MTH 632
Current Papers

Notes

Q: Write $-3i^4$ in the form of $a+ib$.

20 FRI 20 Muharram

A: $a+ib = -3i^4$

$= -3(i^2)^2$

Notes

$= -3(-1)^2$

$a+ib = -3+0i$

Q: Find antiderivative of $\int_{i}^{i/2} e^{\pi z}\,dz$

21 SAT 21 Muharram

Notes A:

$$I = \int_{i}^{i/2} e^{\pi z}\,dz$$

$$= \left.\frac{e^{\pi z}}{\pi}\right|_{i}^{i/2}$$

22 SUN 22 Muharram

$\because e^{i\pi/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$= 0 + i(1) = +i$

Important

$$= \frac{1}{\pi}\left[e^{i\pi/2} - e^{i\pi}\right]$$

$e^{i\pi} = \cos\pi + i\sin\pi$

$= -1 + 0 = -1$

$$= \frac{1}{\pi}\left[i - (-1)\right] = \frac{i+1}{\pi}$$

Notes

23 MON 23 Muharram

Q: $Z_n = \frac{1}{n^3} - i$ Solve it for convergence

A:

$$Z_n = \frac{1}{n^3} - i$$

Notes

24 TUE 24 Muharram

$$\lim_{n\to\infty} Z_n = \lim_{n\to\infty}\left(\frac{1}{n^3} - i\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{n^3}\right) - \lim_{n\to\infty}(i)$$

$$= \frac{1}{\infty} - i = -i < 0$$

(converges)

Notes

25 WED 25 Muharram

Q: Prove that

$$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$

satisfies Laplace's equation.

Notes

26 THU 26 Muharram

A: $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x \quad , \quad \frac{\partial u}{\partial y} = -6xy - 6y$$

$$\frac{\partial^2 u}{\partial x^2} = 6x + 6 \quad , \quad \frac{\partial^2 u}{\partial y^2} = -6x - 6$$

Notes

27 FRI 27 Muharram

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 6 - 6x - 6 = 0$$

Hence, u satisfies Laplace's equation.

Notes

28 SAT 28 Muharram

Q: Evaluate $\bar{z}_1 - \bar{z}_2$ when $z_1 = 5+i$ & $z_2 = -8+3i$.

A:

$$z_1 = 5+i \quad , \quad z_2 = -8+3i$$

Notes

29 SUN 29 Muharram

$$\bar{z}_1 = 5-i \quad , \quad \bar{z}_2 = -8-3i$$

$$\bar{z}_1 - \bar{z}_2 = 5 - i + 8 + 3i$$
$$= 13 + 2i$$

Important

Q: If $\int_c f(z)\,dz$ & $f(z) = \frac{z+2}{z}$ & $z = 2e^{i\theta}$

A: $I = \int_c f(z)\,dz$

September 2019

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④

30 MON
30 Muharram

Week 40

Notes

$$I = \int_0^{\pi} \left(\frac{z+2}{z}\right) dz$$

$\because z = 2e^{i\theta}$

$dz = 2ie^{i\theta} d\theta$

$$= \int_0^{\pi} \left(\frac{z}{z} + \frac{2}{z}\right) dz$$

1 TUE
1 Safar

Notes

$$= \int_0^{\pi} \left(1 + \frac{2}{z}\right) dz$$

$$= \int_0^{\pi} \left(1 + \frac{2}{2e^{i\theta}}\right) 2ie^{i\theta} d\theta$$

2 WED
2 Safar

Notes

$$= \int_0^{\pi} \left(1 + \frac{1}{e^{i\theta}}\right) 2ie^{i\theta} d\theta$$

$$= 2i \int_0^{\pi} \left(\frac{e^{i\theta}+1}{\cancel{e^{i\theta}}}\right) \cdot \cancel{e^{i\theta}} d\theta$$

3 THU
3 Safar

Notes

$$= 2i \int_0^{\pi} (e^{i\theta} + 1) d\theta$$

$$= 2i \left[\int_0^{\pi} e^{i\theta} d\theta + \int_0^{\pi} d\theta\right]$$

⑤

Hijri 1441

Notes

4 FRI
4 Safar

$$I = 2i \left[\frac{e^{i\theta}}{i}\Big|_0^{\pi} + \theta\Big|_0^{\pi}\right]$$

$$= 2i \left[\frac{e^{i\pi} - e^{0}}{i}\right] + 2i(\pi - 0)$$

Notes

$$= 2e^{i\pi} - 2 + 2i\pi$$

$\because e^{i\pi} = \cos\pi + i\sin\pi$

$= -1 + 0 = -1$

$$= 2(-1) - 2 + 2i\pi$$

$$= 2i\pi - 4$$

5 SAT
5 Safar

Q: Poove that

Notes

$$\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz = e + \frac{1}{e}$$

A: L.H.S

6 SUN
6 Safar

$$I = \int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$$

Important

$$= \frac{\sin\frac{z}{2}}{1/2}\Bigg|_0^{\pi+2i}$$

October 2019

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7 MON 7 Safar

$$I = 2 \left| \sin \frac{z}{2} \right|_0^{\pi+2i}$$

$$= 2\left[\sin\left(\frac{\pi+2i}{2}\right) - \sin(0)\right]$$

$$= 2\left[\sin \frac{\pi}{2} + i\right]$$

8 TUE 8 Safar

$$= \cancel{2}\left[\frac{e^{i(\frac{\pi}{2}+i)} - e^{-i(\frac{\pi}{2}+i)}}{\cancel{2}i}\right]$$

$$= \frac{1}{i}\left[e^{i\frac{\pi}{2}-1} - e^{-i\frac{\pi}{2}+1}\right]$$

9 WED 9 Safar

$$= \frac{1}{i}\left[e^{i\frac{\pi}{2}}\cdot e^{-1} - e^{-i\frac{\pi}{2}}\cdot e^{1}\right]$$

$\because e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i = i$

$$= \frac{1}{i}\left[(i)e^{-1} - (-i)e^{1}\right]$$

$e^{-i\frac{\pi}{2}} = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right) = 0 - i = -i$

10 THU 10 Safar

$$= \frac{1}{i}\left(i\cdot\frac{1}{e} + ie\right)$$

$$= \frac{1}{e} + e$$

Hence, Proved

Q. Find df & $df(v_p)$ if $f = (x^2-1)y + (y^2-1)x$

11 FRI 11 Safar

A:

As chain rule

$$d(h(f)) = h'(f)df$$

12 SAT 12 Safar

Product rule

$$d(fg) = gdf + fdg$$

Now,

$$f = (x^2-1)y + (y^2-1)x$$

13 SUN 13 Safar

$$df = (2x\,dx)y + (x^2-1)dy + (2y\,dy)x + (y^2-1)dx$$

$$= 2xy\,dx + (x^2-1)dy + 2xy\,dy + (y^2-1)dx$$

$$df = (2xy + y^2 - 1)dx + (x^2 + 2xy - 1)dy$$

By Tangent vector

$$df(v_p) = [2P_1P_2 + P_2^2 - 1]v_1 + [P_1^2 + 2P_2P_1 - 1]v_2$$

14 MON 14 Safar

Notes

Q: Fint $\hat{t} = \frac{x'(t)}{|x'(t)|}$

where

$$x(t) = a\cos(t)\hat{e}_1 + a\sin(t)\hat{e}_2 + bt\hat{e}_3$$

A:

15 TUE 15 Safar

Notes

$$x(t) = a\cos(t)\hat{e}_1 + a\sin(t)\hat{e}_2 + bt\hat{e}_3$$

$$x'(t) = -a\sin(t)\hat{e}_1 + a\cos(t)\hat{e}_2 + b\hat{e}_3$$

$$|x'(t)| = \sqrt{(a\sin t)^2 + (a\cos t)^2 + (b)^2}$$

$$= \sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2}$$

16 WED 16 Safar

Notes

$$= \sqrt{a^2(\sin^2 t + \cos^2 t) + b^2}$$

$$= \sqrt{a^2(1) + b^2}$$

$$|x'(t)| = \sqrt{a^2+b^2}$$

17 THU 17 Safar

Notes

we know that

$$\hat{t} = x = \frac{x'(t)}{|x'(t)|} = \frac{-a\sin(t)\hat{e}_1 + a\cos(t)\hat{e}_2 + b\hat{e}_3}{\sqrt{a^2+b^2}}$$

18 FRI 18 Safar

Notes

Q: If $u(x,y) = e^y\cos x$ then find $v(x,y)$.

A: $u(x,y) = e^y\cos x$

19 SAT 19 Safar

Notes

$$\frac{\partial u}{\partial x} = -e^y\sin x \quad , \quad \frac{\partial u}{\partial y} = e^y\cos x$$

$$\frac{\partial^2 u}{\partial x^2} = -e^y\cos x \quad , \quad \frac{\partial^2 u}{\partial y^2} = e^y\cos x$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -e^y\cos x + e^y\cos x$$

$$= 0$$

20 SUN 20 Safar

Notes

It is harmonic function.

Now, to find v.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = -e^y\sin x$$

Important

By integrating w.r.t. y

$$v(x,y) = -e^y\sin x + g(x)$$

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21 MON 21 Safar

By taking derivative w.r.t. x

$$\frac{\partial v}{\partial x} = -e^{y}\cos x + g'(x)$$

Put $\frac{\partial v}{\partial x} = -e^{y}\cos x$

22 TUE 22 Safar

$$-e^{y}\cos x = -e^{y}\cos x + g'(x)$$

$$g'(x) = 0$$

$$g(x) = c$$

Hence,

$$v(x,y) = -e^{y}\sin x + c$$

23 WED 23 Safar

Q: Find Singularity for function

$$f(z) = \frac{1}{z^2+2z+2}$$

24 THU 24 Safar

A: Here

$$z^2+2z+2=0$$

By applying quadratic formula

$a=1$, $b=2$, $c=2$

25 FRI 25 Safar

$$z = \frac{(-2)\pm\sqrt{(2)^2-4(1)(2)}}{2(1)}$$

$$= \frac{(-2)\pm\sqrt{4-8}}{2}$$

26 SAT 26 Safar

$$z = \frac{-2\pm\sqrt{-4}}{2}$$

$$= \frac{-2\pm 2i}{2}$$

$$= -1 \pm i$$

27 SUN 27 Safar

Hence, z is singular Point.

Q: Show that

$$\overline{\cos(iz)} = \cos(i\bar{z})$$

Important

A: L.H.S

$$= \overline{\cos(iz)}$$

$\because z = x+iy$

$\bar{z} = x-iy$

28 MON 28 Safar

$$= \overline{\cos(-y+ix)}$$

$\because z = x+iy$

$iz = ix - y$

$\overline{iz} = -ix - y$

$$= \cos y \cosh x - \sin y \sinh x \quad \text{--- (1)}$$

R.H.S

29 TUE 29 Safar

$$= \cos(\overline{iz})$$

$$= \cos(-ix - y)$$

$$= \cos(-(y+ix))$$

$$= \cos(y+ix)$$

30 WED 1 R. Awwal

$$= \cos y \cosh x - \sin y \sinh x \quad \text{--- (2)}$$

from eq (1) & (2)

L.H.S = R.H.S

31 THU 2 R. Awwal

Q: Prove that $\frac{d}{dz}(\cos z) = -\sin z$

A: L.H.S

$$= \frac{d}{dz}(\cos z)$$

1 FRI 3 R. Awwal

$$= \frac{d}{dz} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{d}{dz} \frac{(-1)^n z^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot \cancel{2n}\,(z^{2n-1})}{\cancel{(2n)}(2n-1)!}$$

2 SAT 4 R. Awwal

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (z^{2n-1})}{(2n-1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1} \cdot (-1)^1 (z^{2n-1})}{(2n-1)!}$$

3 SUN 5 R. Awwal

$$= -\sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^{(2n-1)}}{(2n-1)!}$$

$$= -\sin z$$

Important

Hence, Proved

4 MON 6 R. Awwal

Notes

Q: Fernet Serret Formula

Prove that

(2) $\because \bar{n} = \tau \bar{B} - k\bar{t}$

$n = B \times t$

differentiate w.r.t. s

Notes

5 TUE 7 R. Awwal

$\bar{n} = B\bar{t} + t\bar{B}$

As $t = n \times b$

$n = b \times t$

$B = t \times n$

6 WED 8 R. Awwal

Notes

$\bar{n} = k(b \times n) + (-\tau(n \times t))$ $\quad \because \bar{t} = k\bar{n}$, $\bar{b} = -\tau\bar{n}$

$= -k\bar{t} - \tau(-B)$

$= -k\bar{t} + \bar{B}\tau$

$\bar{n} = \bar{B}\tau - k\bar{t}$

Notes

7 THU 9 R. Awwal

(1) Prove that

$\bar{t} = k\bar{n}$

$k = \dfrac{d\theta}{ds}$

$k = \dfrac{d\theta}{|dt|} \cdot \dfrac{|dt|}{ds}$

Notes

8 FRI 10 R. Awwal

$k = \left|\dfrac{d\theta}{dt}\right| \cdot \left|\dfrac{dt}{ds}\right|$ $\quad \because \left|\dfrac{d\theta}{dt}\right| = 1$

$k = \left|\dfrac{dt}{ds}\right|$ $\quad \because t = \dfrac{d\gamma}{ds}$

Notes

9 SAT 11 R. Awwal

$= \left|\dfrac{d\gamma'}{ds}\right| = |\gamma''| = k$

γ'' is collinear with $\hat{n}$

$|\gamma''| = \pm kn$

$\gamma'' = kn$ $\quad \because \bar{t} = \gamma''$

Notes

10 SUN 12 R. Awwal

Hence, $t' = k\bar{n}$

Prove that

(3) $\bar{b} = -\tau\bar{n}$

As

$t \cdot b = 0$

Important

differentiate with r.t b

$t \cdot \bar{b} + \bar{t} \cdot b = 0$ $\quad \because \bar{t} = k\bar{n}$

$t \cdot \bar{b} + k\bar{n} \cdot b = 0$ $\quad \because n \cdot b = 0$

$t \cdot \bar{b} = 0$

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11 MON 13 R. Awwal

$\Rightarrow$ $\bar{b}$ is proper to t

also $\bar{b}$ is collinear with n

$$\bar{b} = \pm \tau n$$

$$\bar{b} = -\tau n$$

12 TUE 14 R. Awwal

b has opposite direction so -ve sign

Q: If x, y & ϕ, ψ then Prove that

$$\cancel{fdg} \; d(fg) = gfd + fdg$$

13 WED 15 R. Awwal

A:

$$d(fg) = \sum \frac{\partial}{\partial x_i}(fg)\,dx_i$$

$$= \sum\left(\frac{\partial f}{\partial x_i} g + \frac{\partial g}{\partial x_i} f\right) dx_i$$

14 THU 16 R. Awwal

$$= \sum \frac{\partial f}{\partial x_i} g\,dx_i + \sum \frac{\partial g}{\partial x_i} f\,dx_i$$

$$= g \sum \frac{\partial f}{\partial x_i} dx_i + f \sum \frac{\partial g}{\partial x_i} dx_i$$

$$= gdf + fdg$$

15 FRI 17 R. Awwal

Q: If $P = (1, 1, 0)$ & $V = (1, 0, -3)$

$f = x^2yz$, then find $V_p[f]$

A:

$$f = x^2yz$$

16 SAT 18 R. Awwal

$$\frac{\partial f}{\partial x} = 2xyz, \quad \frac{\partial f}{\partial y} = x^2 z, \quad \frac{\partial f}{\partial z} = x^2 y$$

at Point $P = (1,1,0)$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,1,0)} = 0, \quad \left.\frac{\partial f}{\partial y}\right|_{(1,1,0)} = 0, \quad \left.\frac{\partial f}{\partial z}\right|_{(1,1,0)} = 1$$

17 SUN 19 R. Awwal

Now

$$V_p[f] = \sum v_i \frac{\partial f}{\partial x_i}(P)$$

$$= 1(0) + 0(0) - 3(1)$$

$$= -3$$

Important

-3 is directional derivative.

Q: Prove that

$$\frac{d}{dz}(\sin^{-1}hz) = \frac{1}{\sqrt{1+z^2}}$$

A: Let

$$w = \sin^{-1}hz$$

$$z = \sinh w$$

Taking derivative ~~w.r.t z~~

$$\frac{d}{dz}(z) = \frac{d}{dz}(\sinh w)$$

$$1 = \frac{d}{dw}(\sinh w) \cdot \frac{dw}{dz}$$

$$1 = -\cosh w \cdot \frac{dw}{dz}$$

$$-\frac{1}{\cosh w} = \frac{dw}{dz}$$

$$\frac{dw}{dz} = \frac{1}{\sqrt{1+(\sinh w)^2}}$$

$$\frac{d}{dz}(\sin^{-1}hz) = \frac{1}{\sqrt{1+z^2}}$$

Q: If $\lim_{n\to\infty} |z_n| = z$ then prove that $||z_n| - |z|| \le |z_n - z|$ and show that $\lim_{n\to\infty} |z_n| = z$

A:

$$\because z_n = (z_n - z) + z$$

$$|z_n| \le |z_n - z| + |z|$$

So, $$|z_n| - |z| \le |z_n - z|$$

$$\because z = (z - z_n) + z_n$$

$$|z| \le |z - z_n| + |z_n|$$

$$-|z_n| + |z| \le |z_n - z|$$

Hence,

$$-|z_n| + |z| \le |z_n - z| \le |z_n| - |z|$$

Important

$$||z_n| - |z|| \le |z_n - z|$$

25 MON 27 R. Awwal

Q: Prove that

$$\frac{d}{dz}(\sin z) = \cos z$$

A:

L.H.S

$$= \frac{d}{dz}(\sin z)$$

$$= \frac{d}{dz}\sum_{n=0}^{\infty}\frac{(-1)^n (z)^{2n+1}}{(2n+1)!}$$

26 TUE 28 R. Awwal

$$= \sum_{n=0}^{\infty}\frac{d}{dz}\frac{(-1)^n z^{2n+1}}{(2n+1)!}$$

27 WED 29 R. Awwal

$$= \sum_{n=0}^{\infty}\frac{(-1)^n z^{2n}\cancel{(2n+1)}}{\cancel{(2n+1)}(2n)!}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n z^{2n}}{2n!}$$

28 THU 30 R. Awwal

$$= \cos z$$

Hence, Proved

29 FRI 1 R. Sani

Q: Show that

$$\sum_{n=1}^{\infty}\bar{z}_n = \bar{S}$$

A:

we know that

30 SAT 2 R. Sani

$$\bar{z}_n = x_n - iy_n$$

$$\& \quad S = X + iY \quad , \quad \bar{S} = X - iY$$

$$\sum_{n=1}^{\infty} x_n = X \quad , \quad \sum_{n=1}^{\infty} y_n = Y$$

1 SUN 3 R. Sani

$$\sum_{n=1}^{\infty}\bar{z}_n = \sum_{n=1}^{\infty}(x_n - iy_n)$$

$$= \sum_{n=1}^{\infty} x_n - i\sum_{n=1}^{\infty} y_n$$

$$= X - iY$$

Important

$$\sum_{n=1}^{\infty}\bar{z}_n = \bar{S}$$

Hence, Proved

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2 MON 4 R. Sani

Q: Evaluate

$$\int_c \frac{\cosh z}{z^4}\, dz$$

$x = \pm 2, \quad y = \pm 2$

3 TUE 5 R. Sani

A: let

$$f(z) = \int_{-2}^{2} \frac{\cosh z}{z^4}\, dz$$

By Applying cauchy integral formula

$$= \int_{-2}^{2} \frac{\cosh z}{(z-0)^{3+1}}\, dz \qquad \because z_0 = 0$$

4 WED 6 R. Sani

$$= \frac{2\pi i}{3!}\left[\frac{d^3}{dz^3}\cosh z\right]_{z=-2}^{2}$$

5 THU 7 R. Sani

$$= \frac{\pi i}{3}\left(\cosh(-2) - \cosh(2)\right)$$

$$= \frac{\pi i}{3}(0)$$

$$= 0$$

6 FRI 9 R. Sani

Q: State and Prove Liouville's Theorem

A:

Statement:

Prove that every entire bounded function is constant.

7 SAT 9 R. Sani

Proof:

$\because$ $f(z)$ is analytic everywhere in complex plane and is bounded.

So, by cauchy inequality theorem

8 SUN 10 R. Sani

$$\left|f^{(n)}(a)\right| \leq \frac{n!}{r^n} M \qquad \text{where } |z-a| = r$$

for $n = 1$ $\qquad \because n \neq 0, \infty^0$ in indeterminate for r^n $r \to \infty$

Important

$$|f'(a)| \leq \frac{M}{r} \qquad \text{If } a = z$$

$$\Rightarrow |f'(z)| \leq \frac{M}{r} \quad — (1)$$

9 MON 11 R. Sani

If $r \to \infty$ then from (1)

we conclude that $f'(z) \to 0$

So, $f(z) =$ constant

10 TUE 12 R. Sani

Q: If $f = 8x^3yz$, $P = (2, 0, 3)$

$v = (1, 1, 1)$

then find $v_p[f]$

A:

11 WED 13 R. Sani

$$f = 8x^3yz$$

$$\frac{\partial f}{\partial x} = 24x^2yz , \quad \frac{\partial f}{\partial y} = 8x^3z , \quad \frac{\partial f}{\partial z} = 8x^3y$$

At Point $(2, 0, 3)$

$$\left.\frac{\partial f}{\partial x}\right|_{(2,0,3)} = 0 , \quad \left.\frac{\partial f}{\partial y}\right|_{(2,0,3)} = 192 , \quad \left.\frac{\partial f}{\partial z}\right|_{(2,0,3)} = 0$$

12 THU 14 R. Sani

So,

$$v_p[f] = 1(0) + 1(192) + 1(0)$$

$$= 192$$

Q: $\log(1-i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}$

13 FRI 15 R. Sani

A:

L.H.S

$= \log(1-i)$

$= \log|1-i| + i\,\mathrm{Arg}(1-i)$

14 SAT 16 R. Sani

$= \ln\sqrt{2} - \frac{\pi}{4}i$

$= \frac{1}{2}\ln 2 - \frac{\pi}{4}i$

$\theta = \tan^{-1}\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$

$|1-i| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

R.H.S

15 SUN 17 R. Sani

Q: $\log(-ei) = 1 - \frac{\pi}{2}i$

L.H.S

$= \log(-ei)$

$= \ln|-ei| + i\,\mathrm{Arg}(-ei)$

$= \ln e - \frac{\pi}{2}i$

$= 1 - \frac{\pi}{2}i$

$\therefore \theta = \tan^{-1}\left(\frac{-1}{0}\right) = \infty$

$= -\frac{\pi}{2}$

$|-ei| = \sqrt{(e)^2+(0)^2} = e$

R.H.S

Important

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16 MON 18 R. Sani

Q: Show that

$$2Re(z) = z+\bar{z}$$

A: R.H.S

$$= z+\bar{z}$$

$$= x+iy+\overline{x+iy}$$

17 TUE 19 R. Sani

$$= x+\cancel{iy}+x-\cancel{iy}$$

$$= 2x$$

$$= 2\,Re(z)$$

18 WED 20 R. Sani

Q: Find Harmonic conjugate

$$u(x,y) = 5x(4-2y)$$

A:

$$u(x,y) = 5x(4-2y)$$

19 THU 21 R. Sani

$$\frac{\partial u}{\partial x} = 20-10y \quad , \quad \frac{\partial u}{\partial y} = -10x$$

$$\frac{\partial^2 u}{\partial x^2} = 0 \quad , \quad \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Fuction is harmonic

20 FRI 22 R. Sani

To find v

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 20-10y$$

By integrating w.r.t y

21 SAT 23 R. Sani

$$v(x,y) = 20y-5y^2+g(x)$$

Taking derivative w.r.t x

$$\frac{\partial v}{\partial x} = g'(x)$$

22 SUN 24 R. Sani

Put $\frac{\partial v}{\partial x} = 10x$

$$g'(x) = 10x$$

$$g(x) = 5x^2+c$$

Important

$$v(x,y) = 20y-5y^2+5x^2+c$$

(28)

DECEMBER 2019 Week 52

Q: Show that

$$\log(1+i)^2 = 2\log(1+i)$$

A:

$$(1+i) = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$= \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$(1+i)^2 = \left(\sqrt{2}\, e^{i\pi/4}\right)^2 = 2e^{i\pi/2}$$

therefore $\mathrm{Arg}(1+i) = \frac{\pi}{4}$ & $\mathrm{Arg}(1+i)^2 = \frac{\pi}{2}$

$$\Rightarrow \log(1+i)^2 = \ln|(1+i)^2| + i\,\mathrm{Arg}(1+i)^2$$

$$= \ln|2e^{i\pi/2}| + i\frac{\pi}{2}$$

$$= \ln\left|2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right| + i\frac{\pi}{2}$$

$$= \ln|2| + i\frac{\pi}{2}$$

$$= 2\left(\frac{1}{2}\ln|2| + i\frac{\pi}{4}\right)$$

$$= 2\left(\ln\sqrt{2} + \frac{i\pi}{4}\right)$$

(29)

Hijri 1441 DECEMBER 2019

$$= 2\log(1+i)$$

Q: show that

$$\log(-1+i)^2 \neq 2\log(-1+i)$$

A:

L.H.S

$$-1+i = \sqrt{2}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\, e^{i3\pi/4}$$

$$(-1+i)^2 = \left(\sqrt{2}\, e^{i3\pi/4}\right)^2 = 2e^{i3\pi/2}$$

therefore $\mathrm{Arg}(-1+i) = \frac{3\pi}{4}$, $\mathrm{Arg}(-1+i)^2 = \frac{3\pi}{2}$

$$\Rightarrow \log(-1+i)^2 = \ln|(-1+i)^2| + i\,\mathrm{Arg}(-1+i)^2$$

$$= \ln|2e^{i3\pi/2}| + i\frac{3\pi}{2}$$

$$= \ln\left|2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right| + i\frac{3\pi}{2}$$

$$= \ln|2| + i\frac{3\pi}{2}$$

$$\neq 2\log(-1+i)$$

30 MON 3 J. Awwal

Q: Prove that $u(x,y) = e^{-y}\sin x$ is harmonic or not?

A:

$$u(x,y) = e^{-y}\sin x$$

$$\frac{\partial u}{\partial x} = e^{-y}\cos x \quad , \quad \frac{\partial u}{\partial y} = -e^{-y}\sin x$$

31 TUE 4 J. Awwal

$$\frac{\partial^2 u}{\partial x^2} = -e^{-y}\sin x \quad , \quad \frac{\partial^2 u}{\partial x^2} = e^{-y}\sin x$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -e^{-y}\sin x + e^{-y}\sin x$$

$$= 0$$

1 WED 5 J. Awwal

Hence, Proved

2 THU 6 J. Awwal

Q: $f(z) = (x-y)^2 + 2i(x+y)$
Find analytic region

A:

$$f(z) = (x-y)^2 + 2i(x+y)$$

3 FRI 7 J. Awwal

let

$$u = (x-y)^2 \quad , \quad v = 2(x+y)$$

$$\frac{\partial u}{\partial x} = 2(x-y) \quad , \quad \frac{\partial v}{\partial x} = 2$$

4 SAT 8 J. Awwal

$$\frac{\partial u}{\partial y} = -2(x-y) \quad , \quad \frac{\partial v}{\partial y} = 2$$

Now, by C.R equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

5 SUN 9 J. Awwal

$$2(x-y) = 2 \quad \& \quad 2 = 2(x-y)$$

$$x - y = 1 \quad \& \quad x - y = 1$$

So, Analytic region is $x-y=1$

Important

6 MON 10 J. Awwal

Q: show that the function $f(z)=\bar{z}$ is nowhere differentiable.

A:

As $z = x+iy$
$\bar{z} = x-iy$

let $u+iv = x-iy$

$u = x$, $v = -y$

7 TUE 11 J. Awwal

$\frac{\partial u}{\partial x} = 1$, $\frac{\partial v}{\partial x} = 0$

8 WED 12 J. Awwal

$\frac{\partial u}{\partial y} = 0$, $\frac{\partial v}{\partial y} = -1$

$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$

CR equations are not satisfied.

So, $f(z) = \bar{z}$ nowhere differentiable.

9 THU 13 J. Awwal

10 FRI 14 J. Awwal

Q: Evaluate $\int_c \frac{e^{-z}}{z-\frac{\pi}{2}i} dz$ by cauchy integral formula where $x=\pm 2$, $y=\pm 2$

A:

$$f(z) = \int_c \frac{e^{-z}}{z-(\frac{\pi}{2}i)} dz$$

11 SAT 15 J. Awwal

$$= 2\pi i \left[e^{-z} \right]_{z=\frac{\pi}{2}i}$$

12 SUN 16 J. Awwal

$$= 2\pi i(-i)$$

$$= 2\pi$$

Important

Q: show that $f(z)=|z|$ is differentiable at $z=0$ but not analytic at $z=0$.

A: $f(z) = |z|$

13 MON 17 J. Awwal

~~$u+iv = |x+iy|$~~

$$= \frac{f(z+h)-f(z)}{h}$$

at $z=0$

$$= \frac{f(h)-f(0)}{h} = \frac{|h|}{h} = 1$$

14 TUE 18 J. Awwal

$\therefore$ $f(z)$ is differentiable.

$$f(z) = |z| = \sqrt{x^2+y^2}$$

$$u = \sqrt{x^2+y^2} \quad , \quad v = 0$$

15 WED 19 J. Awwal

$$\frac{\partial u}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} \quad , \quad \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

$\therefore$ $|z|$ is not analytic

16 THU 20 J. Awwal

17 FRI 21 J. Awwal

Q: Find the arc length

$$S = \int_a^b \left|\frac{d\bar{x}}{dt}\right| dt$$

$$\bar{x}(t) = (\cos t, \sin t, t)$$

where $0 \le t \le 2\pi$

18 SAT 22 J. Awwal

A: $\bar{x}(t) = (\cos t, \sin t, t)$

$$|\bar{x}'(t)| = |-\sin t, \cos t, 1|$$

$$= \sqrt{2}$$

19 SUN 23 J. Awwal

$$\int_0^{2\pi} |\bar{x}'(t)|\, dt = \int_0^{2\pi} \sqrt{2}\, dt$$

$$= \sqrt{2}\int_0^{2\pi} dt$$

Important

$$= \sqrt{2}(t)\Big|_0^{2\pi} = \sqrt{2}(2\pi - 0)$$

$$= \sqrt{2}(2\pi = 2\sqrt{2}\pi$$

M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31									

Notes

20 MON 24 J. Awwal

Q: Prove that

$$u = \frac{1}{2}\log(x^2+y^2)$$

A:

$$u(x,y) = \frac{1}{2}\log(x^2+y^2)$$

Notes

21 TUE 25 J. Awwal

$$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2}, \quad \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}, \quad \frac{\partial^2 u}{\partial y^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

Notes

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{x^2-y^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$= 0$$

22 WED 26 J. Awwal

Function is Harmonic

Now, to find v

Notes

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{x}{x^2+y^2}$$

23 THU 27 J. Awwal

By integrating w.r.t. y

$$v(x,y) = \tan^{-1}\left(\frac{y}{x}\right) + g(x)$$

Address & Telephone INDEX

Taking derivative w.r.t. x

$$\frac{\partial v}{\partial x} = -\frac{y}{x^2+y^2} + g'(x)$$

Put $\frac{\partial v}{\partial x} = -\frac{y}{x^2+y^2}$

$$-\frac{y}{x^2+y^2} = -\frac{y}{x^2+y^2} + g'(x)$$

$$g'(x) = 0$$

$$g(x) = c$$

$$v(x,y) = \tan^{-1}\left(\frac{y}{x}\right) + c$$

Q: $\int_0^\infty e^{-zt}\,dt \qquad Re\,z > 0$

A: $I = \int_0^\infty e^{-zt}\,dt$

$= \lim_{b\to\infty} \int_0^b e^{-zt}\,dt$

$= \lim_{b\to\infty} \left[\frac{e^{-zt}}{-z}\right]_0^b$

$= -\frac{1}{z} \lim_{b\to\infty} \left(e^{-zb} - e^{-z(0)}\right]$

$= -\frac{1}{z} \lim_{b\to\infty} \left(e^{-zb} - e^{0}\right)$

$= -\frac{1}{z}(0-1)$

$= \frac{1}{z}$ when $Re\,z > 0$

Q: $z_n = -2 + i\frac{(-1)^n}{n^2} \quad (n=1,2,\dots)$

$\lim_{n\to\infty} z_n$ converges or not?

A: $\lim_{n\to\infty} z_n = \lim_{n\to\infty}\left(-2 + i\frac{(-1)^n}{n^2}\right)$

$= \lim_{n\to\infty}(-2) + \lim_{n\to\infty}\left[\frac{i(-1)^n}{n^2}\right]$

$= -2 + i\lim_{n\to\infty}\frac{(-1)^n}{n^2}$

$= -2 + i(0)$

$= -2 < 0$

So, z_n converges.

Q: $z_n = 2i - \frac{25}{n!}$ $(n = 0, 1, 2, ---)$

$\lim_{n\to\infty} z_n$ converges or not?

A: $\lim_{n\to\infty} z_n = \lim_{n\to\infty} \left(2i - \frac{25}{n!}\right)$

$= \lim_{n\to\infty} (2i) - \lim_{n\to\infty} \left(\frac{25}{n!}\right)$

$= 2i - \frac{25}{\infty!}$

$= 2i - 0$

$= 2i > 0$

(divergence)

Q: $f(z) = \sin z$ is analytic or not?

A: $\sin z = \sin(x+iy)$

$= \sin x \cos iy + \cos x \sin iy$

$= \sin x \cosh y + i \cos x \sinh y$

$\because u = \sin x \cosh y$, $v = \cos x \sinh y$

$\frac{\partial u}{\partial x} = \cos x \cosh y$, $\frac{\partial v}{\partial x} = -\sin x \sinh y$

$\frac{\partial u}{\partial y} = \sin x \sinh y$, $\frac{\partial v}{\partial y} = +\cos x \cosh y$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

So, C.R equations are satisfied.

Hence, $\sin z$ is analytic

Name & Address Tel/Mob/Fax/E-mail

Q: $u(x,y) = x^3 - 2x + 2$

Find Harmonic conjugate?

A:

$$u(x,y) = x^3 - 2x + 2$$

$$\frac{\partial u}{\partial x} = 3x^2 - 2 \quad , \quad \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial^2 u}{\partial x^2} = 6x \quad , \quad \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 0$$

$$= 6x \neq 0$$

Hence, this function is not Harmonic

Q: Test the analytic function $f(z) = e^z$

A:

$$u + iv = e^{x+iy} \Rightarrow e^x \cdot e^{iy}$$

$$= e^x[\cos y + i \sin y]$$

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$u = e^x \cos y \quad , \quad v = e^x \sin y$

$$\frac{\partial u}{\partial x} = e^x \cos y \quad , \quad \frac{\partial v}{\partial x} = e^x \sin y$$

$$\frac{\partial u}{\partial y} = -e^x \sin y \quad , \quad \frac{\partial v}{\partial y} = e^x \cos y$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

CR equations are satisfied.

Hence, Function is analytic.

Q: $w = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$

write 1- form as $\Sigma(df_i P) v_i$

A: Let $f_1, f_2, f_3 : \mathbb{R}^3 \to \mathbb{R}$ be three functions.

Let

$$\phi = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$$