# FINALTERM EXAMINATION

Fall 2008

MTH501- Linear Algebra

Ref No: 417476
Time: 120 min
Marks: 70

**Student Info**

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| Student ID: | |
| Center: | |
| Exam Date: | |

**For Teacher's Use Only**

| Q No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
|---|---|---|---|---|---|---|---|---|---|
| Marks | | | | | | | | | |
| Q No. | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | |
| Marks | | | | | | | | | |
| Q No. | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | |
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| Marks | | | | | | | | | |

**Question No: 1 ( Marks: 1 ) - Please choose one**

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Reduced echelon form of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ is

► $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

► $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

► $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

► $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

**Question No: 2 ( Marks: 1 ) - Please choose one**

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A matrix that results from applying a single elementary row operation to an identity matrix is called

► **Invertible matrix**

► Singular matrix

► Scalar matrix

► Elementary matrix

**Question No: 3 ( Marks: 1 ) - Please choose one**

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For an n×n matrix $(A^t)^t$ =

► $A^t$

► **A**

► $A^{-1}$

► $(A^{-1})^{-1}$

**Question No: 4 ( Marks: 1 ) - Please choose one**

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What is the largest possible number of pivots a 4×6 matrix can have?

► 4

► 6

► 10

► 0

**Question No: 5 ( Marks: 1 ) - Please choose one**

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The characteristic polynomial of a 5×5 matrix is $\lambda^5 - 4\lambda^4 - 45\lambda^3 = 0$ ,the eigenvalues are

► 0,-5, 9

► 0,0,0,5,9

► 0,0,0,-5,9

► 0,0,5,-9

**Question No: 6 ( Marks: 1 ) - Please choose one**

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Find the characteristic equation of the given matrix

$$\begin{bmatrix} 6 & 8 & 5 & 4 \\ 0 & 2 & -8 & 7 \\ 0 & 0 & 9 & 6 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

► $(6-\lambda)(2-\lambda)(9-\lambda)=0$

► $(6-\lambda)(8-\lambda)(5-\lambda)(4-\lambda)=0$

► $(6-\lambda)^2(2-\lambda)(9-\lambda)=0$

► $(6-\lambda)(6-\lambda)(7-\lambda)(4-\lambda)=0$

**Question No: 7 ( Marks: 1 ) - Please choose one**

A is diagonalizable if $A = PDP^{-1}$ Where

► D is any matrix and P is an invertible matrix

► D is a diagonal matrix and P is any matrix

► D is a diagonal matrix and P is invertible matrix

► D is a invertible matrix and P is any matrix

**from book**
A square matrix *A* is said to be diagonalizable if *A* is similar to a diagonal matrix. i.e. if $A = PDP^{-1}$ for some invertible matrix *P* and some diagonal matrix *D*.

**Question No: 8 ( Marks: 1 ) - Please choose one**

The inverse of an invertible lower triangular matrix is

► lower triangular matrix

► upper triangular matrix

► diagonal matrix

The standard operations on triangular matrices conveniently preserve the triangular form: the sum and product of two upper triangular matrices is again upper triangular. The inverse of an upper triangular matrix is also upper triangular, and of course we can multiply an upper triangular matrix by a constant and it will still be upper triangular. This means that the upper triangular matrices form a subalgebra of the ring of square matrices for any given size. The analogous result holds for lower triangular matrices. Note, however, that the product of a *lower* triangular with an *upper* triangular matrix does *not* preserve triangularity.

**http://en.wikipedia.org/wiki/Triangular_matrix**

**Question No: 9 ( Marks: 1 ) - Please choose one**

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If P is a parallelepiped in $R^3$, then
{volume of T (P)} = |detA|. {volume of P}

- ► Where T is determined by a $2\times2$ matrix A
- ► Where T is determined by a $2\times3$ matrix A
- ► **Where T is determined by a** $3\times3$ **matrix** A not conf.
- ► Where T is determined by a $3\times2$ matrix A

**Question No: 10 ( Marks: 1 ) - Please choose one**

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Let A be a $n\times m$ matrix of rank $r$ then row space of A has dimension

- ► $m-r$
- ► $n-r$
- ► $r$
- ► $nm$

*Main article: Rank (linear algebra)*

The dimension of the row space is called the **rank** of the matrix. This is the same as the maximum number of linearly independent rows that can be chosen from the matrix. For example, the 3 × 3 matrix in the example above has rank two.

The rank of a matrix is also equal to the dimension of the column space. The dimension of the null space is called the **nullity** of the matrix, and is related to the rank by the following equation:

$$\text{rank}(A) + \text{nullity}(A) = n,$$

where *n* is the number of columns of the matrix *A*. The equation above is known as the rank-nullity theorem.

**[edit]**

**Question No: 11 ( Marks: 1 ) - Please choose one**

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The dimension of the vector space $P_4$ is

► 4

► 3

► 5

► 1

The dimension of the vector space $P_4$ is 4. FALSE It's 5.

**Question No: 12 ( Marks: 1 ) - Please choose one**

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Let $u = (3, -2), v = (4, 5)$ .For the weighted Euclidean inner product $\langle u, v \rangle = 4u_1v_1 + 5u_2v_2$ $\langle v, u \rangle =$

► 2

► -2

► 3

► -3

**Question No: 13 ( Marks: 1 ) - Please choose one**

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Let A be $n \times n$ matrix whose entries are real. If $\lambda$ is an eigenvalue of A with x a corresponding eigenvector in $\mathbb{C}^n$, then

► $A\overline{\boldsymbol{x}} = \lambda\overline{\boldsymbol{x}}$

► $A\overline{\boldsymbol{x}} = \overline{\lambda}\overline{\boldsymbol{x}}$

► $A\overline{\boldsymbol{x}} = \overline{\lambda}x$

► $A\overline{\boldsymbol{x}} = \lambda^{-1}\overline{\boldsymbol{x}}$

**Question No: 14 ( Marks: 1 ) - Please choose one**

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Suppose that $\boldsymbol{A} = \begin{bmatrix} 1.25 & -.75 \\ -.75 & 1.25 \end{bmatrix}$ has eigenvalues 2 and 0.5 .Then origin is a

► Saddle point

► Repellor

► Attractor

**Question No: 15 ( Marks: 1 ) - Please choose one**

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Which one is the numerical method used for approximation of dominant eigenvalue of a matrix.

► Power method

► Jacobi's method

► Guass Seidal method

► Gram Schmidt process

**Question No: 16 ( Marks: 1 ) - Please choose one**

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The matrix equation $\boldsymbol{A^T A\hat{x} = A^T b}$ represents a system of linear equations commonly referred to as the

► normal equations for $\boldsymbol{x}$

► normal equations for $\boldsymbol{\hat{x}}$

► normal equations for $\boldsymbol{A}$

► normal equations for $\boldsymbol{b}$

**Question No: 17 ( Marks: 1 ) - Please choose one**

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Let $A$ have eigenvalues 2, 5, 0,-7, and -2. Then the dominant eigenvalue for A is

► $\lambda = 5$

► $\lambda = 0$

► $\lambda = -7$

▶ $\lambda = 2$

**Question No: 18 ( Marks: 1 ) - Please choose one**

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If W is a subspace of $\mathbb{R}^m$, then the transformation $T : \mathbb{R}^m \rightarrow W$ that maps each vector x in $\mathbb{R}^m$ into its orthogonal x in W is called the orthogonal projection of

▶ $\mathbb{R}^m$ in $\mathbb{R}^m$

▶ $\mathbb{R}^m$ in W

▶ $\mathbb{R}^m$ in x

**Question No: 19 ( Marks: 1 ) - Please choose one**

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If $V = \mathbb{R}^n$ , $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ then row reduction of $[c_1 \quad c_2 \quad b_1 \quad b_2]$ to $[I \quad P]$ Produces a matrix P that satisfies

▶ $[x]_B = P[x]_B$ for all x in V

▶ $[x]_C = P[x]_B$ for all x in V

▶ $[x]_B = P[x]_C$ for all x in V

▶ $[x]_B = [x]_C$ for all x in V

**Question No: 20 ( Marks: 1 ) - Please choose one**

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The Casorati matrix for the signals $\boldsymbol{1^k}$, $\boldsymbol{(-2)^k}$ and $\boldsymbol{3^k}$ is

► $\begin{bmatrix} 1^k & (-2)^k & 3^0 \\ 1^{k+1} & (-2)^{k+1} & 3^1 \\ 1^{k+2} & (-2)^{k+2} & 3^2 \end{bmatrix}$

► $\begin{bmatrix} 1^k & (-2)^k & 3^k \\ 1^{k+1} & (-2)^{k+1} & 3^{k+1} \\ 1^{k+2} & (-2)^{k+2} & 3^{k+2} \end{bmatrix}$ not conf.

► $\begin{bmatrix} 1^0 & (-2)^k & 3^k \\ 1^1 & (-2)^{k+1} & 3^{k+1} \\ 1^2 & (-2)^{k+2} & 3^{k+2} \end{bmatrix}$

►

**Question No: 21 ( Marks: 2 )**

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Find the characteristic polynomial and all eigenvalues of the given matrix

$$\begin{bmatrix} a & a \\ a & a \end{bmatrix}$$

**Question No: 22 ( Marks: 2 )**

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Write the Fourier coefficients $a_k \text{ and } b_k$ to the function $f(t) = 2\pi - t$ on the interval $[0, 2\pi]$.

**Question No: 23 ( Marks: 2 )**

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The matrix A is followed by a sequence $\{x_k\}$ produced by the power method. Use these data to estimate the largest eigenvalue of A.

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0.25 \end{bmatrix}, \begin{bmatrix} 1 \\ 0.3158 \end{bmatrix}, \begin{bmatrix} 1 \\ 0.3298 \end{bmatrix}, \begin{bmatrix} 1 \\ 0.3326 \end{bmatrix}$$

**Question No: 24 ( Marks: 3 )**

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If $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ then find an invertible matrix P such that $P^{-1}AP = D(\text{diagonal matrix})$

**Question No: 25 ( Marks: 3 )**

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Check whether the matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{3} \\ \frac{1}{\sqrt{2}} & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$ has orthonormal columns or not?

**Question No: 26** **( Marks: 3 )**

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If A is a $6 \times 4$ matrix, what is the smallest possible dimension of Null A?

**Question No: 27** **( Marks: 5 )**

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Assume that the matrix ***A*** is row equivalent to ***B***. Without calculations, list rank ***A*** and dim Nul ***A***. Then find bases for Col ***A*** and Row ***A.***

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & 0 & -7 \\ 3 & 7 & -2 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 4 & -13 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

**Question No: 28** **( Marks: 5 )**

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Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that the given matrix A has the form $A=PCP^{-1}$.

$$A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$$

with eigenvalue $\lambda = 2 \pm 3i$ and eigenvector

$$v = \begin{bmatrix} 1+3i \\ 2 \end{bmatrix} \text{which corresponds the eigenvalue } \lambda = 2-3i$$

**Question No: 29** **( Marks: 5 )**

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Let $u=\begin{bmatrix}2\\-5\\-1\end{bmatrix}, v=\begin{bmatrix}-7\\-4\\6\end{bmatrix}$ .Compute and compare $u.v, \|u\|^2, \|v\|^2 \text{ and } \|u+v\|^2$. Do not use Pythagorean Theorem.

**Question No: 30 ( Marks: 10 )**

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Find a least squares solution of the inconsistent system $Ax = b$ where $A=\begin{bmatrix}1 & 1\\1 & 2\\1 & 3\end{bmatrix}, b=\begin{bmatrix}0\\0\\6\end{bmatrix}$.

**Question No: 31 ( Marks: 10 )**

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Determine whether the signals $1^k, 2^k$, and $(-2)^k$ are the solutions of the difference equation $y_{k+3} - y_{k+2} - 4y_{k+1} + 4y_k = 0$.

The null space of A is the solution set of the equation
Ax = 0. TRUE
The null space of an m _ n matrix is in Rm. False. It's Rn
The column space of A is the range of the mapping x 7! Ax.
TRUE
If the equation Ax = b is consistent, then Col A is Rm.
FALSE must be consistent for all b
The kernel of a linear transformation is a vector space. TRUE
To show this we show it is a subspace

Col A is the set of a vectors that can be written as Ax for
some x. TRUE Remember that Ax gives a linear combination
of columns of A using x entries as weights.

The null space is a vector space. TRUE

The column space of an m _ n matrix is in Rm TRUE

Col A is the set of all solutions of Ax = b. FALSE It is the set of all b that have solutions.

Nul A is the kernel of the mapping x 7! Ax. TRUE

The range of a linear transformation is a vector space. TRUE
It's a subspace(check), thus vector space.

The set of all solutions of a homogenous linear di_erential
equation is the kernel of a linear transformation. TRUE

A single vector is itself linearly dependent. FALSE unless it in
the zero vector

If H =Spanfb$_1$; : : : ; b$_n$g then fb$_1$; : : : ; b$_n$g is a basis for H.
FALSE They may not be linearly independent.

The columns of an invertible n _ n matrix form a basis for R$_n$
TRUE They are linerly independent and span R$_n$. (why?)

A basis is a spanning set that is as large as possible. FALSE It
is is too large, then it is no longer linearly independent.

In some cases, the linear dependence relations among the
columns of a matrix can be a_ected by certain elementary row
operations on the matrix. FALSE They are not a_ected.

A linearly independent set in a subspace H is a basis for H.
FALSE It may not span.

If a _nite set S of nonzero vectors spans a vector space V, the
some subset is a basis for V. TRUE by Spanning Set Theorem
A basis is a linearly independent set that is as large as
possible. TRUE

The standard method for producing a spanning set for Nul A,
described in this section, sometimes fails to produce a basis.
FALSE It NEVER fails!!!

If B is an echelon form of a matrix A, then the pivot columns
of B form a basis for Col A. FALSE Must look at
corresponding columns in A.

The number of pivot columns of a matrix equals the
dimension of its column space. TRUE Remember these

columns and linearly independent and span the column space.
A plane in $R_3$ is a two dimensional subspace of $R_3$. FALSE
unless the plane is through the origin.

The dimension of the vector space $P_4$ is 4. FALSE It's 5.

If dimV = n and S is a linearly independent set in V, then S
is a basis for V. FALSE S must have exactly n elements.

If a set fv1 : : : vng spans a _nite dimensional vector space V
and if T is a set of more than n vectors in V, then T is
linearly dependent. TRUE The number of linearly independent
vectors that span a set is unique.

$R_2$ is a two dimensional subspace of $R_3$. FALSE Not a subset,
as before.

The number of variables in the equation Ax = 0 equals the
dimension of Nul A. FALSE It's the number of free variables.

A vector space is in_nite dimensional is it is spanned by an
in_nite set. FALSE It must be impossible to span it by a _nite
set.

If dim V = n and if S spans V. then S is a basis for V.
FALSE S must have exactly n elements or be noted as linearly
independent.

The only three dimensional subspace of $R_3$ is $R_3$ itself. TRUE
If spanned by three vectors must be all of $R_3$.