Sample Papers

MTH501-Linear Algebra
Midterm Special 2006

Question # 1:-

Let A= $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ Write 5A.Is det5A=5detA?

Let A = $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$$5A = 5\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 20 & 10 \end{bmatrix}$$

$$\det 5A = ((15\times 10) - (20\times 5))$$

$$\det 5A = 150 - 100 = 50$$

$$\det 5A = 50$$

$$\det A = ((3\times 2) - (1\times 4))$$

$$\det A = 6 - 4 = 2$$

$$5\det A = 5\times 2 = 10$$

Hence proved det5A not equal to 5detA

Question #2:-

Let A = $\begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$ .Show that these matrices do not commute.

Solution:-

$$AB = \begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 14 & 3 \\ -2 & -6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ 29 & -2 \end{bmatrix}$$

$$AB \neq BA$$

Question # 3:-

Use Cramer's rule to solve the system $3x_1 - 2x_2 = 6$, $-5x_1 + 4x_2 = 8$

Note : In order to get full marks do all necessary steps

Solution:-

$$3x_1 - 2x_2 = 6$$

$$-5x_1 + 4x_2 = 8$$

View the System as Ax =b. Using the notation introduced above.

$$A\begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$$

$$A_1(b) = \begin{bmatrix} 6 & -2 \\ 8 & 4 \end{bmatrix}$$

$$A_2(b) = \begin{bmatrix} 3 & 6 \\ -5 & 8 \end{bmatrix}$$

Since detA = 2, the system has a unique solution. By Cramer's rule,

$$x_1 = \frac{\det A_1(b)}{\det A} = \frac{24+16}{2} = 20$$

$$x_2 = \frac{\det A_2(b)}{\det A} = \frac{24+30}{2} = 27$$

Question # 4:-

Find the matrix of linear transformation $T: R^3 \to R^4$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, x_1)$ with respect to the standard basis of R^3?

Note : In order to get full marks do all necessary steps

Solution:-

The standard basis of R^3 is $\{e_1, e_2, e_3\}$

$E_1 = (1, 0, 0)$

$E_2 = (0, 1, 0)$

$E_3 = (0, 0, 1)$

$T(e_1) = T\ (1, 0, 0) = (1, 0, 1, 1)$

$T(e_2) = T\ (0, 1, 0) = (1, 1, 0, 0)$

$T(e_3) = T\ (0, 0, 1) = (0, 1, -1, 0)$

The matrix of linear transformation is

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

If $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & k & 2 \\ 0 & 4 & 2 \end{bmatrix}$ such that $|A| = 0$ then the value of k is

- 8
- **4**
- 0
- None of these

The system of linear equations $\begin{matrix} x + y = 1 \\ x + y = 0 \end{matrix}$ has

- **No solution**
- Infinitely many solutions
- Infinitely many solutions
- d) None of these

Let $A = \begin{bmatrix} 4 & 3 \\ 7 & 2 \\ 9 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix}$ then AB is

$$\begin{bmatrix} 11 & 38 \\ 16 & 47 \\ 18 & 45 \end{bmatrix}$$

- **a)**
- b) A and B are not comfortable for multiplication

$$\begin{bmatrix} 11 & 38 \\ 16 & 47 \\ 8 & 45 \end{bmatrix}$$

- c)
- d) None of these

Determine whether the set S={ $\begin{bmatrix} -4 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ } is a spanning set for R^2 or not.

Note : In order to get full marks do all necessary steps

Let T : $R^2 \to R^3$ be a linear transformation defined by T(x)=Ax , where

A= $\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$,then find an x in R^2 whose image under T is b.Where b= $\begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$

Note : In order to get full marks do all necessary steps

The inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 5 & 1 \end{bmatrix}$ is

$$\begin{bmatrix} 1 & -2 \\ -5 & 1 \end{bmatrix}$$

- (a)

$$\frac{1}{9}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$$

- b)

$$\frac{1}{9}\begin{bmatrix} -1 & 2 \\ 5 & -1 \end{bmatrix}$$

- **(c)**
- **d) None of these**

The set of vectors {(5,0,0), (7,2,-6), (9,4,-8)} is,

- a) Linearly independent
- **b) Linearly dependent**
- c) Basis of R^3