+1)$	1	2	5	10

$$\int_0^3 (x^2+1)dx = \frac{3}{8}h\left(y_0 + y_3 + 3(y_1 + y_2)\right)$$

$$\int_0^3 (x^2+1)dx = \frac{3(1)}{8}\left(1+10+3(2+5)\right)$$

$$\int_0^3 (x^2+1)dx = \frac{3}{8}\left(11+21\right)$$

$$\int_0^3 (x^2+1)dx = \frac{3}{8}(32) = \frac{96}{8} = 12$$

Q. Solve by Gauss Seidel iterative method to 3 decimal place up to

Two iterations

$$8x - y - 2 = 8$$
$$x - 7y + 2z = -4$$
$$2x + y + 9z = 12$$

Answer:

The system is diagonally dominant

$$x=\frac{1}{8}[8+y+2z]$$

$$y=-\frac{1}{7}[-4-x-2z]$$

$$z=\frac{1}{9}[12-2x-y]$$

initial approximantion is $(0,0,0)$

first iteration

$$x_1=\frac{1}{8}[8+0+2(0)]=\frac{8}{8}=1$$

$$y_1=-\frac{1}{7}[-4-1-2(0)]=-\frac{-5}{7}=0.71$$

$$z_1=\frac{1}{9}[12-2(1)-0.71]=\frac{9.29}{9}=1.032$$

sec*ond iteration*

$$x_2=\frac{1}{8}[8+0.71+2(1.032)]=\frac{1}{8}[8+0.71+2.064]=\frac{10.774}{8}=1.34675$$

$$y_2=-\frac{1}{7}[-4-1.34675-2(1.032)]=-\frac{1}{7}[-4-1.34675-2.064]=-\frac{-7.41075}{7}=1.0586$$

$$z_2=\frac{1}{9}[12-2(1.34675)-1.0586]=\frac{1}{9}[12-2.6935-1.0586]=\frac{8.2479}{9}=0.91643$$

Q. Evaluate the integral

$\int_0^4 (x^2+x+2)dx$ **Using Simpson's 1/3 rule. Where** $h=1$

Answer:

x	0	1	2	3	4
Y=(x^2 + x+2)	2	4	8	14	22

$$\int_0^4 (x^2+x+2)dx=\frac{h}{3}\left(y_0+y_4+4(y_1+y_3)+2\left(y_2\right)\right)$$

$$\int_0^4 (x^2+x+2)dx=\frac{1}{3}\left(2+22+4(4+14)+2\left(8\right)\right)$$

$$\int_0^4 (x^2+x+2)dx=\frac{1}{3}\left(24+72+16\right)$$

$$\int_0^4 (x^2+x+2)dx=\frac{1}{3}\left(112\right)=\frac{112}{3}=37.33$$

Q. Find the 2nd derivative of f(x) at x=0.3 using three point equation

x	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	0.125	0.352	0.652	0.756	0.812	0.924

Answer:

$$f''(0.3) = \frac{f(x_3) - 2f(x_2) + f(x_1)}{h^2 = 0.01}$$

$$f''(0.3) = \frac{f(0.4) - 2f(0.3) + f(0.2)}{h^2 = 0.01}$$

$$f''(0.3) = \frac{0.756 - 2(0.652) + 0.352}{h^2 = 0.01}$$

$$f''(0.3) = \frac{0.756 - 1.304 + 0.352}{h^2 = 0.01} = -\frac{0.196}{0.01} = -19.6$$

Final 2013:

Q. Find Newton`s forward difference table from the following data

X	0.0	0.1	0.2	0.3	0.4
F(x)	1	0.9048	0.8187	0.7408	0.6703

…5m

Answer:

X	Y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.0	1				
0.1	0.9048	-0.0952			

0.2	0.8187	-0.0861	0.091		
0.3	0.7408	-0.0779	0.082	-0.009	
0.4	0.6703	-0.0705	0.074	-0.008	0.001

$$p=\frac{x-x_0}{h}=\frac{x-0.0}{0.1}=\frac{x}{0.1}-\frac{0.0}{0.1}=\frac{x}{0.1}-0=\frac{x}{0.1}$$

$$y=y_0+p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_1+\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_2$$

$$x=0.0,\ \ y_0=1,\Delta y_0=-0.0952,\Delta^2 y=0.091,\Delta^3 y=-0.009,\Delta^4 y=0.001$$

$$y=1+\left(\frac{x}{0.1}\right)(-0.0952)+\frac{\frac{x}{0.1}\left(\frac{x}{0.1}-1\right)}{2}(0.091)+\frac{\frac{x}{0.1}\left(\frac{x}{0.1}-1\right)\left(\frac{x}{0.1}-2\right)}{6}(-0.009)+\frac{\frac{x}{0.1}\left(\frac{x}{0.1}-1\right)\left(\frac{x}{0.1}-2\right)\left(\frac{x}{0.1}-3\right)}{24}(0.001)$$

this is required Newton`s interpolating polynomial.

Q5.

Evaluate the integral

$$\int_3^5(\log x+2)dx \ \text{using Simpson's}\ \frac{3}{8}\ \text{rule. take}\ h=1$$

…..3m

Answer:

X	3	4	5
Y=(log x+ 2)	2.4771	2.6020	2.6989

$$\int_3^5(\log x+2)dx=\frac{3}{8}h\left(y_0+y_2+3y_1\right)$$

$$\int_3^5(\log x+2)dx=\frac{3}{8}(1)\left(2.4771+2.6989+3(2.6020)\right)$$

$$\int_3^5(\log x+2)dx=\frac{3}{8}\left(5.176+7.806\right)$$

$$\int_3^5(\log x+2)dx=\frac{3}{8}(12.982)=\frac{38.946}{8}=4.86825$$

Q5.

Evaluate the int *egral*

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx \; u\sin g \; Simpson's \frac{3}{8} rule. take \; h=\frac{\pi}{4}$$ 2m

Answer:

X	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
Y=(cos x+ 2)	3	2.7071	2

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx=\frac{3h}{8}\left(y_0+y_2+3y_1\right)$$

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx=\frac{3\left(\frac{\pi}{4}\right)}{8}\left(3+2+3(2.7071)\right)$$

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx=\frac{\frac{3\pi}{4}}{8}\left(5+8.1213\right)$$

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx=\frac{9.42}{2}(13.1213)=\frac{123.602646}{2}=61.801323$$

Q. if F (h) =256.2354 and $F\left(\frac{h}{2}\right)$ =257.1379, then find $F_1\left(\frac{h}{2}\right)$ using Richardson`s extrapolation limit….2m

Answer:

$$F_1(\frac{h}{2})=\frac{4f(\frac{h}{2})-f(\frac{h}{2})}{4^1-1}$$

$$F_1(\frac{h}{2})=\frac{4(257.1379)-(256.2354)}{3}$$

$$F_1(\frac{h}{2})=\frac{1028.5516-256.2354}{3}$$

$$F_1(\frac{h}{2})=\frac{772.3162}{3}=257.43873$$

7. if in solving given differential equation, 0.1, „, h=1, y3=2 then find the y4 by Adam–moulton's predictor formula .

Answer:

$$P: y_{n+1} = y_n + \frac{h}{24}\left(25y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$$

$$P: y_{3+1=4} = y_3 + \frac{1}{24}\left(25y'_3 - 59y'_{3-1=2} + 37y'_{3-2=1} - 9y'_{3-3=0}\right)$$

$$P: y_4 = y_3 + \frac{1}{24}\left(25y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right)$$

$$P: y_4 = 2 + \frac{1}{24}\left(25(1.3) - 59(1.2) + 37(1.1) - 9(0.1)\right)$$

$$P: y_4 = 2 + \frac{1}{24}\left(32.5 - 70.8 + 40.7 - 0.9\right)$$

$$P: y_4 = 2 + \frac{1}{24}(1.5) = 2 + \frac{1.5}{24} = 2 + 0.0625 = 2.0625$$

12. using Adam-moulton's predictor formula find the predicted value of f(0.4) from ordinary differential equation y'=1+2xy; y(0)=0; h=0.1

x	0	0.1	0.2	0.3
y	0	.1007	.2056	.3199

Answer:

$h = 0.1$

$f(x, y) = 1 + 2xy$

$y_0' = 1 + 2x_0y_0 = 1 + 2(0)(0) = 1$
$y_1' = 1 + 2x_1y_1 = 1 + 2(0.1)(0.1007) = 1.02014$
$y_2' = 1 + 2x_2y_2 = 1 + 2(0.2)(0.2056) = 1.08224$
$y_3' = 1 + 2x_3y_3 = 1 + 2(0.3)(0.3199) = 1.19194$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24}(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$
$$y_4 = 0.3199 + \frac{0.1}{24}(55(1.19194) - 59(1.08224) + 37(1.02014) - 9(1))$$
$$y_4 = 0.446773833$$

Computing y'_4 *for the Corrector Formula*;

$$y'_4 = 1 + 2x_4 y_4 = 1 + 2(0.4)(0.446773833)$$
$$y'_4 = 1.3574190664$$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right)$$
$$y_4 = y_3 + \frac{h}{24}\left(9y'_4 + 19y'_3 - 5y'_2 + y'_1\right)$$
$$y_4 = 0.3199 + \frac{0.1}{24}\left(9(1.3574190664) + 19(1.19194) - 5(1.08224) + 1.02014\right)$$

$$y_4 = 0.446869048$$

Q. **using Adam-moulton's predictor formula find the predicted value of f(0.4) from ordinary differential equation y'=1+xy; y(0)=0; h=0.1**

x	0	0.1	0.2	0.3
y	0	.1007	.2056	.3199

Answer:

$h = 0.1$

$f(x, y) = 1 + xy$

$y_0' = 1 + x_0 y_0 = 1 + (0)(0) = 1$

$y_1' = 1 + x_1 y_1 = 1 + (0.1)(0.1007) = 1.20\bullet 7$

$y_2' = 1 + x_2 y_2 = 1 + (0.2)(0.2056) = 1.04112$

$y_3' = 1 + x_3 y_3 = 1 + (0.3)(0.3199) = 1.\bullet 9597$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24}(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$

$$y_4 = 0.3199 + \frac{0.1}{24}(55(1.\bullet 9597) - 59(1.04112) + 37(1.20\bullet 7) - 9(1))$$

$y_4 = 0.3199 + 0.00416(60.27835 - 61.42608 + 44.4259 - 9) = 0.3199 + 34.27817 = 34.59807$

Computing y'_4 *for the Corrector Formula;*

$$y'_4 = 1 + x_4 y_4 = 1 + (0.4)(34.59807)$$
$$y'_4 = 14.839228$$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})$$
$$y_4 = y_3 + \frac{h}{24}(9y'_4 + 19y'_3 - 5y'_2 + y'_1)$$
$$y_4 = 0.3199 + \frac{0.1}{24}(9(14.839228) + 19(1.09597) - 5(1.04112) + 1.2007)$$

$$y_4 = 0.3199 + 0.00416(133.553052 + 20.82343 - 5.2056 + 1.2007)$$

$$y_4 = 0.3199 + 0.625545781 = 0.945445781$$

Q.2. Evaluate the integral $\int_0^{\frac{\pi}{2}} \cos x dx$ **Using trapezoidal rule**

Take h= $\frac{\pi}{4}$**…..2m**

Answer:

X	0	$\frac{\pi}{4}$
Y= cosx	1	0.7071

Using trapezoidal rule, taking h= $\frac{\pi}{4}$

$$I = \frac{h}{2}(y_0 + y_1)$$

$$I = \frac{\frac{\pi}{4}}{2}(1+0.7071)$$

$$I = \frac{3.14}{2}(1.7071)$$

$$I = \frac{5.360294}{2} = 2.680147$$

Q. obtain numerically the solution of

$y' = x + y^2, y(0) = 2 \, u \sin g \; Euler`s \, method \, to \, find \; y \, at \, x = 1, h = 1.$

Answer:

Here: y(0)=2, x=1, h=1

$$y = y_0 + hf(x_0, y_0)$$

$$y = 2 + 1(1+4) = 2 + 5 = 7$$

Q5.

$$Evaluate \, the \int egral \int_{\frac{\pi}{2}}^{\pi} \sin x dx \, u \sin g \; Simpson's \frac{3}{8} rule. take \, h = \frac{\pi}{4}$$

…..2m

Answer:

X	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Y=sinx	1	0.7071	0

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3h}{8}(y_0 + y_2 + 3y_1)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3\left(\frac{\pi}{4}\right)}{8}(1+0+3(0.7071))$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{\frac{3\pi}{4}}{8}(1+2.1213)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{9.42}{2}(3.1213) = \frac{29.402646}{2} = 14.701323$$

Q. Prove that $E^{\frac{1}{2}} = \mu + \frac{\delta}{2}$

Answer:

$$E^{\frac{1}{2}} = \mu + \frac{\delta}{2}$$

$$E^{\frac{1}{2}} = \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) + \frac{E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2}$$

$$E^{\frac{1}{2}} = \frac{E^{\frac{1}{2}} + E^{-\frac{1}{2}}}{2} + \frac{E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2}$$

$$E^{\frac{1}{2}} = \frac{E^{\frac{1}{2}} + E^{-\frac{1}{2}} + E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2} = \frac{2E^{\frac{1}{2}}}{2} = E^{\frac{1}{2}}$$

Q.2. Evaluate the integral $\int_0^2 \left(x^2 - 3x + 4\right) dx$ Using trapezoidal rule

Take h=1…..2m

Answer:

X	0	1	2
Y= (x^2-3x+4)	4	2	2

Using trapezoidal rule, taking h=1

$$I = \frac{h}{2}\left(y_0 + y_2 + 2\left(y_1\right)\right)$$

$$I = \frac{1}{2}\left(4 + 2 + 2\left(2\right)\right)$$

$$I = \frac{1}{2}\left(6 + 4\right)$$

$$I = \frac{10}{2} = 5$$

Q12. Evaluate the integral

$\int_0^4 (x^2 + 2) dx \, Using \, Simpson's \, \frac{1}{3} \, rule$ ….5m

$take \; h = 1$

Answer:

x	0	1	2	3	4
Y=(x^2 + 2)	2	3	6	11	18

$$\int_0^4 (x^2+2)dx = \frac{h}{3}\left(y_0 + y_4 + 4(y_1 + y_3) + 2\left(y_2\right)\right)$$

$$\int_0^4 (x^2+2)dx = \frac{1}{3}\left(2 + 18 + 4(3+11) + 2\left(6\right)\right)$$

$$\int_0^4 (x^2+2)dx = \frac{1}{3}\left(20 + 56 + 12\right)$$

$$\int_0^4 (x^2+2)dx = \frac{1}{3}\left(88\right) = \frac{88}{3} = 29.33$$

Q. Evaluate the integral $\int_0^3 xdx$ Using Simpson's 3/8 rule. Take h=1 (5)

Answer:

x	0	1	2	3
Y=x	0	1	2	3

$$\int_0^3 xdx = \frac{3h}{8}\left(y_0 + y_3 + 3(y_1 + y_2)\right)$$

$$\int_0^3 xdx = \frac{3}{8}\left(0 + 3 + 3\left(1+2\right)\right)$$

$$\int_0^3 xdx = \frac{3}{8}\left(3+9\right)$$

$$\int_0^3 xdx = \frac{3}{8}\left(12\right) = \frac{36}{8} = 4.5$$

Final 2014

1- **Write three equations for the following system of linear equation for Gauss Seidal Approximation Method (2)**

83x+11y-4z=95
7x+52y++13z=104
3x+8y+29z=71

2- **Evaluate the integral**

$$\int_3^5 (\log x + 2)dx$$

Using trapezoidal rule.
Take h=1 (2)
Answer:

X	3	4	5
Y=(log x+2)	2.4771	2.6020	2.6989

$$\int_3^5 (\log x+2)dx = \frac{h}{2}\left(y_0 + y_2 + 2\left(y_1\right)\right)$$

$$\int_3^5 (\log x+2)dx = \frac{1}{2}\left(2.4771 + 2.6989 + 2\left(2.6020\right)\right)$$

$$\int_3^5 (\log x+2)dx = \frac{1}{2}\left(2.4771 + 2.6989 + 5.204\right)$$

$$\int_3^5 (\log x+2)dx = \frac{1}{2}\left(10.38\right) = \frac{10.38}{2} = 5.19$$

3- Write a formula for finding the value of k_2 in fourth order R-K mrthod?(2)

Answer:

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

4- If $y'=t^2-y$, then find the next two derivatives in terms of t and or/y. (2)

Answer:

w.r.t t 2nd dravative t" = 2t 3rd dravative t"' = 2
w.r.t y. 2nd dravative Y" = 1 3rd dravative Y"' = 0

5- Define intermediate value property? (3)

Answer:

If f(x) is a real valued continuous function in the closed interval a≤ x≤ b if f(a) and f(b) have opposite signs once; that is f(x)=0 has at least one root β such that a≤ b≤ β

Simply

If f(x) = 0 is a polynomial equation and if f(a) and f(b) are of different signs ,then f(x)=0 must have at least one real root between a and b.

Numerical methods for solving either algebraic or transcendental equation are classified into two groups.

6- **If in solving a given differential equation,**

$y_0 = 1, y'_1 = 1.1, y' = 1.2, y'_3 = 1.3, h = 1$ **Then find y_4 by Milne's Predictor formula. (3)**

Answer:

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

$$P: y_4 = y_0 + \frac{4h}{3}\left(2y'_1 - y'_2 + 2y'_3\right)$$

$$P: y_4 = 1 + \frac{4(1)}{3}\left(2(1.1) - 1.2 + 2(1.3)\right)$$

$$P: y_4 = 1 + \frac{4}{3}\left(2.2 - 1.2 + 2.6\right)$$

$$P: y_4 = 1 + \frac{4}{3}\left(3.6\right)$$

$$P: y_4 = 1 + \frac{14.4}{3}$$

$$P: y_4 = \frac{3 + 14.4}{3} = \frac{17.4}{3} = 5.8$$

7- **Evaluate the integral**

$$\int_0^4 x^2 dx$$

Using trapezoidal rule. Take h=1. (3)
Answer:

X	0	1	2	3	4
$Y=x^2$	0	1	4	9	16

$$\int_3^5 (\log x + 2)dx = \frac{h}{2}\left(y_0 + y_4 + 2\left(y_1 + y_2 + y_3\right)\right)$$

$$\int_3^5 (\log x + 2)dx = \frac{1}{2}\left(0 + 16 + 2\left(1 + 4 + 9\right)\right)$$

$$\int_3^5 (\log x + 2)dx = \frac{1}{2}\left(0 + 16 + 28\right)$$

$$\int_3^5 (\log x + 2)dx = \frac{1}{2}\left(44\right) = \frac{44}{2} = 22$$

8- **Obtain numerically the solution of**

$Y'=x^2+2x+y^2$, $y(6)=1$
Using Euler's method to find y at x=1, h=1. (3)
Answer:

$y(6)=1, x_6=1, h=1$

$y_7 = y_6 + hf(x_6, y_6)$

$y_7 = 1+1(x^2+2x+y^2)$

$y_7 = 1+1(1+2+1) = 1+4=5$

9- Using predictor formula find the predicted value of y(0,4) from ordinary differential equation

Y'=x+y; y(0)=1, h=0.1
With the help of the following table (5)

X	0	0.1	0.2	0.3
Y	1	0.9097	0.8375	0.7816

Answer:

Here,

$x_0 = 0 \quad , \quad x_1 = 0.1 \quad , \quad x_2 = 0.2 \quad , \quad x_3 = 0.3 \quad , \quad x_4 = 0.4$

$y_1' = x_1 - y_1 = 0.1 - 0.9097 = -0.8097$
$y_2' = x_2 - y_2 = 0.2 - 0.8375 = -0.6375$
$y_3' = x_3 - y_3 = 0.3 - 0.7816 = -0.4816$

Now, using Predictor Formula

$$y_4 = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$y_4 = 1 + \frac{4*0.1}{3}(-1.9451)$$

$$y_4 = 0.740653333$$

Using the predicted value, we shall now compute the corrected value as;

$$y_4 = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$$

Now,

$$y_4' = x_4 - y_4 = 0.4 - 0.740653333 = -0.340653333$$

10- Use the regula-falsi to solve the equation :

$F(x)=e^x-3x=0$. (Note only 1 iteration is required upto 4 decimal places is required) (5)
Answer:

$$f(x) = e^x - 3x = 0$$

$$f(1) = e^1 - 3(1) = 2.7182 - 3 = -0.2818$$

$$f(2) = e^2 - 3(2) = 7.3890 - 6 = 1.389$$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$x_3 = 2 - \frac{2-1}{1.389 - (-0.2818)}(1.389)$$

$$x_3 = 2 - \frac{1.389}{1.6708} = 2 - 0.8313 = 1.1687$$

11- Evaluate the integral

$\int_0^4 xdx$ **Using Simpson's 1/3 rule (5)**

Answer:

x	0	0.5	1	1.5	2	2.5	3	3.5	4
Y=x	0	0.5	1	1.5	2	2.5	3	3.5	4

$$\int_0^4 xdx = \frac{h}{3}\left(y_0 + y_8 + 4(y_1 + y_3 + y_5 + y_7) + 2\left(y_2 + y_4 + y_6\right)\right)$$

$$h = \frac{b-a}{2n} = \frac{4-0}{2(4)} = \frac{4}{8} = 0.5$$

$$\int_0^4 xdx = \frac{0.5}{3}\left(0 + 4 + 4(0.5 + 1.5 + 2.5 + 3.5) + 2\left(1 + 2 + 3\right)\right)$$

$$\int_0^4 xdx = \frac{0.5}{3}\left(4 + 32 + 12\right)$$

$$\int_0^4 xdx = \frac{0.5}{3}\left(48\right) = \frac{24}{3} = 8$$

12- Find y(1) for the following initial value problem y y'=x, y(0)=2

Using simple Euler's method, taking h=1 (5)

Answer:

$$y(0) = 2, x_0 = 0, h = 1$$

$$y_1 = y_0 + hf\left(x_0, y_0\right)$$

$$y_1 = 2 + 1\left(0 + 2\right)$$

$$y_1 = 2 + 2 = 4$$

what is Simpson's 3/8th rule.

Answer: The general formula for Simpson's 3/8th rule is 3h/8[f0+3(f1+f2)+2f3+(3f4+f5)+2f6+…+3(fn-2+fn-1)+fn] Now if we have to calculate the integral by using this rule then we can simply proceed just write first and last vale and distribute all the remaining values with prefix 3 and 2 Like you have f0,f1,f2,f3,f4 Then the integral can be calculated as 3h/8[f0+f4+3(f1+f2)+2f4] If we have values like f0,f1,f2,f3,f4,f5 Then integral can be calculated as 3h/8[f0+3(f1+f2)+2f3+3f4+f5] Similarly proceeding in this fashion we can calculate the integral in this fashion

Question: what is classic runge-kutta method

Answer: The fourth order Runge-Kutta method is known as the classic formula of classic Runge-Kutta method. yn+1=yn+1/6(k1+2k2+2k3+k4) Where k1=hf(xn,yn) k2=hf(xn+h/2+yn+k1/2) k3=hf(xn+h/2,yn+k'2/2) k4=hf(xn+h,yn+k3)

Question: What is meant by TOL?

Answer: The TOL means the extent of accuracy which is needed for the solution. If you need the accuracy to two places of decimal then the TOL will be 10-2 .similarly the 10-3 means that the accuracy needed to three places of decimal. Suppose you have the root from last iteration 0.8324516 and

0.8324525 if we subtract both and consider absolute value of the difference 0.0000009 now it can be written as 0.09*10-5 so the TOL in this case is 10-5.similarly if we have been provided that you have to for the TOL 10-2 you will check in the same way. In the given equation you will solve the equation by any method and will consider some specific TOL and try to go to that TOL. Some time no TOL is provided and you are asked to perform to some specific no of iterations.

Question: what is meant by uniqueness of LU method.

Answer: An invertible (whose inverse exists) matrix can have LU factorization if and only if all its principal minors(the determinant of a smaller matrix in a matrix) are non zero .The factorization is unique if we require that the diagonal of L or U must have 1's.the matrix has a unique LDU factorization under these condition . If the matrix is singular (inverse does not exists) then an LU factorization may still exist, a square matrix of rank (the rank of a matrix in a field is the maximal no of rows or column) k has an LU factorization if the first k principal minors are non zero. These are the conditions for the uniqueness of the LU decomposition.

Question: how the valu of h is calculated from equally spaced data.

Answer: Consider the following data x y 1 1.6543 2 1.6984 3 2.4546 4 2.9732 5 3.2564 6 3.8765 Here for h=2-1=3-2=1 x y 0.1 1.6543 0.2 1.6984 0.3 2.4546 0.4 2.9732 0.5 3.2564 0.6 3.8765 Here for the calculation of h =0.2-0.1=0.3-0.2=0.1 I think so that you may be able to understand .

what is the relation ship between p=0 and non zero p in interpolation.

Answer: fp=fo+pDfo+1/2(p2-p)D2fo+1/6(p3-3p2+2p)D3fo+1/24(p4-6p3+11p2-6p)D4fo+… This is the interpolation formula For the derivative you will have to take the derivative of this formula w.r.t p and you will get f'p=1/h{Dfo+1/2(2p-1)D2fo+1/6(3p2-6p+2)D3fo+…} Now put here p=0 f'p=1/h{Dfo+1/2(0-1)D2fo+1/6(0-0+2)D3fo+…} So the formula becomes f'p=1/h{Dfo+D2fo/2+D3fo/3+…} so this is the relation ship between non zero and zero p no when you have to calculate p the you use formula p=x-xo/h so this is the impact of x.

Question: What is chopping and Rounding off?

Answer: Chopping and rounding are two different techniques used to truncate the terms needed according to your accuracy needs. In chopping you simply use the mentioned number of digits after the decimal and discard all the remaining terms. Explanation (1/3 - 3/11) + 3/20=(0.333333…-0.27272727..)+0.15=(0.333-0.272)+0.15 This is the three digit chopping.

Question: When the forward and backward interpolation formulae are used?

Answer: In interpolation if we have at the start then we use the forward difference formula and the formula to calculate p is x-x0/h. If the value of x lies at the end then we use Newton's backward formula and formula to calculate the

value of p is x-xn/h. Now I come to your question as in this case the value lies at the end so 6 will be used as the xn. This procedure has been followed by the teacher in the lectures. But some authors also use another technique that is if you calculate the value of p and that is negative then the origin is shifted to that value for which the value of p becomes positive. And then according to that origin the values of differences are used and you need not follow this procedure.

Question: What is forward and backward difference operator and the construction of their table.

Answer: For forward Dfr =fr+1 –fr Df0 = f1-f0 In terms of y Dyr+1=yr+1-yr D stands for the forward difference operator For backward Dfr =fr –fr-1 Df1 = f1-f0 In terms of y Dy1=y1-y0 Here D stands for backwards operator Now the construction of the difference table is based on X Y 1st forward 2nd forward 3rd forward x1 Y1 Y2-Y1=Dy0 x2 Y2 Y3-Y2=Dy1 x3 Y3 Y4-Y3=Dy2 x4 Y4 Now consider the construction of table for the backward table X Y 1st forward 2nd forward 3rd forward x1 Y1 Y2-Y1=Dy1 x2 Y2 Y3-Y2=Dy2 x3 Y3 Y4-Y3=Dy3 x4 Y4 Dear student this is the main difference in the construction of the forward and backwards difference table when you proceed for forward difference table you get in first difference the value Dy0 but in the construction of backwards difference table in the first difference you get Dy1 and in the second difference in the forward difference table you get D2 y0 and in the backward difference table the first value in the second difference is D2 y1. I think so you have made it clear.

Question: What is Jacobi's method?

Answer: Jacobi's Method It is an iterative method and in this method we first of all check either the system is diagonally dominant and, if the system is diagonally dominant then we will calculate the value of first variable from first equation in the form of other variables and from the second equation the value of second variable in the form of other variables and so on. We are provided with the initial approximations and these approximations are used in the first iteration to calculate first approximation of all the variables. The approximations calculated in the first iteration are used in the second iteration to calculate the second approximations and so on.

What is the condition that a root will lie in an interval.

Answer: Suppose that you have a function f(x) and an interval [a,b]and you calculate both f(a)and f(b)if f(a)f(b)<0 then there must exist a root for this function in this interval.In simple words f(a) and f(b)must have opposite signs.

Question: How the divided difference table is constructed?

Answer: x y First difference Second difference Third difference 1 0.2 y0 y1-y0/x1-x0=sy0 sy1- sy0/x2-x0=s2y0 s2y0-s2y1/x3-x0=s3y0 s3y1-s3y0/x4x0 =s4y0 2 0.69 y1 y2-y1/x2-x1=sy1 sy2- sy1/x3-x1=s2y1 s2y2-s2y1/x4-x1=s3y1 3 0.985 y2 y3-y2/x3-x2=sy2 sy3- sy2/x4-x1=s2y2 4 1.2365 y3 y4-y3/x4-

x3=sy3 5 2.3651 y4 This is the complete table showing the central differences . Here s stands for difference

Question: What is Gauss-seidel Method

Answer: It is also an iterative method and in this we also check either the system is diagonally dominant and, if so then we proceed on the same lines as in the Jacobi's method. The difference in both is when we calculate a value of any variable that is instantly replaced by the previous value. If we have initial values like x1=0,x2=0 and x3=0 in the case of three variables in the first iteration we will use all the three values to calculate x1 but to calculate the value of x2 we will use the recent value of x1 which is calculated in the first iteration.

Question: What is partial and full pivoting?

Answer: Partial and full pivoting, In gauss elimination method when you have any of diagonal element aii zero it means the solution does not exist to avoid this we change the equation so that a non zero pivot is achieved .Now you have an argument matrix in which he elements in the first column are 1,3,4 respectively here in case of partial pivoting we will replace first element with the last element it is done by replacing the first equation with last equation it is known as partial pivoting. In full pivoting we change rows and columns but that is not implemented manually it is used in computers.

Question: How the initial vector is choose in power method?

Answer: Choose the initial vector such that the largest element is unity. But no problem with this you will precede with this vector as it is and will not change it. In the given question the initial vector is provided so you have to proceed with the given vector and will normalize this vector and normalizing this vector you will keep in mind that the greatest value should be unity. An eigenvector Vis said to be normalized if the coordinate of largest magnitude is equal to unity Actually this condition is for the normalization of the vectors when you have to normalize the vector you keep in mind that the larget entery in the vector must be 1 and so you take the largest element and divide the remaining values by the greatest value

What is meant by diagonally dominant system?

Answer: A system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 is said to be diagonally dominant if the following condition holds. |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3| => (greater or equal) |a3|+|b3|

Question: State the sufficient condition for the convergence of the system of equation by iterative methods.

Answer: A sufficient condition for convergence of iterative solution to exact solution is |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3|

=> (greater or equal) |a3|+|b3| For the system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 Similarly for the system with more variables we can also construct the same condition

Question: The calculation for numerical analysis should be done in degree or radians.

Answer: All the calculation for numerical analysis should be done in radians not in degrees set your calculator in radians mode and suppose the value of pi=3.14.

Question: How we can identify that Newton forward or backwards interpolation formula is to be used.

Answer: If the value at which we have to interpolate is in the start of the table then we will use Newton's forward interpolation formula if it is at the end of the table then we will use the Newton's backward interpolation formula.

Question: What is meant precision and accuracy?

Answer: Precision and accuracy are two terms which are used in numerical calculations by precision we mean that how the values in different iterations agree to each other or how close are the different values in successive iterations. For example you have performed 3 different iterations and result of all the iteration are 1.32514,1.32516,31.32518 these three values are very precise as these values agree with each other . Accuracy means the closeness to the actual value. Suppose that you have calculated an answer after some iteration and the answer is 2.718245 and the actual answer is 2.718254 and the answer calculated is very accurate but if this answer is 2.72125 then it is not accurate.

What is meant by iterative methods?

Answer: The methods which need one or more iterations are known as iterative methods like bisection method, Newton raphson method, and many other methods.

Question: What is graphically meant by the root of the equation?

Answer: If the graph of a function f(x) =0 cuts the x-axis at a point a then a is known as the root of the equation.

Question: Q. What is the difference between open and bracketing method?

Answer: In open methods we need only one initial approximation of the root that may be any where lying and if it is not very close then we have to perform more iteration and the example of open method is Newton raphson method. In bracketing method we bracket the root and find that interval in which root

lies means we need two initial approximations for the root finding. Bisection method is an example of the bracketing method.

Question: Condition for the existence of solution of the system of equations.

Answer: If the |A| is not equal to zero then the system will have a unique solution if |A|=0 then the system will have no solution

Question: Should the system be diagonally dominant for gauss elimination method?

Answer: The system of equation need not to be diagonally dominant for Gauss elimination method and gauss Jordan method for both the direct method it is not necessary for the system to be diagonally dominant .it should be diagonally dominant for iterative methods like jacobie and gauss seidel method.

What is a trance dental equation?

Answer: An equation is said to be a transcendental equation if it has trigonometric, exponential and logarithmic function or combination of all these functions.

Question: How the value of h is calculated in interpolation?

Answer: There are two types of data in the interpolation one is equally spaced and other is unequally spaced in equally spaced data we need to calculate the value of h that is calculated by subtracting any two consecutive values and taking their absolute value.

Question: What is an algebraic equation?

Answer: An algebraic equation is an equation which is purely polynomial in any variable. Supposed x2+3x+2=0, x4+3x2=0, y3+6y2=0 all are algebraic equations as these are purely polynomial in x and y variable.

Question: What is Descartes rule of signs?

Answer: The number of positive roots of an algebraic equation f(x) =0 can not exceed the no of changes in signs. Similarly the no of negative roots of negative roots of and algebraic equation can not exceed the no of changes in sign of equation f (-x) =0.

Question: What are direct methods?

Answer: The numerical methods which need no information about the initial approximation are known as direct methods like Graffee's root squaring method.

What is direct methods of solving equations?

Answer: Those methods which do not require any information about the initial approximation of root to start the solution are known as direct methods. The examples of direct methods are Graefee root squaring method, Gauss elimination method and Gauss Jordan method.All these methods do not require any type of initial approximation.

Question: What is Iterative method of solving equations?

Answer: These methods require an initial approximation to start. Bisection method, Newton raphson method, secant method, jacobi method are all examples of iterative methods

Question: If an equation is a transcendental, then in which mode the calculations should be done?

Answer: All the calculations in the transcendental equations should be done in the radian mode.

Question: What is the convergence criterion in method of iteration?

Answer: If be a root of f(x) =0 which is equivalent to I be any interval containing the point x= and will converge to the root provided that the initial approximation is chosen in I

Question: When we stop doing iterations when Toll is given?

Answer: Here if TOL is given then we can simply find the value of TOL by subtracting both the consecutive roots and write it in the exponential notation if the required TOL is obtained then we stop.

What is inherent error and give its example.

Answer: It is that quantity of error which is present in the statement of the problem itself, before finding its solution. It arises due to the simplified assumptions made in the mathematical modeling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the problem. e.g. if writing 8375 instead of 8379 in a statement lies in the category of inherent error.

Question: What is local round-off error?

Answer: At the end of computation of a particular problem, the final results in the computer, which is obviously in binary form, should be converted into decimal form-a form understandable to the user-before their print out. Therefore, an additional error is committed at this stage too. This error is

called local round-off error.

Question: What is meant by local truncation error?

Answer: Retaining the first few terms of the series up to some fixed terms and truncating the remaining terms arise to local truncation error.

Question: What is transcendental equation and give two examples.

Answer: An equation is said to be transcendental equation if it has logarithmic, trigonometric and exponential function or combination of all these three. For example it is a transcendental equation as it has an exponential function These all are the examples of transcendental equation.

Question: What is meant by intermediate value property?

Answer: If f(x) is a real valued continuous function in the closed interval if f(a) and f(b) have opposite signs once; that is f(x)=0 has at least one root such that Simply If f(x)=0 is a polynomial equation and if f(a) and f(b) are of different signs ,then f(x)=0 must have at least one real root between a and b.

What is the order of global error in Simpson's 3/8 rule?

Answer: The global error in Simpson's 3/8 rule is of the order of 0(h4).

Question: Which equation models the rate of change of any quantity with respect to another?

Answer: An ordinary differential equation models the rate of change of any quantity with respect to another.

Question: By employing which formula, Adam-Moulton P-C method is derived?

Answer: Adam-Moulton P-C method is derived by employing Newton's backward difference interpolation formula.

Question: What are the commonly used number systems in computers?

Answer: Binary Octal Decimal Hexadecimal

Question: If a system has the base M, then how many different symbols are needed to represent an arbitrary number? Also name those symbols.

Answer: M 0, 1, 2, 3, … , M-1

Is the order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule?

Answer: Yes. The order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule.

Question: What is the order of global error in Trapezoidal rule?
Answer: The global error in Trapezoidal rule is of the order of 0(h2).

Question: What is the formula for finding the width of the interval?
Answer: Width of the interval, h, is found by the formula h=(b-a)/n

Question: What type of region does the double integration give?
Answer: Double integration gives the area of the rectangular region.

Question: Compare the accuracy of Romberg's integration method to trapezoidal and Simpson's rule.
Answer: Romberg's integration method is more accurate to trapezoidal and Simpson's rule

Important Topics

Newton's Forward Diffrence & Backward Difference (Interpolation)
Newton's Forward Diffrence & Backward Difference (Differentiation)
Trapezoidal Rule (***Very Important***)
Simpson's 1/3 Rule
Simpson's 3/8 Rule
Euler's Method (**Important**)
Two Point Formula for Differentiation (**Important**)
Three Point Formula for Integration (**Important**)
Runge-Kutta Second Order Method (**Important**)
Runge-Kutta Fourth Order Method (***Very Important***)
Adam-Moulton Method {Predictor, Corrector and Error} (**Important**)
Milne Method {Predictor, Corrector and Error}

mcqs was mostly from forward backward interpolating and runge kutta and lagrange 1/3 simpson and trapezoidal rule

2 marks question : adam moultan predictor corrector formula

2 marks question : milne predictor corrector formula

3 marks question : formula of k3 of runge kutta

3 marks question : 1/3 simpson question

3 marks : trapezoidal rule question

5 marks : euler related

5 marks: trapezoidal

5 marks: milne predictor corrector formula question in the handouts on page 211(find f(0.4) from ordinary differencial equation)

MTH603-Numerical Analysis

(Solved MCQ's)

LECTURE FROM (23 to 45)

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ALL answers are verified if found any mistake then Correct ACCORDINGLY

Question # 1 Total Marks: 1

The determinant of a diagonal matrix is the product of the diagonal elements.

- True
- False

Question # 2 Total Marks: 1

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

- True
- false

Question # 3 Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.

- True
- False

Question # 4 Total Marks: 1

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

- True
- False

Question # 5 Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has _____zeros along its main diagonal.

- No
- At least one

Question # 6 Total Marks: 1

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to_____.

- Unity
- zero

Question # 7 Total Marks: 1

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

- True
- False

Question # 8 Total Marks: 1

The determinant of a_____matrix is the product of the diagonal elements.

- Diagonal CLOSELY
- Upper triangular
- Lower triangular
- Scalar

yeh charon options theek hain….
You can confirm it from internet…

Question # 9 Total Marks: 1

Eigenvalues of a symmetric matrix are all _____.

- Real
- Zero
- Positive
- Negative

Question # 10 Total Marks: 1

The Power method can be used only to find the eigen value of A that is largest in absolute value—we call this eigen value the dominant eigen value of A.

- True
- False

Question # 11 Total Marks: 1

P-C method is derived by employing

- Newton's backward difference interpolation formula
- Newton's forward difference interpolation formula
- Newton's divided difference interpolation formula
- None of the given choices

Question # 12 Total Marks: 1

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For differences methods we require the set of values.

- True
- False

Question # 13 Total Marks: 1

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

- True
- False

Question # 14 Total Marks: 1

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

- True
- False

Question # 15 Total Marks: 1

Central difference method seems to be giving a better approximation, however it requires more computations.

- True
- False

Question # 16 Total Marks: 1

Iterative algorithms can be more rapid than direct methods.

- True
- False

Question # 17 Total Marks: 1

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Central Difference method is the finite difference method.

- True
- False

Question # 18 Total Marks: 1

Exact solution of 2/3 is not exists.

- TRUE
- FALSE

Question # 19 Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has _____zeros along its main diagonal.

- no
- atleast one

Question # 20 Total Marks: 1

A 3 x 3 identity matrix have three and_____eigen values.

- same
- different

Question # 21 Total Marks: 1

Eigenvalues of a symmetric matrix are all _____.

- real
- complex
- zero
- positive

Question # 22 Total Marks: 1

The Jacobi iteration converges, if A is strictly diagonally dominant.

- TRUE
- FALSE

Question # 23 Total Marks: 1

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Below are all the finite difference methods EXCEPT_____.

- jacobi's method
- newton's backward difference method
- Stirlling formula
- Forward difference method

Question # 24 Total Marks: 1

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).

- TRUE
- FALSE

Question # 25 Total Marks: 1

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

- TRUE
- FALSE

Question # 26 Total Marks: 1

The global error in Simpson's 3/8 rule is of

- $o(h^2)$
- $o(h^3)$
- $o(h^4)$
- None of the given choices

Question # 27 Total Marks: 1

Two matrices with the same characteristic polynomial need not be similar.

- TRUE
- FALSE

Question # 28 Total Marks: 1

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Which of the following period strategic management was considered to be cure for all problems?

- Mid 1950s to mid 1960s
- Mid 1960s to mid 1970s
- Mid 1970s to mid 1980s
- Mid 1980s to mid 1990s

Question # 29 Total Marks: 1

Which of the following is not a pitfall an organization should avoid in strategic planning?
Select correct option:

- Failing to involve key employees in all phases of planning
- Involving all managers rather than delegating planning to a planner
- Top managers not actively supporting the strategic planning process
- Doing strategic planning only to satisfy accreditation or regulatory requirements

Question # 30 Total Marks: 1

which of the following are the factors that concern the nature and direction of the economy in which a firm operates? Select correct option:

- Technological
- Ecological
- Social
- Economic

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Question # 31 Total Marks: 1

Which of the following best describes this statement; “a Systematic and ethical process
for gathering and analyzing information about the competition’s activities and general
business trends to further a business’ own goals”?
Select correct option:

- External assessment
- Industry analysis

- Competitive intelligence program
- Business ethics

Question # 32 Total Marks: 1

According to Porter, which strategy offers products or services to a small range of customers at the lowest price available on the market?
Select correct option:

- Low cost
- Best value
- Cost focus

Question # 33 Total Marks: 1

Long-term objectives includes all of the following EXCEPT:

- Measurable
- Reasonable
- Varying
- Consistent

Question # 34 Total Marks: 1

Which one of the following is NOT is a basic mission of a competitive intelligence program?

- To provide a general understanding of an industry
- To provide a general understanding of a company's competitors
- To identify industry executives who could be hired by the firm
- To identify potential moves a competitor might make that would endanger a firm

Question # 35 Total Marks: 1

While preparing an External Factor Evaluation Matrix, a total score of 0.8 indicates that:

- Firm is taking advantages of strengths and avoiding threats
- Firm is taking no advantage of opportunities and is avoiding threats
- Firm is not taking advantages of opportunities and is not avoiding threats
- Firm is taking advantage of opportunities and is avoiding the threats

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Question No 36 (Marks: 1)

Symbol used for forward differences is

- ▶
- ▶
- ▶
- ▶

Question No: 37 (Marks: 1)

The relationship between central difference operator and the shift operator is given by

- ▶ = E E^1
- ▶ = E + E^1
- ▶ = E^2 + E^2^1
- ▶ _ _

Question No: 38 (Marks: 1)

 = E^1/2 E^1/2

Muller's method requires-------- starting points

- ▶ 1
- ▶ 2
- ▶ 3
- ▶ 4

Question No: 39 (Marks: 1)

If S is an identity matrix, then

- ▶

$$S^{-1} = S$$

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- $S^t = S$
- All are true
- $S^{-1} = S^t$

Question No: 40 (Marks: 1)
If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree----- agreeing with y_x at $x_0, x_1, ..., x_r$

- r+2
- r+1
- r
- r-1

Question No: 41 (Marks: 1)
P in Newton's forward difference formula is defined as

- $p = \left(\frac{x - x_0}{h}\right)$
- $p = \left(\frac{x + x_0}{h}\right)$
- $p = \left(\frac{x + x_n}{h}\right)$
- $p = \left(\frac{x - x_n}{h}\right)$

Question No: 42 (Marks: 1)
Octal number system has the base ----------

- 2
- 8
- 10
- 16

Question No: 43 (Marks: 1)
Newton's divided difference interpolation formula is used when the values of the independent variable are

- Equally spaced
- Not equally spaced

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- Constant
- None of the above

Question No: 44 (Marks: 1)
Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

- 1.5
- 3
- 2
- 1

Question No: 45 (Marks: 1) If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

- $\epsilon(x) = y(x) + P_n(x)$
- $\epsilon(x) = y(x) - P_n(x)$
- $\epsilon(x) = P_n(x) - y(x)$
- $\epsilon(x) = y(x) \cdot P_n(x)$

Question No: 46 (Marks: 1)

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x$. Then $F(x)$ vanishes ------times in the interval I.

- n-1
- n+2

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- n
- n+1

Question No: 47 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

- $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + \ldots)$
- $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + \ldots)$
- $D = \frac{1}{h}(\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \frac{\nabla^5}{5} + \ldots)$
- $D = \frac{1}{h}(\nabla + \frac{\nabla^2}{2!} + \frac{\nabla^3}{3!} + \frac{\nabla^4}{4!} + \frac{\nabla^5}{5!} + \ldots)$

Question No: 48 (Marks: 1) - Please choose one

Finding the first derivative of $f(x)$ at x=0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of---------- will be used.

- Forward difference operator
- Backward difference operator
- Central difference operator
- None of the given choices

Question No: 49 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

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$f'(0.1)$, using two-point equation will be calculated as.....

- -0.5
- 0.5
- 0.75
- -0.75

Question No: 50 (Marks: 1) In Simpson's 1/3 rule, $f(x)$ is of the form

- $ax+b$
- $ax^2 + bx + c$
- $ax^3 + bx^2 + cx + d$
- $ax^4 + bx^3 + cx^2 + dx + e$

Question No: 51 (Marks: 1) - Please choose one

$$I = \int_a^b f(x)dx$$

While integrating , h, width of the interval, is found by the formula----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

Question No: 52 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

- 7
- 8
- 5
- 3

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Question No: 53 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as ……

- 17.5
- 12.5
- 7.5
- -12.5

Question No: 54 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

- 2
- 3
- 5
- 7

Question No: 55 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

- 10
- 11
- 12
- 13

Question No: 56 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- $\frac{(a+b)}{2}$
- $\frac{(a \square b)}{2}$

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- $\frac{(b - a)}{2}$
- None of the given choices

Question No: 57 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- Bisection Method
- Regula Falsi Method
- Secant Method
- None of the given choices

Question No: 58 (Marks: 1) - Please choose one

For the equation $x^3 + 3x - 1 = 0$, the root of the equation lies in the interval.....

- (1, 3)
- (1, 2)
- (0, 1)
- (1, 2)

Question No: 59 (Marks: 1) - Please choose one

Rate of change of any quantity with respect to another can be modeled by

- An ordinary differential equation
- A partial differential equation
- A polynomial equation
- None of the given choices

Question No: 60 (Marks: 1) - Please choose one

If

$\frac{dy}{dx} = f(x, y)$

Then the integral of this equation is a curve in

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- None of the given choices

- xt-plane
- yt-plane
- xy-plane

Question No: 61 (Marks: 1) - Please choose one

In solving the differential equation

$$y' = x + y \; ; y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

- 1.44
- 1.11
- 1.22
- 1.33

Question No: 62 (Marks: 1) - Please choose one

In second order Runge-Kutta method

k_1 is given by

- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

Question No: 63 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

- $k = hf(x + \frac{h}{2}, y + \frac{k_1}{2})$
- $k = hf(x + \frac{h}{3}, y + \frac{k_1}{3})$
- $k = hf(x - \frac{h}{3}, y - \frac{k_1}{3})$

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- $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

Question No: 64 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

- $K_4=hf(x_n+2h, y_n+2K_3)$
- $K_4=hf(x_n-h, y_n-K_3)$
- $K_4=hf(x_n+h, y_n+K_3)$
- None of the given choices

Question No: 65 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

- Newton's backward difference interpolation formula
- Newton's forward difference interpolation formula
- Newton's divided difference interpolation formula
- None of the given choices

Question no. 66 Total Marks: 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit_____or whose antiderivative is not easy to obtain.

- Antiderivative
- Derivatives

Question # 67 Total Marks: 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

- TRUE
- FALSE

Question # 68 Total Marks: 1

An indefinite integral may _____in the sense that the limit defining it may not exist.

- diverge
- converge

Question # 69 Total Marks: 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____

to model the curve in trapezoidal Rule.

- straight lines
- curves
- parabolas
- constant

Question # 70 **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

- TRUE
- FALSE

Question # 71 **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

- FALSE
- TRUE

Question # 72 **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to_____.

- constant
- finite
- infinity
- zero

Question # 73 **Total Marks: 1**

Bisection method is..... method

- Bracketing Method
- Open
- Random
- none

Question # 74 **Total Marks: 1**

Newton Raphson method is .....method

- Bracketing Method
- Open
- Random
- None

Question # 75 **Total Marks: 1**

Eigenvalue is

- Real
- Vector
- odd

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- even

Question # 76 **Total Marks: 1**

For Simpson's 1/3 rule no.of intervals must be

- 2 (even)
- 3
- 5
- 7

Question # 77 **Total Marks: 1**

For Simpson's 1/3 rule valid no.of intervals are

- 1
- 3
- 5
- 8

Question # 78 **Total Marks: 1**

For Simpson's 3/8 rule no.of intervals must be

- 10
- 11
- 12 multiple of three
- 14

Question # 79 **Total Marks: 1**

The determinant of a diagonal matrix is the product of the diagonal elements.

- TRUE
- FALSE

Question # 80 **Total Marks: 1**

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

- TRUE
- FALSE

Question # 81 **Total Marks: 1**

To apply simpson's 3/8 rule, the number of intervals can be

- 4
- 7
- 9 (multiple of three)
- 8

Question # 82 **Total Marks: 1**

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

- TRUE
- FALSE

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Question # 83 **Total Marks: 1**

The Jacobi's method is a method of solving a matrix equation on a matrix that has _____zeros along its main diagonal.

- no
- atleast one

Question #84 **Total Marks: 1**

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to_____.

- unity
- zero

Question # 85 **Total Marks: 1**

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

- TRUE
- FALSE

Question # 86 **Total Marks: 1**

The determinant of a_____matrix is the product of the diagonal elements.

- diagonal
- upper triangular
- lower triangular
- scalar

Question # 87 **Total Marks: 1**

Eigenvalues of a symmetric matrix are all _____.

- real
- zero
- positive
- negative

Question # 88 **Total Marks: 1**

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

- TRUE
- FALSE

Question # 89 **Total Marks: 1**

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

- TRUE
- FALSE

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Question # 90 **Total Marks: 1**

In fourth order Runge-Kutta method, K_1 is given by

- $K_1 = hf(x_n, y_n)$
- $K_1 = 2hf(x_n, y_n)$
- $K_1 = 3hf(x_n, y_n)$
- None of the given choices

Question # 91 **Total Marks: 1**

If there are (n+1) values of y corresponding to (n+1) values of x, then we can represent the function f (x) by a polynomial of degree

- n+2
- n+1
- n
- n-1

Question # 92 **Total Marks: 1**

For differences methods we require the set of values.

- TRUE
- FALSE

Question # 93 **Total Marks: 1**

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

- TRUE
- FALSE

Question # 94 **Total Marks: 1**

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

- TRUE
- FALSE

Question # 95 **Total Marks: 1**

Central difference method seems to be giving a better approximation, however it requires more computations.

- TRUE
- FALSE

Question # 96 **Total Marks: 1**

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The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

- TRUE
- FALSE

Question # 97 **Total Marks: 1**

Iterative algorithms can be more rapid than direct methods.

- FALSE
- TRUE

Question # 98 **Total Marks: 1**

Central Difference method is the finite difference method.

- TRUE
- FALSE

Question # 99 **Total Marks: 1**

A 3 x 3 identity matrix have three and different eigen values.

- TRUE
- FALSE

Question # 100 **Total Marks: 1**

Eigenvalues of a symmetric matrix are all _____.

- Zero
- Real
- positive
- negative

Question # 101 **Total Marks: 1**

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.

- TRUE
- FALSE

Question # 102 **Total Marks: 1**

Exact solution of 2/3 is not exists.

- TRUE
- FALSE

Question # 103 **Total Marks: 1**

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric_____definite matrices A.

- **positive**
- negative

Question #104 **Total Marks: 1**

Differences methods find the_____ solution of the system.

- **numerical**

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- Analytical

Question # 105 **Total Marks: 1**

While deriving Trapezoidal rule, we approximate f (x) in the form_____

- ax+b
- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question #106 **Total Marks: 1**

The Power method can be used only to find the eigen value of A that is largest in absolute value ---------- we call this eigen value the dominant eigen value of A.

- TRUE
- **FALSE**

Question #107 **Total Marks: 1**

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.

- main diagonal
- last column
- last row
- first row

Question #108 **Total Marks: 1**

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

- TRUE
- FALSE

Question # 109 **Total Marks: 1**

A 3 x 3 identity matrix have three and different eigen values.

- TRUE
- **FALSE**

Question # 110 **Total Marks: 1**

Differences methods find the_____ solution of the system.

- **numerical**
- Analytical

Question # 111 **Total Marks: 1**

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

- **TRUE**
- FALSE

Question # 112 **Total Marks: 1**

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If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

- TRUE
- FALSE

Question # 113 **Total Marks: 1**

Eigenvalues of a_____matrix are all real.

- symmetric
- antisymmetric
- rectangular
- triangular

Question # 114 **Total Marks: 1**

By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.Page No.122

- TRUE
- FALSE

Question # 115 **Total Marks: 1**

The dominant eigenvector of a matrix is an eigenvector corresponding to the eigenvalue of largest magnitude (for real numbers, smallest absolute value) of that matrix.

- TRUE
- FALSE

Question # 116 **Total Marks: 1**

Eigenvalues of a symmetric matrix are all _____.

- real
- complex
- zero
- positive

Question # 117 **Total Marks: 1**

Below are all the finite difference methods EXCEPT_____.

- jacobi's method
- newton's backward difference method
- Stirlling formula
- Forward difference method

Question # 118 **Total Marks: 1**

The central difference method is finite method.

- True
- False

Question # 119 **Total Marks: 1**

Power method is applicable if the eigen vectors corresponding to eigen values are linearly _____.

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- independent
- dependent

Question # 120 **Total Marks: 1**

Central difference method seems to be giving a better approximation, however it requires more computations.

- TRUE
- FALSE

Question # 121 **Total Marks: 1**

The Jacobi iteration_____, if A is strictly diagonally dominant.

- converges
- diverges

Question # 122 **Total Marks: 1**

Power method is applicable if the eigen values are_____.

- real and distinct
- real and equal
- positive and distinct
- negative and distinct

Question # 123 **Total Marks: 1**

Determinant of a diagonal matrix is the product of the diagonal elements.

- TRUE
- FALSE

Question # 124 **Total Marks: 1**

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

- TRUE
- FALSE

Question # 125 **Total Marks: 1**

If n x n matrices A and B are similar, then they have the_____eigenvalues (with the same multiplicities).

- **same**
- different

Question # 126 **Total Marks: 1**

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to_____.

- **unity**
- zero

Question # 127 **Total Marks: 1**

Exact solution of 2/3 is not exists.

- TRUE

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- FALSE

Question # 128 **Total Marks: 1**

The Jacobi's method is a method of solving a matrix equation on a matrix that has _____zeros along its main diagonal.

- no
- atleast one

Question # 129 **Total Marks: 1**

A 3 x 3 identity matrix have three and_____eigen values.

- same
- different

Question # 130 **Total Marks: 1**

Eigenvalues of a symmetric matrix are all _____.

- real
- complex
- zero
- positive

Question # 131 **Total Marks: 1**

The Jacobi iteration converges, if A is strictly diagonally dominant.

- TRUE
- FALSE

Question # 132 **Total Marks: 1**

Below are all the finite difference methods EXCEPT_____.

- jacobi's method
- newton's backward difference method
- Stirlling formula
- Forward difference method

Question # 133 **Total Marks: 1**

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).

- TRUE
- FALSE

Question # 134 **Total Marks: 1**

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

- TRUE
- FALSE

Question # 135 **Total Marks: 1**

Two matrices with the same characteristic polynomial need not be similar.

- TRUE
- FALSE

Question # 136 **Total Marks:** 1

The determinant of a_____matrix is the product of the diagonal elements.

- diagonal
- upper triangular
- lower triangular
- scalar

Question # 137 **Total Marks:** 1

The absolute value of a determinant (|detA|) is the product of the absolute values of the eigenvalues of matrix A

- TRUE
- FALSE

Question # 138 **Total Marks:** 1

Central difference method seems to be giving a better approximation, however it requires more computations.

- TRUE
- FALSE

Question # 139 **Total Marks:** 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant
eigenvalue of A.

- TRUE
- FALSE

Question # 140 **Total Marks:** 1

Let A be an n ×n matrix. The number x is an eigenvalue of A if there exists a non-zero vector v such that_____.

- Av = xv
- Ax = xv
- Av + xv=0
- Av = Ax

Question # 141 **Total Marks:** 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

- TRUE
- FALSE

Question # 142 **Total Marks:** 1

Given the following data

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x	0	1	4
y=f(x)	2	1	4

Value of first order divided difference y(0,1) is

- -1
- 1
- 2
- 4

Question # 143 **Total Marks:** 1

Central Difference method is the finite difference method.

- TRUE
- FALSE

Question # 144 **Total Marks:** 1

By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.

- TRUE
- FALSE

Question # 145 **Total Marks: 1**

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

- (x-1)^3
- (x+1)^3
- x^3-1
- x^3+1

Question # 146 **Total Marks: 1**

The Inverse of a matrix can only be found if the matrix is

- Singular
- Non singular
- Scalar
- Diagonal

Question # 147 **Total Marks: 1**

If f (x) contains trigonometric, exponential or logarithmic functions then this equation is known as

- Transcendental equation
- Algebraic
- Polynomial
- Linear

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Question # 148 **Total Marks: 1**

In interpolation is used to represent the δ

- Forward difference
- Central difference
- Backward difference

Question # 149 **Total Marks: 1**

The base of the decimal system is _____

- 10
- 0
- 2
- 8
- None of the above.

Question # 150 **Total Marks: 1**

Bisection method is..... method

- Open Method
- Bracketing Method

Question # 151 **Total Marks: 1**

Which method is not used to solve problems related to integration?

- Runge-Kutta Method
- Simpson's 1/3rd rule
- Trapezoidal rule.

Question # 152 **Total Marks:** 1

Adams – Bashforth is a multistep method.

- True
- False

Question # 153 **Total Marks: 1**

Generally, Adams methods are superior if output at_____points is needed.

- Many
- Two
- Single
- At most

Question # 154 **Total Marks: 1**

In Trapezoidal rule, the integral is computed on each of the subintervals by using linear interpolating formula, i.e. for n = 1 and then summing them up to obtain the desired integral.

- True
- False

Question # 155 **Total Marks: 1**

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A third order ordinary differential equation can be reduced to a system of _____first order ordinary differential equations.

- **0**
- **1**
- **2**
- **3**

Question # 156 **Total Marks: 1**

Euler's Method numerically computes the approximate_____of a function.

- antiderivative
- derivative
- value
- error

Question # 157 **Total Marks:** 1

Multistep method does not improves the accuracy of the answer at each step.

FALSE

TRUE

Question # 158 **Total Marks:** 1

The trapezoidal rule is a numerical method that approximates the value of a _____.

- indefinite integral
- definite integral
- improper integral
- function

Question # 159 **Total Marks:** 1

Simpson's rule is a numerical method that approximates the value of a definite integral by using_____polynomials.

- quadratic
- linear
- cubic
- quartic

Question # 160 **Total Marks: 1**

In Simpson's Rule, we use parabolas to approximate each part of the curve. This proves to be very efficient as compared to Trapezoidal rule.

- True
- False

Question # 161 **Total Marks: 1**

The first langrange polynomial with equally spaced nodes produced the formula for_____.

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- Simpson's rule
- Trapezoidal rule
- Newton's method
- Richardson's method

Question # 162 **Total Marks: 1**

The need of numerical integration arises for evaluating the indefinite integral of a function that has no explicit antiderivative or whose antiderivative is not easy to obtain.

- TRUE
- FALSE

Question # 163 **Total Marks: 1**

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____to model the curve in trapezoidal Rule.

- straight lines
- curves
- parabolas
- constant

Question # 164 **Total Marks: 1**

The Euler method is numerically unstable because of_____convergence of error.

- slow
- fast
- moderate
- no

Question # 165 **Total Marks: 1**

In integrating $\int_1^2 x^2 dx$, by dividing the interval into eight equal parts, width of the interval should be

- 0.125
- 0.250
- 0.500
- 0.825

Question # 166 **Total Marks: 1**

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I.

- 6

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- 9
- 7
- 8

Question # 167 **Total Marks: 1**

The minimum interval in which the root of the equation $x^2-3x+1=0$ is

- (0,1)
- (1,2)
- (2,4)
- (0,2)

Question # 168 **Total Marks: 1**

The quantity of error which is present in the statement of the problem itself, before finding its solution is called

- Inherent error
- Local round off error
- Local truncation error
- Tying error

Question # 169 **Total Marks: 1**

In Regula Falsi Method two points x_n and x_{n+1} are chosen such that f (x_n) and f ($_{n+1}$) have_____signs.

- +ve
- -ve
- Opposite
- Same

Question # 170 **Total Marks: 1**

Regula Falsi Method lies in the category of_____

- Iterative method
- Bracketing method
- Random method
- Graphical method

Question # 171 **Total Marks: 1**

Secant method converges_____than bisection

- Faster
- Slower
- Equally
- None of the given choices

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Question # 172 **Total Marks: 1**

Muller's method is a generalization of:

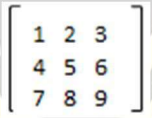
- Bisection method
- Iteration method
- Secant method
- Regula Falsi Method

Question # 173 **Total Marks: 1**

Diagonal dominance of a coefficient matrix is strictly checked in:

- Muller's method
- Bisection method
- Jacobi's method
- Newton-Raphson method

Question # 174 **Total Marks: 1**

It can be verified that for matrix A= 

- $AA^{-1}=I$, I= identity matrix
- $AA^{-1}=D$, D =diagonal matrix
- $AA^{-1}=S$, S =symmetric matrix
- $AA^{-1}=Z$, Z =orthogonal matrix

Question # 175 **Total Marks: 1**

If [A] is an n×n real symmetric matrix, its eigenvalues are real, and there exists an orthogonal matrix [S] such that the diagonal matrix (D) is given by:

- $[D]=[S]^{-1}[A][S]$
- $[D]=[S]^{-1}[S][A]$
- $[D]=[S]^{2}[A][S]$
- None of the above

Question # 176 **Total Marks: 1**

Given the the following data

x	1	2	-4

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f (x)	3	-5	-4

Which formula is useful in finding the interpolating polynomial?

- Newton's backward difference formula
- Lagrange's interpolation formula
- None of the given choices
- Newton's forward difference formula

Question # 177 **Total Marks: 1**

The magnitude of truncation error in Milne's predictor formula is

- $\frac{1}{90}h\Delta^4 y_0'$
- $\frac{1}{90}h\Delta^3 y_0'$
- $\frac{28}{90}h\Delta^3 y_0'$
- $\frac{28}{90}h\Delta^4 y_0'$

Question # 178 **Total Marks: 1**

If f (x)= $5x^6+6x^5-7x^3-9x^2+4x-3$, then its derivative is zero for all x.

- 4th
- 7th
- 6th
- 5th

Question # 179 **Total Marks: 1**

In Richardson's extrapolation method, the extrapolation process is repeated 8until accuracy is achieved, this is called extrapolation to the_____

- Limit
- Function
- Arbitrary value of 'h'
- None of the given choices

Question # 180 **Total Marks: 1**

While deriving Simpson's 3/8 rule, we approximate f (x) in the form_____

- ax+b
- ax^2+bx+c

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- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question # 181 **Total Marks: 1**

When we apply simpson's 3/8 rule, the number of intervals n must be:

- Even
- Odd
- Multiple of 3
- Multiple of 8

Question # 182 **Total Marks: 1**

In integrating $\int_0^1 e^{2x}dx$ by dividing into eight equal parts, width of the interval should be.....

- 0.250
- 0.500
- 0.125
- 0.625

Question # 183 **Total Marks: 1**

Romberg's integration method is_____than Trapezoidal and Simpson's rule.

- None of the given choices
- More accurate
- Less accurate
- Equally accurate

Question # 184 **Total Marks: 1**

A fourth order ordinary differential equation can be reduced to a system of fourth_____ order ordinary differential equations

- First
- Second
- Third
- Fourth

Question # 185 **Total Marks: 1**

Given $\frac{dy}{dt}=\frac{y-t}{y+t}$ with the initial condition y=1.02 at t=0.02. Using Euler's method, y at t= 0.04, h=0.02

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- 3.0392
- 2.0392
- 1.0392
- 0.0392

Question # 186 **Total Marks: 1**

In solving the differential equation

Y' =x+y, y (0.1)= 1.1

h=0.1, By Euler's method y (0.2) is calculated as:

- 1.44
- 1.11
- 1.22
- 1.33

Question # 187 **Total Marks: 1**

To solve the ordinary differential equation

$$3\frac{dy}{dx} + xy^2 = \sin x, y(0) = 5,$$

by Runge-Kutta 2nd order method, you need to rewrite the equation as

- $\frac{dy}{dx} = \sin x - xy^2, y(0) = 5$
- $\frac{dy}{dx} = \frac{1}{3}\left(\sin x - xy^2\right), y(0) = 5$
- $\frac{dy}{dx} = \frac{1}{3}\left(-\cos x - \frac{xy^3}{3}\right), y(0) = 5$
- $\frac{dy}{dx} = \frac{1}{3}\sin x, y(0) = 5$

Question # 188 **Total Marks: 1**

The truncation error in Adam's predictor formula is

$\frac{251}{720}h\nabla^4 y_{n+1}'$

$\frac{251}{720}h\nabla^4 y_n'$

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$\frac{19}{720}h\nabla^4 y_{n+1}'$

$\frac{19}{720}h\nabla^4 y_n'$

Question # 189 **Total Marks: 1**

In solving the differential equation

$Y' = x^2 + 2xy$; $y(0) = 1$

h=0.1, By Euler's method y (0.1) is calculated as

- 1
- 2
- 3
- 4

Question # 190 **Total Marks: 1**

In solving the differential equation

$Y' = x^2 + 2y$; $y(1) = 3$

h=1, By Euler's method y (2) is calculated as

- 4
- 6
- 8
- 10 not sure

Question # 191 **Total Marks: 1**

In integrating $\int_0^{\frac{\pi}{2}} \sin x dx$, by dividing the interval into four equal parts, width of the interval should be:

- π
- $\frac{\pi}{2}$
- $\frac{\pi}{4}$
- $\frac{\pi}{8}$

Question # 192 **Total Marks: 1**

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The minimum interval in which the root of the equation $x^2 - 3x + 1 = 0$ lie is:

- (0,1)
- (1,2)
- (2,4)
- (0,2)

Question # 193 **Total Marks: 1**

If the root the given equation lies between a and b, then the first approximate to the root of the equation by bisection method.....

- $\frac{(a+b)}{2}$
- $\frac{(a-b)}{2}$
- $\frac{(b-a)}{2}$
- None of the given choices

Question # 194 **Total Marks: 1**

If 8+7i is the root of a quadratic equation f (x)=0, then the other root of the equation will be:

- 8-7i
- -8-7i
- -8+7i
- 8

Question # 195 **Total Marks: 1**

If there are two roots of the equation and one root is 2+3i , then the other root will be:

- 2+3i
- 2-3i
- 3+2i
- 3-2i

Question # 196 **Total Marks: 1**

In crout's reduction method the elements of the main diagonal of upper triangular matrix are taken as

- 0

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- 1
- -1
- None of the given choices

Question # 197 **Total Marks: 1**

In Gauss-seidal method, the new approximation calculated is:

- Used in the next iteration for next approximation
- Instantly replaced by the previous one
- Never used again
- None of the given choices

Question # 198 **Total Marks: 1**

In relaxation method, for fast convergence all the terms should be taken to one side and then reordering should be done so that the largest coefficient should appear on the

- End of rows
- Beginning of the rows
- Diagonal
- None of the given choices

Question # 199 **Total Marks: 1**

Every squre_____matrix has an inverse.

- Singular
- Non-Singular
- Zero
- None of the above

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Question # 200 **Total Marks: 1**

If $S=\begin{pmatrix}1 & 2 & 3\\ 6 & 4 & 7\\ 8 & 6 & 5\end{pmatrix}$, then

- $S^{-1}=S^{T}$
- $S^{T}=S$
- $I=S$
- None of the given choices

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Question # 201 **Total Marks: 1**

A 3×3 matrix [A] is said to be orthogonal if

- $[A]^{-1}[A]= I$
- $[A^T][A]=I$
- $[A^T][A]=0$
- None of the given choices

Question # 202 **Total Marks: 1**

Jacobi's method for finding the eigen values can be applied to:

- Real and symmetric matrix
- Real and unsymmetric matrix
- Real and complex unsymmetric matrix
- None of the given choices

Question # 203 **Total Marks: 1**

Lagrange's interpolation formula is used when the values of the independent variable are:

- Equally spaced
- Not equally spaced
- Constant
- None of the above

Question # 204 **Total Marks: 1**

From the following table of values

x	1.3	1.5	1.7	1.9	2.1	2.3
y	2.9648	2.6599	2.3333	1.9922	1.6442	1.2969

Estimation of y' (1.3) will be done using the formula of differential operator in terms of

- Backward difference operator
- Central difference operator
- None of the given choices
- Forward difference operator

Question # 205 **Total Marks: 1**

Relationship between differential operator and backward difference operator is given by

- $hD= \log(1+\nabla)$

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- $hD = -\log(1+\nabla)$

- $hD = \log(1-\nabla)$
- $hD = -\log(1-\nabla)$

Question # 206 **Total Marks: 1**

Shift operator is defined as

- $Ef(x) = f(x-h)$
- $Ef(x) = -f(x-h)$
- $Ef(x) = -f(x+h)$
- $Ef(x) = f(x+h)$

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MTH603

Numerical Analysis

File Version v11.02.02 *Prepared For: Final Term*

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Compiled and Prepared by:

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File Version updated on ***7/24/2011***

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Absolute Error : *76*
Accuracy : *76*
Algebraic equation : *76*
Bisection method : *76*
Bracketing Method : *77*
Crout's Method : *77*
Direct methods : *77*
Gauss Elimination Method : *77*
Gauss Seidel iterative method : *77*
Graeffee's root squaring method : *77*
Intermediate value property : *77*
Inverse of a matrix : *77*
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Open methods : *78*
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Regula –Falsi method : *78*
Relaxation method : *78*
Secant Method : *78*
Significant digits : *79*
Singular matrix : *79*
Transcendental equation : *79*
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Bisection Method *79*
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Regula Falsi Method (Method of False position) *79*
Newton Rophson method *80*
Secant Method *80*
Newton's Formula *80*
Graffee root squaring method *80*
the truncation error (TE) is given by *80*

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Introduction To Numerical Analysis

Course Content:

Number systems, Errors in computation, Methods of solving non-linear equations, Solution of linear system of equations and matrix inversion, Eigen value problems, power method, Jacobi's method, Different techniques of interpolation, Numerical differentiation and integration, Numerical integration formulas, different methods of solving ordinary differential equations.

Each Topic One Solved Question with Steps:Compiled from Past assignment

Preliminary material:

Representation of numbers

Binary Number System (Base 2)=0,1

Used in Computer Internal Operations and Calculation

Octal Number System (Base 8)=0,1,2,3,4,5,6,7

ERROR

Types of error are,

- Inherent errors,
- Local round-off errors
- Local truncation errors

Decimal Number System (Base 10)=0,1,2,3,4,5,6,7,8,9

World Wide used and the most common system

Hexa Number System (Base 16)=0,1,2,3,4,5,6,78,9,A,B,C,D,E,F

Used in Processor Register Addressing

Errors in computations

The initial approximation

It may be found by two methods either by graphical method or
Analytical method
Graphical method

Non-Linear Equations

Bisection Method

Procedure in Detail:

Step,Take two Initial Approximation such that f(x1).F(x2)<0.Means both must have opposite signs.Take their mean by x3=(x1+x2)/2
Next Take two element from which 1 will be x3 and another from x1 or x2 such that both x3 and the other element should have opposite sign.
Repeat the above process to the required numbers of iterations.

Question 1

Find the root of the equation given below by bisection method.

$x^3 - x^2 + x - 7 = 0$

(Note: accuracy up to three decimal places is required.) **Marks: 10**

SOLUTION

Let

$$f(x) = x^3 - x^2 + x - 7$$

Now $f(1) = (1)^3 - (1)^2 + 1 - 7 = -6 < 0$

$f(2) = (2)^3 - (2)^2 + 2 - 7 = -1 < 0$

and $f(3) = (3)^3 - (3)^2 + 3 - 7 = 14 > 0$

As $f(2) f(3) = -14 < 0$

Therefore a real root lies between 2 *and* 3

Iteration 1

Let

$x_0 = 2$ *and* $x_1 = 3$

then

$$x_2 = \left(\frac{x_0 + x_1}{2}\right)$$

$$= \left(\frac{2+3}{2}\right) = 2.5$$

Now $f(2.5) = (2.5)^3 - (2.5)^2 + 2.5 - 7 = 4.875 > 0$

As $f(2) f(2.5) = -1(4.875) = -4.875 < 0$

Therefore a real root lies between 2 *and* 2.5

Iteration 2

$x_0 = 2$ *and* $x_2 = 2.5$

then

$$x_3 = \left(\frac{x_0 + x_2}{2}\right)$$

$$= \left(\frac{2+2.5}{2}\right) = 2.25$$

Now $f(2.25) = (2.25)^3 - (2.25)^2 + 2.25 - 7 = 1.578125 > 0$

As $f(2) f(2.25) = -1(1.578125) = -1.578125 < 0$

Therefore a real root lies between 2 *and* 2.25

Iteration 3

$x_0 = 2 \text{ and } x_3 = 2.25$

then

$$x_4 = \left(\frac{x_0 + x_3}{2}\right)$$

$$= \left(\frac{2 + 2.25}{2}\right) = 2.125$$

$Now\ f(2.125) = (2.125)^3 - (2.125)^2 + 2.125 - 7 = 0.205078 > 0$

$As\ f(2) f(2.125) = -1(0.205078) = -0.205078 < 0$

Therefore a real root lies between 2 *and* 2.125

Iteration 4

$x_0 = 2 \text{ and } x_4 = 2.125$

then

$$x_5 = \left(\frac{x_0 + x_4}{2}\right)$$

$$= \left(\frac{2 + 2.125}{2}\right) = 2.0625$$

$Now\ f(2.0625) = (2.0625)^3 - (2.0625)^2 + 2.0625 - 7 = -0.417725 < 0$

$As\ f(2.0625) f(2.125) = -0.417725(0.205078) = -0.085666 < 0$

Therefore a real root lies between 2.0625 *and* 2.125

Iteration 5

$x_5 = 2.0625 \text{ and } x_4 = 2.125$

then

$$x_6 = \left(\frac{x_5 + x_4}{2}\right)$$

$$= \left(\frac{2.0625 + 2.125}{2}\right) = 2.09375$$

$Now\ f(2.09375) = (2.09375)^3 - (2.09375)^2 + 2.09375 - 7 = -0.111481 < 0$

$As\ f(2.09375) f(2.125) = -0.111481(0.205078) = -0.022862 < 0$

Therefore a real root lies between 2.09375 *and* 2.125

Iteration 6

$x_6 = 2.09375 \text{ and } x_4 = 2.125$

then

$$x_7 = \left(\frac{x_6 + x_4}{2}\right)$$

$$= \left(\frac{2.09375 + 2.125}{2}\right) = 2.109375$$

$Now\ f(2.109375) = (2.109375)^3 - (2.109375)^2 + 2.109375 - 7 = 0.0455 > 0$

$As\ f(2.09375) f(2.109375) = -0.111481(0.0455) = -0.00507 < 0$

Therefore a real root lies between 2.09375 *and* 2.109375

Iteration 7

$x_6 = 2.09375 \, and \, x_7 = 2.109375$

then

$$x_8 = (\frac{x_6 + x_7}{2})$$

$$= (\frac{2.09375 + 2.109375}{2}) = 2.1015625$$

Now $f(2.1015625) = (2.1015625)^3 - (2.1015625)^2 + 2.1015625 - 7 = -0.0333 < 0$

As $f(2.1015625) f(2.109375) = -0.0333(0.0455) = -0.01515 < 0$

Therefore a real root lies between 2.1015625 *and* 2.109375

Iteration 8

$x_8 = 2.1015625 \, and \, x_7 = 2.109375$

then

$$x_9 = (\frac{x_8 + x_7}{2})$$

$$= (\frac{2.1015625 + 2.109375}{2}) = 2.10546875$$

Now $f(2.10546875) = (2.10546875)^3 - (2.10546875^2 + 2.10546875 - 7 = 0.006 > 0$

As $f(2.1015625) f(2.10546875) = -0.0333(0.006) = -0.0002 < 0$

Therefore a real root lies between 2.1015625 *and* 2.10546875

Iteration 9

$x_8 = 2.1015625 \, and \, x_9 = 2.10546875$

then

$$x_{10} = (\frac{x_8 + x_9}{2})$$

$$= (\frac{2.1015625 + 2.10546875}{2}) = 2.103515625$$

Now $f(2.103515625) = (2.103515625)^3 - (2.103515625)^2 + 2.103515625 - 7 = -0.01367 < 0$

As $f(2.103515625) f(2.10546875) = -0.01367(0.006) = -0.00008 < 0$

Therefore a real root lies between 2.103515625 *and* 2.10546875

Iteration 10

$x_{10} = 2.103515625 \, and \, x_9 = 2.10546875$

then

$$x_{11} = (\frac{x_{10} + x_9}{2})$$

$$= (\frac{2.103515625 + 2.10546875}{2}) = 2.104492188$$

Now $f(2.104492188) = (2.104492188)^3 - (2.104492188)^2 + 2.104492188 - 7 = -0.0038 < 0$

As $f(2.104492188) f(2.10546875) = -0.0038(0.006) = -0.0000228 < 0$

Therefore a real root lies between 2.104492188 *and* 2.10546875

Iteration 11

$x_{11} = 2.104492188 \, and \, x_9 = 2.10546875$

then

$$x_{12} = (\frac{x_{11} + x_9}{2})$$

$$= (\frac{2.104492188 + 2.10546875}{2}) = 2.104980469$$

$Now \; f(2.104980469) = (2.104980469)^3 - (2.104980469)^2 + 2.104980469 - 7 = 0.0011 > 0$

$As \; f(2.104492188) \, f(2.104980469) = -0.0038(0.0011) = -0.00000418 < 0$

Therefore a real root lies between 2.104492188 *and* 2.104980469

As the next root lies between 2.104492188 *and* 2.104980469 and these roots are equal up to three decimal places.

So, the required root up to three decimal places is 2.104

Example From Handout at page # 4

Solve $x^3 - 9x + 1 = 0$ for the root between x=2 and x=4 by bisection method

Solution:

Here we are given the interval (2,4) so we need not to carry out intermediate value property to locate initial approximation.

Here $f(x) = x^3 - 9x + 1 = 0$

Now f(2)=-9 and f(4)=29.

Here f(2).f(4)<0 So the root lies between 2 and 4.

$So, x_o = 2, x_1 = 4$

and $x_2 = \frac{x_o + x_1}{2}$ =3 (This formula predicts next iteration)

Now f(3)=1,Here f(2).f(3)<0 So the root lies between 2 and 3.

Repeat above process to required number of iteration.

Step,Take two Initial Approximation such that f(x1).F(x2)<0.Means both must have opposite signs.Take their mean by x3=(x1+x2)/2

Next Take two element from which 1 will be x3 and another from x1 or x2 such that both x3 and the other element should have opposite sign.

Repeat the above process to the required numbers of iterations.

Regula Falsi Method

Formula for the Regula Falsi Method is $x3 = x2 - \frac{x2 - x1}{f(x2) - f(x1)} f(x2)$ or

$$x_{(n+1)} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

Steps: If the interval is given , check whether the root lies in between the interval or not by the principle that states if both number have opposite sign , then the root lies in between interval.

Find the next approximation with the help of the formula.

Question#2 **Marks 10**

Use Regula-Fasli method to compute the root of the equation $f(x)=\cos x-xe^x$ In the interval [0, 1] after third iteration.

Solution:

As $f(x)=\cos x-xe^x$

$f(0)=\cos 0-0e^0=1\succ 0$

$f(1)=\cos 1-1e^1=0.5403-2.718=-2.1779\prec 0$

So root of the eq. lie between 0 *and* 1. *Let* $x_0=0$ *and* $x_1=1$

Thus $f(x_0)=1$ *and* $f(x_1)=-2.1779$

The formula for finding the root of the function f(x) by Regula-Fasli is given by.

$$x_{(n+1)}=x_n-\frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})}f(x_n)$$

For n=1 we have,

$$x_2=x_1-\frac{x_1-x_o}{f(x_1)-f(x_o)}f(x_1)=1-\frac{1-0}{\text{-}2.17797-1}(\text{-}2.17797)=1-\frac{2.17797}{3.17797}=1-0.68533=0.31467$$

$$f(x_2)=\cos(0.31467)-(0.31467)e^{0.31467}=0.950899-(0.31467)(1.369806)=0.519863$$

Since $f(x_1)$ and $f(x_2)$ are of opposite signs, the root lies between x_1 and x_2.
Therefore, for n=2 we have,

$$x_3=x_2-\frac{x_2-x_1}{f(x_2)-f(x_1)}f(x_2)=0.31467-\frac{0.31467-1}{0.519863-(\text{-}2.17797)}(0.519863)$$

$$=0.31467+\frac{0.68533}{2.697833}(0.519863)=0.44673$$

$$f(x_3)=\cos(0.44673)-(0.44673)e^{0.44673}=0.901864-(0.44673)(1.563191)=0.20353$$

Since $f(x_1)$ and $f(x_3)$ are of opposite signs, the root lies between x_1 and x_3.
Therefore, for n=3 we have,

$$x_4=x_3-\frac{x_3-x_1}{f(x_3)-f(x_1)}f(x_2)=0.44673-\frac{0.44673-1}{0.20353-(\text{-}2.17797)}(0.20353)$$

$$=0.44673+\frac{0.54327}{2.3815}(0.20353)=0.493159$$

The required root after 3rd iteration using Regula-Falsi method is 0.493159

Question 2

Use the Regula Falsi (method of false position) to solve the equation $x^3-4x-9=0$

(Note: accuracy up to four decimal places is required) **Marks: 10**

SOLUTION

Let

$f(x) = x^3 - 4x - 9$

Now

$f(0) = (0)^3 - 4(0) - 9 = 0 - 0 - 9 = -9 < 0$

$f(1) = (1)^3 - 4(1) - 9 = 1 - 4 - 9 = -12 < 0$

$f(2) = (2)^3 - 4(2) - 9 = 8 - 8 - 9 = -9 < 0$

$f(3) = (3)^3 - 4(3) - 9 = 27 - 12 - 9 = 6 > 0$

Now, Since $f(2)$ *and* $f(3)$ *are of opposite signs, therefore, the real root lies between* 2 *and* 3

Now,

let $x_1 = 2$ *and* $x_2 = 3$

First iteration

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$= 3 - \frac{3-2}{6-(-9)}(6)$$

$$= 3 - \frac{1}{15}(6)$$

$$= 3 - \frac{2}{5}$$

$$= 2.6$$

$$f(x_3) = f(2.6) = (2.6)^3 - 4(2.6) - 9$$

$$= -1.824$$

Second iteration

$$x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3)$$

$$= 2.6 - \frac{2.6-3}{-1.824-6}(-1.824)$$

$$= 2.69325$$

$$f(x_4) = f(2.69325) = (2.69325)^3 - 4(2.69325) - 9$$

$$= -0.23725$$

Third iteration

$$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_4)$$

$$= 2.69325 - \frac{2.69325 - 2.6}{-0.23725 - (-1.824)}(-0.2061)$$

$$= 2.70536$$

$$f(x_5) = f(2.70536) = (2.70536)^3 - 4(2.70536) - 9$$

$$= -0.02098$$

Fourth iteration

$$x_6 = x_5 - \frac{x_5 - x_4}{f(x_5) - f(x_4)} f(x_5)$$

$$= 2.70536 - \frac{2.70536 - 2.69325}{-0.02098 - (-0.23725)}(-0.02098)$$

$$= 2.70653$$

$$f(x_6) = f(2.70653) = (2.70653)^3 - 4(2.70653) - 9$$

$$= 0.0000368$$

Fifth iteration

$$x_7 = x_6 - \frac{x_6 - x_5}{f(x_6) - f(x_5)} f(x_6)$$

$$= 2.70653 - \frac{2.70653 - 2.70536}{0.0000368 - (-0.02098)}(0.0000368)$$

$$= 2.70653$$

Hence,

The required root of the given equation correct to 4 decimal places is 2.7065

Method of Iterations

Question#1 **Marks 10**

Use the method of iterations to determine the real root of the equation $e^{-x} = 10x$ in the interval [0, 1], correct to four decimal places after four Iterations.

Solution:

Since $e^{-x} = 10x$. *Let* $f(x) = e^{-x} - 10x$. *we see that* $f(0) = 1$ *and* $f(1) = -9.6321$

As $e^{-x} = 10x$, *we can easily find the value of* x, *thus*

$x = \frac{e^{-x}}{10}$, *Let* $\phi(x) = \frac{e^{-x}}{10}$

By taking its derivative we get

$\phi'(x) = -\frac{e^{-x}}{10}$, *we see that* $|\phi'(x)| < 1$ *for all values in* $[0,1]$.

Therefore we can apply method of iterations to the given function.

Take any value within $[0,1]$. *Let*

$x_0 = 0.5$

$x_1 = \phi(0.5) = -\frac{e^{-0.5}}{10} = 0.0607 \quad , \quad f(x_1) = -0.09516$

$x_2 = \phi(0.0607) = -\frac{e^{-0.0607}}{10} = 0.0914 \quad , \quad f(x_2) = 0.00869$

$x_3 = \phi(0.0914) = -\frac{e^{-0.0914}}{10} = 0.0941 \ , \quad f(x_3) = -0.000790477 = -7.90477x10^{-4}$

$x_4 = \phi(0.0941) = -\frac{e^{-0.0941}}{10} = 0.0913 \ , \quad f(x_4) = 0.0000275784 = 2.75784x10^{-5}$

$x_4 = 0.0913$

Example From the Handout at page #

Example

Find a real root of the equation $\cos x = 3x - 1$ correct to seven places of decimal.

Solution

Here it is a transcendental function and all the calculation must be done in the radians mode and value of pi should be 3.14

$f(x) = \cos x - 3x + 1$

$f(0) = \cos 0 - 3(0) + 1 = 1 > 0$

$f(\pi/2) = \cos(1.57) - 3(1.57) + 1 = 0.0007963 - 4.71 + 1 = -3.7092037 < 0$

so a real root lies between 0 *and* $\pi/2$

here $\phi(x) = \frac{1}{3}(\cos x + 1)$

we have $\phi'(x) = -\frac{1}{3}\sin x$

it is clearly less than 1 *as* $\sin$ *is a bounded function and it's values lies between* -1 *and* 1

hence iteration method can be applied

let $x_0 = 0.5$ *be the inital approximation then*

$x_1 = \phi(x_0) = \frac{1}{3}[\cos(0.5) + 1] = 0.6258608$

$x_2 = \phi(x_1) = \frac{1}{3}[\cos(0.6258608) + 1] = 0.6034863$

$x_3 = \phi(x_2) = \frac{1}{3}[\cos(0.6034863) + 1] = 0.6077873$

$x_4 = \phi(x_3) = \frac{1}{3}[\cos(0.6077873) + 1] = 0.6069711$

$x_5 = \phi(x_4) = \frac{1}{3}[\cos(0.6069711) + 1] = 0.6071264$

$x_6 = \phi(x_5) = \frac{1}{3}[\cos(0.6071264) + 1] = 0.6070969$

$$x_7 = \phi(x_6) = \frac{1}{3}[\cos(0.6070969) + 1] = 0.6071025$$

$$x_8 = \phi(x_7) = \frac{1}{3}[\cos(0.6071025) + 1] = 0.6071014$$

$$x_9 = \phi(x_8) = \frac{1}{3}[\cos(0.6071014) + 1] = 0.6071016$$

$$x_{10} = \phi(x_9) = \frac{1}{3}[\cos(0.6071016) + 1] = 0.6071016$$

Newton-Raphson Method

Procedural Detail:
Find the limit if not provided by starting from x=0 to ,…
Find two consecutive numbers for f(x) should have opposite sign.
Use the Newton Raphson Formula to find next approximation .

Question#3 **Marks 10**

Find the real root of the equation $x^3 - 3x - 5 = 0$ using Newton-Raphson method in the interval [2,3] after third iteration.

Solution:

As $f(x) = x^3 - 3x - 5$

$f(2) = 2^3 - 3.2 - 5 = -3 \prec 0$

$f(3) = 3^3 - 3.3 - 5 = 13 \succ 0$

So root of the eq. will lie in $[2,3]$

now $f'(x) = 3x^2 - 3$ *and* $f''(x) = 6x$

$f'(2) = 3.2^2 - 3 = 9$ *and* $f'(3) = 3.3^2 - 3 = 24$

$f''(2) = 6.2 = 12$ *and* $f''(3) = 6.3 = 18$

Since $f(3)$ *and* $f''(3)$*are of same sign.So we choose* $x_0 = 3$

so by Newton's method we have

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{13}{24} = 2.4583$$

$$f(x_1) = (2.4583)^3 - 3(2.4583) - 5 = 14.8561 - 7.3749 - 5 = 2.4812$$

$$f'(x_1) = f'(2.4583) = 3(2.4583)^2 - 3 = 18.1297 - 3 = 15.1297$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.4583 - \frac{2.4812}{15.1297} = 2.2943$$

$$f(x_2) = (2.2943)^3 - 3(2.2943) - 5 = 12.0767 - 6.8829 - 5 = 0.1938$$

$$f'(x_2) = f'(2.2943) = 3(2.2943)^2 - 3 = 15.7914 - 3 = 12.7914$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.2943 - \frac{0.1938}{12.7914} = 2.2791$$

Example From the Handout at page #

Value of e=2.7182

Secant Method

Question 1

Do Four iterations of Secant method, with an accuracy of 3 decimal places to find the root of .

$f(x) = x^3 - 3x + 1 = 0, \quad x_0 = 1, x_1 = 0.5$ **Marks: 10**

Solution:

FORMULA OF SECANT METHOD

$$x_{n+1} = \frac{x_{n-1}(fx_n) - x_n f(x_n - 1)}{(fx_n) - f(x_n - 1)}$$

ITERATION 1

$n = 1$

$f(x_0) = f(1) = -1$

$f(x_1) = f(0.5) = -0.375$

$$x_2 = \frac{x_o(fx_1) - x_1 f(x_o)}{(fx_1) - f(x_o)}$$

$$x_2 = \frac{(1)(-0.375) - (0.5)(-1)}{(-0.375) - (-1)}$$

$$x_2 = \frac{(1)(-0.375) + (0.5)(1)}{(-0.375) + (1)}$$

$$x_2 = \frac{(-0.375) + (0.5)}{0.625}$$

$$x_2 = \frac{0.125}{0.625}$$

$x_2 = 0.2$

ITERATION 2

$$n = 2$$

$$f(x_2) = f(0.2) = 0.408$$

$$x_3 = \frac{x_1(fx_2) - x_2 f(x_1)}{(fx_2) - f(x_1)}$$

$$x_3 = \frac{(0.5)(0.408) - (0.2)(-0.375)}{(0.408) - (-0.375)}$$

$$x_3 = \frac{(0.204) + (0.075)}{(0.408) + (0.375)}$$

$$x_3 = \frac{0.279}{0.783}$$

$$x_3 = 0.3563$$

ITERATION 3

$$n = 3$$

$$f(x_3) = f(0.3563) = -0.02367$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$x_4 = \frac{(0.2) f(0.3563) - (0.3563) f(0.408)}{f(0.3563) - f(0.408)}$$

$$x_4 = \frac{(0.2)(-0.02367) - (0.3563)(0.408)}{(-0.02367) - (0.408)}$$

$$x_4 = \frac{-0.004737 - 0.1453}{-0.43167}$$

$$x_4 = \frac{-0.150034}{-0.43167}$$

$$x_4 = 0.3477$$

ITERATION 4

$n = 4$

$f(x_3) = f(0.3477) = -0.02367$

$x_4 = 0.3477 \quad f(x_4) = -0.00107$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$x_5 = \frac{(0.3563)(-0.00107) - (0.3563)(-0.02367)}{(-0.00107) - (-0.02367)}$$

$$x_5 = \frac{-0.000381 + 0.0823}{-0.00107 + 0.02367}$$

$$x_5 = \frac{0.7849}{0.0226}$$

$x_5 = 0.3473$

$x_5 = 0.3473 \quad f(x_5) = -0.0000096$

Hence, the root after four iterations is **0.347**

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Question 2

Use the secant method to solve the equation $e^x - 3x^2$ for $0 \le x \le 1$. (Perform only 3 iterations.)

Solution

$f(x) = e^x - 3x^2$, $x_0 = 0$, $x_1 = 1$

both are the initial approximations

$f(0) = e^0 - 3(0) = 1$

$f(1) = e^1 - 3(1) = -0.281$

now we calculate the second approximation

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_0) - f(x_1)} = \frac{(0)(-0.281) - (1)1}{-0.281 - 1} = 0.7806$$

$f(0.7806) = e^{0.7806} - 3(0.7806)^2 = 2.1827 - 1.82800 = 0.3546$

now $x_1 = 1$ *and* $x_2 = 0.786$

$f(1) = -0.281 \quad f(0.786) = 0.3546$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \frac{1(0.3546) - (0.786)(-0.281)}{0.3546 + 0.281} = 0.9052$$

now

$x_3 = 0.9052 \qquad f(0.9052) = 2.71^{0.9052} - 3(0.9052)^2 = 0.0074$

$x_2 = 0.786 \qquad f(0.786) = 0.3546$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} = \frac{0.7806(0.0074) - (0.9052)(0.3546)}{0.0074 + 0.3546} = 0.9076$$

so root is 0.9076

Muller's Method

Question 1

Solve the equation $x^3 - 7x^2 + 14x - 6$ by using Muller's method only perform three Iterations.(

$x_0 = 0.5, x_1 = 1, x_2 = 0$)

Solution

1st iteration

$x_0 = 0.5$, $x_1 = 1$ and $x_2 = 0$

$$f(x_0) = f_0 = f(0.5) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6 = -0.625$$
$$f(x_1) = f_1 = f(1) = 1^3 - 7(1)^2 + 14(1) - 6 = 2$$
$$f(x_2) = f_2 = f(0) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$$

$$c = f_0 = -0.625$$
$$h_1 = x_1 - x_0 = 1 - 0.5 = 0.5$$
$$h_2 = x_0 - x_2 = 0.5 - 0 = 0.5$$

$$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$$
$$a = \frac{(0.5)(2) - (0.5 + 0.5)(-0.625) + (0.5)(-6)}{(0.5)(0.5)(0.5 + 0.5)} = \frac{1 - (1)(-0.625) + (0.5)(-6)}{(0.5)(0.5)(1)} = -5.5$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1}$$
$$b = \frac{2 - (-0.625) - (-5.5)(0.5)^2}{0.5} = \frac{2 + 0.625) + (5.5)(0.25)^2}{0.5} = 8$$

$$x = x_0 - \frac{2c}{b - \sqrt{b^2 - 4ac}}$$
$$x = 0.5 - \frac{2(-0.625)}{8 - \sqrt{(8)^2 - (4)(-5.5)(-0.625)}}$$
$$x = 1.8721$$

2nd iteration

$x_0 = 0.5$, $x_1 = 1.8721$ and $x_2 = 1$

$$f(x_0) = f_0 = f(0.5) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6 = -0.625$$
$$f(x_1) = f_1 = f(1) = 1.8721^3 - 7(1.8721)^2 + 14(1.8721) - 6 = 2.228$$

$$f(x_2) = f_2 = f(0) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$$

$$c = f_0 = -0.625$$
$$h_1 = x_1 - x_0 = 1.8721 - 0.5 = 1.3721$$
$$h_2 = x_0 - x_2 = 0.5 - 1 = -0.5$$

$$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$$
$$a = \frac{(-0.5)(2.228) - (1.3721 - 0.5)(-0.625) + (1.3721)(-6)}{(1.3721)(-0.5)(1.3721 - 0.50.5)} = 14.70$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1}$$
$$b = \frac{2.228 - (-0.625) - (14.70)(1.3721)^2}{0.5} = -17.72$$

$$x = x_0 - \frac{2c}{b - \sqrt{b^2 - 4ac}}$$
$$x = 0.5 + \frac{1.25}{12.62 - \sqrt{(17.72)^2 - (4)(14.71)(-0.625)}} = 0.29$$
$$x = 1.8721$$

For 3rd **iteration**

$x_0 = 0.29$, $x_1 = 1.8721$, $x_2 = 1$

For third iteration we will proceed in the same manner.

Graeffe's Root Square Method

Linear Equations

Gaussian Elimination Method

Question 2

Using Gaussian Elimination Method, solve the following system of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$x_1 + 3x_2 + 2x_3 = 13$$
$$3x_1 + x_2 + 3x_3 = 14$$

Solution:

Marks: 10

The Augmented Matrix of the given system of equations is

$$A_b = \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad by \quad R_2 - R_1 \; and \; R_3 - 3R_1$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad by \quad \frac{1}{2}R_2$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \quad by \quad R_3 + (-2)R_2$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad by \quad \frac{-1}{3}R_3$$

Which shows that from the third and second rows

Z=3, y = 2
And from the first row

X+y+2z=9

Using the values of y and z, we get x = 1
Hence the solution of the given system is
X = 1, y = 2, z = 3
Question 1
Using Gaussian Elimination Method, solve the following system of equations

$$2x - y + 2z = 2$$
$$x + 10y - 3z = 5$$
$$x - y - z = 3$$

Marks: 10

Solution:
The Augmented Matrix of the given system of equations is

$$A_b = \begin{bmatrix} 2 & -1 & 2 & 2 \\ 1 & 10 & -3 & 5 \\ 1 & -1 & -1 & 3 \end{bmatrix}$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 2 & -1 & 2 & 2 \\ 1 & -1 & -1 & 3 \end{bmatrix} \quad by \quad R_{12}$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & -21 & 8 & -8 \\ 0 & -11 & 2 & -2 \end{bmatrix} \quad by \quad R_2 - 2R_1, R_3 - R_1$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & 1 & -\frac{8}{21} & \frac{8}{21} \\ 0 & -11 & 2 & -2 \end{bmatrix} \quad by \ \frac{-1}{21} R_2$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & 1 & -\frac{8}{21} & \frac{8}{21} \\ 0 & 0 & -\frac{46}{21} & \frac{46}{21} \end{bmatrix} \quad by \quad R_3 + 11R_2$$

Using Gaussian Elimination method, by backward substitution, we get as follows
From the third row, we get

$$-\frac{46}{21} z = \frac{46}{21}$$

$$\Rightarrow z = -1$$

From the second row, we get

$$y - \frac{8}{21} z = \frac{8}{21}$$

$$\Rightarrow y = \frac{8}{21} + \frac{8}{21} z$$

Putting the value of z, *we get*

$$y = \frac{8}{21} + \frac{8}{21}(-1)$$

$$= \frac{8}{21} - \frac{8}{21}$$

$$= 0$$

And finally from the first row, we get

$$x + 10y - 3z = 5$$

Putting the values of y and z, we get

$$x + 10(0) - 3(-1) = 5$$

$$\Rightarrow x + 3 = 5$$

$$\Rightarrow x = 2$$

So, the solution is

$$x = 2, y = 0, z = -1$$

Guass-Jordan Elimination Method

Question#1 **Marks 10**

By using Gauss –Jordan elimination method, solve the following system of equations,

$$x + y + z = 7$$
$$3x + 3y + 4z = 24$$
$$2x + y + 3z = 16$$

Solution:

The given system in matrix form is

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 24 \\ 16 \end{bmatrix}$$

$$A \quad X = B$$

$$\therefore [A \mid B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 7 \\ 3 & 3 & 4 & 24 \\ 2 & 1 & 3 & 16 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 7 \\ 2 & 1 & 3 & 16 \\ 3 & 3 & 4 & 24 \end{array}\right] \quad R_{23}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 7 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 2 & 9 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to R_1 + R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to R_1 - 2R_3, R_2 \to R_2 - R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad -R_2$$

So,

$x = 3, y = 1, z = 3$

Jacobi's Iterative Method

Solve the following system of equations

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

By Jacobi's iterative method taking the initial starting of solution vector as $(0,0,0)^T$ and perform the first three iterations.

Solution:

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$
$$x = \frac{17 - y + 2z}{20}$$
$$y = \frac{-18 - 3x + z}{20}$$
$$z = \frac{25 - 2x + 3y}{20}$$

starting with $(0,0,0)$

Iteration#01

$$x = \frac{17 - 0 + 2(0)}{20} = \frac{17}{20} = 0.85$$
$$y = \frac{-18 - 3(0) + 0}{20} = \frac{-18}{20} = -0.9$$
$$z = \frac{25 - 2(0) + 3(0)}{20} = \frac{25}{20} = 1.25$$

Iteration#02

$x = 0.85,\ y = -0.9,\ z = 1.25$

$$x = \frac{17 - (-0.9) + 2(1.25)}{20} = \frac{20.4}{20} = 1.02$$
$$y = \frac{-18 - 3(0.85) + 1.25}{20} = \frac{-19.3}{20} = -0.965$$
$$z = \frac{25 - 2(0.85) + 3(-0.9)}{20} = \frac{20.6}{20} = 1.03$$

Iteration#03

$x = 1.02,\ y = -0.965,\ z = 1.03$

$$x=\frac{17-(-0.965)+2(1.03)}{20}=\frac{20.025}{20}=1.00125$$
$$y=\frac{-18-3(1.02)+1.03}{20}=\frac{-20.03}{20}=-1.0015$$
$$z=\frac{25-2(1.02)+3(-0.965)}{20}=\frac{20.065}{20}=1.00325$$

Gauss-Seidel Iteration Method

Question#3 **Marks 10**

Solve Question No. #2 by Gauss-Seidel iterative method and perform first three iterations. What you see the difference after solving the same question by two different iterative methods? Give your comments.

Solution:

The above system of linear equations is diagonally dominant; therefore, Gauss-Seidel iterative method could be applied to find out real roots

$$20x+y-2z=17$$
$$3x+20y-z=-18$$
$$2x-3y+20z=25$$

The above system of equations could be written in the form

$$x=\frac{17-y+2z}{20}$$
$$y=\frac{-18-3x+z}{20}$$
$$z=\frac{25-2x+3y}{20}$$

starting with $(0,0,0)$

Iteration#01

$$x=\frac{17-0+2(0)}{20}=\frac{17}{20}=0.85$$
$$y=\frac{-18-3(0.85)+0}{20}=\frac{-20.55}{20}=-1.0275$$
$$z=\frac{25-2(0.85)+3(-1.0275)}{20}=\frac{20.2175}{20}=1.010875$$

Iteration#02

$x=0.85, y=-1.0275, z=1.010875$

$$x=\frac{17-(-1.0275)+2(1.010875)}{20}=\frac{20.04925}{20}=1.0024625$$
$$y=\frac{-18-3(1.0024625)+1.010875}{20}=\frac{-19.9965125}{20}=-0.999825625$$
$$z=\frac{25-2(1.0024625)+3(-0.999825625)}{20}=\frac{19.995598125}{20}=0.99977990625$$

Iteration#03

$x=1.0024625,\ y=-0.999825625,\ z=0.99977990625$

$$x=\frac{17-(-0.999825625)+2(0.99977990625)}{20}=\frac{19.9904160625}{20}=0.9999708$$
$$y=\frac{-18-3(0.9999708)+0.99977990625}{20}=\frac{-20.00013249375}{20}=-1.0000066247$$
$$z=\frac{25-2(0.9999708)+3(-1.0000066247)}{20}=\frac{20.000038526}{20}=1.0000019263$$

In Gauss Siedal Method, the newly computed values in each iteration are directly involved to find the other value of the system of equation and save memory for computation and hence results are more accurate.

==========

Question 2

Do five iterations to solve the following system of equations by Gauss-Seidal iterative method

$$10x-2y-3z=305$$
$$-2x+10y-2z=154$$
$$-2x-y+10z=120$$

Marks: 10

Solution:

Since, the given system is diagonally dominant; hence we can apply here the Gauss-Seidal method.

From the given system of equations

$$x^{r+1}=\frac{1}{10}\left[305+2y^{r}+3z^{r}\right]$$
$$y^{r+1}=\frac{1}{10}\left[154+2x^{r+1}+2z^{r}\right]$$
$$z^{r+1}=\frac{1}{10}\left[120+2x^{r+1}+y^{r+1}\right]$$

ITERATION 1

For r =0

Taking y=z=0 on right hand side of first equation. In second equation we take z=0 and current value of x. In third equation we take current value of both x and y.

$$x^1 = \frac{1}{10}[305 + 2(0) + 3(0)] = \frac{305}{10} = 30.5$$

$$y^1 = \frac{1}{10}[154 + 2(30.5) + 2(0)] = \frac{1}{10}[154 + 61] = \frac{215}{10} = 21.5$$

$$z^1 = \frac{1}{10}[120 + 2(30.5) + (21.5)] = \frac{1}{10}[120 + 61 + 21.5] = \frac{202.5}{10} = 20.25$$

ITERATION 2

Similar procedure as used in Iteration 1 will be used for Iterations 2, 3, 4 and 5.

$$x^2 = \frac{1}{10}[305 + 2(21.5) + 3(20.25)] = \frac{408.75}{10} = 40.875$$

$$y^2 = \frac{1}{10}[154 + 2(40.875) + 2(20.25)] = \frac{276.25}{10} = 27.625$$

$$z^2 = \frac{1}{10}[120 + 2(40.875) + (27.625)] = \frac{229.375}{10} = 22.938$$

ITERATION 3

$$x^3 = \frac{1}{10}[305 + 2(27.625) + 3(22.938)] = \frac{429.064}{10} = 42.906$$

$$y^3 = \frac{1}{10}[154 + 2(42.906) + 2(22.938)] = \frac{285.688}{10} = 28.569$$

$$z^3 = \frac{1}{10}[120 + 2(42.906) + (28.569)] = \frac{234.381}{10} = 23.438$$

ITERATION 4

$$x^4 = \frac{1}{10}[305 + 2(28.569) + 3(23.438)] = \frac{432.452}{10} = 43.245$$

$$y^4 = \frac{1}{10}[154 + 2(43.245) + 2(23.438)] = \frac{287.366}{10} = 28.737$$

$$z^4 = \frac{1}{10}[120 + 2(43.245) + (28.737)] = \frac{235.227}{10} = 23.523$$

ITERATION 5

$$x^5 = \frac{1}{10}[305 + 2(28.737) + 3(23.523)] = \frac{433.043}{10} = 43.304$$

$$y^5 = \frac{1}{10}[154 + 2(43.304) + 2(23.523)] = \frac{287.654}{10} = 28.765$$

$$z^5 = \frac{1}{10}[120 + 2(43.304) + (28.765)] = \frac{235.373}{10} = 23.537$$

Above Results are summarized in tabular form as

	Variables		
Iterations	**x**	**y**	**z**
1	30.5	21.5	20.25
2	40.875	27.625	22.938
3	42.906	28.569	23.438

4	43.245	28.737	23.523
5	43.304	28.765	23.537

Hence the solution of the given system of equations, after five iterations, is

$x = 43.304$

$y = 28.765$

$z = 23.537$

Relaxation Method

Matrix Inversion

Eigen Value Problems

Power Method

Question 1

Find the largest eigen value of the matrix

$$\begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix}$$

MARKS 10

And the corresponding eigenvector, by Power Method after fourth iteration starting with the initial vector $v^{(0)} = (0,0,1)^T$

SOLUTION

Let $A = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix}$

Choosing an initial vector as $v^{(0)} = (0,0,1)^T$ then

ITERATION 1

$$u^{(1)} = [A]v^{(0)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$$

Now we normalize the resultant vector to get

$$u^{(1)} = 4 \begin{pmatrix} 0 \\ \frac{1}{4} \\ 1 \end{pmatrix} = q_1 v^{(1)}$$

Continuing this procedure for subsequent Iterations , we have

ITERATION 2

$$u^{(2)} = [A]v^{(1)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ 6 \\ 4 \end{pmatrix}$$

$$u^{(2)} = 6 \begin{pmatrix} \frac{1}{24} \\ 1 \\ \frac{2}{3} \end{pmatrix} = q_2 v^{(2)}$$

ITERATION 3

$$u^{(3)} = [A]v^{(2)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} \frac{1}{24} \\ 1 \\ \frac{2}{3} \end{pmatrix} = \begin{pmatrix} 1.167 \\ 20.708 \\ 2.667 \end{pmatrix}$$

$$u^{(3)} = 20.708 \begin{pmatrix} 0.056 \\ 1 \\ 0.129 \end{pmatrix} = q_3 v^{(3)}$$

ITERATION 4

$$u^{(4)} = [A]v^{(3)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0.056 \\ 1 \\ 0.129 \end{pmatrix} = \begin{pmatrix} 1.224 \\ 20.185 \\ 0.516 \end{pmatrix}$$

$$u^{(4)} = 20.185 \begin{pmatrix} 0.061 \\ 1 \\ 0.026 \end{pmatrix} = q_4 v^{(4)}$$

Therefore, the largest eigen value and the corresponding eigen vector accurate to three decimals places are

$\lambda = 20.185$

$$(X) = \begin{pmatrix} 0.061 \\ 1 \\ 0.026 \end{pmatrix}$$

==========

Question 1 **marks 10**

Find the largest eigen value of the matrix

$$\begin{bmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{bmatrix}$$

And the corresponding eigen vector, by Power Method after fourth iteration starting with the initial vector $v^{(0)} = (0,1,0)^T$

SOLUTION

Let $A = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix}$

Choosing an initial vector

$v^{(0)} = (0,1,0)^T$ then

ITERATION 1

$$u^{(1)} = [A]v^{(0)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 10 \end{pmatrix}$$

Now we normalize the resultant vector to get

$$u^{(1)} = 10\begin{pmatrix} 0.5 \\ 0.1 \\ 1 \end{pmatrix} = q_1 v^{(1)}$$

Continuing this procedure for subsequent Iterations, we have

ITERATION 2

$$u^{(2)} = [A]v^{(1)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5.5 \\ 5.6 \\ 4.5 \end{pmatrix}$$

$$u^{(2)} = 5.6\begin{pmatrix} \frac{5.5}{5.6} \\ \frac{5.6}{5.6} \\ \frac{4.5}{5.6} \end{pmatrix} = \begin{pmatrix} 0.982 \\ 1 \\ 0.804 \end{pmatrix} = q_2 v^{(2)}$$

ITERATION 3

$$u^{(3)} = [A]v^{(2)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix} \begin{pmatrix} 0.982 \\ 1 \\ 0.804 \end{pmatrix} = \begin{pmatrix} 12.5 \\ 7.162 \\ 13.394 \end{pmatrix}$$

$$u^{(3)} = 13.394 \begin{pmatrix} 0.933 \\ 0.535 \\ 1 \end{pmatrix} = q_3 v^{(3)}$$

ITERATION 4

$$u^{(4)} = [A]v^{(3)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix} \begin{pmatrix} 0.933 \\ 0.535 \\ 1 \end{pmatrix} = \begin{pmatrix} 10.273 \\ 7.334 \\ 9.283 \end{pmatrix}$$

$$u^{(4)} = 10.273 \begin{pmatrix} 1 \\ 0.714 \\ 0.904 \end{pmatrix} = q_4 v^{(4)}$$

Therefore, the largest eigen value and the corresponding eigen vector accurate to four decimals places are

$\lambda = 10.273$

and

$$(X) = \begin{pmatrix} 1 \\ 0.714 \\ 0.904 \end{pmatrix}$$

Jacobi's Method

Question 1

Using Jacobi's method, find all the eigenvalues and the corresponding eigenvectors of the following matrix,

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Marks: 10

Note: Give results at the end of third rotation.

Solution.

$$Let\ A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$Then,\ A^t = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Hence,

Matrix A is real and Symmetric and Jacobi's method can be applied.

Rotation 1

In Matrix A, all the off-diagonal elements are found be 2.
So, the largest off-diagonal element is found to be
$a_{12} = a_{13} = a_{21} = a_{23} = a_{31} = a_{32} = 2$.

Therefore, we can choose any one of them as the largest element.

Suppose, we choose a_{12} as the largest element

Then, we compute the rotation angle as,

$$\tan 2\theta = \frac{2a_{12}}{a_{11} - a_{22}} = \frac{2 \times 2}{1-1} = \infty$$

Therefore

$$\tan 2\theta = \infty$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \boldsymbol{\theta = \frac{\pi}{4}}$$

Therefore, we construct an orthogonal matrix S_1 such that,

$$S_1 = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now the first rotation gives,

$$D_1 = S_1^{-1} A S_1$$

Here, $S_1^{-1} = = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So,

$D_1 = S_1^{-1} A S_1$

i.e.

$$D_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 2.828 \\ 0 & -1 & 0 \\ 2.828 & 0 & 1 \end{bmatrix}$$

To check that we are right in our calculations, we can see that the sum of the diagonal elements is, $3+(-1)+(1)=3$, which is same as the sum of the diagonal elements of the original matrix A.

Rotation 2

For the second rotation we choose the largest off-diagonal element $d_{13} = d_{31} = 2.828$, then

$$\tan 2\theta = \frac{2d_{13}}{d_{11} - d_{33}} = \frac{2(2.828)}{3-(1)} = \frac{5.657}{2} = 2.828$$

So,

$2\theta = \tan^{-1}(2.828) = 70.526$

$\theta = 35.263$

Therefore, we construct an orthogonal matrix S_2 such that,

$$S_2 = \begin{bmatrix} \cos(35.263) & 0 & -\sin(35.263) \\ 0 & 1 & 0 \\ \sin(35.263) & 0 & \cos(35.263) \end{bmatrix}$$

$$= \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$S_2^{-1} = = \begin{bmatrix} 0.817 & 0 & 0.577 \\ 0 & 1 & 0 \\ -0.577 & 0 & 0.817 \end{bmatrix}$$

Now the rotation 2 gives,

$$D_2 = S_2^{-1} D_1 S_2$$

$$D_2 = \begin{bmatrix} 0.817 & 0 & 0.577 \\ 0 & 1 & 0 \\ -0.577 & 0 & 0.817 \end{bmatrix} \begin{bmatrix} 3 & 0 & 2.828 \\ 0 & -1 & 0 \\ 2.828 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Again to check that we are right in our calculations, we can see that the sum of the diagonal elements is $5+(-1)+(-1)=3$ which is same as the sum of the diagonal elements of the original matrix A.

Rotation 3

We can see that in above iteration that D_2 is a diagonal matrix, so we stop here and take the ***Eigen Values*** as

$$\lambda_1 = 5 \ , \ \lambda_2 = -1 \ , \ \lambda_3 = -1$$

Now the Eigenvectors are the columns vectors of the matrix $S = S_1 S_2$, which are,

$$S = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$= \begin{bmatrix} 0.577 & -0.707 & -0.408 \\ 0.577 & 0.707 & -0.408 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

Therefore, the corresponding ***EigenVectors*** are

$$X_1 = \begin{bmatrix} 0.577 \\ 0.577 \\ 0.577 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -0.707 \\ 0.707 \\ 0 \end{bmatrix}, \quad X_3 = \begin{bmatrix} -0.408 \\ -0.408 \\ 0.817 \end{bmatrix}$$

Interpolation

For a given table of values (,), 0,1,2,..., k k x y k= nwith equally spaced abscissas of a function y= f(x),we define the forward difference operator Δ as follows,

$\Delta y_i = y_{i+1} - y_i, \qquad i = 0,1,\ldots,(n-1)$

These differences are called first differences of the function y and are denoted by the symbol Δy_i Here, Δ is called the first difference operator.

Similarly, rth Difference operator would be $\Delta^r y_i = \Delta^{r-1} y_{i+1} - \Delta^{r-1} y_i$

Leading term =y_o

Leading Diffenrence=Δy

Backward Difference Operators:

$\nabla y_i = y_i - y_{i-1} i = n, \quad (n-1),\ldots,1$

Central Difference is given by,

$\delta y_i = y_{i+(1/2)} - y_{i-(1/2)}$

$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\vdots \quad \vdots \quad \vdots$$
$$\Delta y_{n-1} = y_n - y_{n-1}$$

$$\nabla y_i = y_i - y_{i-1} i = n, \quad (n-1), \ldots, 1$$

OR

$$\nabla y_1 = y_1 - y_0$$
$$\nabla y_2 = y_2 - y_1$$
$$\vdots \quad \vdots \quad \vdots$$
$$\nabla y_n = y_n - y_{n-1}$$

$$\delta y_{1/2} = y_1 - y_0, \qquad \delta y_{3/2} = y_2 - y_1,$$

In General,

$$\delta y_i = y_{i+(1/2)} - y_{i-(1/2)}$$

Higher order differences are defined as follows:

$$\delta^2 y_i = \delta y_{i+(1/2)} - \delta y_{i-(1/2)}$$

$$\delta^n y_i = \delta^{n-1} y_{i+(1/2)} - \delta^{n-1} y_{i-(1/2)}$$

Shift operator, E

Let y = f (x) be a function of x, and let x takes the consecutive values x, x + h, x + 2h, etc. We then define an operator having the property

E f(x) = f(x+h)

Thus, when E operates on f (x), the result is the next value of the function. Here, E is called the shift operator. If we apply the operator E twice on f (x), we get

$E^2 f(x) = E[E f(x)]$

$= E[f(x+h)] = f(x+2h)$

Thus, in general, if we apply the operator 'E' n times on f (x), we get

$E^n f(x) = f(x+nh)$

The inverse operator E is defined as

$$E^{-1} f(x) = f(x-h)$$

Similarly

$$E^{-n} f(x) = f(x-nh)$$

Average Operator, μ;

it is defined as

$$\mu f(x) = \frac{1}{2}\left[f\left(x + \frac{h}{2}\right) + f\left(x - \frac{h}{2}\right)\right]$$

$$= \frac{1}{2}\left[y_{x+(h/2)} + y_{x-(h/2)}\right]$$

Differential Operator, D

it is defined as

$$Df(x) = \frac{d}{dx} f(x) = f'(x)$$
$$D^2 f(x) = \frac{d^2}{dx^2} f(x) = f''(x)$$

$$\Rightarrow \Delta = E - 1$$
$$\Rightarrow \nabla = 1 - E^{-1} = \frac{E-1}{E}$$
$$\delta = E^{1/2} - E^{-1/2}$$
$$\Rightarrow \mu = \frac{1}{2}(E^{1/2} + E^{-1/2})$$
$$hD = \log E$$

Newton Forward Difference Interpolation.

for any real number p, we have the operator E such that

$$E^p f(x) = f(x + ph).$$
$$f(x_0 + ph) = E^p f(x_0) = (1+\Delta)^p f(x_0)$$
$$= \left[1 + p\Delta + \frac{p(p-1)}{2!}\Delta^2 + \frac{p(p-1)(p-2)}{3!}\Delta^3 + \cdots\right] f(x_0)$$
$$f(x_0 + ph) = f(x_0) + p\Delta f(x_0)$$
$$+ \frac{p(p-1)}{2!}\Delta^2 f(x_0) + \frac{p(p-1)(p-2)}{3!}\Delta^3 f(x_0)$$
$$+ \cdots + \frac{p(p-1)\cdots(p-n+1)}{n!}\Delta^n f(x_0) + \text{Error}$$

Forward Differences

Question 2

Construct a forward difference table from the following values of x and y.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y=f(x)	2.0	3.5	-1.7	2.3	4.2	6.5	5.7

Marks:

10

Solution.

Forward-difference table

x	**y**	**Δy**	**Δ^2y**	**Δ^3y**	**Δ^4y**	**Δ^5y**	**Δ^6y**
1.0	2						
		1.5					

1.2	3.5		-6.7				
		-5.2		15.9			
1.4	-1.7		9.2		-27.2		
		4		-11.3		41	
1.6	2.3		-2.1		13.8		-60.8
		1.9		2.5		-19.8	
1.8	4.2		0.4		-6		
		2.3		-3.5			
2.0	6.5		-3.1				
		-0.8					
2.2	5.7						

Backward Differences

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Marks:

10

Solution.

Backward-difference table

x	**y**	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	2						
		1.5					
1.2	3.5		-6.7				
		-5.2		15.9			
1.4	-1.7		9.2		-27.2		
		4		-11.3		41	
1.6	2.3		-2.1		13.8		-60.8
		1.9		2.5		-19.8	
1.8	4.2		0.4		-6		
		2.3		-3.5			

2.0	6.5		-3.1				
		-0.8					
2.2	5.7						

==========

Question 2 **marks 10**

Form a table of forward and backward differences of the function

$f(x) = x^3 - 3x^2 - 5x - 7$ For $x = -1, 0, 1, 2, 3, 4, 5$

SOLUTION

For the given function, the values of y for the given values of x are calculated as

$for\ x = -1$

$f(-1) = (-1)^3 - 3(-1)^2 - 5(-1) - 7 = -6$

$for\ x = 0$

$f(0) = (0)^3 - 3(0)^2 - 5(0) - 7 = -7$

$for\ x = 1$

$f(1) = (1)^3 - 3(1)^2 - 5(1) - 7 = -14$

$for\ x = 2$

$f(2) = (2)^3 - 3(2)^2 - 5(2) - 7 = -21$

$for\ x = 3$

$f(3) = (3)^3 - 3(3)^2 - 5(3) - 7 = -22$

$for\ x = 4$

$f(4) = (4)^3 - 3(4)^2 - 5(4) - 7 = -11$

$for\ x = 5$

$f(5) = (5)^3 - 3(5)^2 - 5(5) - 7 = 18$

So, the table of values of x and y is

x	-1	0	1	2	3	4	5
$y = f(x)$	-6	-7	-14	-21	-22	-11	18

Forward difference table

Forward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
-1	-6						
		-1					

0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Backward difference table

Backward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Divided Differences

Question 1 **marks 10**

For the following table of values, estimate $f(2.5)$ using Newton's forward difference interpolation formula.

x	1	2	3	4	5	6	7	8
$y = f(x)$	1	8	27	64	125	216	343	512

SOLUTION

Forward difference table

Forward difference table for the given values of x and y is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1	1						
		7					
2	8		12				
		19		6			
3	27		18		0		
		37		6		0	
4	64		24		0		0
		61		6		0	
5	125		30		0		
		91		6			
6	216		36				
		127					
7	343		42				
		169					
8	512						

Newton's forward difference interpolation formula is given by

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$
$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots + \frac{p(p-1)(p-2).....(p-n+1)}{n!}\Delta^n y_0$$

Here

$$p = \frac{x - x_0}{h} = \frac{2.5-1}{1} = \frac{1.5}{1} = 1.5$$

And

$$y_0 = 1, \Delta y_0 = 7, \Delta^2 y_0 = 12, \Delta^3 y_0 = 6, \Delta^4 y_0 = \Delta^5 y_0 = \Delta^6 y_0 = 0$$

So, by putting the above values in Newton's forward difference interpolation formula, We have

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$
$$= 1 + 1.5(7) + \frac{1.5(1.5-1)}{2!}(12) + \frac{1.5(1.5-1)(1.5-2)}{3!}(6)$$
$$= 1 + 10.5 + 4.5 - 0.375$$
$$= 15.625$$

i.e.

$y_{2.5} = 15.625$

==========

Question 2 **marks 10**

Compute $f(1.5)$ for the following data by using Newton's divided difference interpolation formula.

x	1	2	4	5	8
$f(x)$	5	14	28	45	92

SOLUTION

The divided difference table for the given data is given by

x	y	1st D.D.	2nd D.D.	3rd D.D.	4th D.D.
1	5				
		9			
2	14		-2/3		
		7		1	
4	28		10/3		-29/126
		17		-11/18	
5	45		-1/3		
		47/3			
8	92				

Newton's Divided Difference formula is

$$f(x) = y_0 + (x-x_0)y[x_0,x_1] + (x-x_0)(x-x_1)y[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)y[x_0,x_1,x_2,x_3]$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)y[x_0,x_1,x_2,x_3,x_4]$$

$$f(1.5) = 5 + (1.5 - 1)\,9 + (1.5\text{-}1)(1.5\text{-}2)(-\frac{2}{3}) + (1.5\text{-}1)\,(1.5\text{-}2)(1.5\text{-}4)1$$
$$+(1.5\text{-}1)\,(1.5\text{-}2)(1.5\text{-}4)(1.5\text{-}5)(-\frac{29}{126})$$
$$=5+4.5+0.1667+0.625+0.5035$$
$$=10.7952$$

Question 3 **marks 10**

Find $y'(6)$ *and* $y''(6)$ from the following table of values.

x	1	2	3	4	5	6
y	3	9	17	27	40	55

SOLUTION

Backward difference table

Backward difference table for the given values of x and y is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1	3					
		6				
2	9		2			
		8		0		
3	17		2		1	
		10		1		-3
4	27		3		-2	
		13		-1		
5	40		2			
		15				
6	55					

By backward difference formula for first derivative, we have

$$y_n' = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4}\right)$$

$$y_n' = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \frac{\nabla^5 y_n}{5}\right)$$

$$y'(6) = \frac{1}{1}\left(15 + \frac{2}{2} + \frac{-1}{3} + \frac{-2}{4} + \frac{-3}{5}\right)$$

$$= 15 + 1 - 0.333 - 0.5 - 0.6$$

$$= 14.567$$

By backward difference formula for second derivative, we have

$$y_n'' = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n\right)$$

Therefore

$$y''(6) = \frac{1}{1^2}[2 + (-1) + \frac{11}{12}(-2) + \frac{5}{6}(-3)]$$

$$= 2 - 1 - 1.833 - 2.5$$

$$= -3.333$$

Question#1 **Marks 10**

Find an equation of a cubic curve which passes through the points (4,-43), (7, 83), (9,327) and (12, 1053) using Divided Difference Formula.

Solution:

Newton's divided difference table is

X	Y	1st D.D	2nd D.D	3rd D.D
4	-43			
7	83	42		
9	327	122	16	
12	1053	242	24	1

Newton's Divided Difference formula is

$$y = f(x) = y_0 + (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2] + ...$$
$$+ (x - x_0)(x - x_1)....(x - x_{n-1})y[x_0, x_1, ...x_n]$$

now putting values in formula

$$y = f(x) = -43 + (x-4)(42) + (x-4)(x-7)(16) + (x-4)(x-7)(x-9)(1)$$
$$= -43 + (x-4)\{42 + 16x - 112 + x^2 - 16x + 63\}$$
$$= -43 + (x-4)(x^2 - 7)$$
$$= -43 + x^3 - 7x - 4x^2 + 28$$
$$= x^3 - 4x^2 - 7x - 15$$

Hence it is required polinomial

Question 2 **marks 10**

Form a table of forward and backward differences of the function

$f(x) = x^3 - 3x^2 - 5x - 7$ For $x = -1, 0, 1, 2, 3, 4, 5$

SOLUTION

For the given function, the values of y for the given values of x are calculated as

for $x = -1$

$f(-1) = (-1)^3 - 3(-1)^2 - 5(-1) - 7 = -6$

for $x = 0$

$f(0) = (0)^3 - 3(0)^2 - 5(0) - 7 = -7$

for $x = 1$

$f(1) = (1)^3 - 3(1)^2 - 5(1) - 7 = -14$

for $x = 2$

$f(2) = (2)^3 - 3(2)^2 - 5(2) - 7 = -21$

for $x = 3$

$f(3) = (3)^3 - 3(3)^2 - 5(3) - 7 = -22$

for $x = 4$

$f(4) = (4)^3 - 3(4)^2 - 5(4) - 7 = -11$

for $x = 5$

$f(5) = (5)^3 - 3(5)^2 - 5(5) - 7 = 18$

So, the table of values of x and y is

x	-1	0	1	2	3	4	5
$y = f(x)$	-6	-7	-14	-21	-22	-11	18

Forward difference table

Forward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Backward difference table

Backward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11					

				6			
4	-11		18				
		29					
5	18						

Question 1 **Marks 10**

Find the interpolating polynomial for the following data by

a) Newton's Forward Difference Formula

b)Lagrange's Formula

Hence show that both the methods give raise to the same polynomial.

x	0	1	2	3
y	1	3	7	13

Solution:

By Newton's Forward Difference Formula

The Newton's Forward difference table is given as

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		2		
1	3		2	
		4		0
2	7		2	
		6		
3	13			

We have Newton's forward difference interpolation formula as

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots \qquad (1)$$

Since, the third and higher order differences are zero; Therefore, Newton's forward difference Interpolation formula reduces to

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 \qquad (2)$$

Here,

$x_0 = 0,\ y_0 = 1,\ \Delta y_0 = 2,\ \Delta^2 y_0 = 2,\ h = 1$

And p is given by

$$p=\frac{x-x_0}{h}=\frac{x-0}{1}=x$$

Substituting these values in Eq.(2), we have

$$y=1+x(2)+\frac{x(x-1)}{2}(2)$$
$$=1+2x+x(x-1)$$
$$=1+2x+x^2-x$$
$$=x^2+x+1$$

Hence, the required polynomial is

$$y=x^2+x+1.$$

b) By Lagrange's Formula

Lagrange's Interpolation formula is given by

$$y=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$
$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

By putting the values in the formula, we get

$$y=\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}(1)+\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}(3)$$
$$+\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}(7)+\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}(13)$$

$$y=\frac{x^3-6x^2+11x-6}{(-1)(-2)(-3)}+\frac{3x^3-15x^2+18x}{(1)(-1)(-2)}$$
$$+\frac{7(x^3-4x^2+3x)}{(2)(1)(-1)}+\frac{13(x^3-3x^2+2x)}{(3)(2)(1)}$$

$$y=\frac{x^3-6x^2+11x-6}{-6}+\frac{3x^3-15x^2+18x}{2}$$
$$+\frac{7x^3-28x^2+21x}{-2}+\frac{13x^3-39x^2+26x}{6}$$

$$y=\frac{x^3-6x^2+11x-6}{-6}+\frac{3x^3-15x^2+18x}{2}+\frac{7x^3-28x^2+21x}{-2}+\frac{13x^3-39x^2+26x}{6}$$
$$=\frac{-x^3+6x^2-11x+6+9x^3-45x^2+54x-21x^3+84x^2-63x+13x^3-39x^2+26x}{6}$$

$$y = \frac{-x^3 + 9x^3 - 21x^3 + 13x^3 + 6x^2 - 45x^2 + 84x^2 - 39x^2 - 11x + 54x - 63x + 26x + 6}{6}$$

$$= \frac{6x^2 + 6x + 6}{6}$$

$$= \frac{6(x^2 + x + 1)}{6}$$

$$= x^2 + x + 1$$

Thus we have the equation

$$y = x^2 + x + 1$$

This is the same polynomial as obtained in Newton's Forward difference interpolating formula. Hence it is proved that both the methods give rise to the same polynomial

Langrange's Interpolation

Find interpolation polynomial by Lagrange's Formula, with the help of following table,

x	1	2	3	5
f(x)	0	7	26	124

And hence find value of f (6).

Solution:

Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$y = f(x) = \frac{(x-2)(x-3)(x-5)}{(1-2)(1-3)(1-5)}(0) + \frac{(x-1)(x-3)(x-5)}{(2-1)(2-3)(2-5)}(7) + \frac{(x-1)(x-2)(x-5)}{(3-1)(3-2)(3-5)}(26)$$
$$+ \frac{(x-1)(x-2)(x-3)}{(5-1)(5-2)(5-3)}(124)$$

$$= 0 + \frac{7\left(x^3-9x^2+23x-15\right)}{3} - \frac{26\left(x^3-8x^2+17x-10\right)}{4} + \frac{124\left(x^3-6x^2+11x-6\right)}{24}$$

$$= \frac{7\left(x^3-9x^2+23x-15\right)}{3} - \frac{13\left(x^3-8x^2+17x-10\right)}{2} + \frac{31\left(x^3-6x^2+11x-6\right)}{6}$$

$$= \frac{14\left(x^3-9x^2+23x-15\right) - 39\left(x^3-8x^2+17x-10\right) + 31\left(x^3-6x^2+11x-6\right)}{6}$$

$$= \frac{1}{6}\left(14x^3-126x^2+322x-210-39x^3+312x^2-663x+390+31x^3-186x^2+341x-186\right)$$

$$= \frac{1}{6}\left(6x^3-6\right)$$

$y = f(x) = x^3 - 1$

now we have to find $f(6)$ *so*

$$f(6) = x^3 - 1$$
$$= (6)^3 - 1$$
$$= 216 - 1$$
$$= 215$$

Differentiation Using Difference Operators

Question#3 **Marks 10**

Find $f'(2.5)$ *and* $f''(2.5)$ from the following table,

x	1.3	1.6	1.9	2.2	2.5
f(x)	2.4	2.9	3.2	4.7	6.4

Solution:

Since x=2.5 occur at the end of the table, it is appropriate to use the backward difference formula for the derivation.

Backward difference table is as

x	$f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.3	2.4				
1.6	2.9	0.5			
1.9	3.2	0.3	-0.2		
2.2	4.7	1.5	1.2	1.4	
2.5	6.4	1.7	0.2	-1	-2.4

using the backward difference formula for $y'(x)$ and $y''(x)$, we have

$$y'_n = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4}\right)$$

now putting value from backward diff .table we have

$$y'_{2.5} = \frac{1}{0.3}\left(1.7 + \frac{0.2}{2} + \frac{(-1)}{3} + \frac{(-2.4)}{4}\right)$$

$$y'_{2.5} = \frac{1}{0.3}(1.7 + 0.1 - 0.3333 - 0.6)$$

$$y'_{2.5} = \frac{1}{0.3}(0.8667)$$

$$y'_{2.5} = 2.8890$$

$$y''_n = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n\right)$$

$$y''_{2.5} = \frac{1}{(0.3)^2}\left(0.2 - 1 + \frac{11}{12}(-2.4)\right)$$

$$y''_{2.5} = \frac{1}{(0.09)}(0.2 - 1 - 2.2)$$

$$y''_{2.5} = \frac{1}{(0.09)}(-3)$$

$$y''_{2.5} = -33.33$$

Numerical Integration

Trapezoidal Rule

Question 1
Marks 10

Evaluate the integral $\int_0^1 e^{2x}dx$
Using (I) Trapezoidal rule
(2) Simpson's 1/3 rule
By dividing the interval of integration into eight equal parts. Also calculate the percentage error from its true values in both cases.

Solution:

a) By Trapezoidal Rule

Since the no. of intervals are eight, so,
$n=8$
And $a=0, b=1$
So, the width of the interval is given by

$$h=\frac{b-a}{n}$$
$$=\frac{1-0}{8}$$
$$=\frac{1}{8}$$
$$=0.125$$

Since the integrand is given by
$y=f(x)=e^{2x}$,
Therefore, the table of values will be

x	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
$y=f(x)$	1	1.2840	1.6487	2.1170	2.7183	3.4903	4.4817	5.7546	7.3891

Now applying the Trapezoidal rule, we have

$$\int_0^1 e^{2x}dx=\frac{h}{2}\left[y_0+2\left(y_1+y_2+y_3+y_4+y_5+y_6+y_7\right)+y_8\right]$$
$$=\frac{0.125}{2}\left[1+2\left(1.2840+1.6487+2.1170+2.7183+3.4903+4.4817+5.7546\right)+7.3891\right]$$
$$=0.0625\left[8.3891+2\left(21.4946\right)\right]$$
$$=0.0625\left(8.3891+42.9892\right)$$
$$=0.0625\left(51.3783\right)$$
$$=3.2111$$

Therefore
$$\int_0^1 e^{2x}dx=3.2111$$

b) By Simpson's Rule

Applying Simpson's rule, we have

$$\int_0^1 e^{2x}\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6) + y_8\right]$$

$$= \frac{0.125}{3}\left[1 + 4[1.2840 + 2.1170 + 3.4903 + 5.7546] + 2[1.6487 + 2.7183 + 4.4817] + 7.3891\right]$$

$$= \frac{0.125}{3}\left[8.3891 + 4(12.6459) + 2(8.8487)\right]$$

$$= \frac{0.125}{3}(8.3891 + 50.5836 + 17.6974)$$

$$= \frac{0.125}{3}(76.6701)$$

$$= 3.1946$$

Therefore

$$\int_0^1 e^{2x}\,dx = 3.1946$$

True value

Bu integration, we have

$$\int_0^1 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^1$$

$$= \frac{1}{2}\left[e^{2x}\right]_0^1$$

$$= \frac{1}{2}\left[e^2 - e^0\right]$$

$$= \frac{1}{2}\left[7.389056099 - 1\right]$$

$$= 3.1945$$

Therefore the true value is

$$\int_0^1 e^{2x}\,dx = 3.1945$$

Percentage error in Trapezoidal rule

$$\text{Error} = |Exact\ value - Computed\ value|$$

So,

$$Error = |3.1945 - 3.2111|$$

$$= |-0.0166|$$

$$= 0.0166$$

And % error is given by

$$\%\,error = \frac{error}{exact\ value} \times 100\%$$
$$= \frac{0.0166}{3.1945} \times 100\%$$
$$= 0.52\%$$

Percentage error in Simpson's 1/3 rule

Error = $|Exact\ value - Computed\ value|$

$$Error = |3.1945 - 3.1946|$$
$$= |-0.0001|$$
$$= 0.0001$$

And % error is given by

$$\%\,error = \frac{error}{exact\ value} \times 100\%$$
$$= \frac{0.0001}{3.1945} \times 100\%$$
$$= 0.002\%$$

Question #2 **Marks 10**

Evaluate the following double integral,

$$\int_1^2 \int_1^2 \frac{1}{x^3 + y^3} dxdy$$

By using trapezoidal rule, with h = k = 0.25.

Solution:

Taking X = 1, 1.25, 1.50, 1.75, 2.00 and y = 1, 1.25, 1.50, 1.75, 2.00, the following table is obtained using the integrand

$$f(x, y) = \frac{1}{x^3 + y^3}$$

X , Y	1.00	1.25	1.50	1.75	2.00
1.00	0.5	0.3386	0.2286	0.1572	0.1111
1.25	0.3386	0.2560	0.1877	0.1368	0.1005
1.50	0.2286	0.1877	0.1481	0.1145	0.0879
1.75	0.1572	0.1368	0.1145	0.0933	0.0749
2.00	0.1111	0.1005	0.0879	0.0749	0.0625

Now using trapezoidal, keeping variable x fixed and changing the variable y.

$$\int_1^2 f(1, y)dy = \frac{0.25}{2}[0.5 + 2(0.3386 + 0.2286 + 0.1572) + 0.1111]$$

= 0.2575

$$\int_1^2 f(1.25, y)dy = \frac{0.25}{2}[0.3386 + 2(0.2560 + 0.1877 + 0.1368) + 0.1005]$$

= 0.02

$$\int_1^2 f(1.50, y)dy = \frac{0.25}{2}[0.2286 + 2(0.1877 + 0.1481 + 0.1145) + 0.0879]$$

= 0.1521

$$\int_1^2 f(1.75, y)dy = \frac{0.25}{2}[0.1572 + 2(0.1368 + 0.1145 + 0.0933) + 0.0749]$$

= 0.1152

$$\int_1^2 f(2.00, y)dy = \frac{0.25}{2}[0.1111 + 2(0.1005 + 0.0879 + 0.0749) + 0.0625]$$

= 0.0875

$$\int_1^2 \int_1^2 \frac{1}{x^3 + y^3} dxdy = \frac{h}{2}[f(1, y) + 2\{f(1.25, y) + f(1.50, y) + f(1.75, y)\} + f(2.00, y)]$$

$$= \frac{0.25}{2}[2575 + 2(0.2 + 0.1521 + 0.1152) + 0.0875]$$

= 0.1600

Simpson's 1/3 and 3/8 rules

A Simpson's 1/3 Rule:

Question: Evaluate the following integral, $\int_{1.0}^{1.8} \frac{e^x + e^{-x}}{2} dx$

Using Simpson's 1/3 rule by taking h=0.2 and e=2.7183

Solution:

First of all table of the function is created,

x	$y = f(x) = \frac{e^x + e^{-x}}{2}$
1.0	1.543
1.2	1.811
1.4	2.151
1.6	2.577
1.8	3.107

Simpson's $\frac{1}{3}$*rd rule for the given function is*

$$I = \frac{h}{3}\left[f_0 + 4(f_1 + f_3) + 2f_2 + f_4\right]$$

$$I = \frac{0.2}{3}\left[1.543 + 4(1.811 + 2.577) + 2(2.151) + 3.107\right]$$

$$I = \frac{0.2}{3}\left[1.543 + 17.552 + 4.302 + 3.107\right]$$

$$I = \frac{0.2}{3}\left[26.504\right] = 1.7669$$

Question 1 **marks 10**

A river is 80 feet wide. Depth d in feet at a distance of x feet from one bank is given by the following table.

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross-section by Simpson's 1/3 rule.

SOLUTION

Let $y = d$, depth in feet, then the table is given by

x	0	10	20	30	40	50	60	70	80
$y = d$	0	4	7	9	12	15	14	8	3

Here $h = 10$

Let A be the area of the cross-section, then the area of the cross-section is given by

$$A = \int_0^{80} y dx$$

And the area of cross-section by Simpson's 1/3 rule is given by

$$A = \int_0^{80} y dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4 + y_6) + y_8]$$

$$= \frac{10}{3}[0 + 4(4 + 9 + 15 + 8) + 2(7 + 12 + 14) + 3]$$

$$= \frac{10}{3}[0 + 4(36) + 2(33) + 3]$$

$$= \frac{10}{3}[0 + 144 + 66 + 3]$$

$$= \frac{10}{3}(213)$$

$$= 710\, sq.ft$$

A Simpson's 3/8 Rule:
Use Simpson's 3/8 rule to estimate the number of square feet of hand in given lots when x and y are measured in feet (Use accuracy up to three places of decimal)

x	0	100	200	300	400	500	600	700	800	900	1000
y	125	125	120	112	90	90	95	88	75	35	0

Solution

Simpson's 3/8th rule may be written as

$$I=\frac{3h}{8}[f_0+3(f_1+f_2)+2f_3+3(f_4+f_5)+2f_6+3(f_7+f_8)+2f_9+f_{10}]$$

$$I=\frac{3(100)}{8}[125+3(125+120)+2(112)+3(90+90)+2(95)+3(88+75)+2(35)+0]$$

$$I=\frac{3(100)}{8}[125+3(245)+2(112)+3(180)+2(95)+3(163)+2(35)]$$

$$I=(37.5)[125+735+224+540+190+489+70]$$

$$I=(37.5)[2375]=88987.5$$

Differential Equations

Taylor Series Method

Question 1 **marks 10**

Apply Taylor's Series algorithm of order 2 on the initial value problem

$$y'=x^2+y^2 \;;\quad y(0)=1 \;;\quad h=0.05$$

in the interval [0,0.1].

Sol.

As $y'=x^2+y^2$ *Since* $x_0=0$ *and* $y_0=1$

so $y'_0=x_0^2+y_0^2=0+1=1$

Now $y_0''=2x_0+2y_0y_0'=2(0)+2(1)(1)=2$

As Taylor's Series method algorithm of order 2 is

$$y(t)=y(t_0)+(t-t_0)y'(t_0)+\frac{(t-t_0)^2}{2!}y''(t_0)$$

$$y(t)=y(t_0)+hy'(t_0)+\frac{h^2}{2!}y''(t_0)$$

$$y(.05)=y_1=1+.05(1)+\frac{(.05)^2}{2!}(2)=1.0525$$

Now we find y_2, *so* $y'_1=x_1^2+y_1^2=(.05)^2+(1.0525)^2=.0025+1.1077=1.11025$

Now $y_1''=2x_1+2y_1y_1=2(.05)+2(1.0525)(1.11025)=.1+2.337=2.437$

Now $y(t)=y_2=y(t_1)+hy'(t_1)+\frac{h^2}{2!}y''(t_1)$

$$y(0.1)=y_2=1.0525+(.05)(1.11025)+\frac{(.05)^2}{2!}(2.437)$$

$$=1.0525+.0555+.003=1.111$$

Question 2 **marks 10**

Given $y'=3x^2+2y$ and $y(0)=2$, find by Taylor's series $y(0.1)$ and $y(0.2)$ taking $h=0.1$.

SOLUTION

First few derivatives from the given differential equation are as follows

$$y' = 3x^2 + 2y$$
$$y'' = 6x + 2y'$$
$$y''' = 6 + 2y''$$
$$y^{(iv)} = 2y'''$$

The initial condition is given as

$x_0 = 0$, $y_0 = 2$

So, using the given initial condition, we have

$$y_0' = 3x_0^2 + 2y_0 = 3(0)^2 + 2(2) = 4$$
$$y_0'' = 6x_0 + 2y_0' = 6(0) + 2(4) = 8$$
$$y_0''' = 6 + 2y_0'' = 6 + 2(8) = 22$$
$$y_0^{(iv)} = 2y_0''' = 2(22) = 44$$
$$y_0^{(v)} = 2y_0^{(iv)} = 2(44) = 88$$

Now, using Taylor's series method up to the fifth term, we have

$$y(x) = y_0 + (x - x_0)y_0' + \frac{(x - x_0)^2}{2}y_0'' + \frac{(x - x_0)^3}{6}y_0''' + \frac{(x - x_0)^4}{24}y_0^{IV} + \frac{(x - x_0)^5}{120}y_0^{V}$$

Substituting the above values of the derivatives, and the initial condition, we obtain

$$y(0.1) = 2 + (0.1 - 0_0)(4) + \frac{(0.1 - 0)^2}{2}(8) + \frac{(0.1 - 0)^3}{6}(22) + \frac{(0.1 - 0)^4}{24}(44) + \frac{(0.1 - 0)^5}{120}(88)$$
$$y(0.1) = 2 + 0.4 + 0.04 + 0.0037 + 0.00018 + 0.00000067$$
$$y(0.1) = 2.44388067 \cong 2.4439$$

So, $y(0.1) = 2.4439$

Now, we have

$x_1 = 0.1$, $y_1 = 2.4439$

Using the above condition derivatives are calculated as follows

$$y_1' = 3x_1^2 + 2y_1 = 3(0.1)^2 + 2(2.4439) = 4.9178$$
$$y_1'' = 6x_1 + 2y_1' = 6(0.1) + 2(4.9178) = 10.4356$$
$$y_1''' = 6 + 2y_1'' = 6 + 2(10.4356) = 26.8712$$
$$y_1^{(iv)} = 2y_1''' = 2(26.8712) = 53.7424$$
$$y_1^{(v)} = 2y_1^{(iv)} = 2(53.7424) = 107.4848$$

Substituting the value of y_1 and its derivatives into Taylor's series expansion we get, after retaining terms up to fifth derivative only, we have

$$y(0.2) = y_1 + (x - x_1)y_1' + \frac{(x-x_1)^2}{2}y_1'' + \frac{(x-x_1)^3}{6}y_1''' + \frac{(x-x_1)^4}{24}y_1^{IV} + \frac{(x-x_1)^5}{120}y_1^{V}$$

$$y(0.2) = 2.4439 + (0.2-0.1)(4.9178) + \frac{(0.2-0.1)^2}{2}(10.4356) + \frac{(0.2-0.1)^3}{6}(26.8712) + \frac{(0.2-0.1)^4}{24}(53.7424) + \frac{(0.2-0.1)^5}{120}(107.4848)$$

$$y(0.2) = 2.4439 + 0.49178 + 0.052178 + 0.004785 + 0.000224 + 0.000009$$

$$= 2.992876 \cong 2.9929$$

So, $y(0.2) = 2.9929$

Euler Method

Question 2 **marks 10**

Use Euler's method to approximate y when x=1, given that

$$\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1, taking\ h = 0.2$$

Sol.

$$\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1, taking\ h = 0.2$$

$$here\ y_{m+1} = y_m + hf(x_m, y_m)$$

$$f(x,y) = \frac{y-x}{y+x}$$

$$x_0 = 0 \ , \ y_0 = 1$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + (0.2)\frac{1-0}{1} = 1.2$$

$x_1 = 0.2 \quad , \quad y_1 = 1.2$

$y_2 = y_1 + hf(x_1, y_1)$

$= 1.2 + (0.2)\dfrac{1.2 - 0.2}{1.2 + 0.2} = 1.2 + (0.2)(0.7142) = 1.2 + 0.14284 = 1.34284$

$x_2 = 0.4 \quad , \quad y_2 = 1.34284$

$y_3 = y_2 + hf(x_2, y_2)$

$= 1.34284 + (0.2)\dfrac{1.34284 - 0.4}{1.34284 + 0.4} = 1.34284 + (0.2)(0.10819) = 1.34284 + 0.10819 = 1.45103$

$x_3 = 0.6 \quad , \quad y_3 = 1.45103$

$y_4 = y_3 + hf(x_3, y_3)$

$= 1.45103 + (0.2)\dfrac{1.45103 - 0.6}{1.45103 + 0.6} = 1.45103 + (0.2)(0.40327) = 1.45103 + 0.08065 = 1.531684$

$x_4 = 0.8 \quad , \quad y_4 = 1.531684$

$y_5 = y_4 + hf(x_3, y_3)$

$= 1.531684 + (0.2)\dfrac{1.531684 - 0.8}{1.531684 + 0.8} = 1.531684 + (0.2)(0.31380) = 1.531684 + 0.06276 = 1.594444$

$x_5 = 1 \quad , \quad y_5 = 1.594444$

$y_6 = y_5 + hf(x_5, y_5)$

$= 1.594444 + (0.2)\dfrac{1.594444 - 1}{1.594444 + 1} = 1.594444 + (0.2)(0.22912) = 1.594444 + 0.045824 = 1.640268$

Euler Modified Method:

Question 1 **marks 10**

Given that $y' = \log(x + y)$ with $y(0) = 1$, Use Modified Euler's method to find $y(0.2)$, taking $h = 0.2$

SOLUTION

The given differential equation is

$$\frac{dy}{dx} = y' = \log(x + y), \quad y(0) = 1$$

,

So, here $x_0 = 0,\ y_0 = 1$,
$h = 0.2$

At first, we use Euler's method to get

$$y_1^{(1)} = y_0 + hf(x_0, y_0)$$
$$= y_0 + h\log(x_0 + y_0)$$
$$= 1 + 0.2[\log(0+1)]$$
$$= 1 + 0.2\log(1)$$
$$= 1 + 0.2(0)$$
$$= 1 + 0$$
$$= 1$$

So $x_1 = 0.2,\ y_1^{(1)} = 1$

Then, we use modified Euler's method to find

$$y(0.2) = y_1 = y_0 + h\frac{f(x_0, y_0) + f\left(x_1, y_1^{(1)}\right)}{2}$$
$$= 1 + (0.2)\frac{\log(0+1) + \log(0.2+1)}{2}$$
$$= 1 + (0.2)\frac{\log(1) + \log(1.2)}{2}$$
$$= 1 + (0.1)[\log(1) + \log(1.2)]$$
$$= 1 + (0.1)(0 + 0.0792)$$
$$= 1 + 0.1(0.0792)$$
$$= 1 + 0.00792$$
$$= 1.00792$$

Runge-Kutta Method

Question 2 **marks 10**

Given $y' = x + 2y + 1, y(0) = 2$,

Find $y(1)$ and $y(2)$ using the Runge-Kutta method of fourth order taking $h = 1$

SOLUTION

Given differential equation is

$y' = x + 2y + 1, y(0) = 2$

Here $h = 1$ and $x_0 = 0, y_0 = 2$

The Fourth-Order Runge-Kutta Method is described as :-

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where,

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

First Iteration:-

$$x_0 = 0 \quad , \quad y_0 = 2$$

$$k_1 = hf(x_0, y_0)$$
$$= 1f(0,2)$$
$$= 1[0 + 2(2) + 1]$$
$$= 1(5)$$
$$= 5$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$
$$= 1f(0 + \frac{1}{2}, 2 + \frac{5}{2})$$
$$= f(0.5, 4.5)$$
$$= 0.5 + 2(4.5) + 1$$
$$= 10.5$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$= 1f(0 + \frac{1}{2}, 2 + \frac{10.5}{2})$$
$$= f(0.5, 7.25)$$
$$= 0.5 + 2(7.25) + 1$$
$$= 16$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$
$$= 1f(0 + 1, 2 + 16)$$
$$= f(1,18)$$
$$= 1 + 2(18) + 1$$
$$= 38$$

Now

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 2 + \frac{1}{6}(5 + 2(10.5) + 2(16) + 38)$$

$$= 2 + \frac{1}{6}(5 + 21 + 32 + 38)$$

$$= 2 + \frac{1}{6}(96)$$

$$= 18$$

Second Iteration:-

$$x_1 = 1,\ y_1 = 18, h = 1$$

$$k_1 = hf(x_1, y_1)$$

$$= 1 f(1,18)$$

$$= 1[1 + 2(18) + 1]$$

$$= 1(38)$$

$$= 38$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 1 f(1 + \frac{1}{2}, 18 + \frac{38}{2})$$

$$= f(1.5, 37)$$

$$= 1.5 + 2(37) + 1$$

$$= 76.5$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 1 f(1 + \frac{1}{2}, 18 + \frac{76.5}{2})$$

$$= f(1.5, 56.25)$$

$$= 1.5 + 2(56.25) + 1$$

$$= 115$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= 1 f(1 + 1, 18 + 115)$$

$$= f(2, 133)$$

$$= 2 + 2(133) + 1$$

$$= 269$$

So,

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 18 + \frac{1}{6}(38 + 2(76.5) + 2(115) + 269)$$

$$= 18 + \frac{1}{6}(38 + 153 + 230 + 269)$$

$$= 18 + \frac{1}{6}(690)$$

$$= 18 + 115$$

$$= 133$$

So,

$$y(1) = 18$$

$$y(2) = 133$$

Milne's Predictor Corrector Method

Question 2
Marks 10

Find the solution of

$$y' = y(x + y), \quad y(0) = 1$$

Using Milne's P-C method at $x = 0.4$ given that $y(0.1) = 1.11689$, $y(0.2) = 1.27739$ and $y(0.3) = 1.50412$

Solution:

Here,

$x_0 = 0 \quad , \quad x_1 = 0.1 \quad , \quad x_2 = 0.2 \quad , \quad x_3 = 0.3 \quad , \quad x_4 = 0.4$

and $y_0 = 1, y_1 = 1.11689, y_2 = 1.27739, y_3 = 1.50412$

$y' = (x + y)y$

$y_1' = (x_1 + y_1)y_1 = (0.1 + 1.11689)(1.11689) = 1.3591323$

$y_2' = (x_2 + y_2)y_2 = (0.2 + 1.27739)(1.27739) = 1.8872032$

$y_3' = (x_3 + y_3)y_3 = (0.2 + 1.50412)(1.50412) = 2.713613$

Now, using Predictor Formula

$$y_4 = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$y_4 = 1 + \frac{4*0.1}{3}[2(1.3591323) - 1.8872032 + 2(2.713613)]$$

$$y_4 = 1.8344383$$

Adam Moultan's Predictor Corrector Method

Question **marks 10**

Using Adam-Moulton Predictor-Corrector Formula find f(0.4) from Ordinary Differential Equation

$$y' = 1 + 2xy \quad ; \quad y(0) = 0 \; ; \quad h = 0.1$$

with the help of following table.

X	0	0.1	0.2	0.3
Y	0	0.1007	0.2056	0.3199

Solution: Here,

$h = 0.1 \qquad f(x,y) = 1 + 2xy$

$y_0' = 1 + 2x_0y_0 = 1 + 2(0)(0) = 1$

$y_1' = 1 + 2x_1y_1 = 1 + 2(0.1)(0.1007) = 1.02014$

$y_2' = 1 + 2x_2y_2 = 1 + 2(0.2)(0.2056) = 1.08224$

$y_3' = 1 + 2x_3y_3 = 1 + 2(0.3)(0.3199) = 1.19194$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24}(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$

$$y_4 = 0.3199 + \frac{0.1}{24}(55(1.19194) - 59(1.08224) + 37(1.02014) - 9(1))$$

$y_4 = 0.446773833$

Computing y'_4 *for the Corrector Formula*;

$y'_4 = 1 + 2x_4y_4 = 1 + 2(0.4)(0.446773833)$

$y'_4 = 1.3574190664$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})$$

$$y_4 = y_3 + \frac{h}{24}(9y'_4 + 19y'_3 - 5y'_2 + y'_1)$$

$$y_4 = 0.3199 + \frac{0.1}{24}(9(1.3574190664) + 19(1.19194) - 5(1.08224) + 1.02014)$$

$y_4 = 0.446869048$

FAQ updated version.

Question: What is Bracketing method?

Answer: Methods such as bisection method and the false position method of finding roots of a nonlinear equation f(x) = 0 require bracketing of the root by two guesses. Such

methods are called bracketing methods. These methods are always convergent since they are based on reducing the interval between the two guesses to zero in on the root.

Question: What is an Open method?

Answer: In the Newton-Raphson method, the root is not bracketed. Only one initial guess of the root is needed to get the iterative process started to find the root of an equation. Hence, the method falls in the category of open methods.

Question: Explain Muller's method briefly.

Answer: In Muller's method, f (x) = 0 is approximated by a second degree polynomial; that is by a quadratic equation that fits through three points in the vicinity of a root. The roots of this quadratic equation are then approximated to the roots of the equation f (x) 0.This method is iterative in nature and does not require the evaluation of derivatives as in Newton-Raphson method. This method can also be used to determine both real and complex roots of f (x) = 0.

Question: Explain the difference between the linear and non-linear equations.

Answer: Linear Equation An algebraic equation is said to be linear in which each term is either a constant or the product of a constant and the first power of a single variable. One or more variables can be involved in the linear equations. e.g. x+3y+z=0 2x-y+4z=7 etc. Non-Linear Equation An equation is said to Non-Linear equation if it is not linear. Equations involving the power of the variable 2 or higher, transcendental, logarithmic and trigonometric equations etc lie in the category of Non-Linear equations. e.g. x2+5x+3=0 sinx+3y+9=0 xlogx-7x+4y=2 etc.

Question: Explain which value is to be choosed as X0 in N-R method.

Answer: If, for a given function, f(a)*f(b)<0, then any value between a and b inclusive can be chosen as x0.

Question: Define iterative method of solving linear equations with two examples.

Answer: Under iterative methods, the initial approximate solution is assumed to be known and is improved towards the exact solution in an iterative way. We consider Jacobi, Gauss-Seidel and relaxation methods under iterative methods.

Question: Define Pivoting.

Answer: The Gaussian elimination method fails if any one of the pivot elements becomes zero. In such a situation, we rewrite the equations in a different order to avoid zero pivots. Changing the order of equations is called pivoting.

Question: Write the two steps of solving the linear equations using Gaussian Elimination method.

Answer: In this method, the solution to the system of equations is obtained in two stages.

i) the given system of equations is reduced to an equivalent upper triangular form using elementary transformations

ii) the upper triangular system is solved using back substitution procedure

Question: Describe Gauss-Jordan elimination method briefly

Answer: This method is a variation of Gaussian elimination method. In this method, the elements above and below the diagonal are simultaneously made zero. That is a given system is reduced to an equivalent diagonal form using elementary transformations. Then the solution of the resulting diagonal system is obtained. Sometimes, we normalize the pivot row with respect to the pivot element, before elimination. Partial pivoting is also used whenever the pivot element becomes zero.

Question: Describe briefly Crout's reduction method.

Answer: Here the coefficient matrix [A] of the system of equations is decomposed into the product of two matrices [L] and [U], where [L] is a lower-triangular matrix and [U] is an upper-triangular matrix with 1's on its main diagonal.

Question: Describe briefly the Jacobi's method of solving linear equations.

Answer: This is an iterative method, where initial approximate solution to a given system of equations is assumed and is improved towards the exact solution in an iterative way.

Question: What is the difference between Jacobi's method and Gauss Seidal method?

Answer: The difference between jacobi's method and gauss Seidel method is that in jacobi's method the approximation calculated are used in the next iteration for next approximation but in Gauss-seidel method the new approximation calculated is instantly replaced by the previous one.

Question: What is the basic idea of Relaxation method?

Answer: We can improve the solution vector successively by reducing the largest residual to zero at that iteration. This is the basic idea of relaxation method.

Question: How the fast convergence in the relaxation method is achieved?

Answer: To achieve the fast convergence of the procedure, we take all terms to one side and then reorder the equations so that the largest negative coefficients in the equations appear on the diagonal.

Question: Which matrix will have an inverse?

Answer: Every square non-singular matrix will have an inverse.

Question: What are the popular methods available for finding the inverse of a matrix?

Answer: Gauss elimination and Gauss-Jordan methods are popular among many methods available for finding the inverse of a matrix.

Question: Explain Gaussian Elimination Method for finding the inverse of a matrix.

Answer: In this method, if A is a given matrix, for which we have to find the inverse; at first, we place an identity matrix, whose order is same as that of A, adjacent to A which we call an augmented matrix. Then the inverse of A is computed in two stages. In the first stage, A is converted into an upper triangular form, using Gaussian elimination method In the second stage, the above upper triangular matrix is reduced to an identity matrix by row transformations. All these operations are also performed on the adjacently placed identity matrix. Finally, when A is transformed into an identity matrix, the adjacent matrix gives the inverse of A. In order to increase the accuracy of the result, it is essential to employ partial pivoting

Question: What are the steps for finding the largest eigen value by power method.

Answer:

Procedure Step 1: Choose the initial vector such that the largest element is unity.
Step 2: The normalized vector is pre-multiplied by the matrix [A].
Step 3: The resultant vector is again normalized.

Question: What is the method for finding the eigen value of the least magnitude of the matrix [A]?

Answer: For finding the eigen value of the least magnitude of the matrix [A], we have to apply power method to the inverse of [A].

Question: What is interpolation?

Answer: The process of estimating the value of y, for any intermediate value of x, lying inside the table of values of x is called interpolation.

Question: What is extrapolation?

Answer: The method of computing the value of y, for a given value of x, lying outside the table of values of x is known as extrapolation.

Question: What happens when shift operator E operates on the function.

Answer: When shift operator E operates on the function it results in the next value of the function.Ef(x)=f(x+h)

Question: What is the basic condition for the data to apply Newton's interpolation methods?

Answer: To apply Newton's interpolation methods, data should be equally spaced.

Question: When is the Newton's forward difference interpolation formula used?

Answer: Newton's forward difference interpolation formula is mainly used for interpolating the values of y near the beginning of a set of tabular values and for extrapolating values of y, a short distance backward from y0.

Question: When is the Newton's forward difference interpolation formula used?

Answer: Newton's forward difference interpolation formula is mainly used for interpolating the values of y near the beginning of a set of tabular values and for extrapolating values of y, a short distance backward from y0.

Question: When the Newton's backward difference interpolation formula is used?

Answer: For interpolating the value of the function y = f (x) near the end of table of values, and to extrapolate value of the function a short distance forward from yn, Newton's backward Interpolation formula is used.

Question: What is the formula for finding the value of p in Newton's forward difference interpolation formula?

Answer: P=(x-x0)/h

Question: What is the formula for finding the value of p in Newton's backward difference interpolation formula?

Answer: P=(x-xn)/h

Question: If the values of the independent variable are not equally spaced then which formula should be used for interpolation?

Answer: If the values of the independent variable are not given at equidistant intervals, then the Lagrange's interpolation formula should be used.

Question: To use Newton's divided difference interpolation formula, what should the values of independent variables be?

Answer: To use Newton's divided difference interpolation formula, the values of independent variables should not be equally spaced.

Question: Which difference formula is symmetric function of its arguments?

Answer: Divided difference formula is symmetric function of its arguments.

Question: Is the interpolating polynomial found by Lagrange's and Newton's divided difference formulae is same?

Answer: Yes. The interpolating polynomial found by Lagrange's and Newton's divided difference formulae is one and the same.

Question: Which formula involves less number of arithmetic operations? Newton or Lagrange's?

Answer: Newton's formula involves less number of arithmetic operations than that of Lagrange's.

Question: When do we need Numerical Methods for differentiation and integration?

Answer: If the function is known and simple, we can easily obtain its derivative (s) or can evaluate its definite integral However, if we do not know the function as such or the function is complicated and is given in a tabular form at a set of points x0,x1,…,xn, we use only numerical methods for differentiation or integration of the given function.

Question: If the value of the independent variable at which the derivative is to be found appears at the beginning of the table of values, then which formula should be used?

Answer: If the value of the independent variable at which the derivative is to be found appears at and near the beginning of the table, it is appropriate to use formulae based on forward differences to find the derivatives.

Question: If the value of the independent variable at which the derivative is to be found occurs at the end of the table of values, then which formula should be used?

Answer: If the value of the independent variable at which the derivative is to be found occurs at the end of the table of values, it is appropriate to use formulae based on backward differences to find the derivatives.

Question: Why we need to use RICHARDSON'S EXTRAPOLATION METHOD?

Answer: To improve the accuracy of the derivative of a function, which is computed by starting with an arbitrarily selected value of h, Richardson's extrapolation method is often employed in practice.

Question: To apply Simpson's 1/3 rule, what should the number of intervals be?

Answer: To apply Simpson's 1/3 rule, the number of intervals must be even.

Question: To apply Simpson's 3/8 rule, what should the number of intervals be?

Answer: To apply Simpson's 3/8 rule, the number of intervals must be multiple of 3.

Question: What is the order of global error in Simpson's 1/3 rule?

Answer: The global error in Simpson's 1/3 rule is of the order of 0(h4).

Question: Is the order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule?

Answer: Yes. The order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule.

Question: What is the order of global error in Trapezoidal rule?

Answer: The global error in Trapezoidal rule is of the order of 0(h2).

Question: What is the formula for finding the width of the interval?

Answer: Width of the interval, h, is found by the formula h=(b-a)/n

Question: What type of region does the double integration give?

Answer: Double integration gives the area of the rectangular region.

Question: Compare the accuracy of Romberg's integration method to trapezoidal and Simpson's rule.

Answer: Romberg's integration method is more accurate to trapezoidal and Simpson's rule

Question: What is the order of global error in Simpson's 3/8 rule?

Answer: The global error in Simpson's 3/8 rule is of the order of 0(h4).

Question: Which equation models the rate of change of any quantity with respect to another?

Answer: An ordinary differential equation models the rate of change of any quantity with respect to another.

Question: By employing which formula, Adam-Moulton P-C method is derived?

Answer: Adam-Moulton P-C method is derived by employing Newton's backward difference interpolation formula.

Question: What are the commonly used number systems in computers?

Answer: Binary Octal Decimal Hexadecimal

Question: If a system has the base M, then how many different symbols are needed to represent an arbitrary number? Also name those symbols.

Answer: M 0, 1, 2, 3, … , M-1

Question: What is inherent error and give its example.

Answer: It is that quantity of error which is present in the statement of the problem itself, before finding its solution. It arises due to the simplified assumptions made in the mathematical modeling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the problem. e.g. if writing 8375 instead of 8379 in a statement lies in the category of inherent error.

Question: What is local round-off error?

Answer: At the end of computation of a particular problem, the final results in the computer, which is obviously in binary form, should be converted into decimal form-a form understandable to the user-before their print out. Therefore, an additional error is committed at this stage too. This error is called local round-off error.

Question: What is meant by local truncation error?

Answer: Retaining the first few terms of the series up to some fixed terms and truncating the remaining terms arise to local truncation error.

Question: What is transcendental equation and give two examples.

Answer: An equation is said to be transcendental equation if it has logarithmic, trigonometric and exponential function or combination of all these three. For example it is a transcendental equation as it has an exponential function These all are the examples of transcendental equation.

Question: What is meant by intermediate value property?

Answer: If f(x) is a real valued continuous function in the closed interval if f(a) and f(b) have opposite signs once; that is f(x)=0 has at least one root such that Simply If f(x)=0 is a polynomial equation and if f(a) and f(b) are of different signs ,then f(x)=0 must have at least one real root between a and b.

Question: What is direct methods of solving equations?

Answer: Those methods which do not require any information about the initial approximation of root to start the solution are known as direct methods. The examples of

direct methods are Graefee root squaring method, Gauss elimination method and Gauss Jordan method. All these methods do not require any type of initial approximation.

Question: What is Iterative method of solving equations?

Answer: These methods require an initial approximation to start. Bisection method, Newton raphson method, secant method, jacobi method are all examples of iterative methods

Question: If an equation is a transcendental, then in which mode the calculations should be done?

Answer: All the calculations in the transcendental equations should be done in the radian mode.

Question: What is the convergence criterion in method of iteration?

Answer: If be a root of f(x) =0 which is equivalent to I be any interval containing the point x= and will converge to the root provided that the initial approximation is chosen in I

Question: When we stop doing iterations when Toll is given?

Answer: Here if TOL is given then we can simply find the value of TOL by subtracting both the consecutive roots and write it in the exponential notation if the required TOL is obtained then we stop.

Question: How the value of h is calculated in interpolation?

Answer: There are two types of data in the interpolation one is equally spaced and other is unequally spaced. In equally spaced data we need to calculate the value of h that is calculated by subtracting any two consecutive values and taking their absolute value.

Question: What is an algebraic equation?

Answer: An algebraic equation is an equation which is purely polynomial in any variable. Supposed x2+3x+2=0, x4+3x2=0, y3+6y2=0 all are algebraic equations as these are purely polynomial in x and y variable.

Question: What is Descartes rule of signs?

Answer: The number of positive roots of an algebraic equation f(x) =0 can not exceed the no of changes in signs. Similarly the no of negative roots of negative roots of and algebraic equation can not exceed the no of changes in sign of equation f (-x) =0.

Question: What are direct methods?

Answer: The numerical methods which need no information about the initial approximation are known as direct methods like Graffee's root squaring method.

Question: What is meant by iterative methods?

Answer: The methods which need one or more iterations are known as iterative methods like bisection method, Newton raphson method, and many other methods.

Question: What is graphically meant by the root of the equation?

Answer: If the graph of a function f(x) =0 cuts the x-axis at a point a then a is known as the root of the equation.

Question: Q. What is the difference between open and bracketing method?

Answer: In open methods we need only one initial approximation of the root that may be any where lying and if it is not very close then we have to perform more iteration and the example of open method is Newton raphson method. In bracketing method we bracket the root and find that interval in which root lies means we need two initial

approximations for the root finding. Bisection method is an example of the bracketing method.

Question: Condition for the existence of solution of the system of equations.

Answer: If the |A| is not equal to zero then the system will have a unique solution if |A|=0 then the system will have no solution

Question: Should the system be diagonally dominant for gauss elimination method?

Answer: The system of equation need not to be diagonally dominant for Gauss elimination method and gauss Jordan method for both the direct method it is not necessary for the system to be diagonally dominant .it should be diagonally dominant for iterative methods like jacobie and gauss seidel method.

Question: What is meant by diagonally dominant system?

Answer: A system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 is said to be diagonally dominant if the following condition holds. |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3| => (greater or equal) |a3|+|b3|

Question: State the sufficient condition for the convergence of the system of equation by iterative methods.

Answer: A sufficient condition for convergence of iterative solution to exact solution is |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3| => (greater or equal) |a3|+|b3| For the system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 Similarly for the system with more variables we can also construct the same condition

Question: The calculation for numerical analysis should be done in degree or radians.

Answer: All the calculation for numerical analysis should be done in radians not in degrees set your calculator in radians mode and suppose the value of pi=3.14.

Question: How we can identify that Newton forward or backwards interpolation formula is to be used.

Answer: If the value at which we have to interpolate is in the start of the table then we will use Newton's forward interpolation formula if it is at the end of the table then we will use the Newton's backward interpolation formula.

Question: What is meant precision and accuracy?

Answer: Precision and accuracy are two terms which are used in numerical calculations by precision we mean that how the values in different iterations agree to each other or how close are the different values in successive iterations. For example you have performed 3 different iterations and result of all the iteration are 1.32514,1.32516,31.32518 these three values are very precise as these values agree with each other . Accuracy means the closeness to the actual value. Suppose that you have calculated an answer after some iteration and the answer is 2.718245 and the actual answer is 2.718254 and the answer calculated is very accurate but if this answer is 2.72125 then it is not accurate.

Question: What is the condition that a root will lie in an interval.

Answer: Suppose that you have a function f(x) and an interval [a,b]and you calculate both f(a)and f(b)if f(a)f(b)<0 then there must exist a root for this function in this interval.In simple words f(a) and f(b)must have opposite signs.

Question: How the divided difference table is constructed?

Answer: x y First difference Second difference Third difference 1 0.2 y0 y1-y0/x1-x0=sy0 sy1- sy0/x2-x0=s2y0 s2y0-s2y1/x3-x0=s3y0 s3y1-s3y0/x4x0 =s4y0 2 0.69 y1 y2-y1/x2-x1=sy1 sy2- sy1/x3-x1=s2y1 s2y2-s2y1/x4-x1=s3y1 3 0.985 y2 y3-y2/x3-x2=sy2 sy3- sy2/x4-x1=s2y2 4 1.2365 y3 y4-y3/x4-x3=sy3 5 2.3651 y4 This is the complete table showing the central differences . Here s stands for difference

Question: What is Gauss-seidel Method

Answer: It is also an iterative method and in this we also check either the system is diagonally dominant and, if so then we proceed on the same lines as in the Jacobi's method. The difference in both is when we calculate a value of any variable that is instantly replaced by the previous value. If we have initial values like x1=0,x2=0 and x3=0 in the case of three variables in the first iteration we will use all the three values to calculate x1 but to calculate the value of x2 we will use the recent value of x1 which is calculated in the first iteration.

Question: What is partial and full pivoting?

Answer: Partial and full pivoting, In gauss elimination method when you have any of diagonal element aii zero it means the solution does not exist to avoid this we change the equation so that a non zero pivot is achieved .Now you have an argument matrix in which he elements in the first column are 1,3,4 respectively here in case of partial pivoting we will replace first element with the last element it is done by replacing the first equation with last equation it is known as partial pivoting. In full pivoting we change rows and columns but that is not implemented manually it is used in computers.

Question: How the initial vector is choose in power method?

Answer: Choose the initial vector such that the largest element is unity. But no problem with this you will precede with this vector as it is and will not change it. In the given question the initial vector is provided so you have to proceed with the given vector and will normalize this vector and normalizing this vector you will keep in mind that the greatest value should be unity. An eigenvector Vis said to be normalized if the coordinate of largest magnitude is equal to unity Actually this condition is for the normalization of the vectors when you have to normalize the vector you keep in mind that the larget entery in the vector must be 1 and so you take the largest element and divide the remaining values by the greatest value.

Question: what is the relation ship between p=0 and non zero p in interpolation.

Answer: fp=fo+pDfo+1/2(p2-p)D2fo+1/6(p3-3p2+2p)D3fo+1/24(p4-6p3+11p2-6p)D4fo+… This is the interpolation formula For the derivative you will have to take the derivative of this formula w.r.t p and you will get f'p=1/h{Dfo+1/2(2p-1)D2fo+1/6(3p2-6p+2)D3fo+…} Now put here p=0 f'p=1/h{Dfo+1/2(0-1)D2fo+1/6(0-0+2)D3fo+…} So the formula becomes f'p=1/h{Dfo+D2fo/2+D3fo/3+…} so this is the relation ship between non zero and zero p no when you have to calculate p the you use formula p=x-xo/h so this is the impact of x.

Question: What is chopping and Rounding off?

Answer: Chopping and rounding are two different techniques used to truncate the terms needed according to your accuracy needs. In chopping you simply use the mentioned number of digits after the decimal and discard all the remaining terms. Explanation (1/3 - 3/11) + 3/20=(0.333333…-0.27272727..)+0.15=(0.333-0.272)+0.15 This is the three digit chopping.

Question: When the forward and backward interpolation formulae are used?

Answer: In interpolation if we have at the start then we use the forward difference formula and the formula to calculate p is x-x0/h. If the value of x lies at the end then we use Newton's backward formula and formula to calculate the value of p is x-xn/h. Now I come to your question as in this case the value lies at the end so 6 will be used as the xn. This procedure has been followed by the teacher in the lectures. But some authors also use another technique that is if you calculate the value of p and that is negative then the origin is shifted to that value for which the value of p becomes positive. And then according to that origin the values of differences are used and you need not follow this procedure.

Question: What is forward and backward difference operator and the construction of their table.

Answer: For forward Dfr =fr+1 –fr Df0 = f1-f0 In terms of y Dyr+1=yr+1-yr D stands for the forward difference operator For backward Dfr =fr –fr-1 Df1 = f1-f0 In terms of y Dy1=y1-y0 Here D stands for backwards operator Now the construction of the difference table is based on X Y 1st forward 2nd forward 3rd forward x1 Y1 Y2-Y1=Dy0 x2 Y2 Y3-Y2=Dy1 x3 Y3 Y4-Y3=Dy2 x4 Y4 Now consider the construction of table for the backward table X Y 1st forward 2nd forward 3rd forward x1 Y1 Y2-Y1=Dy1 x2 Y2 Y3-Y2=Dy2 x3 Y3 Y4-Y3=Dy3 x4 Y4

Dear student this is the main difference in the construction of the forward and backwards difference table when you proceed for forward difference table you get in first difference the value Dy0 but in the construction of backwards difference table in the first difference you get Dy1 and in the second difference in the forward difference table you get D2 y0 and in the backward difference table the first value in the second difference is D2 y1. I think so you have made it clear.

Question: What is Jacobi's method?

Answer: Jacobi's Method It is an iterative method and in this method we first of all check either the system is diagonally dominant and, if the system is diagonally dominant then we will calculate the value of first variable from first equation in the form of other variables and from the second equation the value of second variable in the form of other variables and so on. We are provided with the initial approximations and these approximations are used in the first iteration to calculate first approximation of all the variables. The approximations calculated in the first iteration are used in the second iteration to calculate the second approximations and so on.

Question: what is Simpson's 3/8th rule.

Answer: The general formula for Simpson's 3/8th rule is 3h/8[f0+3(f1+f2)+2f3+(3f4+f5)+2f6+…+3(fn-2+fn-1)+fn] Now if we have to calculate the integral by using this rule then we can simply proceed just write first and last vale and distribute all the remaining values with prefix 3 and 2 Like you have f0,f1,f2,f3,f4 Then the integral can be calculated as 3h/8[f0+f4+3(f1+f2)+2f4] If we have values like f0,f1,f2,f3,f4,f5 Then integral can be calculated as 3h/8[f0+3(f1+f2)+2f3+3f4+f5] Similarly proceeding in this fashion we can calculate the integral in this fashion

Question: what is classic runge-kutta method

Answer: The fourth order Runge-Kutta method is known as the classic formula of classic Runge-Kutta method. yn+1=yn+1/6(k1+2k2+2k3+k4) Where k1=hf(xn,yn) k2=hf(xn+h/2+yn+k1/2) k3=hf(xn+h/2,yn+k'2/2) k4=hf(xn+h,yn+k3)

Question: What is meant by TOL?

Answer: The TOL means the extent of accuracy which is needed for the solution. If you need the accuracy to two places of decimal then the TOL will be 10-2 .similarly the 10-3 means that the accuracy needed to three places of decimal. Suppose you have the root from last iteration 0.8324516 and 0.8324525 if we subtract both and consider absolute value of the difference 0.0000009 now it can be written as 0.09*10-5 so the TOL in this case is 10-5.similarly if we have been provided that you have to for the TOL 10-2 you will check in the same way. In the given equation you will solve the equation by any method and will consider some specific TOL and try to go to that TOL. Some time no TOL is provided and you are asked to perform to some specific no of iterations.

Question: what is meant by uniqueness of LU method.

Answer: An invertible (whose inverse exists) matrix can have LU factorization if and only if all its principal minors(the determinant of a smaller matrix in a matrix) are non zero .The factorization is unique if we require that the diagonal of L or U must have 1's.the matrix has a unique LDU factorization under these condition . If the matrix is singular (inverse does not exists) then an LU factorization may still exist, a square matrix of rank (the rank of a matrix in a field is the maximal no of rows or column) k has an LU factorization if the first k principal minors are non zero. These are the conditions for the uniqueness of the LU decomposition.

Question: how the valu of h is calculated from equally spaced data.

Answer: Consider the following data x y 1 1.6543 2 1.6984 3 2.4546 4 2.9732 5 3.2564 6 3.8765 Here for h=2-1=3-2=1 x y 0.1 1.6543 0.2 1.6984 0.3 2.4546 0.4 2.9732 0.5 3.2564 0.6 3.8765 Here for the calculation of h =0.2-0.1=0.3-0.2=0.1 I think so that you may be able to understand .

Glossary (Updated Version)

Absolute Error :

The absolute error is used to denote the actual value of a quantity less it's rounded value if x and x* are respectively the rounded and actual values of a quantity, then absolute error is defined by AE=|x-x*|

Accuracy :

The Extent of the closeness between the actual value and estimated value is known as accuracy. Suppose you have taken readings like 2.1234,2.1236and 2.1238 and 2.45,2.52,2.63 Now if the actual root is 2.65 then the second values are more accurate and not precise but first set of values is precise but not accurate as it differs from the actual value.

Algebraic equation :

The equation f(x)=0 is known as algebraic equation if it is purely a polynomial in x. Like f(x)=3 ,$f(x)=x^3-3x-4$ are some examples of algebraic equations and the if variable is changed to y x or any one then under the same conditions it will be algebraic.

Bisection method :

It is a bracketing method which is used to locate the root of an equation and it make use of intermediate value property to locate the interval in which the root lies, formula which is used is (a+b)/2 where a and b are points such that f(a)f(b)<0.

Bracketing Method :

The iterative methods which require two initial approximation for it's first iteration are known as bracketing methods. Bisection method is the example of the bracketing method as it requires a interval for the approximation of the root.

Crout's Method :

This method is used to solve the system of the equations by decomposing the system into two matrices L and U where L is a lower triangular matrix and U is an upper triangular matrix. In U all the elements in the main diagonal are 1.

Descartes rule of signs : The no of positive roots of an algebraic equation f(x)=0 with real coefficients cannot exceed the no of changes in the in sign of the coefficients in the polynomial f(x)=0,similarly the no of negative roots of f(x)=0 can not exceed the no of changes in the sign of the coefficients of f(-x)=0

Direct methods :

These are the methods which do not need the knowledge of the initial approximation are known as direct methods.

Gauss Elimination Method :

It is a direct method which is used to solve a system of equations

Gauss Seidel iterative method :

It is an iterative method in which an initial approximation is given. First of all system should be checked either it is diagonally dominant or not if it is not then it is made diagonally dominant. Secondly the first variable is calculated in terms of other variables from first equation and Second variable from second equation and so on .the previous value of the variable is replaced by new value instantly as it is obtained. This is the difference in the gauss seidel and jacobie iterative method.

Graeffee's root squaring method :

This method is used to find all the roots of the polynomial equation.

Intermediate value property :

If for an equation f(x)=0 for two values a and b we have such that $f(a)f(b)<0$ then there must exist a root between a and b in the interval [a,b].

Inverse of a matrix :

The matrix B is said to be inverse of a matrix A if the product of and B is I identity matrix.

Iterative methods :

Iterative methods are those type of methods which always require an initial approximation to start an iteration

Jacobie's iterative method: :

It is an iterative method in which an initial approximation is given. First of all system should be checked either it is diagonally dominant or not if it is not then it is made diagonally dominant. Secondly the first variable is calculated in terms of other variables from first equation and Second variable from second equation and so on after first iteration in the second iteration all the variables are replaced by the previous one.

Muller's Method :

In Muller's method f(x)=0 is approximated by a second degree polynomial, that is by quadratic equation that fits through three points in the vicinity of a root .Then roots of this quadratic equation are then approximated to the roots of equation f(x)=0

Newton Raphson Method. :

It is an open method which is used to locate a root of the equation, it needs only one initial approximation for it's first iteration. xn+1=xn-(fxn/f'xn)

Non singular matrix :

A matrix is said to be non singular if the determinant of the matrix is non zero. |A| is not equal to zero and the inverse of non singular exists.

Open methods :

The methods which require only one initial approximation to start the first iteration, for example the Newton's Raphsom method is known as the open method as it requires only one initial approximation.

Pivoting :

If any of the pivot elements in gauss elimination become zero then this method fails so to Avoid such type of situation equation are rearranged to get rid of zero pivot element, this procedure is known as pivoting.

Polynomial:

An expression of the form $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ where n is a positive integer and $a_0, a_1, a_2 + \ldots . a_n$ are real constants, such type of expression is called an nth degree polynomial in x if $a_o \neq 0$

Precision :

The extent of closeness of different measurement taken to estimate a certain value. Suppose you have done different iterations to measure the root of an equation and take different values as 2.1234, 2.1236, and 2.1238 these all the values are very close to each other so these are very precise.

Regula –Falsi method :

It is also an iterative method and is a bracketing method and use intermediate value property to get it's initial guess. The formula used for this is xn+1=xn-(xn-xn-1/f(xn)-f(xn-1))

Relative Error : It is the ratio of the absolute error to the actual value of the quantity. Thus RE=AE=AE\|x*|

Relaxation method :

It is also an iterative method and in this method you solve the system of equation by making the greatest residual to zero.

Root : If you have an equation f(x)=0 then the no a is said to be the root of the equation if f(a)=0 Suppose you have an equation f(x)=x2-4 the 2 is a root of the equation as f(2)=4-4=0

Secant Method :

The secant method is also an open method and it takes two initial values for it's first approximation, the formula used for this is known as xn+1={xn-1f(xn)-xnf(xn-1)}/{f(xn)-f(xn-1)}

Significant digits :

A significant digit in an approximate no is a digit, which gives reliable information about the size of the number. In other words a significant digit is used to express accuracy, that Is how many digits in the no have meaning.

Singular matrix :

A matrix s said to be a singular matrix if the determinant of the matrix is zero |A|=0,the inverse of the singular matrix do not exist.

Transcendental equation :

An equation f(x) =0 is said to be transcendental equation if it contains trigonometric, logarithmic and exponential functions

Truncation Error :

It is defined as the replacement of one series by another series with fewer terms. The error arising by this approximation is known as truncation error.

Important Formula For MTH603

Bisection Method

$$x2 = \frac{x0 + x1}{2}$$

Muller Method

$$x = \frac{x0 - 2c}{+b - \sqrt{b^2 - 4ac}}$$

in this formula x0,x1, x2 will be and U will just put the values according to the given formulas.

$$a = \frac{h2\,f1 - (h1 + h2)\,f0 + h1\,f2}{h1h2(h1 + h2)}$$

$$b = \frac{f1 = f0 - ah1^2}{h1}$$

$$c = f(x0)$$

$$h1 = x1 - x0$$

$$h2 = x0 - x2$$

Regula Falsi Method (Method of False position)

$$x3 = x2 - \frac{x2 - x1}{f(x2) - f(x1)}\,f(x2)$$

Newton Rophson method

$$x1 = x0 - \frac{f(x0)}{f'(x0)}$$

Secant Method

$$x2 = \frac{x0\,f(x1) - x1\,f(x0)}{f(x1) - f(x0)}$$

Newton's Formula

$$x1 = \frac{1}{2}(x0 + {}^{n}\!/_{x0})$$

in this formula
x0= 2 perfect square near to 12 such like 9 and 16

Graffee root squaring method

$$f(x)\,f(-x) = a_3^{\,2}x^6 - (a_2^{\,2} - 2a_1a_3)x^4 + (a_1^{\,2} - 2a_0a_2)x^2 - x_0^2$$

the truncation error (TE) is given by

$$\text{TE} \le \frac{x^{2n+2}}{(2n+2)!}$$

The TE is independent of the computer used.

Short Questions Paper

Set-01

Approximate the integral $\int_{1}^{1.6} \frac{2x}{x^2+1}\,dx$ using Simpson's 1/3 rule and calculate the percentage error. (Take result up to 4 decimal places)

Note: In order to get full marks do all necessary steps.

Construct a forward difference table for the following values

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

Note : In order to get full marks do all necessary steps.

Solve the system

$$4x+3y=24$$
$$3x+4y-z=30$$
$$-y+4z=-24$$

by Gauss Seidal Method, taking $(0, 0, 0)^t$ as initial approximation(Two iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Let $f(x)=x+\frac{2}{x}$, use cubic Lagrange interpolation based on the nodes $x_0=0.5, x_1=1, x_2=2 \text{ and } x_3=2.5$ to approximate *f(1.5)* and *f(1.3)*.

Note : In order to get full marks do all necessary stepsSolution

Approximate the Dominant Eigenvalue and corresponding Eigenvector for the matrix

$$\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

by using Power Method. Start with $X_0=(1,1,1)^t$. (Five iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

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Set-02

Question No: 31 (Marks: 2)

If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32 (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2)dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

Evaluate the integral

$$\int_3^5 (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36 (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37 (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38 (Marks: 5)

Evaluate the integral

$$\int_0^3 (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39 (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$ taking $h = 0.1$

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Set-03

Question:

Answer:

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Multiple Choice Question

Set-01

Question No: 1 (Marks: 1) - Please choose one

Symbol used for forward differences is

- ∇

- Δ

- δ

- μ

Question No: 2 (Marks: 1) - Please choose one

The relationship between central difference operator and the shift operator is given by

- $\delta = \mathrm{E} - \mathrm{E}^{-1}$

- $\delta = \mathrm{E} + \mathrm{E}^{-1}$

- $\delta = \mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}}$

- $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}}$

Question No: 3 (Marks: 1) - Please choose one

Muller's method requires --------starting points

- 1

- 2

- 3

- 4

Question No: 4 (Marks: 1) - Please choose one

If S is an identity matrix, then

- ▶ $S^{-1} = S$
- ▶ $S^t = S$
- ▶ $All\ are\ true$
- ▶ $S^{-1} = S^t$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_0, x_1, ..., x_r$

- ▶ r+2
- ▶ r+1
- ▶ r
- ▶ r-1

Question No: 6 (Marks: 1) - Please choose one

P in Newton's forward difference formula is defined as

- ▶ $p = (\frac{x - x_0}{h})$
- $p = (\frac{x + x_0}{h})$
- ▶

$$p = (\frac{x + x_n}{h})$$

►

$$p = (\frac{x - x_n}{h})$$

►

Question No: 7 (Marks: 1) - Please choose one

_____ Octal number system has the base ----------

► 2

► 8

► 10

► 16

Question No: 8 (Marks: 1) - Please choose one

_____ Newton's divided difference interpolation formula is used when the values of the independent variable are

► Equally spaced

► Not equally spaced

► Constant

► None of the above

Question No: 9 (Marks: 1) - Please choose one

_____ Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

► 1.5

► 3

► 2

► 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

► $\varepsilon(x) = y(x) + P_n(x)$

► $\varepsilon(x) = y(x) - P_n(x)$

► $\varepsilon(x) = P_n(x) - y(x)$

► $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11 (Marks: 1) - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I.

- n-1
- n+2
- n
- n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

- $D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}+\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}+...)$
- $D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2}+\frac{\Delta^3}{3}+\frac{\Delta^4}{4}+\frac{\Delta^5}{5}+...)$
- $D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\frac{\Delta^5}{5}-...)$
- $D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}-\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}-...)$

Question No: 13 (Marks: 1) - Please choose one

Finding the first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

► Forward difference operator

► **Backward difference operator**

► Central difference operator

► None of the given choices

Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

► -0.5

► 0.5

► 0.75

► **-0.75**

Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

► $ax+b$

► ax^2+bx+c

- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question No: 16 (Marks: 1) - Please choose one

While $I=\int_a^b f(x)dx$ integrating , h , width of the interval, is found by the formula-----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

Question No: 17 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

- 7
- 8
- **5**
- 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as ……

► 17.5

► 12.5

► **7.5**

► -12.5

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

► 2

► 3

► **5**

► 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

► 10

► 11

► **12**

► 13

Question No: 21 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- $\frac{(a+b)}{2}$
- $\frac{(a-b)}{2}$
- $\frac{(b-a)}{2}$
- None of the given choices

Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- Bisection Method
- Regula Falsi Method
- Secant Method
- None of the given choices

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

- (1, 3)

► (1, 2)

► (0, 1)

► (1, 2)

Question No: 24 (Marks: 1) - Please choose one

_____ Rate of change of any quantity with respect to another can be modeled by

► An ordinary differential equation

► A partial differential equation

► A polynomial equation

► None of the given choices

Question No: 25 (Marks: 1) - Please choose one

_____ If

$$\frac{dy}{dx} = f(x, y)$$

Then the integral of this equation is a curve in

► None of the given choices

► xt-plane

► yt-plane

► xy-plane

Question No: 26 (Marks: 1) - Please choose one

In solving the differential equation

$$y' = x + y \;;\; y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

- 1.44
- 1.11
- 1.22
- 1.33

Question No: 27 (Marks: 1) - Please choose one

In second order Runge-Kutta method

k_1 is given by

- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

Question No: 28 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

▶ $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

▶ $k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$

▶ $k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$

▶ $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

Question No: 29 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

▶ $k_3 = hf(x_n + 2h, y_n + 2k_3)$

▶ $k_3 = hf(x_n - h, y_n - k_3)$

▶ $k_3 = hf(x_n + h, y_n + k_3)$

▶ None of the given choices

Question No: 30 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

▶ Newton's backward difference interpolation formula

▶ Newton's forward difference interpolation formula

▶ Newton's divided difference interpolation formula

► None of the given choices

==========

Set-02

/wEPDwUKMTY2I

/wEWBgK+3MqI/

Question # 1 of 8 (Start time: 01:24:49 PM) **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

- antiderivative
- derivatives

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLy4ZCpA

Time Left 59 sec(s)

Question # 2 of 8 (Start time: 01:26:18 PM) **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLlwO3cC

Time Left 79 sec(s)

Question # 3 of 8 (Start time: 01:27:35 PM) **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

diverge

converge

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKMu86b

Time Left 85 sec(s)

Question # 5 of 8 (Start time: 01:30:21 PM) **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas
- constant

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgK6oIaxB

Time Left 83 sec(s)

Question # 6 of 8 (Start time: 01:31:18 PM) **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

Select correct option:

- TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgKPg8yyI

Time Left 84 sec(s)

Question # 7 of 8 (Start time: 01:32:33 PM) **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

Select correct option:

FALSE

TRUE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKK6eLRI

Time Left 84 sec(s)

Question # 8 of 8 (Start time: 01:33:57 PM) **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

Select correct option:

- constant
- finite
- infinity
- zero

Click here to Save Answer & Move to Next Question

==========

Set-03

No.	Questions	Mks
1	Find value of given data by Adam Moultan's method	10
2	Find value of given data by Dividend Difference Composite method	10
3	Draw backward difference tables for given Data	5+5

x			
y			

x			
y			

4	Write Simpson's 1/3 formula	2
5	Find value by Euler's Method	3
6	Find value of K1 by 2nd Order R-K method	2
7	Convergence is used when ----------	1
8	Bisection method is …….. method • Bracketing Method • Open • Random • none http://vustudents.ning.com/	1
9	Newton Raphson method is …….. method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real • Vector • odd • even	1
11	Find value of y'(1) by Euler's Method taking h=1	2
12	Find value of y'(3) from given table. x | | | | y | | | |	2
13	Find value of y'(0.3) by Lagrange's Method x | | | | y | | | |	3
14	For Simpson's 1/3 rule no.of intervals must be • 1 • 3 • 5 • 8	1

15	For Simpson's 1/3 rule valid no.of intervals are • 1 • 3 • 5 • 8	1
16	For Simpson's 3/8 rule no.of intervals must be • 10 • 11 • 12 • 14 http://vustudents.ning.com/	1
17	Find the value of y'(1) from given forward difference table	2

x	y	Δy	Δy^2	Δy^3

Numerical analysis mth603 paper

Numerical Analysis numerical paper 2009
The paper was very easy.
The mcq's were really easy.
Most of the mcq's were from the last 5 lecture.
and from jacobi's method and other's.
then the logical mcq's like
s inverse * s = I
it was repeated twice.
one 5 mark question was from newton's rapson method
one 10 mark question was from lecture 11 page 69 example
best of luck

==========

Set-04

/w EPDw UKMTY2
/w EWBgKPm46n

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 1 of 10 (Start time: 04:40:08 PM) Total Marks: 1

The determinant of a diagonal matrix is the product of the diagonal elements.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWBgKin4OcE

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 2 of 10 (Start time: 04:40:58 PM) Total Marks: 1

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWBgKx743v

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 3 of 10 (Start time: 04:41:55 PM) Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY
/w EWBgKR8tba

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 4 of 10 (Start time: 04:42:30 PM) Total Marks: 1

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY
/w EWBgKjyL19A

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 5 of 10 (Start time: 04:42:58 PM) Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

Select correct option:

- no
- atleast one

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY
/w EWBgKw w uK

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 6 of 10 (Start time: 04:43:20 PM) Total Marks: 1

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

Select correct option:

- unity
- zero

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY
/w EWBgLdv/PBE

Time Left 55 sec(s)

Quiz Start Time: 04:40 PM

Question # 7 of 10 (Start time: 04:43:39 PM) Total Marks: 1

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWCgL0zNjNE

Time Left 90 sec(s)

Quiz Start Time: 04:40 PM

Question # 8 of 10 (Start time: 04:45:08 PM) Total Marks: 1

The determinant of a _____ matrix is the product of the diagonal elements.

Select correct option:

- diagonal
- upper triangular
- lower triangular
- scalar

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWCgKw hsW

Time Left 89

sec(s)

Quiz Start Time: 04:40 PM

Question # 9 of 10 (Start time: 04:46:35 PM) Total Marks: 1

Eigenvalues of a symmetric matrix are all _____.

Select correct option:

- real
- zero
- positive
- negative

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2
/w EWBgLX34ivA

Time Left 89 sec(s)

Quiz Start Time: 04:40 PM

Question # 10 of 10 (Start time: 04:47:33 PM) Total Marks: 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

==========

Set-05

/w EPDw UKMTY2

/w EWBgKqiMqh0

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 1 of 10 (Start time: 04:50:37 PM) Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWCgLw ndfn

Time Left 90 sec(s)

Quiz Start Time: 04:50 PM

Question # 2 of 10 (Start time: 04:50:51 PM) Total Marks: 1

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

Select correct option:

- (x-1)^3

- (x+1)^3
- x^3-1
- x^3+1

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKJ5b6A

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 3 of 10 (Start time: 04:51:41 PM) Total Marks: 1

For differences methods we require the set of values.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKd3KGC

Time Left 88 sec(s)

Quiz Start Time: 04:50 PM

Question # 4 of 10 (Start time: 04:51:57 PM) Total Marks: 1

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY:

/w EWBgLC663Y

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 5 of 10 (Start time: 04:52:22 PM) Total Marks: 1

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgL71Ojw

Time Left 88 sec(s)

Quiz Start Time: 04:50 PM

Question # 6 of 10 (Start time: 04:53:38 PM) Total Marks: 1

Central difference method seems to be giving a better approximation, however it requires more computations.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgLTr6GgI

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 7 of 10 (Start time: 04:54:56 PM) Total Marks: 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY

/w EWBgKr97ef

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 8 of 10 (Start time: 04:55:31 PM) Total Marks: 1

Iterative algorithms can be more rapid than direct methods.

Select correct option:

FALSE

TRUE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgL/r4KyC

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 9 of 10 (Start time: 04:55:49 PM) Total Marks: 1

Central Difference method is the finite difference method.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgLonf2RE

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 10 of 10 (Start time: 04:56:20 PM) Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

==========

Set-06

/wEPDwUKMTY2I

/wEWBgK+3MqI/

Question # 1 of 8 (Start time: 01:24:49 PM) Total Marks: 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

antiderivative

derivatives

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLy4ZCpA

Time Left 59 sec(s)

Question # 2 of 8 (Start time: 01:26:18 PM) **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLlwO3cC

Time Left 79 sec(s)

Question # 3 of 8 (Start time: 01:27:35 PM) **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

- diverge
- converge

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKMu86b

Time Left 85 sec(s)

Question # 5 of 8 (Start time: 01:30:21 PM) **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas

constant

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgK6oIaxB

Time Left 83 sec(s)

Question # 6 of 8 (Start time: 01:31:18 PM) **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgKPg8yyI

Time Left 84 sec(s)

Question # 7 of 8 (Start time: 01:32:33 PM) **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

Select correct option:

- FALSE
- TRUE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKK6eLRI

Time Left 84 sec(s)

Question # 8 of 8 (Start time: 01:33:57 PM) **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

Select correct option:

- constant
- finite
- infinity
- zero

Click here to Save Answer & Move to Next Question

==========

Set-07

Question # 1 of 10 (Start time: 10:22:24 AM) Total Marks: 1

Eigenvalues of a symmetric matrix are all _____.
Select correct option:

real
zero
positive
negative

Question # 2 of 10 (Start time: 10:23:07 AM) Total Marks: 1

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.
Select correct option:

TRUE
FALSE

Question # 3 of 10 (Start time: 10:23:55 AM) Total Marks: 1

Exact solution of 2/3 is not exists.
Select correct option:

TRUE
FALSE

Question # 4 of 10 (Start time: 10:24:53 AM) Total Marks: 1

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric _____ definite matrices A.
Select correct option:

positive
negative

Question # 5 of 10 (Start time: 10:26:04 AM) Total Marks: 1

Differences methods find the _____ solution of the system.

Select correct option:

numerical
Analytical

Question # 6 of 10 (Start time: 10:26:49 AM) Total Marks: 1

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.
Select correct option:

(x-1)^3
(x+1)^3 **i m not sure about this answer**
x^3-1
x^3+1

Question # 7 of 10 (Start time: 10:28:08 AM) Total Marks: 1

The Power method can be used only to find the eigen value of A that is largest in absolute value—----------we call this eigen value the dominant eigen value of A.
Select correct option:

TRUE
FALSE

Question # 8 of 10 (Start time: 10:29:33 AM) Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.
Select correct option:

main diagonal
last column
last row
first row **i m not sure about this answer**

Question # 9 of 10 (Start time: 10:30:33 AM) Total Marks: 1

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.
Select correct option:

TRUE
FALSE

Question # 10 of 10 (Start time: 10:31:28 AM) Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.
Select correct option:

TRUE
FALSE

==========

Set-08

Differences methods find the _____ solution of the system.

Select correct option:

- **numerical**
- Analytical

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

- **TRUE**
- FALSE

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

Select correct option:

- **TRUE**
- FALSE

Eigenvalues of a _____ matrix are all real.

Select correct option:

- **symmetric**
- antisymmetric
- rectangular
- triangular

By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.

Select correct option:

- **TRUE**
- FALSE

The dominant eigenvector of a matrix is an eigenvector corresponding to the eigenvalue of largest magnitude (for real numbers, smallest absolute value) of that matrix.

Select correct option:

- **TRUE**
- FALSE

Eigenvalues of a symmetric matrix are all _____ .

Select correct option:

- **real**
- complex
- zero
- positive

Below are all the finite difference methods EXCEPT _____.

Select correct option:

- jacobi's method
- newton's backward difference method
- **Stirlling formula**
- Forward difference method

Quiz#10

The central difference method is finite method.

Select correct option:

True

False

Power method is applicable if the eigen vectors corresponding to eigen values are linearly _____.

Select correct option:

- **independent**
- dependent

Central difference method seems to be giving a better approximation, however it requires more computations.

Select correct option:

- **TRUE**
- FALSE

The Jacobi iteration _____, if A is strictly diagonally dominant.

Select correct option:

- **converges**
- diverges

Power method is applicable if the eigen values are _____.

Select correct option:

- **real and distinct**
- real and equal
- positive and distinct
- negative and distinct

determinant of a diagonal matrix is the product of the diagonal elements.

Select correct option:

- **TRUE**
- FALSE

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

Select correct option:

- **TRUE**
- FALSE

If n x n matrices A and B are similar, then they have the _____ eigenvalues (with the same multiplicities).

Select correct option:

- **same**
- different

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

Select correct option:

- **unity**
- zero

Set-09

Exact solution of 2/3 is not exists.

TRUE

FALSE

The Jacobi's method is

a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

no

atleast one

A 3 x 3 identity matrix have three and _____eigen values.

same

different

Eigenvalues of a symmetric matrix are all _____ .

real

complex

zero

positive

The Jacobi iteration converges, if A is strictly diagonally dominant.

TRUE

FALSE

Below are all the finite difference methods EXCEPT _____.

jacobi's method

newton's backward difference method

Stirlling formula

Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).

TRUE

FALSE

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

TRUE

FALSE

The characteristics polynomial of a 3x 3

identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

(x-1)^3

(x+1)^3

x^3-1

x^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE

FALSE

==========

Set-10

/wEPDwUKMTY2I

/wEWBgLDz9rUC

MC090400760 : Imran Shahzad

Time Left 67 sec(s)

Quiz Start Time: 05:13 PM

Question # 1 of 10 (Start time: 05:13:27 PM) **Total Marks:** 1

The determinant of a _____ matrix is the product of the diagonal elements.

Select correct option:

- diagonal
- upper triangular
- lower triangular
- scalar

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBALlstSEAv

MC090400760 : Imran Shahzad

Time Left 44 sec(s)

Quiz Start Time: 05:13 PM

Question # 2 of 10 (Start time: 05:14:53 PM) **Total Marks:** 1

The absolute value of a determinant (|detA|) is the product of the absolute values of the eigenvalues of matrix A

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAL85ba6D

MC090400760 : Imran Shahzad

Time Left 78 sec(s)

Quiz Start Time: 05:13 PM

Question # 3 of 10 (Start time: 05:16:13 PM) **Total Marks:** 1

Central difference method seems to be giving a better approximation, however it requires more computations.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBALvyM2J(

MC090400760 : Imran Shahzad

Time Left 66 sec(s)

Quiz Start Time: 05:13 PM

Question # 4 of 10 (Start time: 05:17:36 PM) **Total Marks:** 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

- (•) TRUE
- ( ) FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLv7oHdI

MC090400760 : Imran Shahzad

Time Left 67 sec(s)

Quiz Start Time: 05:13 PM

Question # 5 of 10 (Start time: 05:18:12 PM) **Total Marks:** 1

Let A be an n ×n matrix. The number x is an eigenvalue of A if there exists a non-zero vector v such that _____.

Select correct option:

- ( ) Av = xv
- ( ) Ax = xv

- Av + xv=0
- Av = Ax

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAKOofDw

MC090400760 : Imran Shahzad

Time Left 40 sec(s)

Quiz Start Time: 05:13 PM

Question # 6 of 10 (Start time: 05:19:22 PM) **Total Marks:** 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

Select correct option:

- TRUE
- /wEPDwUKMTY2I

/wEWBAKyoPefC

MC090400760 : Imran Shahzad

Time Left 78 sec(s)

Quiz Start Time: 05:13 PM

Question # 8 of 10 (Start time: 05:21:26 PM) **Total Marks:** 1

The Jacobi iteration converges, if A is strictly diagonally dominant.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgK0iY31C

MC090400760 : Imran Shahzad

Time Left 74 sec(s)

Quiz Start Time: 05:13 PM

/wEPDwUKMTY2I

/wEWBAKG4seDI

MC090400760 : Imran Shahzad

Time Left 47 sec(s)

Quiz Start Time: 05:13 PM

Question # 9 of 10 (Start time: 05:22:47 PM) Total Marks: 1

Central Difference method is the finite difference method.

Total Marks: 1

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAKw0/X2E

MC090400760 : Imran Shahzad

Time Left 34 sec(s)

Quiz Start Time: 05:13 PM

Question # 10 of 10 (Start time: 05:23:47 PM) Total Marks: 1

By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

(Start time: 05:20:47 PM)

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

Select correct option:

- (x-1)^3
- (x+1)^3
- x^3-1
- x^3+1

Click here to Save Answer & Move to Next Question

==========

Set-11

Contents:

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5. Spring 2010 FinalTerm OPKST MTH603 mc080408680.doc

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6. my MTH603 quiz.doc
7. my.mth603.doc
8. quizz mth603(2).doc
9. quizz mth603.doc

/w EPDw UKMTY:

/w EWBgKPm46n

Question # 1 of 10 (Start time: 04:40:08 PM) Total Marks: 1

The determinant of a diagonal matrix is the product of the diagonal elements.

1. True
2. False

Question # 2 of 10 (Start time: 04:40:58 PM) Total Marks: 1

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

1. True
2. false

A 3 x 3 identity matrix have three and different eigen values.

1. True
2. False

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

1. True

2. False

The Jacobi's method is a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

1. No
2. At least one

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

1. Unity
2. zero

/w EPDw UKMTY2

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

1. True
2. False

/w EWBgKin4OcB

/w EPDw UKMTY2

The determinant of a _____ matrix is the product of the diagonal elements.

1. Diagonal
2. Upper triangular
3. Lower triangular
4. Scalar

Waisay main nay is ka answer Diagnol keea tha….par yeh charon options theek hain….
You can confirm it from internet…

Jab main yeh MCQ kar raha tha tou tab hi mujhay is par shak ho raha tha….kyun k upper aur lower triangular matrices tou linear algebra mein bhi bahut ziada bataye gaye tou yeh property wahan se hi yaad thi…

Eigenvalues of a symmetric matrix are all _____.

1. Real
2. Zero
3. Positive
4. Negative /w EPDw UKMTY2

/w EWBgKR8tbaA

The Power method can be used only to find the eigen value of A that is largest in absolute value—we call this eigen value the dominant eigen value of A.

1. True
2. False

/w EPDw UKMTY2

/w EWBgKjyL19A

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

1. (x-1)^3
2. (x+1)^3
3. x^3-1
4. x^3+1

For differences methods we require the set of values.

1. True
2. False

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

1. True
2. False

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

1. True
2. False

a ko agar aap lambda se replace kar dain tou baat clear ho jaye gi….labda ki jagah a use keea gaya hai tou is liay yeh working thora sa confuse karti hai…

Central difference method seems to be giving a better approximation, however it requires more computations.

1. True
2. False

Iterative algorithms can be more rapid than direct methods.

1. True (main nay true hi keea tha, aap isay dekh lena)
2. False

Central Difference method is the finite difference method.

1. True
2. False

/w EPDw UKMTY2

MTH603 Numerical Analysis

Mid Term Examination - Spring 2006
Time Allowed: 90 Minutes

Question No. 1

Mar

ks : 10

Use bisection method to find the solution for
(Perform only three iterations.)

Question No. 2

$2 + \cos(e^{x} - 2) - e^{x} = 0$

on interval [0.5, 1.5]

Marks : 2

Bisection and false position methods are also known as bracketing method and are always

- Divergent
- Convergent

Question No. 3

Mar

ks : 10

Use Gauss Elimination method to solve the following system.

$4x_1 - x_2 + x_3 = 8$

$$2x_1 + 5x_2 + 2x_3 = 3$$
$$x_1 + 2x_2 + 4x_3 = 11$$

Question No. 4 **Mar**

ks : 2

The Inverse of a matrix can only be found if the matrix is

- Singular
- Non singular
- Scalar
- Diagonal

Question No. 5 **Mar**

ks : 2

If f (x) contains trigonometric, exponential or logarithmic functions then this equation is known as

- Transcendental equation
- Algebraic
- Polynomial
- Linear

Question No. 6 **Mar**

ks : 2

In interpolation is used to represent the δ

- Forward difference
- Central difference
- Backward difference

Question No. 7 **Mar**

ks : 2

The base of the decimal system is _____

- 10
- 0
- 2
- 8
- None of the above.

Question No. 8

Use Newton's Raphson Method to find the solution for (Perform only three iterations.)

Question No. 9

Marks : 10

$x^3 + 3x^2 - 1 = 0$ on [-3,-2].

Marks : 10

Approximate f(0.05) by using any of the interpolation technique.

x	0.0	0.2	0.4	0.6	0.8
F(x)	1.000	1.22140	1.49182	1.82212	2.22554

$$[(f_0 + f_n) + 2\sum_{i=1}^{n-1} f_i]$$

is known as

- Simpson's 1/3 rd Rule
- Simpson's 3/8 rule
- Trapezoidal rule

Question No: 2 (Marks: 2) - Please choose one

Bisection method is …….. method

- Open Method
- Bracketing Method

Question No: 3 (Marks: 2) - Please choose one

Which method is not used to solve problems related to integration?

- Runge-Kutta Method
- Simpson's 1/3rd rule
- Trapezoidal rule.

Question No: 7 (Marks: 10)

Interpolate the value of 0.25 using Newton's forward difference formula.

x	0.2	0.3	0.4	0.5	0.6
F(x)	0.2304	0.2788	0.3222	0.3617	0.3979

(Perform all the necessary calculation missing calculation and steps may deduct marks.)

Exact solution of 2/3 is not exists.
TRUE
FALSE

The Jacobi's method is
a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

no
atleast one

A 3 x 3 identity matrix have three and _____eigen values.
same
different

Eigenvalues of a symmetric matrix are all _____ .
real
complex
zero
positive

The Jacobi iteration converges, if A is strictly diagonally dominant.
TRUE
FALSE

Below are all the finite difference methods EXCEPT _____.

jacobi's method
newton's backward difference method
Stirlling formula
Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).
TRUE
FALSE

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

TRUE
FALSE

The characteristics polynomial of a 3x 3
identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

(x-1)^3
(x+1)^3
x^3-1
x^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE
FALSE

Top

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i have attempt mth 603 today..... my paper was power method 5 marks , from vactor 3 marks, Newton_Repshon's forwrd distnce formula 5 marks.,define extra polation 2 marks ,, and most of MCQ's from , Newton_Repshon's method ,or 18 to 22 lec's

question 29: Distinguish between Related and unrelated diversification with example 5marks
question 30 Five porters Model 5 marks
question 31: conecntric diversification two example 3marks
question 32 significant of R&D for an organization 3marks

Which of the following period strategic management was considered to be cure for all problems?
Mid 1950s to mid 1960s
Mid 1960s to mid 1970s
Mid 1970s to mid 1980s
Mid 1980s to mid 1990s

Which of the following is not a pitfall an organization should avoid in strategic planning?
Select correct option:
Failing to involve key employees in all phases of planning
Involving all managers rather than delegating planning to a planner
Top managers not actively supporting the strategic planning process
Doing strategic planning only to satisfy accreditation or regulatory requirements

which of the following are the factors that concern the nature and direction of the economy in which a firm operates?
Select correct option:
Technological
Ecological
Social
Economic

Which of the following best describes this statement; “a Systematic and ethical process for gathering and analyzing information about the competition’s activities and general business trends to further a business’ own goals”?
Select correct option:
External assessment

Industry analysis
Competitive intelligence program
Business ethics

According to Porter, which strategy offers products or services to a small range of customers at the lowest price available on the market?
Select correct option:
Low cost
Best value
Cost focus
Differentiation

Long-term objectives includes all of the following EXCEPT:

Measurable

Reasonable

Varying

Consistent

Which one of the following is NOT is a basic mission of a competitive intelligence program?

To provide a general understanding of an industry

To provide a general understanding of a company's competitors

To identify industry executives who could be hired by the firm

To identify potential moves a competitor might make that would endanger a firm

While preparing an External Factor Evaluation Matrix, a total score of 0.8 indicates that:

Firm is taking advantages of strengths and avoiding threats

Firm is taking no advantage of opportunities and is avoiding threats

Firm is not taking advantages of opportunities and is not avoiding threats

Firm is taking advantage of opportunities and is avoiding the threats

Top

Use the Regula-Falsi method to compute a real root of the equation

$x^3 - 9x + 1 = 0$, **(Perform two iterations for each part)**

(i) if the root lies between 2 and 4
(ii) if the root lies between 2 and 3.

Solve the following system of equations by Crout's method

$$x_1 + 2x_2 + x_3 = 2$$
$$2x_1 + x_2 - 10x_3 = 0.182$$
$$2x_1 + 3x_2 - x_3 = 2$$

Find the numerical solution of

$$55x - 25z = -200$$
$$-37y - 4z = -250$$
$$-25x - 4y + 29z = 100$$

Using Gauss-Siedel Mehtod with $(1,1,1)$ as the initial value. Perform two iterations only.

Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$ by the bisection method.
(Perform four iterations only)

Top

Question # 1 of 8 (Start time: 08:34:31 PM) **Total Marks:** 1

Adams – Bashforth is a multistep method.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 2 of 8 (Start time: 08:35:01 PM)

Generally, Adams methods are superior if output at _____ points is needed.

Select correct option:

- many

- two
- single
- at most

Click here to Save Answer & Move to Next Question

Question # 3 of 8 (Start time: 08:36:02 PM) **Total Marks:** 1

In Trapezoidal rule, the integral is computed on each of the sub-intervals by using linear interpolating formula, i.e. for n = 1 and then summing them up to obtain the desired integral.

Total Marks: 1

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

Question # 4 of 8 (Start time: 08:36:52 PM)

Euler's Method numerically computes the approximate _____ of a function.

Select correct option:

- antiderivative
- derivative
- value
- error

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 5 of 8 (Start time: 08:37:25 PM)

Multistep method does not improves the accuracy of the answer at each step.

Select correct option:

- FALSE
- TRUE

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 6 of 8 (Start time: 08:37:56 PM)

The trapezoidal rule is a numerical method that approximates the value of a _____.

Select correct option:

- indefinite integral
- definite integral
- improper integral
- function

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 7 of 8 (Start time: 08:38:47 PM)

Simpson's rule is a numerical method that approximates the value of a definite integral by using _____ polynomials.

Select correct option:

- quadratic
- linear
- cubic
- quartic

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 8 of 8 (Start time: 08:39:23 PM)

In Simpson's Rule, we use parabolas to approximate each part of the curve. This proves to be very efficient as compared to Trapezoidal rule.

Select correct option:

- TRUE
- FALSE

Question # 3 of 8 (Start time: 09:46:11 PM) Total Marks: 1

The first langrange polynomial with equally spaced nodes produced the formula for _____.

Select correct option:

- Simpson's rule
- Trapezoidal rule
- Newton's method
- Richardson's method

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2 /wEWBgLgsdHjB

MC090406505 : Sumera Naz

Time Left 12 sec(s)

Quiz Start Time: 09:44 PM

Question # 4 of 8 (Start time: 09:46:48 PM) Total Marks: 1

The need of numerical integration arises for evaluating the indefinite integral of a function that has no explicit antiderivative or whose antiderivative is not easy to obtain.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2 /w EWCgKl9pWZE

MC090406505 : Sumera Naz

Time Left 73 sec(s)

Quiz Start Time: 09:44 PM

Question # 5 of 8 (Start time: 09:48:13 PM) Total Marks: 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas
- constant

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2 /w EWCgLo7tKAA

MC090406505 : Sumera Naz Time Left 69 sec(s)

Quiz Start Time: 09:44 PM

Question # 6 of 8 (Start time: 09:48:35 PM) Total Marks: 1

The Euler method is numerically unstable because of _____ convergence of error.

Select correct option:

- slow
- fast
- moderate
- no

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2 /w EWBgLyjI72BA

MC090406505 : Sumera Naz Time Left 74 sec(s)

Quiz Start Time: 09:44 PM

Question # 8 of 8 (Start time: 09:49:41 PM) Total Marks: 1

Adams – Bashforth is a multistep method.

Select correct option:

- TRUE
- FALSE

Click here to Save Answ er & Move to Next Question

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/wEPDwUKMTY2N /wEWBgK+3MqIA

Question # 1 of 8 (Start time: 01:24:49 PM) **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

- antiderivative
- derivatives

/wEPDwUKMTY2N /wEWBgLy4ZCpA

Question # 2 of 8 (Start time: 01:26:18 PM) **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

- TRUE
- FALSE

/wEPDwUKMTY2N /wEWBgLlwO3cC

Question # 3 of 8 (Start time: 01:27:35 PM) **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

- diverge
- converge

/wEPDwUKMTY2N /wEWCgKMu86bD

Question # 5 of 8 (Start time: 01:30:21 PM) **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas
- constant

/wEPDwUKMTY2N /wEWBgK6oIaxB

Question # 6 of 8 (Start time: 01:31:18 PM) **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or ∞ or -∞ or, in some cases, as both endpoints approach limits.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N

/wEWBgKPg8yyD

Question # 7 of 8 (Start time: 01:32:33 PM) **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

Select correct option:

- FALSE
- TRUE

/wEPDwUKMTY2N /wEWCgKK6eLRD

Question # 8 of 8 (Start time: 01:33:57 PM) **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

Select correct option:

- constant
- finite
- infinity
- zero

Top

/wEPDwUKMTY2N /wEWBgK+3MqIA

Question # 1 of 8 (Start time: 01:24:49 PM) **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

- antiderivative
- derivatives

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWBgLy4ZCpA

Time Left 59 sec(s)

Question # 2 of 8 (Start time: 01:26:18 PM) **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

- TRUE
- FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWBgLlwO3cC

Time Left 79 sec(s)

Question # 3 of 8 (Start time: 01:27:35 PM) **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

- diverge
- converge

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWCgKMu86bD

Time Left 85 sec(s)

Question # 5 of 8 (Start time: 01:30:21 PM) **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

- straight lines
- curves
- parabolas
- constant

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWBgK6oIaxB

Time Left 83 sec(s)

Question # 6 of 8 (Start time: 01:31:18 PM) **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWBgKPg8yyD

Time Left 84 sec(s)

Question # 7 of 8 (Start time: 01:32:33 PM) **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

Select correct option:

FALSE

TRUE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N /wEWCgKK6eLRD

Time Left 84 sec(s)

Question # 8 of 8 (Start time: 01:33:57 PM) **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

Select correct option:

constant

finite

- infinity
- zero

Click here to Save Answer & Move to Next Question

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MTH603-Numerical Analysis
SEMESTER FALL 2005
www.vupages.com

Approximate the integral $\int_{1}^{1.6} \frac{2x}{x^2+1} dx$ using Simpson's 1/3 rule and calculate the percentage error. (Take result up to 4 decimal places)

Note: In order to get full marks do all necessary steps.

Construct a forward difference table for the following values

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

Note : In order to get full marks do all necessary steps.

Solve the system

$4x+3y=24$

$3x+4y-z=30$

$-y+4z=-24$

by Gauss Seidal Method, taking $(0, 0, 0)^t$ as initial approximation(Two iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Let $f(x) = x + \frac{2}{x}$, use cubic Lagrange interpolation based on the nodes $x_0 = 0.5, x_1 = 1, x_2 = 2$ *and* $x_3 = 2.5$ to approximate *f(1.5)* and *f(1.3)*.

Note : In order to get full marks do all necessary stepsSolution

Approximate the Dominant Eigenvalue and corresponding Eigenvector for the matrix

$$\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

by using Power Method. Start with $X_0 = (1,1,1)^t$. (Five iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Top

NUMERICAL ANALYSIS

Paper Final Term (Held: 25th Feb 2010)

No.	Questions	Mks
1	Find value of given data by Adam Moultan's method	10
2	Find value of given data by Dividend Difference Composite method	10
3	Draw backward difference tables for given Data x | | | | y | | | | x | | | | y | | | |	5+5
4	Write Simpson's 1/3 formula	2
5	Find value by Euler's Method	3
6	Find value of K1 by 2nd Order R-K method	2
7	Convergence is used when ----------	1
8	Bisection method is …….. method • Bracketing Method • Open • Random	1

	• none http://vustudents.ning.com/	
9	Newton Raphson method is …….. method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real • Vector • odd • even	1
11	Find value of y'(1) by Euler's Method taking h=1	2
12	Find value of y'(3) from given table. x | | | | y | | | |	2
13	Find value of y'(0.3) by Lagrange's Method x | | | | y | | | |	3
14	For Simpson's 1/3 rule no.of intervals must be • 1 • 3 • 5 • 8	1
15	For Simpson's 1/3 rule valid no.of intervals are • 1 • 3 • 5 • 8	1
16	For Simpson's 3/8 rule no.of intervals must be • 10 • 11 • 12 • 14 http://vustudents.ning.com/	1
17	Find the value of y'(1) from given forward difference table x | y | Δy | Δy^2 | Δy^3 | | | | | | | | | | | | | | | | | | | |	2

Numerical analysis mth603 paper
Numerical Analysis numerical paper 2009

The paper was very easy.
The mcq's were really easy.
Most of the mcq's were from the last 5 lecture.
and from jacobi's method and other's.
then the logical mcq's like
s inverse * s = I
it was repeated twice.
one 5 mark question was from newton's rapson method
one 10 mark question was from lecture 11 page 69 example
best of luck

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NUMERICAL ANALYSIS

Paper Final Term (Held: 25th Feb 2010)

No.	Questions	Mks
1	Find value of given data by Adam Moultan's method	10
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6	Find value of K1 by 2nd Order R-K method	2
7	Convergence is used when ----------	1
8	Bisection method is …….. method • Bracketing Method • Open • Random • none http://vustudents.ning.com/	1
9	Newton Raphson method is …….. method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real	1

<table>
<tr><td></td><td>• Vector
• odd
• even</td><td></td></tr>
<tr><td>11</td><td>Find value of y'(1) by Euler's Method taking h=1</td><td>2</td></tr>
<tr><td>12</td><td>Find value of y'(3) from given table.
<table><tr><td>x</td><td></td><td></td><td></td></tr><tr><td>y</td><td></td><td></td><td></td></tr></table></td><td>2</td></tr>
<tr><td>13</td><td>Find value of y'(0.3) by Lagrange's Method
<table><tr><td>x</td><td></td><td></td><td></td></tr><tr><td>y</td><td></td><td></td><td></td></tr></table></td><td>3</td></tr>
<tr><td>14</td><td>For Simpson's 1/3 rule no.of intervals must be
• 1
• 3
• 5
• 8</td><td>1</td></tr>
<tr><td>15</td><td>For Simpson's 1/3 rule valid no.of intervals are
• 1
• 3
• 5
• 8</td><td>1</td></tr>
<tr><td>16</td><td>For Simpson's 3/8 rule no.of intervals must be
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http://vustudents.ning.com/</td><td>1</td></tr>
<tr><td>17</td><td>Find the value of y'(1) from given forward difference table
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</table>

Numerical analysis mth603 paper
Numerical Analysis numerical paper 2009
The paper was very easy.
The mcq's were really easy.
Most of the mcq's were from the last 5 lecture.
and from jacobi's method and other's.
then the logical mcq's like
s inverse * s = I
it was repeated twice.
one 5 mark question was from newton's rapson method
one 10 mark question was from lecture 11 page 69 example
best of luck

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FINALTERM EXAMINATION
Spring 2010
MTH603- Numerical Analysis (Session - 2)

Ref No: 1508683
Time: 90 min
Marks: 60

Question No: 1 (Marks: 1) - Please choose one

Symbol used for forward differences is

- ∇
- Δ
- δ
- μ

Question No: 2 (Marks: 1) - Please choose one

The relationship between central difference operator and the shift operator is given by

- $\delta = \mathrm{E} - \mathrm{E}^{-1}$
- $\delta = \mathrm{E} + \mathrm{E}^{-1}$
- $\delta = \mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}}$
- $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}}$

Question No: 3 (Marks: 1) - Please choose one

Muller's method requires --------starting points

- 1
- 2
- 3
- 4

Question No: 4 (Marks: 1) - Please choose one

If S is an identity matrix, then

- $S^{-1} = S$
- $S^t = S$
- $All\ are\ true$
- $S^{-1} = S^t$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with

y_x at $x_{0,}x_1,...,x_r$

- ► r+2
- ► r+1
- ► r
- ► r-1

Question No: 6 (Marks: 1) - Please choose one

_____ P in Newton's forward difference formula is defined as

- ► $p = (\frac{x - x_0}{h})$
- ► $p = (\frac{x + x_0}{h})$
- ► $p = (\frac{x + x_n}{h})$
- ► $p = (\frac{x - x_n}{h})$

Question No: 7 (Marks: 1) - Please choose one

_____ Octal number system has the base ----------

- ► 2
- ► 8
- ► 10
- ► 16

Question No: 8 (Marks: 1) - Please choose one

_____ Newton's divided difference interpolation formula is used when the values of the independent variable are

- ► Equally spaced
- ► Not equally spaced
- ► Constant
- ► None of the above

Question No: 9 (Marks: 1) - Please choose one

_____ Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

- 1.5
- 3
- 2
- 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

- $\varepsilon(x) = y(x) + P_n(x)$
- $\varepsilon(x) = y(x) - P_n(x)$
- $\varepsilon(x) = P_n(x) - y(x)$
- $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11 (Marks: 1) - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I .

- n-1
- n+2
- n
- n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

- $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + ...)$
- $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + ...)$
- $D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - ...)$
- $D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} - ...)$

Question No: 13 (Marks: 1) - Please choose one

Finding the

first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

- Forward difference operator
- Backward difference operator
- Central difference operator
- None of the given choices

Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

- -0.5
- 0.5
- 0.75
- -0.75

Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

- $ax+b$
- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question No: 16 (Marks: 1) - Please choose one

While integrating $I=\int_a^b f(x)dx$, h, width of the interval, is found by the formula-----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

Question No: 17 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

- 7
- 8
- 5
- 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f^{//}(0.2)$, using three-point equation will be calculated as ……

- 17.5
- 12.5
- 7.5
- -12.5

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

- 2
- 3
- 5
- 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

- 10
- 11
- 12
- 13

Question No: 21 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- $\frac{(a+b)}{2}$
- $\frac{(a-b)}{2}$
- $\frac{(b-a)}{2}$
- None of the given choices

Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- Bisection Method
- Regula Falsi Method
- Secant Method
- None of the given choices

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

- (1, 3)
- (1, 2)
- (0, 1)
- (1, 2)

Question No: 24 (Marks: 1) - Please choose one

Rate of change of any quantity with respect to another can be modeled by

- An ordinary differential equation
- A partial differential equation
- A polynomial equation
- None of the given choices

Question No: 25 (Marks: 1) - Please choose one

If $\frac{dy}{dx} = f(x, y)$

Then the integral of this equation is a curve in

- None of the given choices
- xt-plane
- yt-plane
- xy-plane

Question No: 26 (Marks: 1) - Please choose one

In solving the differential equation

$$y' = x + y \;;\; y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

- 1.44
- 1.11
- 1.22
- 1.33

Question No: 27 (Marks: 1) - Please choose one

In second order Runge-Kutta method

k_1 is given by

- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

Question No: 28 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

- $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
- $k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

Question No: 29 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

- $k_3 = hf(x_n + 2h, y_n + 2k_3)$
- $k_3 = hf(x_n - h, y_n - k_3)$
- $k_3 = hf(x_n + h, y_n + k_3)$
- None of the given choices

Question No: 30 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

- Newton's backward difference interpolation formula
- Newton's forward difference interpolation formula
- Newton's divided difference interpolation formula
- None of the given choices

Question No: 31 (Marks: 2)

If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32 (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2)dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

Evaluate the integral

$$\int_3^5 (\log x + 2) dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36 (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37 (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38 (Marks: 5)

Evaluate the integral

$$\int_0^3 (x^2 + x) dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39 (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$ taking $h = 0.1$

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Set-12

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Set-13

==========

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Mth603 Solved MCQS for Final Term Exam

Exact solution of 2/3 is not exists.
TRUE
FALSE

The Jacobi's method is A method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

No
At least one

A 3 x 3 identity matrix have three and _____eigen values.
Same
Different

Eigenvalues of a symmetric matrix are all _____ .
Real
Complex
Zero
Positive

The Jacobi iteration converges, if A is strictly diagonally dominant.
TRUE
FALSE

Below are all the finite difference methods EXCEPT _____.

Jacobi's method
Newton's backward difference method
Stirlling formula
Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).
TRUE
FALSE

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix, the eigenvalues of A are the diagonal entries of A.

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TRUE
FALSE

The characteristics polynomial of a 3x 3 Identity matrix is _____, if x is the Eigen values of the given 3 x 3 identity matrix. Where symbol ^ shows power.

(X-1)^3
(x+1)^3
X^3-1
X^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE
FALSE

Bisection method is a

Bracketing method
Open method

Regula Falsi means

Method of Correct position
Method of unknown position
Method of false position
Method of known position

Eigenvalues of a symmetric matrix are all _____.
Select correct option:

Real
Zero
Positive
Negative

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.
Select correct option:

TRUE
FALSE

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Exact solution of 2/3 is not exists.
Select correct option:

TRUE
FALSE

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric _____ definite matrices A.
Select correct option:

Positive
Negative

Differences methods find the _____ solution of the system.
Select correct option:

Numerical
Analytical

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this Eigenvalue the dominant eigenvalue of A.
Select correct option:

TRUE
FALSE

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.
Select correct option:

Main diagonal
Last column
Last row
First row

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If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.
Select correct option:

TRUE
FALSE

A 3 x 3 identity matrix have three and different Eigen values.
Select correct option:

TRUE
FALSE

Newton Raphson method falls in the category of

Bracketing method
Open Method
Iterative Method
Indirect Method

Newton Raphson method is also known as
Tangent Method
Root method
Open Method
Iterative Method

Secant Method uses values for approximation

1
3
2
4

Secant Method is than bisection method for finding root
Slow
Faster

In Newton Raphson method

Root is bracketed
Root is not bracketed

Regula falsi method and bisection method are both

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Convergent
Divergent

In bisection method the two points between which the root lies are

Similar to each other
Different
Not defined
Opposite

In which methods we do not need initial approximation to start
Indirect Method
Open Method
Direct Method
Iterative Method

Root may be

Complex
Real
Complex or real
None

In Regula falsi method we choose points that have signs

2 points opposite signs
3 points opposite signs
2 points similar signs
None of the given

In a bounded function values lie between
1 and -1
1 and 2
0 and 1
0 and -2

Newton Raphson method is a method which when it leads to division of number close to zero
Diverges
Converges

Which of the following method is modified form of Newton Raphson Method?
Regula falsi method
Bisection method
Secant method

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Jacobi's Method

Which 1 of the following is generalization of Secant method?
Muller's Method
Jacobi's Method
Bisection Method
N-R Method

Secant Method needs starting points

2
3
4
1

Near a simple root Muller's Method converges than the secant method

Faster
Slower

If S is an identity matrix, then

$S^{-1} = S$
$S^t = S$
$S^{-1} = S^t$
All are true

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree --- agreeing with y_x at $x_0, x_1, \ldots, x_r$

r+2
r+1
R
R-1

P in Newton's forward difference formula is defined as

$$p = \left(\frac{x - x_0}{h}\right)$$

$$p = \left(\frac{x + x_0}{h}\right)$$

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$$p = (\frac{x + x_n}{h})$$

$$p = (\frac{x - x_n}{h})$$

Octal numbers has the base

10
2
8
16

Newton's divided difference interpolation formula is used when the values of the independent variable are

Equally spaced

Not equally spaced

Constant
None of the above

Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

1.5

3
2

1

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

$$\varepsilon(x) = y(x) + P_n(x)$$

$$\varepsilon(x) = y(x) - P_n(x)$$

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$\varepsilon(x) = y(x) \times P_n(x)$

$\varepsilon(x) = P_n(x) + y(x)$

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I.

N-1
N+2
N
N+1

Differential operator in terms of forward difference operator is given by

$$D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + ...)$$

$$D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + ...)$$

$$D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - ...)$$

$$D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} - ...)$$

Finding the first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

Forward difference operator
Backward difference operator
Central difference operator
All of the given choices

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For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

-0.5
0.5
0.75
-0.75

In Simpson's 1/3 rule, $f(x)$ is of the form

- $ax+b$
- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

$$I=\int_a^b f(x)dx$$

While integrating , h, width of the interval, is found by the formula-----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$

None of the given choices

To apply Simpson's 1/3 rule, valid number of intervals are.....

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7
8
5
3

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$ **, using three-point equation will be calculated as ……**

17.5
12.5
7.5
-12.5

To apply Simpson's 1/3 rule, the number of intervals in the following must be

2
3
5
7

To apply Simpson's 3/8 rule, the number of intervals in the following must be

10
11
12
13

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

$\frac{(a+b)}{2}$
$\frac{(a-b)}{2}$

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$\frac{(b-a)}{2}$

None of the given choices

.....lies in the category of iterative method.

Bisection Method
Regula Falsi Method
Secant Method
All of the given choices

For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

(1, 3)
(1, 2)
(0, 1)
(1, 2)

Rate of change of any quantity with respect to another can be modeled by

An ordinary differential equation
A partial differential equation

A polynomial equation

None of the given choices

If
$\frac{dy}{dx}=f(x,y)$

Then the integral of this equation is a curve in

None of the given choices

Xt-plane
Yt-plane
Xy-plane

In solving the differential equation

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$$y' = x + y \;;\; y(0.1) = 1.1$$

$h = 0.1$, **By Euler's method** $y(0.2)$ **is calculated as**

1.44
1.11
1.22
1.33

In second order Runge-Kutta method
k_1 is given by

$k_1 = hf(x_n, y_n)$
$k_1 = 2hf(x_n, y_n)$
$k_1 = 3hf(x_n, y_n)$
None of the given choices

In fourth order Runge-Kutta method, k_2 is given by

$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
$k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$
$k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$
$k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

In fourth order Runge-Kutta method, k_4 **is given by**

$k_3 = hf(x_n + 2h, y_n + 2k_3)$
$k_3 = hf(x_n - h, y_n - k_3)$
$k_3 = hf(x_n + h, y_n + k_3)$
None of the given choices

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Adam-Moulton P-C method is derived by employing

Newton's backward difference interpolation formula
Newton's forward difference interpolation formula
Newton's divided difference interpolation formula
None of the given choices

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain

Derivatives
Antiderivative

If $|A| \neq 0$ then system will have a

Definite solution
Unique solution
Correct solution
No solution

If $|A| = 0$ then

There is a unique solution
There exists a complete solution
There exists no solution
None of the above options

Direct method consists of method

2
3
5
4

We consider Jacobi's method Gauss Seidel Method and relaxation method as

Direct method
Iterative method
Open method
All of the above

In Gauss Elimination method Solution of equation is obtained in

3 stages
2 stages
4 stages
5 stages

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Gauss Elimination method fails if any one of the pivot values becomes
Greater
Small
Zero
None of the given

Changing the order of the equation is known as

Pivoting
Interpretation

Full pivoting is than partial pivoting
Easy
More complicated

The following is the variation of Gauss Elimination method

Jacobi's method
Gauss Jordan Elimination method

Courts reduction method is also known as Cholesky Reduction method
True
False

Jacobi's method is also known as method of Simultaneous displacement
True
False
Gauss Seidel method is also known as method of Successive displacement
False
True
In Jacobi's method approximation calculated is used for
Nothing
Calculating the next approximation
Replaced by previous one
All above

In Gauss Seidel method approximation calculated is replaced by previous one
True
False

Relaxation method is derived by
South well
Not defined

Power method is applicable for only
Real metrics

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Symmetric
Unsymmetrical
Both symmetric and real

The process of eliminating value of y for intermediate value of x is know as interpolation
True
False

In Richardson's extrapolation method, we usually use two different step sizes ……and …… to yield a higher order method.

h, h/2
h, h/3
h, h/4
None

In Simpson's 3/8 rule, we divide the interval of integration into n sub-intervals. Where n is divisible by.....

3
4
5
None

1-Generally, Adams methods are superior if output at many points is needed.

- True
- False

2- Euler's method is only useful for a few steps and small step sizes; however Euler's method together with Richardson extrapolation may be used to increase the _____.

- **order and accuracy**
- divergence

3- The first lngrange polynomial with equally spaced nodes produced the formula for _____.

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- Simpson's rule
- **Trapezoidal rule**
- Newton's method
- Richardson's method

4- The need of numerical integration arises for evaluating the indefinite integral of a function that has no explicit antiderivative or whose antiderivative is not easy to obtain.

- TRUE
- **FALSE**

5- The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule

- **straight lines**
- curves
- parabolas
- constant

6- The Euler method is numerically unstable because of _____ convergence of error.
- Slow
- Fast
- Moderate
- No

7- Adams – Bashforth is a multistep method.

- **True**
- False

8- The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

- Antiderivative
- **Derivatives**

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9- In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

- **True**
- False

10- An indefinite integral may _____ in the sense that the limit defining it may not exist.

- **Diverge**
- Converge

11- The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

- **straight lines**
- curves
- parabolas
- constant

12- An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

- True
- False

13- Euler's Method numerically computes the approximate derivative of a function.

- True
- False

14- If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

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- Constant
- Finite
- Infinity
- zero

Question : While solving a system of linear equations, which of the following approach is economical for the computer memory?
Select correct option:

Direct
Iterative
Analytical
Graphical

Question : The basic idea of relaxation method is to reduce the largest residual to ……
Select correct option:

One
Two
Zero
None of the given choices

Question: The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.
Select correct option:
main diagonal
last column
last row
first row

Question: If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.
Select correct option:
TRUE
FALSE

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Question : A 3 x 3 identity matrix have three and different eigen values.
Select correct option:
TRUE
FALSE

Question : Which of the following is a reason due to which the LU decomposition of the system of linear equations; x+y = 1, x+y =2 is not possible?
Select correct option:
Associated coefficient matrix is singular
All values of l's and u's can't be evaluated
Determinant of coefficient matrix is zero
All are equivalent

Question : Gauss - Jordan Method is similar to …….
Select correct option:
Gauss–Seidel method
Iteration's method
Relaxation Method
Gaussian elimination method

Question : While using Relaxation method, which of the following is the largest Residual for 1st iteration on the system; 2x+3y = 1, 3x +2y = - 4 ?
Select correct option:
-4
3
2
1

Question : Gauss–Seidel method is also known as method of …….
Select correct option:
Successive displacement
Iterations
False position
None of the given choices

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Question : Jacobi's Method is a/an……
Select correct option:
Iterative method
Direct method

Question : The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.
Select correct option:
(x-1)^3
(x+1)^3
x^3-1
x^3+1

Question : The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.
Select correct option:
TRUE
FALSE

Question: In …… method, a system is reduced to an equivalent diagonal form using elementary transformations.
Select correct option:
Jacobi's
Gauss-Seidel
Relaxation
Gaussian elimination

Question : The linear equation: 2x+0y-2=0 has -------- solution/solutions.
Select correct option:

unique
no solution
infinite many
finite many

Question : Under elimination methods, we consider, Gaussian elimination and ……methods.
Select correct option:

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Gauss-Seidel
Jacobi
Gauss-Jordan elimination
None of the given choices

Question : Which of the following method is not an iterative method?
Select correct option:

Jacobi's method
Gauss-Seidel method
Relaxation methods
Gauss-Jordan elimination method

Question : An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.
Select correct option:
TRUE
FALSE

Question : Exact solution of 2/3 is not exists.
Select correct option:
TRUE

FALSE

Question : When the condition of diagonal dominance becomes true in Jacobi's Method. Then its means that the method is …….
Select correct option:

Stable
Unstable
Convergent
Divergent

Question : Gauss–Seidel method is similar to …….
Select correct option:

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Iteration's method
Regula-Falsi method
Jacobi's method
None of the given choices

Question : Sparse matrices arise in computing the numerical solution of …….
Select correct option:

Ordinary differential equations
Partial differential equations
Linear differential equations
Non-linear differential equations

Question : While solving by Gauss-Seidel method, which of the following is the first Iterative solution for the system; x-2y =1, x+4y=4 ?
Select correct option:

(1, 0.75)
(0,0)
(1,0)
(0,1)

Question: While solving a system of linear equations by Gauss Jordon Method, after all the elementary row operations if there lefts also zeros on the main diagonal then which of the is true about the system?
Select correct option:

System may have unique solutions
System has no solution
System may have multiple numbers of finite solutions
System may have infinite many solutions

Question: Numerical methods for finding the solution of the system of equations are classified as direct and …… methods
Select correct option:

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Indirect
Iterative
Jacobi
None of the given choices

Question : If the Relaxation method is applied on the system; 2x+3y = 1, 3x +2y = - 4, then largest residual in 1st iteration will reduce to -------.
Select correct option:
zero
4
-1
-1

Question : While using Relaxation method, which of the following is the Residuals for 1st iteration on the system; 2x+3y = 1, 3x +2y =4 ?
Select correct option:
(2,3)
(3,-2)
(-2,3)
(1,4)

Question : If the order of coefficient matrix corresponding to system of linear equations is 3*3 then which of the following will be the orders of its decomposed matrices; 'L' and 'U'?
Select correct option:
Order of 'L' = 3*1, Order of 'U' = 1*3
Order of 'L' = 3*2, Order of 'U' = 2*3
Order of 'L' = 3*3, Order of 'U' = 3*3
Order of 'L' = 3*4, Order of 'U' = 4*3

Question : While solving the system; x–2y = 1, x+4y = 4 by Gauss-Seidel method, which of the following ordering is feasible to have good approximate solution?
Select correct option:
x+4y = 1, x-2y = 4
x+2y = 1, x- 4y =4
x+4y = 4, x–2y = 1
no need to reordering

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Question : Full pivoting, in fact, is more ……than the partial pivoting.
Select correct option:

Easiest
Complicated

Question : Gauss–Seidel method is also known as method of …….
Select correct option:
Successive displacement
Iterations
False position
None of the given choices

Question : For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

- (1, 3)
- (1, 2)
- (0, 1)
- (1, 2)

Question :-.....lies in the category of iterative method.

- Bisection Method
- Regula Falsi Method
- Secant Method
- all of the given choices

Question : Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.
True
1. false

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Question: A 3 x 3 identity matrix have three and different eigen values.
1. True
False
Question : If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).
1. True
False
Question : The Jacobi's method is a method of solving a matrix equation on a matrix that has ____zeros along its main diagonal.
No
1. At least one
Question : An eigenvector V is said to be normalized if the coordinate of largest magnitude is
equal to _____.
Unity
1. zero

Question : If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- ▶ $\frac{(a+b)}{2}$
- ▶ $\frac{(a-b)}{2}$
- ▶ $\frac{(b-a)}{2}$
- ▶ None of the given choices

Question : To apply Simpson's 3/8 rule, the number of intervals in the following must be

- ▶ 10

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► 11
► 12
► 13

Question : The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric_____ definite matrices A.
Select correct option:
positive
negative
Question : Differences methods find the _____ solution of the system.
Select correct option:
numerical
Analytical
Question : To apply Simpson's 1/3 rule, the number of intervals in the following must be

► 2 (**Simpson**"s **1/3 rule must use** an even **number** of elements')
► 3
► 5
► 7

Question : The Power method can be used only to find the eigenvalue of A that is largest in absolute value we call this eigenvalue the dominant eigenvalue of A.
Select correct option:

TRUE
FALSE

Question : The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.
Select correct option:

main diagonal
last column
last row
first row

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Question : Bisection and false position methods are also known as bracketing method and are always
Divergent
Convergent

Question : The Inverse of a matrix can only be found if the matrix is
Singular
Every square non-singular matrix will have an inverse.
Scalar
Diagonal
Question : In interpolation is used to represent the δ

Forward difference
Central difference
Backward difference

Question : The base of the decimal system is _____
10
0
2
8
None of the above.

Question : Bisection method is ..... method
► Open Method
► **Bracketing Method**
Question : Exact solution of 2/3 is not exists.
TRUE
FALSE
Question : The Jacobi's method is a method of solving a matrix equation on a matrix that has ____zeros along its main diagonal.
No
atleast one

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Question: A 3 x 3 identity matrix have three and _____eigen values.

same

different

Question : Eigenvalues of a symmetric matrix are all _____ .

real

complex

zero

positive

Question : The Jacobi iteration converges, if A is strictly diagonally dominant.

TRUE

FALSE

Question : Below are all the finite difference methods EXCEPT _____.

jacobi's method

newton's backward difference method

Stirlling formula

Forward difference method

Question: If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).

TRUE

FALSE

Question : If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

TRUE

FALSE

Question: The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

(x-1)^3

(x+1)^3

x^3-1

x^3+1

Question : Two matrices with the same characteristic polynomial need not be similar.

TRUE

FALSE

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Question : The determinant of a diagonal matrix is the product of the diagonal elements.

True

1. False

Qusetion : The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

True

1. False

Question : The determinant of a _____ matrix is the product of the diagonal elements.

Diagonal

1. Upper triangular
2. Lower triangular
3. Scalar

Question : For differences methods we require the set of values.

True

False

Question : If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

True

False

Question : Central difference method seems to be giving a better approximation, however it requires more computations.

True

False

Question : Iterative algorithms can be more rapid than direct methods.

True

1. False

Question : Central Difference method is the finite difference method.

True

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1. False

Question : Back substitution procedure is used in **……**
Select correct option:
Gaussian Elimination Method
Jacobi's method
Gauss-Seidel method
None of the given choices
Question : The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

True
False1 .

Question: The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.
•
main diagonal
last column
last row
first row
Question : .An eigenvector V is said to be normalized if the coordinate of largest magnitude is equalto _____.
Unity
Zero
Question : An eigenvector V is said to be normalized if the coordinate of largest magnitude is equalto zero.

TRUE
FALSE
Question : .The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

True

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False

Question : The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric_____ definite matrices A.

Pos I t ive
Negative

Q u e s t i o n : .T h e d e t e r m i n a n t o f a d i a g o n a l m a t r i x i s t h e p r o d u c t o f t h e d i a g o n a l e l e m e n t s.

True
False1

Question : Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

True
False

Question : Power method is applicable if the eigen values are _____.

real and distinct
real and equal
positive and distinct
negative and distinct

Question : Simpson's rule is a numerical method that approximates the value of a definite integral by using polynomials.

Quadratic
Linear
Cubic
Quartic

Q u e s t i o n : .I n S i m p s o n ' s R u l e , w e u s e p a r a b o l a s t o a p p r o x i m a t i n g e a c h p a r t o f t h e c u r v e . T h i s p r o v e s to be very efficient as compared to Trapezoidal rule.

True

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False
Question : The predictor-corrector method an implicit method. (multi-step methods)

True
False
Question : Generally, Adams methods are superior if output at many points is needed.

True
False

Question : The Trapezoidal rule is a numerical method that approximates the value of a._____.

Indefinite integral
Definite integral
Improper integral
Function
Question : The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Anti deri vat ive
Derivatives.
Question : .An indefinite integral may _____ in the sense that the limit defining it may not exist.

diverge
Converge
Question : An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or ∞ or -∞ or, in some cases, as both endpoints approach limits.

TRUE
FALSE
Question : Euler's Method numerically computes the approximate derivative of a function.

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TRUE
FALSE
Question :.Euler's Method numerically computes the approximate _____ of a function.

Antiderivative
Derivative
Error
Value
Question: If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.
Chose the correct option :
Constant
Finite
Infinity
Zero

Question : Euler's Method numerically computes the approximate derivative of a function.
•
TRUE
•
FALSE

Question: .The Jacobi iteration _____, if A is strictly diagonally dominant.

converges
Diverges
Question :. Two matrices with the same characteristic polynomial need not be similar.

TRUE
fALSE

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Question :. Differences methods find the _____ solution of the system.
Nu me rical
Analytica
Question : .By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.

TRUE
FALSE
Question : The absolute value of a determinant (|detA|) is the product of the absolute values of the eigen values of matrix A

TRUE
FALSE

Question : Eigenvectors of a symmetric matrix are orthogonal, but only for distinct eigenvalues.

TRUE
FALSE

.
Question : Let A be an n ×n matrix. The number x is an eigenvalue of A if there exists a non-zerovector v such that _____.

Av = xv
Ax = xv not shore
Av + xv=0
Av = Ax1
Question : In Jacobi's Method, the rate of convergence is quite _____ compared with other methods.

slow
Fast

Question : .Numerical solution of 2/3 up to four decimal places is _____.

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0.667
0.6666
0.6667
0.666671 .

Question : Symbol used for forward differences is

Δ Correct
δ
μ

Question : .The relationship between central difference operator and the shift operator is given by

$\delta = E - E^{-1}$

$\delta = E + E^{-1}$

$\delta = E^{1/2} + E^{1/2}$

$\delta = E^{1/2} - E^{1/2}$

Question : Muller's method requires -------- starting points

1

2

3

Question : By using determinants, we can easily check that the solution of the given system of linear equation _____ and it is _____.

Select correct option:
exits, unique
exists, consistent
trivial, unique
nontrivial, inconsistent

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Question : Two matrices with the _____ characteristic polynomial need not be similar.

Select correct option:
same
different

Question : In …… method, the elements above and below the diagonal are simultaneously made zero.

Select correct option:
Jacobi's
Gauss-Seidel
Gauss–Jordon Elimination
Relaxation

Question : Which of the following is equivalent form of the system of equations in matrix form; AX=B ?

Select correct option:
XA = B
X = B(Inverse of A)
X =(Inverse of A)B
BX = A

Question : If the determinant of a matrix A is not equal to zero then the system of equations will have…….

Select correct option:
a unique solution
many solutions
infinite many solutions
None of the given choices

Question : Sparse matrix is a matrix with …….

Select correct option:
Some elements are zero
Many elements are zero
Some elements are one

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Many elements are one

Question : An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.
Select correct option:

TRUE
FALSE

Question # 1 of 10 (Start time: 11:14:39 PM) Total Marks: 1
The Jacobi iteration _____, if A is strictly diagonally dominant.

Select correct option:
converges
diverges

Question # 2 of 10 (Start time: 11:16:04 PM) Total Marks: 1
The Jacobi's method is a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

Select correct option:
No
atleast one

Question # 3 of 10 (Start time: 11:17:14 PM) Total Marks: 1
Power method is applicable if the eigen vectors corresponding to eigen values are linearly _____.

Select correct option:
independent
dependent

Question # 4 of 10 (Start time: 11:17:42 PM) Total Marks: 1
Power method is applicable if the eigen values are _____.

Select correct option:
real and distinct
real and equal
positive and distinct
negative and distinct

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Question # 7 of 10 (Start time: 11:19:55 PM) Total Marks: 1
The determinant of a diagonal matrix is the product of the diagonal elements.

Select correct option:
TRUE
FALSE

Question # 8 of 10 (Start time: 11:21:14 PM) Total Marks: 1
For differences methods we require the set of values.

Select correct option:
TRUE
FALSE

Question # 10 of 10 (Start time: 11:23:55 PM) Total Marks: 1
Two matrices with the _____ characteristic polynomial need not be similar.

Select correct option:
Same
different

Question # 1 of 10 Total Marks: 1
While using Relaxation method, which of the following is the Residuals for 1st iteration on the system; 2x+3y = 1, 3x +2y =4 ?

Select correct option:
(2,3)
(3,-2)
(-2,3)
(1,4)

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Question # 2 of 10 (Start time: 11:14:32 PM) Total Marks: 1
Sparse matrices arise in computing the numerical solution of …….

Select correct option:
Ordinary differential equations
Partial differential equations

Linear differential equations
Non-linear differential equations

Question # 3 of 10 (Start time: 11:15:18 PM) Total Marks: 1
In …… method, the elements above and below the diagonal are simultaneously made zero.

Select correct option:
Jacobi's
Gauss-Seidel
Gauss–Jordon Elimination
Relaxation

Question # 5 of 10 (Start time: 11:17:54 PM) Total Marks: 1
Which of the following is equivalent form of the system of equations in matrix form; AX=B ?

Select correct option:
XA = B
X = B(Inverse of A)
X =(Inverse of A)B
BX = A

Question # 7 of 10 (Start time: 11:20:24 PM) Total Marks: 1
If the determinant of a matrix A is not equal to zero then the system of equations will have……

Select correct option:
A unique solution
many solutions
infinite many solutions
None of the given choices

Question # 8 of 10 (Start time: 11:21:37 PM) Total Marks: 1
Sparse matrix is a matrix with …….

Select correct option:
Some elements are zero

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Many elements are zero
Some elements are one
Many elements are one

Question # 4 of 10 (Start time: 11:31:21 PM) Total Marks: 1
Back substitution procedure is used in …….

Select correct option:
Gaussian Elimination Method
Jacobi's method
Gauss-Seidel method
None of the given choices

Question # 5 of 10 (Start time: 11:32:12 PM) Total Marks: 1
The linear equation: 2x+0y-2=0 has -------- solution/solutions.

Select correct option:
unique
no solution
infinite many
finite many

Question # 8 of 10 (Start time: 11:35:30 PM) Total Marks: 1
For a system of linear equations, the corresponding coefficient matrix has the value of determinant; |A| = 0, then which of the following is true?

Select correct option:
The system has unique solution
The system has finite multiple solutions
The system has infinite may solutions
The system has no solution

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Question # 9 of 10 (Start time: 11:36:21 PM) Total Marks: 1
For the system; 2x+3y = 1, 3x +2y = - 4, if the iterative solution is (0,0) and 'dxi = 2' is the increment in 'y' then which of the following will be taken as next iterative solution?

Select correct option:
(2,0)
(0,3)

(0,2)
(1,-4)

Question # 2 of 10 (Start time: 11:42:14 PM)Total Marks: 1
Which of the following method is not an iterative?

Select correct option:
Gauss–Seidel method
Iteration's method
Relaxation Method
Gauss Jordan method

Question # 3 of 10 (Start time: 11:43:46 PM)Total Marks: 1
Sparse matrix is a matrix with …….

Select correct option:
Some elements are zero
Many elements are zero
Some elements are one
Many elements are one

Question # 4 of 10 (Start time: 11:44:33 PM)Total Marks: 1
While using Relaxation method, which of the following is the Residuals for 1st iteration on the system; 2x+3y = 1, 3x +2y =4

Select correct option:
(2,3)
(3,-2)
(-2,3)
(1,4)

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Question # 6 of 10 (Start time: 11:47:15 PM)Total Marks: 1
Relaxation Method is a/an …….

Select correct option:

Direct method
Iterative method

Question # 9 of 10 (Start time: 11:50:33 PM)Total Marks: 1
Full pivoting, in fact, is more ……..than the partial pivoting.

Select correct option:
Easiest
Complicated

Question # 10 of 10 (Start time: 11:51:55 PM)Total Marks: 1
Gauss–Seidel method is also known as method of …….

Select correct option:
Successive displacement
Iterations
False position
None of the given choices

Question # 2 of 10 (Start time: 11:31:28 PM) Total Marks: 1
Iterative algorithms can be more rapid than direct methods.

Select correct option:
FALSE
TRUE

Question # 3 of 10 (Start time: 11:32:02 PM) Total Marks: 1
Below are all the finite difference methods EXCEPT _____.

Select correct option:
jacobi's method
newton's backward difference method
Stirling formula
Forward difference method

Question # 2 of 10 Total Marks: 1
Sparse matrices arise in computing the numerical solution of …….

Select correct option:
Ordinary differential equations
Partial differential equations
Linear differential equations

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Non-linear differential equations

Question # 9 of 10
If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

Select correct option:
TRUE
FALSE

Question # 10 of 10
An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.

Select correct option:
TRUE
FALSE

Question No: 1 (Marks: 1) - Please choose one
Symbol used for forward differences is

- ∇
- Δ
- δ
- μ

Question No: 2 (Marks: 1) - Please choose one
The relationship between central difference operator and the shift operator is given by

- $\delta = E - E^{-1}$
- $\delta = E + E^{-1}$
- $\delta = E^{\frac{1}{2}} + E^{-\frac{1}{2}}$
- $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

Question No: 3 (Marks: 1) - Please choose one
Muller's method requires --------starting points

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- 1
- 2
- **3**
- 4

Question No: 4 (Marks: 1) - Please choose one

If S is an identity matrix, then

- $S^{-1} = S$
- $S^{t} = S$
- $All\ are\ true$
- $S^{-1} = S^{t}$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_0, x_1, \ldots, x_r$

- r+2
- r+1
- **r**
- r-1

Question No: 6 (Marks: 1) - Please choose one

P in Newton's forward difference formula is defined as

- $p = (\frac{x - x_0}{h})$
- $p = (\frac{x + x_0}{h})$
- $p = (\frac{x + x_n}{h})$
- $p = (\frac{x - x_n}{h})$

Question No: 7 (Marks: 1) - Please choose one

Octal number system has the base ----------

- 2
- **8**

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► 10
► 16

Question No: 8 (Marks: 1) - Please choose one

Newton's divided difference interpolation formula is used when the values of the independent variable are

► Equally spaced

► **Not equally spaced**

► Constant
► None of the above

Question No: 9 (Marks: 1) - Please choose one

Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

► 1.5

► **3**
► 2

► 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

► $\varepsilon(x) = y(x) + P_n(x)$

► $\varepsilon(x) = y(x) - P_n(x)$

► $\varepsilon(x) = P_n(x) - y(x)$

► $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11 (Marks: 1) - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I.

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- n-1
- **n+2**
- n
- n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

- $D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}+\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}+...)$
- $D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2}+\frac{\Delta^3}{3}+\frac{\Delta^4}{4}+\frac{\Delta^5}{5}+...)$
- $D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\frac{\Delta^5}{5}-...)$
- $D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}-\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}-...)$

Question No: 13 (Marks: 1) - Please choose one

Finding the first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

- Forward difference operator
- **Backward difference operator**
- Central difference operator
- None of the given choices

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Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

- -0.5
- **0.5**
- 0.75
- -0.75

Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

- $ax+b$
- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question No: 16 (Marks: 1) - Please choose one

While integrating $I=\int_a^b f(x)dx$, h, width of the interval, is found by the formula-----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

Question No: 17 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

- 7
- **8**
- 5
- 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

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$f''(0.2)$, using three-point equation will be calculated as ……

- ► 17.5
- ► 12.5
- ► 7.5
- ► **-12.5**

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

- ► **2**
- ► 3
- ► 5
- ► 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

- ► 10
- ► 11
- ► **12**
- ► 13

Question No: 21 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- ► $\frac{(a+b)}{2}$
- ► $\frac{(a-b)}{2}$
- ► $\frac{(b-a)}{2}$
- ► None of the given choices

Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- ► Bisection Method

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► Regula Falsi Method
► Secant Method
► **All the given choices**

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

► (1, 3)
► (1, 2)
► **(0, 1)**
► (1, 2)

Question No: 24 (Marks: 1) - Please choose one

Rate of change of any quantity with respect to another can be modeled by

► **An ordinary differential equation**
► A partial differential equation
► A polynomial equation
► None of the given choices

Question No: 25 (Marks: 1) - Please choose one

If
$\frac{dy}{dx}=f(x,y)$
Then the integral of this equation is a curve in

► None of the given choices
► xt-plane
► **yt-plane**
► **xy-plane**

Question No: 26 (Marks: 1) - Please choose one

In solving the differential equation
$y'=x+y \;;\; y(0.1)=1.1$
$h=0.1$, By Euler's method $y(0.2)$ is calculated as

► 1.44
► 1.11
► **1.22**

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► 1.33

Question No: 27 (Marks: 1) - Please choose one

In second order Runge-Kutta method

k_1 is given by

► $k_1 = hf(x_n, y_n)$
► $k_1 = 2hf(x_n, y_n)$
► $k_1 = 3hf(x_n, y_n)$
► None of the given choices

Question No: 28 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

► $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
► $k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$
► $k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$
► $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

Question No: 29 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

► $k_3 = hf(x_n + 2h, y_n + 2k_3)$
► $k_3 = hf(x_n - h, y_n - k_3)$
► $k_3 = hf(x_n + h, y_n + k_3)$
► None of the given choices

Question No: 30 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

► **Newton's backward difference interpolation formula**
► Newton's forward difference interpolation formula
► Newton's divided difference interpolation formula

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► None of the given choices

Mth603 Solved MCQS for Final Term Exam

Solved by Mermaid with reference of book

Exact solution of 2/3 is not exists.
TRUE
FALSE

The Jacobi's method is
A method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

No
At least one

A 3 x 3 identity matrix have three and _____eigen values.
Same
Different

Eigenvalues of a symmetric matrix are all _____ .
Real
Complex
Zero
Positive

The Jacobi iteration converges, if A is strictly diagonally dominant.
TRUE
FALSE

Below are all the finite difference methods EXCEPT _____.

Jacobi's method
Newton's backward difference method
Stirling formula
Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).
TRUE
FALSE

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If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix, the eigenvalues of A are the diagonal entries of A.

TRUE
FALSE

The characteristics polynomial of a 3x 3 Identity matrix is _____, if x is the Eigen values of the given 3 x 3 identity matrix. Where symbol ^ shows power.

(X-1)^3
(x+1)^3
X^3-1
X^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE
FALSE

Bisection method is a

Bracketing method
Open method

Regula Falsi means

Method of Correct position
Method of unknown position
Method of false position
Method of known position

Eigenvalues of a symmetric matrix are all _____.
Select correct option:

Real
Zero
Positive
Negative

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.
Select correct option:

TRUE
FALSE

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Exact solution of 2/3 is not exists.
Select correct option:

TRUE
FALSE

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric _____ definite matrices A.
Select correct option:

Positive
Negative

Differences methods find the _____ solution of the system.
Select correct option:

Numerical
Analytical

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this Eigenvalue the dominant eigenvalue of A.
Select correct option:

TRUE
FALSE

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.
Select correct option:

Main diagonal
Last column
Last row
First row

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If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.
Select correct option:

TRUE
FALSE

A 3 x 3 identity matrix have three and different Eigen values.
Select correct option:

TRUE
FALSE

Newton Raphson method falls in the category of

Bracketing method
Open Method
Iterative Method
Indirect Method

Newton Raphson method is also known as
Tangent Method
Root method
Open Method
Iterative Method

Secant Method uses values for approximation

1
3
2
4

Secant Method is than bisection method for finding root
Slow
Faster

In Newton Raphson method

Root is bracketed
Root is not bracketed

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Regula falsi method and bisection method are both

Convergent
Divergent

In bisection method the two points between which the root lies are

Similar to each other
Different
Not defined
Opposite

In which methods we do not need initial approximation to start
Indirect Method
Open Method
Direct Method
Iterative Method

Root may be

Complex
Real
Complex or real
None

In Regula falsi method we choose points that have signs

2 points opposite signs
3 points opposite signs
2 points similar signs
None of the given

In a bounded function values lie between
1 and -1
1 and 2
0 and 1
0 and -2

Newton Raphson method is a method which when it leads to division of number close to zero
Diverges
Converges

Which of the following method is modified form of Newton Raphson Method?
Regula falsi method

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Bisection method
Secant method
Jacobi's Method

Which 1 of the following is generalization of Secant method?
Muller's Method
Jacobi's Method
Bisection Method
N-R Method

Secant Method needs starting points

2
3
4
1

Near a simple root Muller's Method converges than the secant method

Faster
Slower

If S is an identity matrix, then

$S^{-1} = S$
$S^t = S$
$S^{-1} = S^t$
All are true

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_0, x_1, \ldots, x_r$

r+2
r+1
R
R-1

P in Newton's forward difference formula is defined as

$$p = \left(\frac{x - x_0}{h}\right)$$

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$$p = (\frac{x + x_0}{h})$$

$$p = (\frac{x + x_n}{h})$$

$$p = (\frac{x - x_n}{h})$$

Octal numbers has the base

10
2
8
16

Newton's divided difference interpolation formula is used when the values of the independent variable are

Equally spaced

Not equally spaced

Constant
None of the above

Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

1.5
3
2
1

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

$$\varepsilon(x) = y(x) + P_n(x)$$

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$$\varepsilon(x) = y(x) - P_n(x)$$

$$\varepsilon(x) = y(x) \times P_n(x)$$

$$\varepsilon(x) = P_n(x) + y(x)$$

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I.

N-1
N+2
N
N+1

Differential operator in terms of forward difference operator is given by

$$D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + ...)$$

$$D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + ...)$$

$$D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - ...)$$

$$D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} - ...)$$

Finding the first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

Forward difference operator
Backward difference operator
Central difference operator
All of the given choices

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For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

-0.5
0.5
0.75
-0.75

In Simpson's 1/3 rule, $f(x)$ is of the form

- $ax+b$
- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

$$I=\int_a^b f(x)dx$$

While integrating , h , width of the interval, is found by the formula-----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

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To apply Simpson's 1/3 rule, valid number of intervals are.....

7
8
5
3

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as .....

17.5
12.5
7.5
-12.5

To apply Simpson's 1/3 rule, the number of intervals in the following must be

2
3
5
7

To apply Simpson's 3/8 rule, the number of intervals in the following must be

10
11
12
13

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

$\frac{(a+b)}{2}$

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$\frac{(a-b)}{2}$
$\frac{(b-a)}{2}$
None of the given choices

.....lies in the category of iterative method.

Bisection Method
Regula Falsi Method
Secant Method
All of the given choices

For the equation $x^3+3x-1=0$, the root of the equation lies in the interval.....

(1, 3)
(1, 2)
(0, 1)
(1, 2)

Rate of change of any quantity with respect to another can be modeled by

An ordinary differential equation
A partial differential equation

A polynomial equation

None of the given choices

If
$\frac{dy}{dx}=f(x,y)$
Then the integral of this equation is a curve in

None of the given choices

Xt-plane
Yt-plane
Xy-plane

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In solving the differential equation

$$y' = x + y \;;\; y(0.1) = 1.1$$

$h = 0.1$, **By Euler's method** $y(0.2)$ **is calculated as**

1.44
1.11
1.22
1.33

In second order Runge-Kutta method
k_1 is given by

$k_1 = hf(x_n, y_n)$
$k_1 = 2hf(x_n, y_n)$
$k_1 = 3hf(x_n, y_n)$
None of the given choices

In fourth order Runge-Kutta method, k_2 is given by

$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
$k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$
$k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$
$k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

In fourth order Runge-Kutta method, k_4 **is given by**

$k_3 = hf(x_n + 2h, y_n + 2k_3)$
$k_3 = hf(x_n - h, y_n - k_3)$

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$k_3 = hf(x_n + h, y_n + k_3)$

None of the given choices

Adam-Moulton P-C method is derived by employing

Newton's backward difference interpolation formula
Newton's forward difference interpolation formula
Newton's divided difference interpolation formula
None of the given choices

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain

Derivatives
Antiderivative

If $|A| \neq 0$ then system will have a

Definite solution
Unique solution
Correct solution
No solution

If $|A| = 0$ then

There is a unique solution
There exists a complete solution
There exists no solution
None of the above options

Direct method consists of method

2
3
5
4

We consider Jacobi's method Gauss Seidel Method and relaxation method as

Direct method
Iterative method
Open method
All of the above

In Gauss Elimination method Solution of equation is obtained in

3 stages
2 stages

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4 stages
5 stages

Gauss Elimination method fails if any one of the pivot values becomes
Greater
Small
Zero
None of the given

Changing the order of the equation is known as

Pivoting
Interpretation

Full pivoting is than partial pivoting
Easy
More complicated

The following is the variation of Gauss Elimination method

Jacobi's method
Gauss Jordan Elimination method

Courts reduction method is also known as Cholesky Reduction method
True
False

Jacobi's method is also known as method of Simultaneous displacement
True
False
Gauss Seidel method is also known as method of Successive displacement
False
True
In Jacobi's method approximation calculated is used for
Nothing
Calculating the next approximation
Replaced by previous one
All above

In Gauss Seidel method approximation calculated is replaced by previous one
True
False

Relaxation method is derived by
South well
Not defined

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Power method is applicable for only
Real metrics
Symmetric
Unsymmetrical
Both symmetric and real

The process of eliminating value of y for intermediate value of x is know as interpolation
True
False

Question No: 31 (Marks: 2)

If $F(h)=256.2354$ and $F(\frac{h}{2})=257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32 (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x+2)dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

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Evaluate the integral

$$\int_{3}^{5} (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36 (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37 (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38 (Marks: 5)

Evaluate the integral

$$\int_{0}^{3} (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39 (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$ taking $h = 0.1$

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Question No: 50 (Marks: 5)

Solve $\sin x = 1 + x^3$ by using the Newton Raphson method up to one iteration (Note: Accuracy up to four decimal places is required)

Answer:

$f(x) = \sin x = 1 + x^3$

$f(x) = \sin x - 1 - x^3$

$f(1) = \sin 1 - 1 - 1^3 = 0.0174 - 1 - 1 = -1.9826$

$f(2) = \sin 2 - 1 - 2^3 = 0.0348 - 1 - 8 = -8.9652$

$f'(1) = -\cos x - 3x^2 = -\cos 1 - 3(1)^2 = -0.9998 - 3 = -3.9998$

$f'(2) = -\cos x - 3x^2 = -\cos 2 - 3(2)^2 = -0.9993 - 12 = -12.9993$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = -3.9998 - \frac{-8.9652}{-12.9993} = -3.9998 - 0.6896 = -4.6894$$

Q: Evaluate the integral $\int_0^1 (x+1)\,dx$ By Trapezoidal Rule, h=0.2

Answer:

X	0	0.2	0.4	0.6	0.8	1
Y= (x+1)	1.0	1.2	1.4	1.6	1.8	2

Using trapezoidal rule, taking h=0.2

$$I = \frac{h}{2}\left(y_0 + y_5 + 2\left(y_1 + y_2 + y_3 + y_4\right)\right)$$

$$I = \frac{0.2}{2}\left(1.0 + 2 + 2(1.2 + 1.4 + 1.6 + 1.8)\right)$$

$$I = \frac{0.2}{2}\left(3 + 12\right)$$

$$I = \frac{3}{2} = 1.5$$

Q: Evaluate the integral $\int_3^5 (\sin x + \cos x)\,dx$ By Simpson`s 3/8 rule. Take the interval h=1.

Answer:

X	3	4	5
Y=(sin x+ cos x)	1.0509	1.0673	1.0833

$$\int_3^5 (\sin x + \cos x)dx = \frac{3h}{8}\left(y_0 + y_2 + 3\left(y_1\right)\right)$$

$$\int_3^5 (\sin x + \cos x)dx = \frac{3(1)}{8}\left(1.0509 + 1.0833 + 3(1.0673)\right)$$

$$\int_3^5 (\sin x + \cos x)dx = \frac{3}{8}\left(2.1342 + 3.2019\right)$$

$$\int_3^5 (\sin x + \cos x)dx = \frac{3}{8}(5.3361) = \frac{16.0083}{8} = 2.0010375$$

Q: Using Milne's method find y(0.1) dy/dx=x-y/x+y, Y(0)=1.

Answer:

$$\frac{dy}{dx} = \frac{x-y}{x+y},\ \ y(0) = 1,$$

$$find\ y(0.1)$$

$$y_0' = \frac{x-y}{x+y} = \frac{0-1}{0+1} = -\frac{1}{1} = -1$$

$$now, usingPredictor\ formula$$

$$y_1 = y_0 + \frac{4h}{3}\left(2y_0'\right)$$

Q: y'=2t+3y then find next two derivatives w.r.t t or y.

Answer:

w.r.t t 2nd dravative Y'' = 2 3rd dravative Y''' = 0
w.r.t y. 2nd dravative Y'' = 3 3rd dravative Y''' = 0

Q: Make backward difference table from;

X	**0**	**3**	**4**
Y	**4**	**7**	**9**

Q:Solve by Gauss-elimination method:
1 0 8/3 x 8

0 -1 1/3 y = 0

0 0 -1/3 z 1

Q: Find y by Euler's method when x=1, dy/dx=1/x+y, y(0)=1, h=1
Answer:

$$y_1 = y_0 + hf\left(x_0, y_0\right)$$

$$y_1 = 1 + 1\left(\frac{1}{x_0 + y_0}\right) = 1 + 1\left(\frac{1}{1+1}\right) = 1 + 0.5 = 1.5$$

Q: Evaluate the integral by Simpson's 1/3 rule , $\int_0^{\frac{\pi}{4}} (\sin x + 1)\, dx$ **taking interval of** $\frac{\pi}{4}$**.Answer:**

X	0	$\frac{\pi}{4}$
Y=(sin x+ 1)	1	1.7071

$$\int_3^5 (\sin x + 1)dx = \frac{h}{3}(y_0 + y_1)$$

$$\int_3^5 (\sin x + 1)dx = \frac{\frac{\pi}{4}}{3}(1 + 1.7071)$$

$$\int_3^5 (\sin x + 1)dx = \frac{3.14}{12}(2.7071) = \frac{8.500294}{12} = 0.7083$$

2012 paper1:

Q.1. State the sufficient condition of convergence of the iterative solution to the exact solution? 2m

Answer: page 72

A sufficient condition for convergence of the iterative solution to the exact solution is $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq 1}}^{n} |a_{ii}|$, i=1, 2...n When this condition (diagonal dominance) is true, Jacobi's method converges

Q.2. Evaluate the integral $\int_3^5 (\log x + 1)dx$ Using trapezoidal rule

Take h=1…..2m

Answer:

X	3	4	5
Y= logx+1	0.6020	0.6989	0.7781

Using trapezoidal rule, taking h=1

$$I = \frac{h}{2}(y_0 + y_3 + 2y_1)$$

$$I = \frac{1}{2}(0.6020 + 0.7781 + 2(0.6989))$$

$$I = \frac{1}{2}(1.3801 + 1.3978)$$

$$I = \frac{2.7779}{2} = 1.38895$$

Q3. Write the formula for finding the value of k_1 in the fourth order R-K Method. 2m

Answer: page 205

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

Q4. Write Adam-Moulton's Predictor formula for finding the solution of a differential equation. 2m

Answer: page 223

$$P: y_{n+1} = y_n + \frac{h}{24}\left(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$$

Q5. Obtain numerically the solution of y` = x^2 +2x+y^2, y (0) =1 Using Euler's method to find y at x=1, h=1 …. 3m

Answer:

$Y_1 = y_0 + hf(x_0, y_0)$

$Y_1 = 1 + (1)\ (1+1)$

$Y_1 = 1+2=3$

Q6. If f(2)=-2.6146 and f(3)=4.7610, then find the first approximation using the Regula-Falsi method? 3m

Answer:

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$x_3 = 3 - \frac{3-2}{4.7610 - (-2.6146)}(4.7610)$$

$$x_3 = 3 - \frac{4.7610}{7.3756}$$

$$x_3 = 3 - \frac{4.7610}{7.3756} = 3 - 0.6455 = 2.3545$$

Q7. If in a solving a giving differential equations

$y_0 = 1, y_1' = 1.1, y_2' = 1.2, y_3' = 1.3, h = 1$

then find y_4 *by Mi* ln *e* Pr *edictor formula* ……3m

Answer:

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

$$P: y_4 = y_0 + \frac{4h}{3}\left(2y'_1 - y'_2 + 2y'_3\right)$$

$$P: y_4 = 1 + \frac{4(1)}{3}\left(2(1.1) - 1.2 + 2(1.3)\right)$$

$$P: y_4 = 1 + \frac{4}{3}\left(2.2 - 1.2 + 2.6\right)$$

$$P: y_4 = 1 + \frac{4}{3}\left(3.6\right)$$

$$P: y_4 = 1 + \frac{14.4}{3}$$

$$P: y_4 = \frac{3 + 14.4}{3} = \frac{17.4}{3} = 5.8$$

Q8. Find F(h), using Richardson's extrapolation limit, while finding

$y'(0.01)\, to\, the\ function\ y = \frac{1}{x}\, with\, h = 0.005$ ……..3m

Answer:

$$F(h) = \frac{y(x+h) - y(x-h)}{2h}$$

$$F(h) = \frac{\frac{1}{0.01 + 0.005} - \frac{1}{0.01 - 0.005}}{2(0.005)}$$

$$F(h) = \frac{\frac{1}{0.015} - \frac{1}{0.005}}{0.01}$$

$$F(h) = \frac{66.66 - 200}{0.01}$$

$$F(h) = \frac{-133.34}{0.01} = -13334$$

Q.9

$Evaluate\, the \int egral$

$$\int_0^3 (x^2 + x)dx$$

$u \sin g\ Simpson's \frac{3}{8} rule$ take h=1……5m

Answer:

X	0	1	2	3
$Y=(x^2+x)$	0	2	6	12

$$\int_0^3 (x^2+x)dx = \frac{3}{8}h\left[y_0 + y_3 + 3(y_1+y_2)\right]$$

$$\int_0^3 (x^2+x)dx = \frac{3(1)}{8}\left[0+12+3(2+6)\right]$$

$$\int_0^3 (x^2+x)dx = \frac{3}{8}\left[12+24\right] = \frac{3}{8}(36) = \frac{108}{8} = 13.5$$

Q10. Construct a backward difference table from the following values of x and y…..5m

x	-1	0	1	2	3
Y=f(x)	10	2	10	62	80

Answer:

x	Y=f(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
-1	10				
0	2	-8			
1	10	8	16		
2	62	52	44	28	
3	80	18	-34	-78	-106

Q11. From the following table of values, construct forward difference table…..5m

X	1.00	1.05	1.10	1.15	1.20
y	1.000	1.0247	1.0488	1.0724	1.0954

Answer:

x	Y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.00	1.000				
1.05	1.0247	0.0247			
1.10	1.0488	0.0241	0.0006		
1.15	1.0724	0.0236	-0.0005	-0.0011	
1.20	1.0954	0.023	-0.0006	-0.0001	0.0000

Q12. Use Runge-Kutta Method of order four to find the values of $k_1k_2k_3k_4$ for the initial value problem

$y' = \frac{1}{2}(2x^3 + y)$, $y(1)=2$ taking $h=0.1$…….5m

Answer:

here $f(x,y)=\frac{1}{2}\left(2x^3+y\right),\quad h=0.1,\quad x_1=1,\quad y_1=2$

$$K_1=hf(x_1,y_1)=0.1\left(\frac{1}{2}\left(2\left(1^3+2\right)\right)\right)$$

$$K_1=hf(x_1,y_1)=0.1\left(\frac{6}{2}\right)=\frac{0.6}{2}=0.3$$

$$K_2=hf\left(x_1+\frac{h}{2},y_1+\frac{k_1}{2}\right)=0.1\left(1+\frac{0.1}{2},2+\frac{0.3}{2}\right)$$

$$K_2=hf\left(x_1+\frac{h}{2},y_1+\frac{k_1}{2}\right)=0.1\left(1.05,2+0.15\right)$$

$$K_2=hf\left(x_1+\frac{h}{2},y_1+\frac{k_1}{2}\right)=0.1\left(1.05,2.15\right)=0.1\left(1.05+2.15\right)=0.32$$

$$K_3=hf\left(x_1+\frac{h}{2},y_1+\frac{k_2}{2}\right)=0.1\left(1+\frac{0.1}{2},2+\frac{0.32}{2}\right)$$

$$K_3=hf\left(x_1+\frac{h}{2},y_1+\frac{k_2}{2}\right)=0.1\left(1.05,2+0.16\right)$$

$$K_3=hf\left(x_1+\frac{h}{2},y_1+\frac{k_2}{2}\right)=0.1\left(1.05,2.16\right)=0.1\left(1.05+2.16\right)=0.321$$

$$K_4=hf\left(x_1+h,y_1+k_3\right)=0.1\left(1+0.1,2+0.321\right)$$

$$K_4=hf\left(x_1+h,y_1+k_3\right)=0.1\left(1.1,2.321\right)=0.1\left(0.1+2.21575\right)=0.3421$$

Now,

$$y_2=y_1+\frac{1}{6}\left(k_1+2k_2+2k_3+k_4\right)$$

$$y_2=2+\frac{1}{6}\left(0.3+2\left(0.32\right)+2\left(0.321\right)+0.3421\right)$$

$$y_2=2+\frac{1}{6}\left(0.3+0.64+0.642+0.3421\right)=2+\frac{1.9241}{6}=2+0.320683=2.320683$$

2012 Paper2

Q1. Obtain numerically the solution of $y'=x+y^2, y(0)=2$ Using Euler`s method to find y at x=1, h=1….2m

Answer:

$h=1, x=1 \text{ and } y(0)=2$

$$y=y_0+hf(x_0,y_0)$$

$$y=2+1(1+2)$$

$$y=2+3=5$$

Q2. If f(0)=3 and f(1)=9, then the next approximate value of the function using secant method….2m

Answer:

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(0)9 - (1)3}{9 - 3}$$

$$x_2 = \frac{0-3}{9-3} = \frac{-3}{6} = -0.5$$

Q3. Write a backward difference formula of $D^2f(x)$….2m

Answer:

$$y_n'' = D^2 y_n = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right)$$

Q4. Write a formula for Simpson's 1/3 rule……2m

Answer:

$$\int_{x_0}^{x_{2N}} f(x)dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots + y_{2N-1}) + 2(y_2 + y_4 + \cdots + y_{2N-2}) + y_{2N}] + \text{Error term}$$

Q5.

$Evaluate\ the\ \text{int}egral$

$$\int_3^5 (\log x + x)dx$$

…..3m

$$use\ Simpson's\ \frac{3}{8}\ rule$$

$take\ h = 1$

Answer:

X	3	4	5
Y=(log x+ x)	3.4771	4.6020	5.6989

$$\int_3^5 (\log x + x)dx = \frac{3}{8}h\left(y_0 + y_2 + 3y_1\right)$$

$$\int_3^5 (\log x + x)dx = \frac{3}{8}(1)\left(3.4771 + 5.6989 + 3(4.6020)\right)$$

$$\int_3^5 (\log x + x)dx = \frac{3}{8}\left(9.176 + 13.806\right)$$

$$\int_3^5 (\log x + x)dx = \frac{3}{8}(22.982) = \frac{68.946}{8} = 8.61825$$

Q6. Write the two steps of solving the linear equations using Gaussian Elimination method….3m

Answer:

In this method, the solution to the system of equations is obtained in two stages.
I) the given system of equations is reduced to an equivalent upper triangular form using elementary transformations
ii) The upper triangular system is solved using back substitution procedure

Q.7

Use Runge – Kutta Method of order four to find the values of

k_1 *and* k_2 *for the* int *ial value problem* ….3m

$y' = yx^2, y(0) = 1, h = 0.1$

Answer:

$f(x, y) = yx^2, y(0) = 1 and\ h = 0.1, \quad x_0 = 0$

$k_1 = hf(x_0, y_0) = 0.1\left(1\left(0^2\right)\right) = 0$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.1\left(0 + \frac{0.1}{2}, 1 + \frac{0}{2}\right) = 0.1(0.05, 1) = 0.1(0.05 + 1) = 0.105$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2)$

$y_1 = 1 + \frac{1}{6}(0 + 2(0.105)) = 1 + \frac{0.21}{6} = 1.035$

Q.8

Evaluate the int *egral*

$\int_0^3 (x^2 + x)dx$ …..3m

$U \sin g\ T\ (apezoidal)\ rule$

$take\ h = 1$

Asnwer:

x	0	1	2	3
Y=(x² + x)	0	2	6	12

$\int_0^3 (x^2 + x)dx = \frac{h}{2}(y_0 + y_3 + 2(y_1 + y_2))$

$\int_0^3 (x^2 + x)dx = \frac{1}{2}(0 + 12 + 2(2 + 6))$

$\int_0^3 (x^2 + x)dx = \frac{1}{2}(28)$

$\int_0^3 (x^2 + x)dx = \frac{28}{2} = 14$

Q.9

Evaluate the integral

$\int_0^3 (x+1)dx$ …..5m

Using Simpson's $\frac{3}{8}$ *rule*

Answer:

x	0	1	2	3
Y=(x + 1)	1	2	3	4

$$\int_0^3 (x+1)dx = \frac{3}{8}h\left(y_0 + y_3 + 3(y_1 + y_2)\right)$$

$$h = \frac{b-a}{3n} = \frac{3-0}{3(3)} = \frac{3}{9} = 0.3$$

$$\int_0^3 (x^2 + x)dx = \frac{3}{8}(0.3)\left(1 + 4 + 3(2+3)\right)$$

$$\int_0^3 (x^2 + x)dx = \frac{0.9}{8}\left(5 + 15\right)$$

$$\int_0^3 (x^2 + x)dx = \frac{0.9}{8}(20) = \frac{18}{8} = 2.25$$

Q10. Find the dominant Eigen-value and the corresponding eigenvector of the matrix

$\begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix}$ *by Power Method with vector with vector* $v^0 = (1,1,1)^T$ *as the initial vector*

….5 m

Answer:

vector $v^0 = (1,1,1)^T$ *as the initial vector. Then, compute first iteration*

$$u^1 = [A]v^0 \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 8+1+2 \\ 0+10-1 \\ 6+2+15 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ 23 \end{pmatrix}$$

$$u^1 = 23 \begin{pmatrix} \frac{11}{23} \\ \frac{9}{23} \\ 1 \end{pmatrix} = q_1 v^1$$

sec*ond iteration*

$$u^2 = [A]v^1 \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{pmatrix} \frac{11}{23} \\ \frac{9}{23} \\ 1 \end{pmatrix} = \begin{bmatrix} \frac{88}{23} + \frac{9}{23} + 2 = \frac{143}{23} \\ 0 + \frac{90}{23} - 1 = \frac{67}{23} \\ \frac{66}{23} + \frac{18}{23} + 15 = \frac{429}{23} \end{bmatrix}$$

$$u^2 = 23 \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = q_2 v^2$$

third iteration

$$u^3 = [A]v^2 \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = \begin{bmatrix} 8+1+2 \\ 0+10-1 \\ 6+2+15 \end{bmatrix} \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = \begin{bmatrix} 1573 \\ 603 \\ 9867 \end{bmatrix}$$

$$u^3 = 9867 \begin{bmatrix} \frac{1573}{9867} = 0.0159 \\ \frac{603}{9867} = 0.0611 \\ 1.0 \end{bmatrix} = 9867 \begin{bmatrix} 0.0159 \\ 0.0611 \\ 1.0 \end{bmatrix} q_3 v^2$$

The l arg *est eign – value and the corresponding eignvector as accurate to two decimals are*

$$\lambda = 9867 \; and \; (X) = \begin{bmatrix} 0.01 \\ 0.06 \\ 1.0 \end{bmatrix}$$

Q11. From the following table of values, construct backward difference table….5m

X	**0.1**	**0.2**	**0.3**	**0.4**
F(x)	**1.10517**	**1.22140**	**1.34986**	**1.49182**

Answer:

x	F(x)	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
0.1	1.10517			
0.2	1.22140	0.11623		
0.3	1.34986	0.12846	0.01223	
0.4	1.49182	0.14196	0.0135	0.0012 7

Q12. Evaluate the integral

$\int_0^4 (x^2+1)dx \; U\,\text{si}\,ng\; Simpson's \frac{1}{3} rule$

$take\ h = 1$

….5m

Answer:

x	0	1	2	3	4
Y=(x^2 + 1)	1	2	5	10	17

$$\int_0^4 (x^2+1)dx = \frac{h}{3}\left(y_0 + y_4 + 4(y_1 + y_3) + 2\left(y_2\right)\right)$$

$$\int_0^4 (x^2+1)dx = \frac{1}{3}\left(1+17+4(2+10)+2\left(5\right)\right)$$

$$\int_0^4 (x^2+1)dx = \frac{1}{3}\left(18+48+10\right)$$

$$\int_0^4 (x^2+1)dx = \frac{1}{3}\left(76\right) = \frac{76}{3} = 25.33$$

2 Marks Questions:

Q. Evaluate the integral

$\int_0^{\pi/2} \cos 2x dx$ **Using Simpson's 1/3 rule. Where** $h=\pi/4$

Answer:

x	0	1	2	3	4
Y= cos 2x	0	0.999 3	0.9975	0.9945	0.9902

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{h}{3}\left(y_0 + y_4 + 4(y_1 + y_3) + 2\left(y_2\right)\right)$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{\frac{\pi}{4}}{3}\left(0 + 0.9902 + 4(0.9993 + 0.9945) + 2\left(0.9975\right)\right)$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{3.14}{12}\left(0.9902 + 7.9752 + 1.995\right)$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{3.14}{12}\left(10.9604\right) = \frac{34.415656}{12} = 2.8679$$

Q. Difference between Jacobi's Method and Gauss Seidel Method.

Answer:

The **Jacobi method** is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal. Each diagonal element is solved for, and an approximate value plugged in. The process is then iterated until it converges. This algorithm is a stripped-down version of the Jacobi transformation method of matrix diagonalization.

The Gauss-Seidel Method

With the Jacobi method, the values of obtained in the th iteration remain unchanged until the entire th iteration has been calculated. With the Gauss-Seidel method, we use the new values as soon as they are known. For example, once we have computed from the first equation, its value is then used in the second equation to obtain the new and so on.

Q. if $y' = t + 3y$, then find next two derivation in term of t and/or y.

Answer:

w.r.t t 2nd dravative t'' = 1 3rd dravative t''' = 0
w.r.t y. 2nd dravative Y'' = 2 3rd dravative Y''' = 0

3 Marks Questions:

Q. Evaluate the integral

$\int_{\pi/2}^{\pi} \sin x dx$ **Using Simpson's 3/8 rule. Where** $h = \pi / 4$

Answer:

X	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Y=sinx	1.0	0.7071	0.0

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3h}{8}\left(y_0 + y_2 + 3\left(y_1\right)\right)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3\left(\frac{\pi}{4}\right)}{8}\left(1.0 + 0.0 + 3(0.7071)\right)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{\frac{3\pi}{4}}{8}\left(1.0 + 2.1213\right)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{2.355}{8}\left(3.1213\right) = \frac{7.3506615}{8} = 0.9188$$

Q. Use Ruge-Kutta method of order four to find value of k_1 and k_2 for initial value problem. $y' = 1 + xy,\ y(0) = 2,\quad h = 0.2$

Answer:

here $f\left(x, y\right) = 1 + xy,\quad h = 0.2,\quad x_0 = 0,\quad y_0 = 2$

$$K_1 = hf\left(x_0, y_0\right) = 0.2\left(1 + \left(0\right)\left(2\right)\right)$$

$$K_1 = hf\left(x_0, y_0\right) = 0.2\left(1 + 0\right) = 0.2\left(1\right) = 0.2$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\left(0 + \frac{0.2}{2}, 2 + \frac{0.2}{2}\right)$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\left(0.1, 2 + 0.1\right)$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\left(0.1, 2.1\right) = 0.2\left(0.1 + 2.1\right) = 0.44$$

Now,

$$y_1 = y_0 + \frac{1}{6}\left(k_1 + 2k_2\right)$$

$$y_1 = 2 + \frac{1}{6}\left(0.2 + 2\left(0.44\right)\right)$$

$$y_1 = 2 + \frac{1}{6}\left(0.2 + 0.88\right) = 2 + \frac{1.08}{6} = 2 + 0.18 = 2.18$$

Q. Find the residuals by relation method

$$6x_1 - 3x_2 + x_3 = 11$$
$$2x_1 + x_2 - 8x_3 = -15$$
$$x_1 - 7x_2 + x_3 = 10$$

With starting vector (0,0,0)

Answer:

Re-arrange the equation:

$6x_1-3x_2+x_3=11$

$X_1-7x_2+x_3=10$

$2x_1+x_2-8x_3=-15$

Thus, we get

$R_1=11-6x_1+3x_2-x_3=11-6(0)+3(0)-(0)=11$

$R_2=10-x_1+7x_2-x_3=10-(0)+7(0)-(0)=10$

$R_3=-15-2x_1-3x_2+8x_3=-15-2(0)-3(0)+8(0)=-15$

The largest residual in magnitude is R_3

$Dx_3=\frac{R_3}{a_{33}}=\frac{-15}{8}=1.875$

Similarly, we find the new residuals of large magnitude and relax it to zero, and so on.

5 Marks Questions:

Q. Evaluate the integral

$$\int_0^3 (x^2+1)\,dx$$

Using Simpson's 3/8 rule. Where $h=1$

Answer:

x	0	1	2	3
$Y=(x^2+1)$	1	2	5	10

$$\int_0^3 (x^2+1)dx = \frac{3}{8}h\left(y_0 + y_3 + 3(y_1 + y_2)\right)$$

$$\int_0^3 (x^2+1)dx = \frac{3(1)}{8}\left(1+10+3(2+5)\right)$$

$$\int_0^3 (x^2+1)dx = \frac{3}{8}\left(11+21\right)$$

$$\int_0^3 (x^2+1)dx = \frac{3}{8}(32) = \frac{96}{8} = 12$$

Q. Solve by Gauss Seidel iterative method to 3 decimal place up to

Two iterations

$$8x - y - 2 = 8$$
$$x - 7y + 2z = -4$$
$$2x + y + 9z = 12$$

Answer:

The system is diagonally dominant

$$x = \frac{1}{8}\left[8 + y + 2z\right]$$

$$y = -\frac{1}{7}\left[-4 - x - 2z\right]$$

$$z = \frac{1}{9}\left[12 - 2x - y\right]$$

initial approximantion is $(0, 0, 0)$

first iteration

$$x_1 = \frac{1}{8}\left[8 + 0 + 2(0)\right] = \frac{8}{8} = 1$$

$$y_1 = -\frac{1}{7}\left[-4 - 1 - 2(0)\right] = -\frac{-5}{7} = 0.71$$

$$z_1 = \frac{1}{9}\left[12 - 2(1) - 0.71\right] = \frac{9.29}{9} = 1.032$$

sec*ond iteration*

$$x_2 = \frac{1}{8}\left[8 + 0.71 + 2(1.032)\right] = \frac{1}{8}\left[8 + 0.71 + 2.064\right] = \frac{10.774}{8} = 1.34675$$

$$y_2 = -\frac{1}{7}\left[-4 - 1.34675 - 2(1.032)\right] = -\frac{1}{7}\left[-4 - 1.34675 - 2.064\right] = -\frac{-7.41075}{7} = 1.0586$$

$$z_2 = \frac{1}{9}\left[12 - 2(1.34675) - 1.0586\right] = \frac{1}{9}\left[12 - 2.6935 - 1.0586\right] = \frac{8.2479}{9} = 0.91643$$

Q. Evaluate the integral

$\int_0^4 (x^2 + x + 2)dx$ **Using Simpson's 1/3 rule. Where** $h = 1$

Answer:

x	0	1	2	3	4
Y=(x^2 + x+2)	2	4	8	14	22

$$\int_0^4 (x^2 + x + 2)dx = \frac{h}{3}\left(y_0 + y_4 + 4(y_1 + y_3) + 2\left(y_2\right)\right)$$

$$\int_0^4 (x^2 + x + 2)dx = \frac{1}{3}\left(2 + 22 + 4(4 + 14) + 2\left(8\right)\right)$$

$$\int_0^4 (x^2 + x + 2)dx = \frac{1}{3}\left(24 + 72 + 16\right)$$

$$\int_0^4 (x^2 + x + 2)dx = \frac{1}{3}\left(112\right) = \frac{112}{3} = 37.33$$

Q. Find the 2nd derivative of f(x) at x=0.3 using three point equation

x	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	0.125	0.352	0.652	0.756	0.812	0.924

Answer:

$$f''(0.3) = \frac{f(x_3) - 2f(x_2) + f(x_1)}{h^2 = 0.01}$$

$$f''(0.3) = \frac{f(0.4) - 2f(0.3) + f(0.2)}{h^2 = 0.01}$$

$$f''(0.3) = \frac{0.756 - 2(0.652) + 0.352}{h^2 = 0.01}$$

$$f''(0.3) = \frac{0.756 - 1.304 + 0.352}{h^2 = 0.01} = -\frac{0.196}{0.01} = -19.6$$

Final 2013:

Q. Find Newton`s forward difference table from the following data

X	0.0	0.1	0.2	0.3	0.4
F(x)	1	0.9048	0.8187	0.7408	0.6703

…5m

Answer:

X	Y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.0	1				
0.1	0.9048	-0.0952			

0.2	0.8187	-0.0861	0.091		
0.3	0.7408	-0.0779	0.082	-0.009	
0.4	0.6703	-0.0705	0.074	-0.008	0.001

$$p=\frac{x-x_0}{h}=\frac{x-0.0}{0.1}=\frac{x}{0.1}-\frac{0.0}{0.1}=\frac{x}{0.1}-0=\frac{x}{0.1}$$

$$y=y_0+p\Delta y_0+\frac{p(p-1)}{2!}\Delta^2 y_0+\frac{p(p-1)(p-2)}{3!}\Delta^3 y_1+\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_2$$

$$x_0=0.0,\ \ y_0=1,\Delta y_0=-0.0952,\Delta^2 y=0.091,\Delta^3 y=-0.009,\Delta^4 y=0.001$$

$$y=1+\left(\frac{x}{0.1}\right)(-0.0952)+\frac{\frac{x}{0.1}\left(\frac{x}{0.1}-1\right)}{2}(0.091)+\frac{\frac{x}{0.1}\left(\frac{x}{0.1}-1\right)\left(\frac{x}{0.1}-2\right)}{6}(-0.009)+\frac{\frac{x}{0.1}\left(\frac{x}{0.1}-1\right)\left(\frac{x}{0.1}-2\right)\left(\frac{x}{0.1}-3\right)}{24}(0.001)$$

this is required Newton`s int *erpolating polynomial*.

Q5.

Evaluate the int *egral*

$$\int_3^5(\log x+2)dx\ u\sin g\ Simpson's\frac{3}{8}rule.take\ h=1$$

…..3m

Answer:

X	3	4	5
Y=(log x+ 2)	2.4771	2.6020	2.6989

$$\int_3^5(\log x+2)dx=\frac{3}{8}h\left(y_0+y_2+3y_1\right)$$

$$\int_3^5(\log x+2)dx=\frac{3}{8}(1)\left(2.4771+2.6989+3(2.6020)\right)$$

$$\int_3^5(\log x+2)dx=\frac{3}{8}\left(5.176+7.806\right)$$

$$\int_3^5(\log x+2)dx=\frac{3}{8}(12.982)=\frac{38.946}{8}=4.86825$$

Q5.

Evaluate the int *egral*

$\int_0^{\frac{\pi}{2}}(\cos x+2)dx$ *u sin g Simpson's* $\frac{3}{8}$ *rule. take* $h=\frac{\pi}{4}$ …..2m

Answer:

X	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
Y=(cos x+ 2)	3	2.7071	2

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx=\frac{3h}{8}\left(y_0+y_2+3y_1\right)$$

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx=\frac{3\left(\frac{\pi}{4}\right)}{8}\left(3+2+3(2.7071)\right)$$

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx=\frac{\frac{3\pi}{4}}{8}\left(5+8.1213\right)$$

$$\int_0^{\frac{\pi}{2}}(\cos x+2)dx=\frac{9.42}{2}(13.1213)=\frac{123.602646}{2}=61.801323$$

Q. if F (h) =256.2354 and $F\left(\frac{h}{2}\right)$ **=257.1379, then find** $F_1\left(\frac{h}{2}\right)$ **using Richardson`s extrapolation limit….2m**

Answer:

$$F_1(\frac{h}{2})=\frac{4f(\frac{h}{2})-f(\frac{h}{2})}{4^1-1}$$

$$F_1(\frac{h}{2})=\frac{4(257.1379)-(256.2354)}{3}$$

$$F_1(\frac{h}{2})=\frac{1028.5516-256.2354}{3}$$

$$F_1(\frac{h}{2})=\frac{772.3162}{3}=257.43873$$

7. if in solving given differential equation, 0.1, ,,, h=1, y3=2 then find the y4 by Adam–moulton's predictor formula .

Answer:

$$P: y_{n+1} = y_n + \frac{h}{24}\left(25y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$$

$$P: y_{3+1=4} = y_3 + \frac{1}{24}\left(25y'_3 - 59y'_{3-1=2} + 37y'_{3-2=1} - 9y'_{3-3=0}\right)$$

$$P: y_4 = y_3 + \frac{1}{24}\left(25y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right)$$

$$P: y_4 = 2 + \frac{1}{24}\left(25(1.3) - 59(1.2) + 37(1.1) - 9(0.1)\right)$$

$$P: y_4 = 2 + \frac{1}{24}\left(32.5 - 70.8 + 40.7 - 0.9\right)$$

$$P: y_4 = 2 + \frac{1}{24}(1.5) = 2 + \frac{1.5}{24} = 2 + 0.0625 = 2.0625$$

12. using Adam-moulton's predictor formula find the predicted value of f(0.4) from ordinary differential equation y'=1+2xy; y(0)=0; h=0.1

x	**0**	**0.1**	**0.2**	**0.3**
y	**0**	**.1007**	**.2056**	**.3199**

Answer:

$h = 0.1$

$f(x, y) = 1 + 2xy$

$y_0' = 1 + 2x_0 y_0 = 1 + 2(0)(0) = 1$
$y_1' = 1 + 2x_1 y_1 = 1 + 2(0.1)(0.1007) = 1.02014$
$y_2' = 1 + 2x_2 y_2 = 1 + 2(0.2)(0.2056) = 1.08224$
$y_3' = 1 + 2x_3 y_3 = 1 + 2(0.3)(0.3199) = 1.19194$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24}\left(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24}\left(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right)$$
$$y_4 = 0.3199 + \frac{0.1}{24}\left(55(1.19194) - 59(1.08224) + 37(1.02014) - 9(1)\right)$$
$$y_4 = 0.446773833$$

Computing y'_4 *for the Corrector Formula*;

$$y'_4 = 1 + 2x_4 y_4 = 1 + 2(0.4)(0.446773833)$$
$$y'_4 = 1.3574190664$$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right)$$
$$y_4 = y_3 + \frac{h}{24}\left(9y'_4 + 19y'_3 - 5y'_2 + y'_1\right)$$
$$y_4 = 0.3199 + \frac{0.1}{24}\left(9(1.3574190664) + 19(1.19194) - 5(1.08224) + 1.02014\right)$$

$$y_4 = 0.446869048$$

Q. **using Adam-moulton's predictor formula find the predicted value of f(0.4) from ordinary differential equation y'=1+xy; y(0)=0; h=0.1**

x	**0**	**0.1**	**0.2**	**0.3**
y	**0**	**.1007**	**.2056**	**.3199**

Answer:

$h = 0.1$

$f(x, y) = 1 + \ xy$

$y_0' = 1 + \ x_0 y_0 = 1 + \ (0)(0) = 1$

$y_1' = 1 + \ x_1 y_1 = 1 + \ (0.1)(0.1007) = 1.20$

$y_2' = 1 + \ x_2 y_2 = 1 + \ (0.2)(0.2056) = 1.04$

$y_3' = 1 + \ x_3 y_3 = 1 + \ (0.3)(0.3199) = 1.09597$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24}\left(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24}\left(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right)$$

$$y_4 = 0.3199 + \frac{0.1}{24}\left(55(1.09597) - 59(1.04112) + 37(1.2007) - 9(1)\right)$$

$y_4 = 0.3199 + 0.00416(60.27835 - 61.42608 + 44.4259 - 9) = 0.3199 + 34.27817 = 34.59807$

Computing y'_4 *for the Corrector Formula*;

$$y'_4 = 1 + \ x_4 y_4 = 1 + \ (0.4)(34.59807)$$
$$y'_4 = 14.839228$$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})$$
$$y_4 = y_3 + \frac{h}{24}(9y'_4 + 19y'_3 - 5y'_2 + y'_1)$$
$$y_4 = 0.3199 + \frac{0.1}{24}(9(14.839228) + 19(1.09597) - 5(1.04112) + 1.2007)$$

$$y_4 = 0.3199 + 0.00416(133.553052 + 20.82343 - 5.2056 + 1.2007)$$

$$y_4 = \ 0.3199 + 0.625545781 = 0.945445781$$

Q.2. Evaluate the integral $\int_0^{\frac{\pi}{2}} \cos x dx$ **Using trapezoidal rule**

Take h= $\frac{\pi}{4}$**…..2m**

Answer:

X	0	$\frac{\pi}{4}$
Y= cosx	1	0.7071

Using trapezoidal rule, taking h= $\frac{\pi}{4}$

$$I = \frac{h}{2}(y_0 + y_1)$$

$$I = \frac{\frac{\pi}{4}}{2}(1+0.7071)$$

$$I = \frac{3.14}{2}(1.7071)$$

$$I = \frac{5.360294}{2} = 2.680147$$

Q. obtain numerically the solution of

$y' = x + y^2, y(0) = 2 \, u \sin g \; Euler`s method to \; find \; y \; at \; x = 1, h = 1.$

Answer:

Here: y(0)=2, x=1, h=1

$$y = y_0 + hf\,(x_0, y_0)$$

$$y = 2 + 1(1+4) = 2 + 5 = 7$$

Q5.

$$Evaluate \, the \int egral \int_{\frac{\pi}{2}}^{\pi} \sin x dx \, u \sin g \; Simpson's \frac{3}{8} rule. take \, h = \frac{\pi}{4}$$

…..2m

Answer:

X	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Y=sinx	1	0.7071	0

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3h}{8}(y_0 + y_2 + 3y_1)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3\left(\frac{\pi}{4}\right)}{8}(1+0+3(0.7071))$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{\frac{3\pi}{4}}{8}(1+2.1213)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{9.42}{2}(3.1213) = \frac{29.402646}{2} = 14.701323$$

Q. Prove that $E^{\frac{1}{2}} = \mu + \frac{\delta}{2}$

Answer:

$$E^{\frac{1}{2}} = \mu + \frac{\delta}{2}$$

$$E^{\frac{1}{2}} = \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) + \frac{E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2}$$

$$E^{\frac{1}{2}} = \frac{E^{\frac{1}{2}} + E^{-\frac{1}{2}}}{2} + \frac{E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2}$$

$$E^{\frac{1}{2}} = \frac{E^{\frac{1}{2}} + E^{-\frac{1}{2}} + E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2} = \frac{2E^{\frac{1}{2}}}{2} = E^{\frac{1}{2}}$$

Q.2. Evaluate the integral $\int_0^2 \left(x^2 - 3x + 4\right) dx$ **Using trapezoidal rule**

Take h=1…..2m

Answer:

X	0	1	2
Y= (x^2-3x+4)	4	2	2

Using trapezoidal rule, taking h=1

$$I = \frac{h}{2}\left(y_0 + y_2 + 2\left(y_1\right)\right)$$

$$I = \frac{1}{2}\left(4 + 2 + 2\left(2\right)\right)$$

$$I = \frac{1}{2}\left(6 + 4\right)$$

$$I = \frac{10}{2} = 5$$

Q12. Evaluate the integral

$\int_0^4 (x^2 + 2) dx \, U \sin g \; Simpson's \frac{1}{3} rule$ ….5m

$take \; h = 1$

Answer:

x	0	1	2	3	4
Y=(x^2 + 2)	2	3	6	11	18

$$\int_0^4 (x^2+2)dx = \frac{h}{3}\left(y_0 + y_4 + 4(y_1 + y_3) + 2\left(y_2\right)\right)$$

$$\int_0^4 (x^2+2)dx = \frac{1}{3}\left(2 + 18 + 4(3+11) + 2\left(6\right)\right)$$

$$\int_0^4 (x^2+2)dx = \frac{1}{3}\left(20 + 56 + 12\right)$$

$$\int_0^4 (x^2+2)dx = \frac{1}{3}\left(88\right) = \frac{88}{3} = 29.33$$

Q. Evaluate the integral $\int_0^3 xdx$ Using Simpson's 3/8 rule. Take h=1 (5)

Answer:

x	0	1	2	3
Y=x	0	1	2	3

$$\int_0^3 xdx = \frac{3h}{8}\left(y_0 + y_3 + 3(y_1 + y_2)\right)$$

$$\int_0^3 xdx = \frac{3}{8}\left(0 + 3 + 3\left(1+2\right)\right)$$

$$\int_0^3 xdx = \frac{3}{8}\left(3 + 9\right)$$

$$\int_0^3 xdx = \frac{3}{8}\left(12\right) = \frac{36}{8} = 4.5$$

Final 2014

1- Write three equations for the following system of linear equation for Gauss Seidal Approximation Method (2)

83x+11y-4z=95
7x+52y++13z=104
3x+8y+29z=71

2- Evaluate the integral

$$\int_3^5 (\log x + 2)dx$$

Using trapezoidal rule.
Take h=1 (2)
Answer:

X	3	4	5
Y=(log x+2)	2.4771	2.6020	2.6989

$$\int_3^5 (\log x+2)dx = \frac{h}{2}\left(y_0 + y_2 + 2\left(y_1\right)\right)$$

$$\int_3^5 (\log x+2)dx = \frac{1}{2}\left(2.4771 + 2.6989 + 2\left(2.6020\right)\right)$$

$$\int_3^5 (\log x+2)dx = \frac{1}{2}\left(2.4771 + 2.6989 + 5.204\right)$$

$$\int_3^5 (\log x+2)dx = \frac{1}{2}\left(10.38\right) = \frac{10.38}{2} = 5.19$$

3- Write a formula for finding the value of k_2 in fourth order R-K mrthod?(2)

Answer:

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

4- If $y'=t^2-y$, then find the next two derivatives in terms of t and or/y. (2)
Answer:

w.r.t t 2nd dravative t'' = 2t 3rd dravative t''' = 2
w.r.t y. 2nd dravative Y'' = 1 3rd dravative Y''' = 0

5- Define intermediate value property? (3)
Answer:
If f(x) is a real valued continuous function in the closed interval a≤ x≤ b if f(a) and f(b) have opposite signs once; that is f(x)=0 has at least one root β such that a≤ b≤ β
Simply
If f(x) = 0 is a polynomial equation and if f(a) and f(b) are of different signs ,then f(x)=0 must have at least one real root between a and b.
Numerical methods for solving either algebraic or transcendental equation are classified into two groups.

6- If in solving a given differential equation,

$y_0 = 1, y'_1 = 1.1, y' = 1.2, y'_3 = 1.3, h = 1$ **Then find y_4 by Milne's Predictor formula. (3)**

Answer:

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

$$P: y_4 = y_0 + \frac{4h}{3}\left(2y'_1 - y'_2 + 2y'_3\right)$$

$$P: y_4 = 1 + \frac{4(1)}{3}\left(2(1.1) - 1.2 + 2(1.3)\right)$$

$$P: y_4 = 1 + \frac{4}{3}\left(2.2 - 1.2 + 2.6\right)$$

$$P: y_4 = 1 + \frac{4}{3}\left(3.6\right)$$

$$P: y_4 = 1 + \frac{14.4}{3}$$

$$P: y_4 = \frac{3 + 14.4}{3} = \frac{17.4}{3} = 5.8$$

7- Evaluate the integral

$$\int_0^4 x^2 dx$$

Using trapezoidal rule. Take h=1. (3)
Answer:

X	0	1	2	3	4
$Y=x^2$	0	1	4	9	16

$$\int_3^5 (\log x + 2)dx = \frac{h}{2}\left(y_0 + y_4 + 2\left(y_1 + y_2 + y_3\right)\right)$$

$$\int_3^5 (\log x + 2)dx = \frac{1}{2}\left(0 + 16 + 2\left(1 + 4 + 9\right)\right)$$

$$\int_3^5 (\log x + 2)dx = \frac{1}{2}\left(0 + 16 + 28\right)$$

$$\int_3^5 (\log x + 2)dx = \frac{1}{2}\left(44\right) = \frac{44}{2} = 22$$

8- Obtain numerically the solution of

$Y'=x^2+2x+y^2$, y(6)=1
Using Euler's method to find y at x=1, h=1. (3)
Answer:

$y(6)=1, x_6=1, h=1$

$y_7 = y_6 + hf\left(x_6, y_6\right)$

$y_7 = 1+1\left(x^2+2x+y^2\right)$

$y_7 = 1+1(1+2+1) = 1+4=5$

9- Using predictor formula find the predicted value of y(0,4) from ordinary differential equation

Y'=x+y; y(0)=1, h=0.1
With the help of the following table (5)

X	**0**	**0.1**	**0.2**	**0.3**
Y	**1**	**0.9097**	**0.8375**	**0.7816**

Answer:

Here,

$x_0 = 0 \quad , \quad x_1 = 0.1 \quad , \quad x_2 = 0.2 \quad , \quad x_3 = 0.3 \quad , \quad x_4 = 0.4$

$y_1' = x_1 - y_1 = 0.1 - 0.9097 = -0.8097$

$y_2' = x_2 - y_2 = 0.2 - 0.8375 = -0.6375$

$y_3' = x_3 - y_3 = 0.3 - 0.7816 = -0.4816$

Now, using Predictor Formula

$$y_4 = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right)$$

$$y_4 = 1 + \frac{4*0.1}{3}(-1.9451)$$

$$y_4 = 0.740653333$$

Using the predicted value, we shall now compute the corrected value as;

$$y_4 = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right)$$

Now,

$$y_4' = x_4 - y_4 = 0.4 - 0.740653333 = -0.340653333$$

10- Use the regula-falsi to solve the equation :

F(x)=e^x-3x=0. (Note only 1 iteration is required upto 4 decimal places is required) (5)

Answer:

$$f(x) = e^x - 3x = 0$$

$$f(1) = e^1 - 3(1) = 2.7182 - 3 = -0.2818$$

$$f(2) = e^2 - 3(2) = 7.3890 - 6 = 1.389$$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$x_3 = 2 - \frac{2-1}{1.389 - (-0.2818)}(1.389)$$

$$x_3 = 2 - \frac{1.389}{1.6708} = 2 - 0.8313 = 1.1687$$

11- Evaluate the integral

$\int_0^4 x\,dx$ **Using Simpson's 1/3 rule (5)**

Answer:

x	0	0.5	1	1.5	2	2.5	3	3.5	4
Y=x	0	0.5	1	1.5	2	2.5	3	3.5	4

$$\int_0^4 x dx = \frac{h}{3}\left(y_0 + y_8 + 4(y_1 + y_3 + y_5 + y_7) + 2\left(y_2 + y_4 + y_6\right)\right)$$

$$h = \frac{b-a}{2n} = \frac{4-0}{2(4)} = \frac{4}{8} = 0.5$$

$$\int_0^4 x dx = \frac{0.5}{3}\left(0 + 4 + 4(0.5 + 1.5 + 2.5 + 3.5) + 2\left(1 + 2 + 3\right)\right)$$

$$\int_0^4 x dx = \frac{0.5}{3}\left(4 + 32 + 12\right)$$

$$\int_0^4 x dx = \frac{0.5}{3}\left(48\right) = \frac{24}{3} = 8$$

12- Find y(1) for the following initial value problem y y'=x, y(0)=2

Using simple Euler's method, taking h=1 (5)

Answer:

$$y(0) = 2, x_0 = 0, h = 1$$

$$y_1 = y_0 + hf\left(x_0, y_0\right)$$

$$y_1 = 2 + 1\left(0 + 2\right)$$

$$y_1 = 2 + 2 = 4$$

what is Simpson's 3/8th rule.

Answer: The general formula for Simpson's 3/8th rule is 3h/8[f0+3(f1+f2)+2f3+(3f4+f5)+2f6+…+3(fn-2+fn-1)+fn] Now if we have to calculate the integral by using this rule then we can simply proceed just write first and last vale and distribute all the remaining values with prefix 3 and 2 Like you have f0,f1,f2,f3,f4 Then the integral can be calculated as 3h/8[f0+f4+3(f1+f2)+2f4] If we have values like f0,f1,f2,f3,f4,f5 Then integral can be calculated as 3h/8[f0+3(f1+f2)+2f3+3f4+f5] Similarly proceeding in this fashion we can calculate the integral in this fashion

Question: what is classic runge-kutta method

Answer: The fourth order Runge-Kutta method is known as the classic formula of classic Runge-Kutta method. yn+1=yn+1/6(k1+2k2+2k3+k4) Where k1=hf(xn,yn) k2=hf(xn+h/2+yn+k1/2) k3=hf(xn+h/2,yn+k'2/2) k4=hf(xn+h,yn+k3)

Question: What is meant by TOL?

Answer: The TOL means the extent of accuracy which is needed for the solution. If you need the accuracy to two places of decimal then the TOL will be 10-2 .similarly the 10-3 means that the accuracy needed to three places of decimal. Suppose you have the root from last iteration 0.8324516 and

0.8324525 if we subtract both and consider absolute value of the difference 0.0000009 now it can be written as 0.09*10-5 so the TOL in this case is 10-5.similarly if we have been provided that you have to for the TOL 10-2 you will check in the same way. In the given equation you will solve the equation by any method and will consider some specific TOL and try to go to that TOL. Some time no TOL is provided and you are asked to perform to some specific no of iterations.

Question: what is meant by uniqueness of LU method.

Answer: An invertible (whose inverse exists) matrix can have LU factorization if and only if all its principal minors(the determinant of a smaller matrix in a matrix) are non zero .The factorization is unique if we require that the diagonal of L or U must have 1's.the matrix has a unique LDU factorization under these condition . If the matrix is singular (inverse does not exists) then an LU factorization may still exist, a square matrix of rank (the rank of a matrix in a field is the maximal no of rows or column) k has an LU factorization if the first k principal minors are non zero. These are the conditions for the uniqueness of the LU decomposition.

Question: how the valu of h is calculated from equally spaced data.

Answer: Consider the following data x y 1 1.6543 2 1.6984 3 2.4546 4 2.9732 5 3.2564 6 3.8765 Here for h=2-1=3-2=1 x y 0.1 1.6543 0.2 1.6984 0.3 2.4546 0.4 2.9732 0.5 3.2564 0.6 3.8765 Here for the calculation of h =0.2-0.1=0.3-0.2=0.1 I think so that you may be able to understand .

what is the relation ship between p=0 and non zero p in interpolation.

Answer: fp=fo+pDfo+1/2(p2-p)D2fo+1/6(p3-3p2+2p)D3fo+1/24(p4-6p3+11p2-6p)D4fo+… This is the interpolation formula For the derivative you will have to take the derivative of this formula w.r.t p and you will get f'p=1/h{Dfo+1/2(2p-1)D2fo+1/6(3p2-6p+2)D3fo+…} Now put here p=0 f'p=1/h{Dfo+1/2(0-1)D2fo+1/6(0-0+2)D3fo+…} So the formula becomes f'p=1/h{Dfo+D2fo/2+D3fo/3+…} so this is the relation ship between non zero and zero p no when you have to calculate p the you use formula p=x-xo/h so this is the impact of x.

Question: What is chopping and Rounding off?

Answer: Chopping and rounding are two different techniques used to truncate the terms needed according to your accuracy needs. In chopping you simply use the mentioned number of digits after the decimal and discard all the remaining terms. Explanation (1/3 - 3/11) + 3/20=(0.333333…-0.27272727..)+0.15=(0.333-0.272)+0.15 This is the three digit chopping.

Question: When the forward and backward interpolation formulae are used?

Answer: In interpolation if we have at the start then we use the forward difference formula and the formula to calculate p is x-x0/h. If the value of x lies at the end then we use Newton's backward formula and formula to calculate the

value of p is x-xn/h. Now I come to your question as in this case the value lies at the end so 6 will be used as the xn. This procedure has been followed by the teacher in the lectures. But some authors also use another technique that is if you calculate the value of p and that is negative then the origin is shifted to that value for which the value of p becomes positive. And then according to that origin the values of differences are used and you need not follow this procedure.

Question: What is forward and backward difference operator and the construction of their table.

Answer: For forward Dfr =fr+1 –fr Df0 = f1-f0 In terms of y Dyr+1=yr+1-yr D stands for the forward difference operator For backward Dfr =fr –fr-1 Df1 = f1-f0 In terms of y Dy1=y1-y0 Here D stands for backwards operator Now the construction of the difference table is based on X Y 1st forward 2nd forward 3rd forward x1 Y1 Y2-Y1=Dy0 x2 Y2 Y3-Y2=Dy1 x3 Y3 Y4-Y3=Dy2 x4 Y4 Now consider the construction of table for the backward table X Y 1st forward 2nd forward 3rd forward x1 Y1 Y2-Y1=Dy1 x2 Y2 Y3-Y2=Dy2 x3 Y3 Y4-Y3=Dy3 x4 Y4 Dear student this is the main difference in the construction of the forward and backwards difference table when you proceed for forward difference table you get in first difference the value Dy0 but in the construction of backwards difference table in the first difference you get Dy1 and in the second difference in the forward difference table you get D2 y0 and in the backward difference table the first value in the second difference is D2 y1. I think so you have made it clear.

Question: What is Jacobi’s method?

Answer: Jacobi’s Method It is an iterative method and in this method we first of all check either the system is diagonally dominant and, if the system is diagonally dominant then we will calculate the value of first variable from first equation in the form of other variables and from the second equation the value of second variable in the form of other variables and so on. We are provided with the initial approximations and these approximations are used in the first iteration to calculate first approximation of all the variables. The approximations calculated in the first iteration are used in the second iteration to calculate the second approximations and so on.

What is the condition that a root will lie in an interval.

Answer: Suppose that you have a function f(x) and an interval [a,b]and you calculate both f(a)and f(b)if f(a)f(b)<0 then there must exist a root for this function in this interval.In simple words f(a) and f(b)must have opposite signs.

Question: How the divided difference table is constructed?

Answer: x y First difference Second difference Third difference 1 0.2 y0 y1-y0/x1-x0=sy0 sy1- sy0/x2-x0=s2y0 s2y0-s2y1/x3-x0=s3y0 s3y1-s3y0/x4x0 =s4y0 2 0.69 y1 y2-y1/x2-x1=sy1 sy2- sy1/x3-x1=s2y1 s2y2-s2y1/x4-x1=s3y1 3 0.985 y2 y3-y2/x3-x2=sy2 sy3- sy2/x4-x1=s2y2 4 1.2365 y3 y4-y3/x4-

x3=sy3 5 2.3651 y4 This is the complete table showing the central differences . Here s stands for difference

Question: What is Gauss-seidel Method

Answer: It is also an iterative method and in this we also check either the system is diagonally dominant and, if so then we proceed on the same lines as in the Jacobi's method. The difference in both is when we calculate a value of any variable that is instantly replaced by the previous value. If we have initial values like x1=0,x2=0 and x3=0 in the case of three variables in the first iteration we will use all the three values to calculate x1 but to calculate the value of x2 we will use the recent value of x1 which is calculated in the first iteration.

Question: What is partial and full pivoting?

Answer: Partial and full pivoting, In gauss elimination method when you have any of diagonal element aii zero it means the solution does not exist to avoid this we change the equation so that a non zero pivot is achieved .Now you have an argument matrix in which he elements in the first column are 1,3,4 respectively here in case of partial pivoting we will replace first element with the last element it is done by replacing the first equation with last equation it is known as partial pivoting. In full pivoting we change rows and columns but that is not implemented manually it is used in computers.

Question: How the initial vector is choose in power method?

Answer: Choose the initial vector such that the largest element is unity. But no problem with this you will precede with this vector as it is and will not change it. In the given question the initial vector is provided so you have to proceed with the given vector and will normalize this vector and normalizing this vector you will keep in mind that the greatest value should be unity. An eigenvector Vis said to be normalized if the coordinate of largest magnitude is equal to unity Actually this condition is for the normalization of the vectors when you have to normalize the vector you keep in mind that the larget entery in the vector must be 1 and so you take the largest element and divide the remaining values by the greatest value

What is meant by diagonally dominant system?

Answer: A system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 is said to be diagonally dominant if the following condition holds. |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3| => (greater or equal) |a3|+|b3|

Question: State the sufficient condition for the convergence of the system of equation by iterative methods.

Answer: A sufficient condition for convergence of iterative solution to exact solution is |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3|

=> (greater or equal) |a3|+|b3| For the system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 Similarly for the system with more variables we can also construct the same condition

Question: The calculation for numerical analysis should be done in degree or radians.

Answer: All the calculation for numerical analysis should be done in radians not in degrees set your calculator in radians mode and suppose the value of pi=3.14.

Question: How we can identify that Newton forward or backwards interpolation formula is to be used.

Answer: If the value at which we have to interpolate is in the start of the table then we will use Newton's forward interpolation formula if it is at the end of the table then we will use the Newton's backward interpolation formula.

Question: What is meant precision and accuracy?

Answer: Precision and accuracy are two terms which are used in numerical calculations by precision we mean that how the values in different iterations agree to each other or how close are the different values in successive iterations. For example you have performed 3 different iterations and result of all the iteration are 1.32514,1.32516,31.32518 these three values are very precise as these values agree with each other . Accuracy means the closeness to the actual value. Suppose that you have calculated an answer after some iteration and the answer is 2.718245 and the actual answer is 2.718254 and the answer calculated is very accurate but if this answer is 2.72125 then it is not accurate.

What is meant by iterative methods?

Answer: The methods which need one or more iterations are known as iterative methods like bisection method, Newton raphson method, and many other methods.

Question: What is graphically meant by the root of the equation?

Answer: If the graph of a function f(x) =0 cuts the x-axis at a point a then a is known as the root of the equation.

Question: Q. What is the difference between open and bracketing method?

Answer: In open methods we need only one initial approximation of the root that may be any where lying and if it is not very close then we have to perform more iteration and the example of open method is Newton raphson method. In bracketing method we bracket the root and find that interval in which root

lies means we need two initial approximations for the root finding. Bisection method is an example of the bracketing method.

Question: Condition for the existence of solution of the system of equations.

Answer: If the |A| is not equal to zero then the system will have a unique solution if |A|=0 then the system will have no solution

Question: Should the system be diagonally dominant for gauss elimination method?

Answer: The system of equation need not to be diagonally dominant for Gauss elimination method and gauss Jordan method for both the direct method it is not necessary for the system to be diagonally dominant .it should be diagonally dominant for iterative methods like jacobie and gauss seidel method.

What is a trance dental equation?

Answer: An equation is said to be a transcendental equation if it has trigonometric, exponential and logarithmic function or combination of all these functions.

Question: How the value of h is calculated in interpolation?

Answer: There are two types of data in the interpolation one is equally spaced and other is unequally spaced in equally spaced data we need to calculate the value of h that is calculated by subtracting any two consecutive values and taking their absolute value.

Question: What is an algebraic equation?

Answer: An algebraic equation is an equation which is purely polynomial in any variable. Supposed x2+3x+2=0, x4+3x2=0, y3+6y2=0 all are algebraic equations as these are purely polynomial in x and y variable.

Question: What is Descartes rule of signs?

Answer: The number of positive roots of an algebraic equation f(x) =0 can not exceed the no of changes in signs. Similarly the no of negative roots of negative roots of and algebraic equation can not exceed the no of changes in sign of equation f (-x) =0.

Question: What are direct methods?

Answer: The numerical methods which need no information about the initial approximation are known as direct methods like Graffee's root squaring method.

What is direct methods of solving equations?

Answer: Those methods which do not require any information about the initial approximation of root to start the solution are known as direct methods. The examples of direct methods are Graefee root squaring method, Gauss elimination method and Gauss Jordan method.All these methods do not require any type of initial approximation.

Question: What is Iterative method of solving equations?

Answer: These methods require an initial approximation to start. Bisection method, Newton raphson method, secant method, jacobi method are all examples of iterative methods

Question: If an equation is a transcendental, then in which mode the calculations should be done?

Answer: All the calculations in the transcendental equations should be done in the radian mode.

Question: What is the convergence criterion in method of iteration?

Answer: If be a root of f(x) =0 which is equivalent to I be any interval containing the point x= and will converge to the root provided that the initial approximation is chosen in I

Question: When we stop doing iterations when Toll is given?

Answer: Here if TOL is given then we can simply find the value of TOL by subtracting both the consecutive roots and write it in the exponential notation if the required TOL is obtained then we stop.

What is inherent error and give its example.

Answer: It is that quantity of error which is present in the statement of the problem itself, before finding its solution. It arises due to the simplified assumptions made in the mathematical modeling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the problem. e.g. if writing 8375 instead of 8379 in a statement lies in the category of inherent error.

Question: What is local round-off error?

Answer: At the end of computation of a particular problem, the final results in the computer, which is obviously in binary form, should be converted into decimal form-a form understandable to the user-before their print out. Therefore, an additional error is committed at this stage too. This error is

called local round-off error.

Question: What is meant by local truncation error?

Answer: Retaining the first few terms of the series up to some fixed terms and truncating the remaining terms arise to local truncation error.

Question: What is transcendental equation and give two examples.

Answer: An equation is said to be transcendental equation if it has logarithmic, trigonometric and exponential function or combination of all these three. For example it is a transcendental equation as it has an exponential function These all are the examples of transcendental equation.

Question: What is meant by intermediate value property?

Answer: If f(x) is a real valued continuous function in the closed interval if f(a) and f(b) have opposite signs once; that is f(x)=0 has at least one root such that Simply If f(x)=0 is a polynomial equation and if f(a) and f(b) are of different signs ,then f(x)=0 must have at least one real root between a and b.

What is the order of global error in Simpson's 3/8 rule?

Answer: The global error in Simpson's 3/8 rule is of the order of 0(h4).

Question: Which equation models the rate of change of any quantity with respect to another?

Answer: An ordinary differential equation models the rate of change of any quantity with respect to another.

Question: By employing which formula, Adam-Moulton P-C method is derived?

Answer: Adam-Moulton P-C method is derived by employing Newton's backward difference interpolation formula.

Question: What are the commonly used number systems in computers?

Answer: Binary Octal Decimal Hexadecimal

Question: If a system has the base M, then how many different symbols are needed to represent an arbitrary number? Also name those symbols.

Answer: M 0, 1, 2, 3, … , M-1

Is the order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule?

Answer: Yes. The order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule.

Question:	What is the order of global error in Trapezoidal rule?
Answer:	The global error in Trapezoidal rule is of the order of 0(h2).

Question:	What is the formula for finding the width of the interval?
Answer:	Width of the interval, h, is found by the formula h=(b-a)/n

Question:	What type of region does the double integration give?
Answer:	Double integration gives the area of the rectangular region.

Question:	Compare the accuracy of Romberg's integration method to trapezoidal and Simpson's rule.
Answer:	Romberg's integration method is more accurate to trapezoidal and Simpson's rule

Important Topics

Newton's Forward Diffrence & Backward Difference (Interpolation)
Newton's Forward Diffrence & Backward Difference (Differentiation)
Trapezoidal Rule (***Very Important***)
Simpson's 1/3 Rule
Simpson's 3/8 Rule
Euler's Method (**Important**)
Two Point Formula for Differentiation (**Important**)
Three Point Formula for Integration (**Important**)
Runge-Kutta Second Order Method (**Important**)
Runge-Kutta Fourth Order Method (***Very Important***)
Adam-Moulton Method {Predictor, Corrector and Error} (**Important**)
Milne Method {Predictor, Corrector and Error}

mcqs was mostly from forward backward interpolating and runge kutta and lagrange 1/3 simpson and trapezoidal rule

2 marks question : adam moultan predictor corrector formula

2 marks question : milne predictor corrector formula

3 marks question : formula of k3 of runge kutta

3 marks question : 1/3 simpson question

3 marks : trapezoidal rule question

5 marks : euler related

5 marks: trapezoidal

5 marks: milne predictor corrector formula question in the handouts on page 211(find f(0.4) from ordinary differencial equation)

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FINALTERM EXAMINATION
Spring 2010
MTH603- Numerical Analysis (Session - 2)

Question No: 1 (Marks: 1) - Please choose one

Symbol used for forward differences is

- ∇
- Δ
- δ
- μ

Question No: 2 (Marks: 1) - Please choose one

The relationship between central difference operator and the shift operator is given by

- $\delta = \mathrm{E} - \mathrm{E}^{-1}$
- $\delta = \mathrm{E} + \mathrm{E}^{-1}$
- $\delta = \mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}}$
- $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}}$

Question No: 3 (Marks: 1) - Please choose one

Muller's method requires --------starting points

- 1
- 2
- 3
- 4

Question No: 4 (Marks: 1) - Please choose one

If S is an identity matrix, then

- $S^{-1} = S$
- $S^{t} = S$
- $All\ are\ true$

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► $S^{-1} = S^t$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_0, x_1, ..., x_r$

► r+2
► r+1
► r
► r-1

Question No: 6 (Marks: 1) - Please choose one

P in Newton's forward difference formula is defined as

$p = (\frac{x - x_0}{h})$

►

$p = (\frac{x + x_0}{h})$

►

$p = (\frac{x + x_n}{h})$

►

$p = (\frac{x - x_n}{h})$

►

Question No: 7 (Marks: 1) - Please choose one

Octal number system has the base ----------

► 2
► 8
► 10
► 16

Question No: 8 (Marks: 1) - Please choose one

Newton's divided difference interpolation formula is used when the values of the independent variable are

► Equally spaced

► Not equally spaced

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► Constant
► None of the above

Question No: 9 (Marks: 1) - Please choose one

Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

► 1.5

► 3
► 2

► 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

► $\varepsilon(x) = y(x) + P_n(x)$
► $\varepsilon(x) = y(x) - P_n(x)$
► $\varepsilon(x) = P_n(x) - y(x)$
► $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11 (Marks: 1) - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \overline{x}$. Then $F(x)$ vanishes ------times in the interval I.

► n-1
► n+2
► n
► n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

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$$D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + ...)$$

►

$$D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + ...)$$

►

$$D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - ...)$$

►

$$D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} - ...)$$

►

Question No: 13 (Marks: 1) - Please choose one

Finding the first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

► Forward difference operator
► Backward difference operator
► Central difference operator
► None of the given choices

Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

► -0.5
► 0.5

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► 0.75
► -0.75

Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

► $ax+b$
► ax^2+bx+c
► ax^3+bx^2+cx+d
► $ax^4+bx^3+cx^2+dx+e$

Question No: 16 (Marks: 1) - Please choose one

$$I=\int_a^b f(x)dx$$

While integrating , h, width of the interval, is found by the formula-----.

► $\frac{b-a}{n}$
► $\frac{b+a}{n}$
► $\frac{a-b}{n}$
► None of the given choices

Question No: 17 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

► 7
► 8
► 5
► 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7

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$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as ……

- ► 17.5
- ► 12.5
- ► 7.5
- ► -12.5

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

- ► 2
- ► 3
- ► 5
- ► 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

- ► 10
- ► 11
- ► 12
- ► 13

Question No: 21 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- ► $\frac{(a+b)}{2}$
- ► $\frac{(a-b)}{2}$
- ► $\frac{(b-a)}{2}$
- ► None of the given choices

Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

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- ► Bisection Method
- ► Regula Falsi Method
- ► Secant Method
- ► None of the given choices

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3 + 3x - 1 = 0$, the root of the equation lies in the interval.....

- ► (1, 3)
- ► (1, 2)
- ► (0, 1)
- ► (1, 2)

Question No: 24 (Marks: 1) - Please choose one

Rate of change of any quantity with respect to another can be modeled by

- ► An ordinary differential equation
- ► A partial differential equation
- ► A polynomial equation
- ► None of the given choices

Question No: 25 (Marks: 1) - Please choose one

If

$$\frac{dy}{dx} = f(x, y)$$

Then the integral of this equation is a curve in

- ► None of the given choices
- ► xt-plane
- ► yt-plane
- ► xy-plane

Question No: 26 (Marks: 1) - Please choose one

In solving the differential equation

$$y' = x + y \ ; \ y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

- ► 1.44

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- 1.11
- 1.22
- 1.33

Question No: 27 (Marks: 1) - Please choose one

In second order Runge-Kutta method
k_1 is given by

- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

Question No: 28 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

- $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
- $k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

Question No: 29 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

- $k_3 = hf(x_n + 2h, y_n + 2k_3)$
- $k_3 = hf(x_n - h, y_n - k_3)$
- $k_3 = hf(x_n + h, y_n + k_3)$
- None of the given choices

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Question No: 30 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

- ► Newton's backward difference interpolation formula
- ► Newton's forward difference interpolation formula
- ► Newton's divided difference interpolation formula
- ► None of the given choices

Question No: 31 (Marks: 2)

If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32 (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2) dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

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Evaluate the integral

$$\int_{3}^{5} (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36 (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37 (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38 (Marks: 5)

Evaluate the integral

$$\int_{0}^{3} (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39 (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$$

taking $h = 0.1$

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FINALTERM EXAMINATION
MTH603- Numerical Analysis (Session - 2)

Question No: 1 (Marks: 1) - Please choose one

Symbol used for forward differences is

- ∇
- Δ
- δ
- μ

Question No: 2 (Marks: 1) - Please choose one

The relationship between central difference operator and the shift operator is given by

- $\delta = \mathrm{E} - \mathrm{E}^{-1}$
- $\delta = \mathrm{E} + \mathrm{E}^{-1}$
- $\delta = \mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}}$
- $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}}$

Question No: 3 (Marks: 1) - Please choose one

Muller's method requires --------starting points

- 1
- 2
- 3
- 4

Question No: 4 (Marks: 1) - Please choose one

If S is an identity matrix, then

- $S^{-1} = S$
- $S^{t} = S$
- $All\ are\ true$

- $S^{-1} = S^{t}$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_0, x_1, ..., x_r$

- r+2
- r+1
- r
- r-1

Question No: 6 (Marks: 1) - Please choose one

P in Newton's forward difference formula is defined as

- $p = (\frac{x - x_0}{h})$
- $p = (\frac{x + x_0}{h})$
- $p = (\frac{x + x_n}{h})$
- $p = (\frac{x - x_n}{h})$

Question No: 7 (Marks: 1) - Please choose one

Octal number system has the base ----------

- 2
- 8
- 10
- 16

Question No: 8 (Marks: 1) - Please choose one

Newton's divided difference interpolation formula is used when the values of the independent variable are

- Equally spaced
- Not equally spaced

- Constant
- None of the above

Question No: 9 (Marks: 1) - Please choose one

Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

- 1.5
- 3
- 2
- 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

- $\varepsilon(x) = y(x) + P_n(x)$
- $\varepsilon(x) = y(x) - P_n(x)$
- $\varepsilon(x) = P_n(x) - y(x)$
- $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11 (Marks: 1) - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \overline{x}$. Then $F(x)$ vanishes ------times in the interval I.

- n-1
- n+2
- n
- n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

$$D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}+\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}+...)$$

►

$$D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2}+\frac{\Delta^3}{3}+\frac{\Delta^4}{4}+\frac{\Delta^5}{5}+...)$$

►

$$D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\frac{\Delta^5}{5}-...)$$

►

$$D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}-\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}-...)$$

►

Question No: 13 (Marks: 1) - Please choose one

Finding the first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

► Forward difference operator
► Backward difference operator
► Central difference operator
► None of the given choices

Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

► -0.5
► 0.5

- 0.75
- -0.75

Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

- $ax+b$
- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

Question No: 16 (Marks: 1) - Please choose one

$$I=\int_a^b f(x)dx$$

While integrating , h, width of the interval, is found by the formula-----.

- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
- None of the given choices

Question No: 17 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

- 7
- 8
- 5
- 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7

$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as ……

- ► 17.5
- ► 12.5
- ► 7.5
- ► -12.5

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

- ► 2
- ► 3
- ► 5
- ► 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

- ► 10
- ► 11
- ► 12
- ► 13

Question No: 21 (Marks: 1) - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- ► $\frac{(a+b)}{2}$
- ► $\frac{(a-b)}{2}$
- ► $\frac{(b-a)}{2}$
- ► None of the given choices

Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- ► Bisection Method
- ► Regula Falsi Method
- ► Secant Method
- ► None of the given choices

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3 + 3x - 1 = 0$, the root of the equation lies in the interval.....

- ► (1, 3)
- ► (1, 2)
- ► (0, 1)
- ► (1, 2)

Question No: 24 (Marks: 1) - Please choose one

Rate of change of any quantity with respect to another can be modeled by

- ► An ordinary differential equation
- ► A partial differential equation
- ► A polynomial equation
- ► None of the given choices

Question No: 25 (Marks: 1) - Please choose one

If

$$\frac{dy}{dx} = f(x, y)$$

Then the integral of this equation is a curve in

- ► None of the given choices
- ► xt-plane
- ► yt-plane
- ► xy-plane

Question No: 26 (Marks: 1) - Please choose one

In solving the differential equation

$$y' = x + y \;;\; y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

- ► 1.44

- 1.11
- 1.22
- 1.33

Question No: 27 (Marks: 1) - Please choose one

In second order Runge-Kutta method
k_1 is given by

- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

Question No: 28 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

- $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
- $k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$
- $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

Question No: 29 (Marks: 1) - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

- $k_3 = hf(x_n + 2h, y_n + 2k_3)$
- $k_3 = hf(x_n - h, y_n - k_3)$
- $k_3 = hf(x_n + h, y_n + k_3)$
- None of the given choices

Question No: 30 (Marks: 1) - Please choose one

Adam-Moulton P-C method is derived by employing

- ► Newton's backward difference interpolation formula
- ► Newton's forward difference interpolation formula
- ► Newton's divided difference interpolation formula
- ► None of the given choices

Question No: 31 (Marks: 2)

If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32 (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2) dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

Evaluate the integral

$$\int_{3}^{5} (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

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Using Simpson's 3/8 rule

Take h=1

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$$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$$

taking $h = 0.1$

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Question # 1 of 10 (Start time: 04:40:08 PM) Total Marks: 1

The determinant of a diagonal matrix is the product of the diagonal elements.

1. True
2. False

Question # 2 of 10 (Start time: 04:40:58 PM) Total Marks: 1

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

1. True
2. false

A 3 x 3 identity matrix have three and different eigen values.

1. True
2. False

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

1. True
2. False

The Jacobi's method is a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

1. No
2. At least one

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

1. Unity
2. zero

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

1. True

2. False

The determinant of a _____ matrix is the product of the diagonal elements.

1. Diagonal
2. Upper triangular
3. Lower triangular
4. Scalar

Waisay main nay is ka answer Diagnol keea tha....par yeh charon options theek hain....
You can confirm it from internet...

Jab main yeh MCQ kar raha tha tou tab hi mujhay is par shak ho raha tha....kyun k upper aur lower triangular matrices tou linear algebra mein bhi bahut ziada bataye gaye tou yeh property wahan se hi yaad thi...

Eigenvalues of a symmetric matrix are all _____.

1. Real
2. Zero
3. Positive
4. Negative

The Power method can be used only to find the eigen value of A that is largest in absolute value—we call this eigen value the dominant eigen value of A.

1. True
2. False

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

1. (x-1)^3
2. (x+1)^3
3. x^3-1
4. x^3+1

For differences methods we require the set of values.

1. True
2. False

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

1. True
2. False

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

1. True
2. False

a ko agar aap lambda se replace kar dain tou baat clear ho jaye gi....labda ki jagah a use keea gaya hai tou is liay yeh working thora sa confuse karti hai...

Central difference method seems to be giving a better approximation, however it requires more computations.

1. True
2. False

Iterative algorithms can be more rapid than direct methods.

1. True (main nay true hi keea tha, aap isay dekh lena)
2. False

Central Difference method is the finite difference method.

1. True
2. False

MTH603 Numerical Analysis

Exact solution of 2/3 is not exists.

TRUE
FALSE

The Jacobi's method is
a method of solving a matrix equation on a matrix that has ____ zeros along its main diagonal.

no
atleast one

A 3 x 3 identity matrix have three and _____eigen values.
same
different

Eigenvalues of a symmetric matrix are all _____ .
real
complex
zero
positive

The Jacobi iteration converges, if A is strictly diagonally dominant.
TRUE
FALSE

Below are all the finite difference methods EXCEPT _____.

jacobi's method
newton's backward difference method
Stirlling formula
Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).
TRUE
FALSE

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

TRUE
FALSE

The characteristics polynomial of a 3x 3
identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where

symbol ^ shows power.

(x-1)^3
(x+1)^3
x^3-1
x^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE
FALSE

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i have attempt mth 603 today..... my paper was power method 5 marks , from vactor 3 marks, Newton_Repshon's forwrd distnce formula 5 marks.,define extra polation 2 marks ,, and most of MCQ's from , Newton_Repshon's method ,or 18 to 22 lec's

question 29: Distinguish between Related and unrelated diversification with example 5marks
question 30 Five porters Model 5 marks
question 31: conecntric diversification two example 3marks
question 32 significant of R&D for an organization 3marks

Which of the following period strategic management was considered to be cure for all problems?
Mid 1950s to mid 1960s
Mid 1960s to mid 1970s
Mid 1970s to mid 1980s
Mid 1980s to mid 1990s

Which of the following is not a pitfall an organization should avoid in strategic planning?
Select correct option:
Failing to involve key employees in all phases of planning
Involving all managers rather than delegating planning to a planner
Top managers not actively supporting the strategic planning process
Doing strategic planning only to satisfy accreditation or regulatory requirements

which of the following are the factors that concern the nature and direction of the economy in which a firm operates?
Select correct option:
Technological
Ecological
Social
Economic

Which of the following best describes this statement; "a Systematic and ethical process for gathering and analyzing information about the competition's activities and general business trends to further a business' own goals"?
Select correct option:
External assessment
Industry analysis
Competitive intelligence program
Business ethics

According to Porter, which strategy offers products or services to a small range of customers at the lowest price available on the market?
Select correct option:
Low cost
Best value
Cost focus
Differentiation

Long-term objectives includes all of the following EXCEPT:

Measurable

Reasonable

Varying

Consistent

Which one of the following is NOT is a basic mission of a competitive intelligence program?

To provide a general understanding of an industry

To provide a general understanding of a company's competitors

To identify industry executives who could be hired by the firm

To identify potential moves a competitor might make that would endanger a firm

While preparing an External Factor Evaluation Matrix, a total score of 0.8 indicates that:

Firm is taking advantages of strengths and avoiding threats

Firm is taking no advantage of opportunities and is avoiding threats

Firm is not taking advantages of opportunities and is not avoiding threats

Firm is taking advantage of opportunities and is avoiding the threats

Top

Use the Regula-Falsi method to compute a real root of the equation

$x^3 - 9x + 1 = 0$, **(Perform two iterations for each part)**

(i) if the root lies between 2 and 4
(ii) if the root lies between 2 and 3.

Solve the following system of equations by Crout's method

$$x_1 + 2x_2 + x_3 = 2$$
$$2x_1 + x_2 - 10x_3 = 0.182$$
$$2x_1 + 3x_2 - x_3 = 2$$

Find the numerical solution of

$$55x - 25z = -200$$
$$-37y - 4z = -250$$
$$-25x - 4y + 29z = 100$$

Using Gauss-Siedel Mehtod with $(1,1,1)$ as the initial value. Perform two iterations only.

Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$ by the bisection method.
(Perform four iterations only)

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Approximate the integral $\int_{1}^{1.6} \frac{2x}{x^2+1} dx$ using Simpson's 1/3 rule and calculate the percentage error. (Take result up to 4 decimal places)

Note: In order to get full marks do all necessary steps.

Construct a forward difference table for the following values

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

Note : In order to get full marks do all necessary steps.

Solve the system

$4x+3y=24$

$3x+4y-z=30$

$-y+4z=-24$

by Gauss Seidal Method, taking $(0, 0, 0)^t$ as initial approximation(Two iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Let $f(x)=x+\frac{2}{x}$, use cubic Lagrange interpolation based on the nodes $x_0=0.5, x_1=1, x_2=2$ *and* $x_3=2.5$ to approximate *f(1.5)* and *f(1.3)*.

Note : In order to get full marks do all necessary stepsSolution

Approximate the Dominant Eigenvalue and corresponding Eigenvector for the matrix

$$\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

by using Power Method. Start with $X_0=(1,1,1)^t$. (Five iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

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FINALTERM EXAMINATION
MTH603- Numerical Analysis (Session - 2)

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- $\delta = \mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}}$
- $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}}$

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Muller's method requires --------starting points

- 1
- 2
- 3
- 4

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If S is an identity matrix, then

- $S^{-1} = S$
- $S^{t} = S$
- $All\ are\ true$

- $S^{-1} = S^{t}$

Question No: 5 (Marks: 1) - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_0, x_1, ..., x_r$

- r+2
- r+1
- r
- r-1

Question No: 6 (Marks: 1) - Please choose one

P in Newton's forward difference formula is defined as

- $p = (\frac{x - x_0}{h})$
- $p = (\frac{x + x_0}{h})$
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- 16

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- Not equally spaced

- Constant
- None of the above

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Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

- 1.5
- 3
- 2
- 1

Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

- $\varepsilon(x) = y(x) + P_n(x)$
- $\varepsilon(x) = y(x) - P_n(x)$
- $\varepsilon(x) = P_n(x) - y(x)$
- $\varepsilon(x) = y(x) \times P_n(x)$

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Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \overline{x}$. Then $F(x)$ vanishes ------times in the interval I.

- n-1
- n+2
- n
- n+1

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$$D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}+\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}+...)$$

►

$$D=\frac{1}{h}(\Delta+\frac{\Delta^2}{2}+\frac{\Delta^3}{3}+\frac{\Delta^4}{4}+\frac{\Delta^5}{5}+...)$$

►

$$D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\frac{\Delta^5}{5}-...)$$

►

$$D=\frac{1}{h}(\Delta-\frac{\Delta^2}{2!}+\frac{\Delta^3}{3!}-\frac{\Delta^4}{4!}+\frac{\Delta^5}{5!}-...)$$

►

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Differential operator in terms of ----------will be used.

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For the given table of values

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► 0.5

- 0.75
- -0.75

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- ax^2+bx+c
- ax^3+bx^2+cx+d
- $ax^4+bx^3+cx^2+dx+e$

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- $\frac{b-a}{n}$
- $\frac{b+a}{n}$
- $\frac{a-b}{n}$
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To apply Simpson's 1/3 rule, valid number of intervals are.....

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- 8
- 5
- 3

Question No: 18 (Marks: 1) - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7

$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as ……

- ► 17.5
- ► 12.5
- ► 7.5
- ► -12.5

Question No: 19 (Marks: 1) - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

- ► 2
- ► 3
- ► 5
- ► 7

Question No: 20 (Marks: 1) - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

- ► 10
- ► 11
- ► 12
- ► 13

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If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

- ► $\frac{(a+b)}{2}$
- ► $\frac{(a-b)}{2}$
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Question No: 22 (Marks: 1) - Please choose one

.....lies in the category of iterative method.

- ► Bisection Method
- ► Regula Falsi Method
- ► Secant Method
- ► None of the given choices

Question No: 23 (Marks: 1) - Please choose one

For the equation $x^3 + 3x - 1 = 0$, the root of the equation lies in the interval.....

- ► (1, 3)
- ► (1, 2)
- ► (0, 1)
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If

$$\frac{dy}{dx} = f(x, y)$$

Then the integral of this equation is a curve in

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In solving the differential equation

$$y' = x + y \;;\; y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

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- 1.22
- 1.33

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- $k_1 = hf(x_n, y_n)$
- $k_1 = 2hf(x_n, y_n)$
- $k_1 = 3hf(x_n, y_n)$
- None of the given choices

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In fourth order Runge-Kutta method, k_2 is given by

- $k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$
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Adam-Moulton P-C method is derived by employing

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$$\int_{0}^{3} (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

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Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$$

taking $h = 0.1$

AA
dear fellows
dis iz my paper of math603

more than 5 questn about
simpson 3/8 3 mark 2 mark 2 mark 3 mark etc
trapezal

R-K method 5 marks
2nd derivativ backward diff formula 2 marks
P:C method 5 marks

now about mcqs
Eulaer method fst 2nd derivativ say bht saray mcqs aye thay
calculatn waly
or baki sab simpson trapzel R-K methd

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Rana Muhammad Safdar (rana.sfdr@gmail.com)

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Collected and Composed by Bilal Farooq (bilal.zaheem@gmail.com)

FINALTERM EXAMINATION
Spring 2010
MTH603- Numerical Analysis (Session - 2)

Question No: 1 (Marks: 1) - Please choose one

Symbol used for forward differences is

- ∇
- Δ
- δ
- μ

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The relationship between central difference operator and the shift operator is given by

- $\delta = E - E^{-1}$
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Question No: 3 (Marks: 1) - Please choose one

Muller's method requires --------starting points

- 1
- 2
- 3
- 4

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If S is an identity matrix, then

- $S^{-1} = S$
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- $All\ are\ true$
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If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_{0,}x_1,...,x_r$

- r+2
- r+1
- r
- r-1

Question No: 6 (Marks: 1) - Please choose one

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- $p = (\frac{x - x_0}{h})$
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- $p = (\frac{x + x_n}{h})$
- $p = (\frac{x - x_n}{h})$

Question No: 7 (Marks: 1) - Please choose one

Octal number system has the base ----------

- 2
- 8
- 10
- 16

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Question No: 8 (Marks: 1) - Please choose one

Newton's divided difference interpolation formula is used when the values of the independent variable are

- ► Equally spaced
- ► Not equally spaced
- ► Constant
- ► None of the above

Question No: 9 (Marks: 1) - Please choose one

Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

- ► 1.5
- ► 3
- ► 2
- ► 1

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Question No: 10 (Marks: 1) - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

- ► $\varepsilon(x) = y(x) + P_n(x)$
- ► $\varepsilon(x) = y(x) - P_n(x)$
- ► $\varepsilon(x) = P_n(x) - y(x)$
- ► $\varepsilon(x) = y(x) \times P_n(x)$

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Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \bar{x}$. Then $F(x)$ vanishes ------times in the interval I.

- n-1
- n+2
- n
- n+1

Question No: 12 (Marks: 1) - Please choose one

Differential operator in terms of forward difference operator is given by

- $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + ...)$
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Question No: 13 (Marks: 1) - Please choose one

Finding the first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------will be used.

- Forward difference operator
- Backward difference operator
- Central difference operator
- None of the given choices

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Question No: 14 (Marks: 1) - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.....

- ► -0.5
- ► 0.5
- ► 0.75
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Question No: 15 (Marks: 1) - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

- ► $ax+b$
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$$I=\int_a^b f(x)dx$$

While integrating , h, width of the interval, is found by the formula-----.

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To apply Simpson's 1/3 rule, valid number of intervals are.....

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x	02	0.3	0.4	0.5	0.6	0.7
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$f''(0.2)$, using three-point equation will be calculated as ……

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- ► 7.5
- ► -12.5

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To apply Simpson's 1/3 rule, the number of intervals in the following must be

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- ► 3
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Question No: 20 (Marks: 1) - Please choose one

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- ► 11
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▶

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▶

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FINALTERM EXAMINATION Spring 2010
MTH603- Numerical Analysis (Session - 2)

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► $k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$

►

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$$\int_0^{\frac{\pi}{2}} (\cos x + 2)dx$$

Using Simpson's 3/8 rule

$$\frac{\pi}{4}$$

Take h=

Question No: 33 (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34 (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35 (Marks: 3)

Evaluate the integral

$$\int_3^5 (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36 (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37 (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38 (Marks: 5)

Evaluate the integral

$$\int_0^3 (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39 (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$$y' = \frac{1}{2}(2x^3 + y),\ y(1) = 2$$

taking $h = 0.1$

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