DATE: Mth:304 DATE:

lecture # 23

→ **Moment force about an axis:**

The tendency of a force to rotate a rigid body about an axis is called moment of force about an axis.

→ **Quiz:** The moment measures, M of a force about a point provides a **measure** of the tendency rotation. (Sometimes called torque)

$$M = \vec{F} \cdot \vec{d} \Rightarrow M_A = F \cdot d$$

→ Unit of a moment: ✓

SI-unit: Nm (Newton meter)

→ $M = \vec{F} \cdot \vec{d} \Rightarrow N(m) \Rightarrow \boxed{Nm}$ ✓

→ Properties of Moment:

* It has only magnitude & Sense.

$M_o = \pm F \cdot d$ ✓

گھڑی وار (anti clock wise) ↺ — مثبت +ve

گھڑی وار (Clock-Wise) ↻ — منفی -ve ✓

$M_o = -\vec{F} \cdot d$

→ Moment = force X Perpendicular distance

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Perpenecular (⊥) fo distance

→ It is b/w line of action of force & center of rotation.

موومینٹم کو ہم (Subtract) اور (add) بھی کر سکتے ہیں۔

→ If two or more moment acting on an object then overall moment is difference b/w ↻ & ↺.

اگر 2 یا 2 سے زیادہ موومینٹم کسی Object پر عمل کر رہے ہوں تو اوورآل (overall) difference کلاک وائز اور اینٹی کلاک وائز کے درمیان ہوگا۔

اگر کلاک وائز موومینٹم زیادہ ہو اور اینٹی کلاک وائز کم ہو تو ہم کلاک - وائز میں سے اینٹی کلاک وائز کو (-) کر دیں گے اور جو کلاک وائز موومینٹ حاصل ہوگا۔

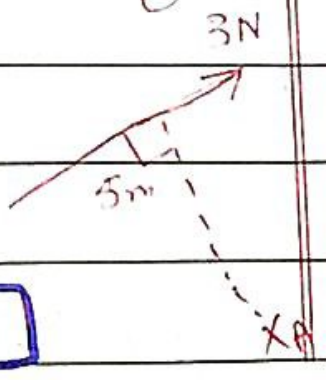
Note :-

→ Exp: calculate moment about A for each of following force.

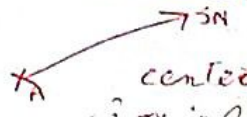
Moment = Force X distance

$M_A = \vec{F} \cdot \vec{d}$

$M_A = 3N \times 5m \Rightarrow 15Nm$



Similarly:

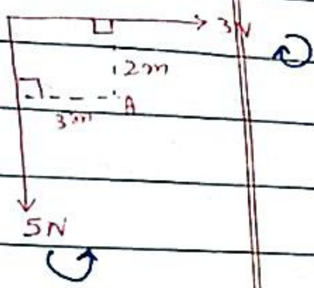


تو سو فیصد جو فورس ہوگا ان کا centeo object کے point (A) پہ جو force ہوگا اور وہاں پر گزرتا ہے تو صفر ہو جائے گی

$M_A = (0) \times d \Rightarrow (0)$

line of action of force
جو فورس point سے گزرتی ہے تو Zero ہوگا جب

Exp#2 (1)



جہاں پر فورس کی سمت بنتی ہے وہاں سے ہم direction بناتی clock OR anti-clock wise سوچ کر لیں

Sol:

$\circlearrowleft$ $Moment_A = 5N \times 3m = +15Nm$

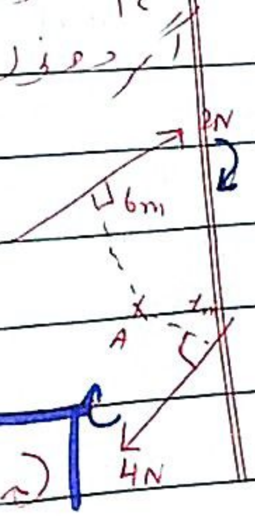
$\circlearrowright$ $Moment_A = 3N \times 2m = 6Nm$

Resultant moment: $15Nm - 6Nm = 9Nm$ $\circlearrowleft$

کیونکہ 15 بڑا ہے 6 سے یہاں پر anti clock زیادہ ہے اس لیے

جب 1 anticlock اور دوسری clock wise ہو تو ہم زیادہ میں سے کم کو (−) کریں گے اور اگر دونوں clockwise ہوں تو ہم (+) کریں گے۔

(2) clockwise forces دونوں ہیں تو:



$Moment_A = 2N \times 6m = 12Nm$

$Moment_A = 4N \times 1m = 4Nm$

Resultant moment:

$12Nm + 4Nm = 16Nm$

→ Non-perpendicular moment

اس میں force (distance) کو perpendicular لینا ہوتا ہے

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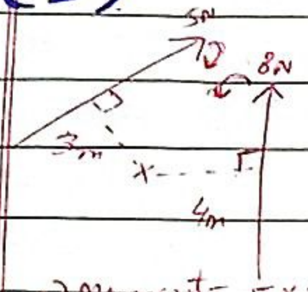
(1)



Sol: ~~Moment_A = 5×3 = 15Nm~~

↻ Moment_A = 8×4 = 32Nm

Resultant: **32 − 15 = 17Nm ↺**

(2)

↺ Moment_A = 5×4 = **20Nm**

↻ Moment_A = ~~7×6 = 42Nm~~

✓ 7 Sin 30° × 6m

$7(\frac{1}{2})6 = $ **21Nm**

یہاں sin 30° اس لیے

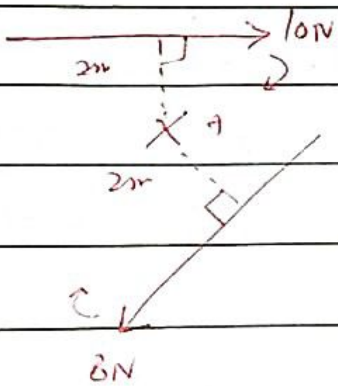
30° angle دیا ہے

vertically لیا ہے

Resultant =

21 − 20 = 1Nm (↻)

(3)

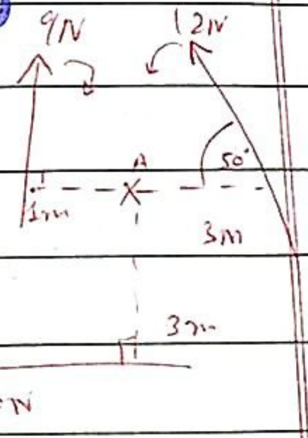


↺ moment = 8×2 = 16Nm

↻ 〃 = 10×2 = 20

→ Resultant: 20 − 16 = **4Nm (↻)**

(4)



↺ moments = (10×3) + (9×1) ⇒ **39Nm**

↻ ~~moment = 12 Sin 50 × 3 =~~ **27.6Nm**

→ Resultant =

39 − 27.6 = 11.4Nm ↺ ✓

Lect #24

→ Cross product:

← 3-D میں کراس پراڈکٹ find کیا جاتا ہے

→ A determinant of order-2:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Exp.

$$\begin{vmatrix} 2 & 1 \\ -6 & 4 \end{vmatrix} = (2\times 4) - (-6\times 1)$$
$$= 8 + 6 = 14$$

→ A determinant of order-3

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \Rightarrow a_1(b_2c_3 - c_2b_3) - a_2(b_1c_3 - c_1b_3) + a_3(b_1c_2 - c_1b_2)$$

Exp

$$\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 5 & 4 & 2 \end{vmatrix} = 1(0-4) - 2(6+5) + (-1)(12-0) \Rightarrow = -4 - 22 - 12$$
$$= \boxed{-38}$$

→ **Exp**

If $a(1,3,4)$ & $b(2,7,-5)$

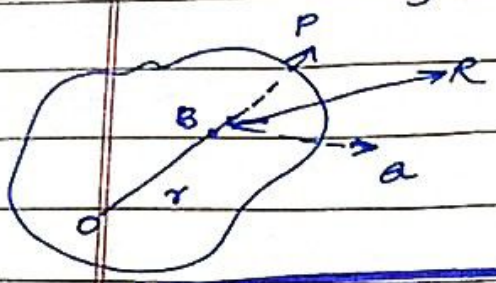
$a \times b$?

Sol

$$a\times b = \begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{vmatrix} \Rightarrow i(-15-28) - j(-5-8) + k(7-6)$$

$$= i(-43) - j(-13) + k(1) \Rightarrow \boxed{-43i + 13j + k}$$

DAY: Varignon's theorem DATE:



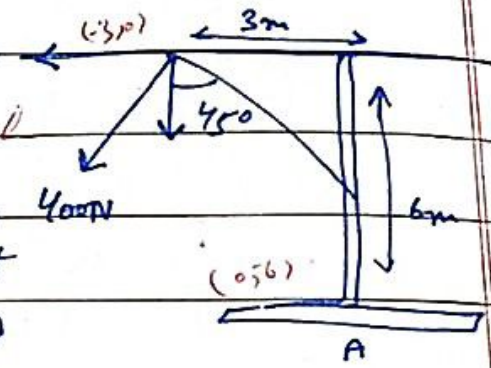
$M_o = r \times R$

$R = (P + Q)$

$M_o = r \times R = r \times (P + Q)$

$M_o = Rd = -Pp + Qq$

Exp



→ Scalar approach

$\text{moment}_A = (400 \cos 45° \times 6) + (400 \sin 45° \times 3)$ ↻

$= 2545°\ 2\ N.m$ Clockwise

→ Vector approach

$\vec{F} = 400 \cos 45 (-i) + 400 \sin 45° (-j) =$

$-282.8\,i - 282.8\,j$

$\vec{r} = (x_2 - x_1)\,i + (y_2 - y_1)\,j = (0+3)\,i + (0+6)\,j$

$\vec{r} = 3\vec{i} + 6\vec{j}$

$$M_o = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 3 & 6 & 0 \\ 282.8 & -282.8 & 0 \end{vmatrix} = 2545.2\,k \ \text{Nm}$$

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Lect#25

Net Moment OR Resultant moment:

The Sum of all moments produced by all forces is called Net or resultant moment.

Counterclock wise OR anti-clock wise = +ve

Clock Wise = -ve

Moment & Equilibrium:

1st condition of Equilibrium:

Net external forces is must be zero.

$\Sigma F = 0$, $\Sigma F_x = 0$ $\Sigma F_y = 0$

Horizontal forces = 0 Vertically forces = 0

یہ ہمیں Translational equilibrium کا 1st eq. of equilibrium دیتا ہے

2nd Condition of equilibrium

The Sum of all moments must be zero

$\Sigma M_i = 0$

Total clockwise moment = Total anti-clock wise moment.

اگر کوئی بورڈ یا شہتیر ہو تو اسکا وزن (weight) درمیان میں ہوگا mid

Quiz

DAY: DATE:

جس سپورٹ پوائنٹ پر rod کے pivoted ہو گا

اسی پوائنٹ پر reaction force ہو گی۔

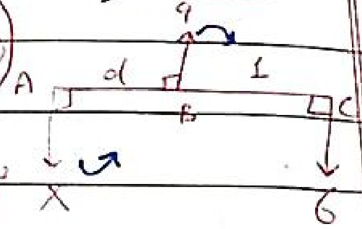
اگر rod دو یا دو سے زیادہ پر supports ہو گا

تو reaction forces بھی act کریں گی۔ ہر پوائنٹ پر الگ الگ

light rod کی small mass ہو گی جس کو

ignore کر دیں گے۔

(1) → **Exmp**) Calculate size of force X and value of d



Resolving forces vertically:

$X + 6 = 9 \Rightarrow \boxed{x = 3}$

→ taking moment about 'C'

Note: جب کسی پوائنٹ کے about moment لیتے ہیں اگر اسی پوائنٹ پر کوئی force act کر رہی ہو تو اس کا moment (0) ہو گا۔

2nd condition کے مطابق استعمال کریں گے (taking moment about 'c')

$C = C_a$ کہ ظاہر ہے

$9 \times 1 = X(d+1)$

$X = 3$,

$9 \times 1 = 3(d+1) \Rightarrow 9 = 3d + 3 \Rightarrow 9 - 3 = 3d$

$6 = 3d \Rightarrow \boxed{d = 2}$

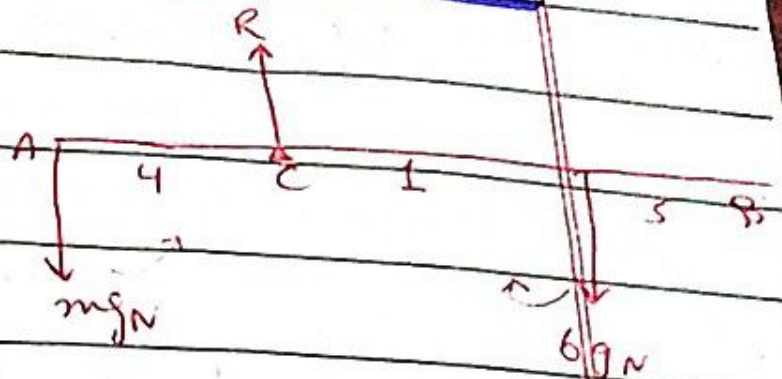
(2) calculate mass?

taking moment about 'C'

$mg \times 4 = 6 \times 1$

$4m = 6 \Rightarrow m = 6/4$

$m = 3/2 \Rightarrow$ **1.5 kg**



(3) Calculate **distance**

taking moments about 'C'

↑ moments: $10 \times (3-d)$

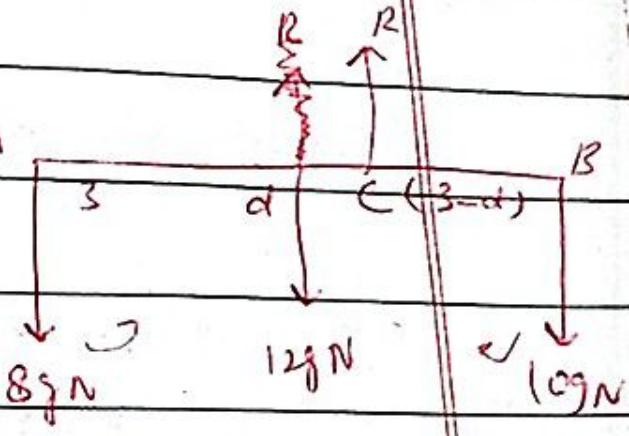
$30 - 10d$

↺ moment = $8g \times (3+d) + 12d$

$24 + 8d + 12d$

$24 + 20d$

$\Rightarrow$

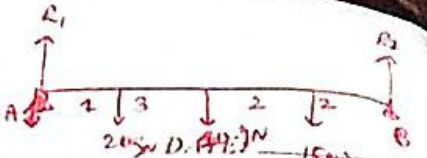


② $G = \curvearrowright$ (2nd condition)

$30 - 10d = 24 + 20d$

$30 = 6 + 30d \Rightarrow d = 0.2$ m

(4) calculate forces



Resolving forces vertically:

$R_1 + R = 45\,kN$

Taking moment about 'A'

$20 \times 1 + 10 \times 4 + 15 \times 6 = R_2 \times 8$

$150 = 8R_2 \Rightarrow \boxed{R_2 = 18.75\,kN}$

So, $R_1 = 45 - R_2 \Rightarrow = 45 - 18.75$

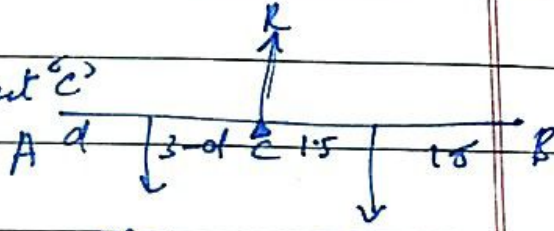
$\boxed{R_2 = 26.25\,kN}$

"Next # 26"

(5) find distance b/w A & center of mass of beam

Sol

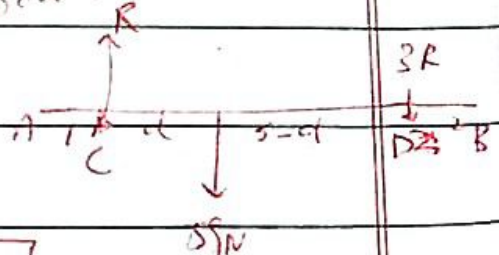
⇒ Taking moment about 'C'



$5g(3-d) = 2g \times 1.5$

$15 - 5d = 3 \Rightarrow 5d = 12 \Rightarrow \boxed{d = 2.5\,m}$

(6) calculate distance b/w A & center of mass of rod.



→ Resolving forces vertically:

$3R + R = 5g$

$4R = 5g \Rightarrow \boxed{R = 1.25\,N}$

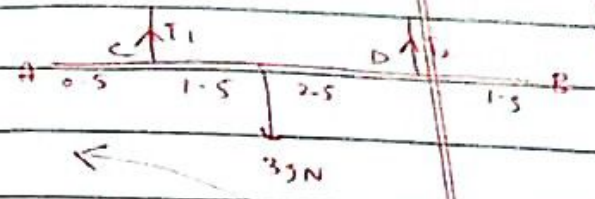
② taking moment about D

$d = 4.75m$

⑦ → calculate (tension) in string C & D

Sol Resolving forces vertically,

$T_1 + T_2 = 39N$



→ Taking moment about 'C'

$39 \times 1.5 = T_2 \times 4$

$4.5g = 4T_2 \Rightarrow (T_2 = 1.125g)$

$T_1 = 39N - T_2 \Rightarrow T_1 = 39 - 1.125 \Rightarrow T_1 = 1.875gN$

→ So, tension in string at 'C' is 1.875gN &

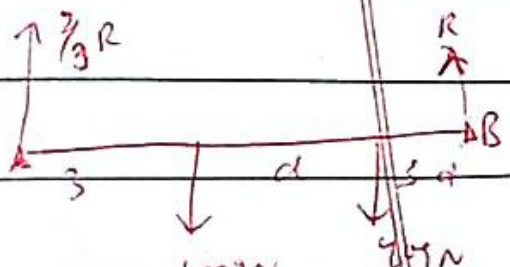
D is 1.125g

⑧

→ (a): Calculate magnitude, in newtons of force exerted by support at 'A'

(b): distance AC.

→ Resolving force :-



$\frac{2}{3}R + R = 100 + 40$

$\frac{5}{3}R = 140gN \rightarrow R = 84N$

Taking moment about A

force at A = 56N

Force at A = $\frac{2}{3} \times 84$

$56 \times 9.8 = 548.8N$

Taking resolving moment at A (distance of j)

$(100 \times 3) + 40(3+d) = 84 \times (6)$

$40d = 84 \Rightarrow d = 2.1m$

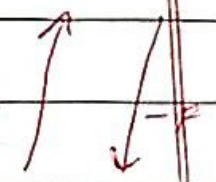
$d = 5.1m$

Lect #28

→ Moment of Couple

Two forces f and -f have same magnitude but opposite direction make couple.

⇒ Moment of couple.



$\vec{M} = \vec{r}_A \vec{F} + \vec{r}_B(-\vec{F})$

$\vec{M} = (\vec{r}_A - \vec{r}_B)\vec{F}$

$\vec{M} = \vec{r} \times \vec{F}$

$\vec{M} = rF\sin\theta = Fd$

→ Addition couples

$\vec{M}_1 = \vec{r} \times \vec{F}_1$, $\vec{M}_2 = \vec{r} \times F_2$

$\vec{M}_1 = r \times R$, $\vec{M}_2 = r \times R$

$\vec{M}_1 = r \times (F_1 + F_2)$,

$\vec{M} = \vec{M}_L$

$\vec{M}_o = \vec{r} \times \vec{F}$

$M_o = Fd(s+s)F$

$r_x F + s F$

$M^o + \vec{s} \times \vec{F}$