See Register also

Summary of Lecture 23 – ELECTROSTATICS II

1. In the last lecture we learned how to calculate the electric field if there are any number of point charges. But how to calculate this when charges are continuously distributed over some region of space? For this, we need to break up the region into little pieces so that each piece is small enough to be like a point charge. So, $\vec{E} = \Delta\vec{E}_1 + \Delta\vec{E}_2 + \Delta\vec{E}_3 + \cdots$, or $\vec{E} = \sum \Delta\vec{E}_i$ is the total electric field. Remember that $\vec{E}$ is a vector that can be resolved into components, $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$. In the limit where the pieces are small enough, we can write it as an integral, $\vec{E} = \int d\vec{E}$ (or $E_x = \int dE_x$, $E_y = \int dE_y$, $E_z = \int dE_z$)

2. Charge Density: when the charges are continuously distributed over a region - a line, the surface of a material, or inside a sphere - we must specify the *charge density*. Depending upon how many dimensions the region has, we define:

   (a) For linear charge distribution: $dq = \lambda ds$

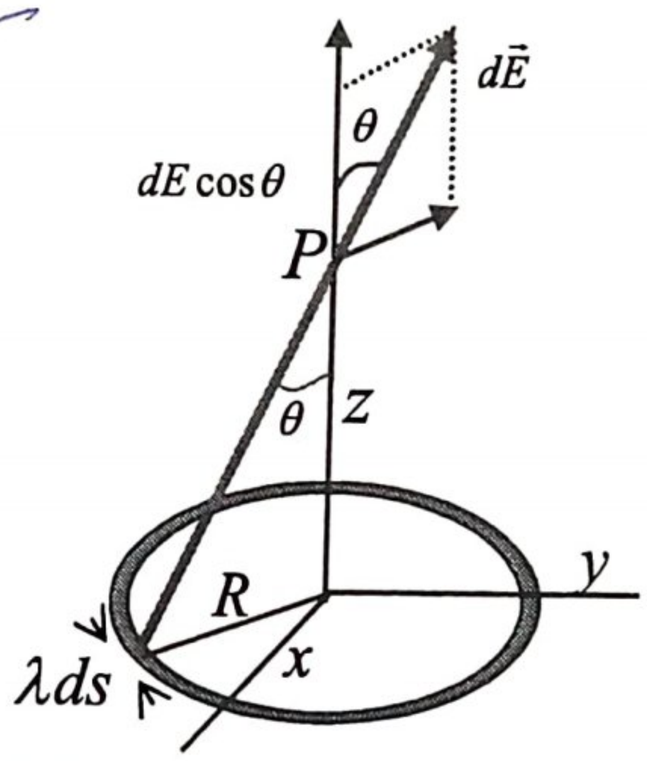
   (b) For surface charge distribution: $dq = \sigma dA$

   (c) For volume charge distribution: $dq = \rho dV$

The dimensions of λ, σ, ρ are determined from the above definitions.

3. As an example of how we work out the electric field coming from a continuous charge distribution, let us work out the electric field from a uniform ring of charge at the point P.

Imp for Numericals.



See on Register.

The small amount of charge λds gives rise to an electric field whose magnitude is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2} = \frac{\lambda ds}{4\pi\varepsilon_0\left(z^2 + R^2\right)}$$

The component in the z direction is $dE_z = dE\cos\theta$ with $\cos\theta = \frac{z}{r} = \frac{z}{\left(z^2 + R^2\right)^{1/2}}$.

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So $dE_z = \dfrac{z\lambda ds}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}$. Since s, which is the arc length, does not depend upon z or R

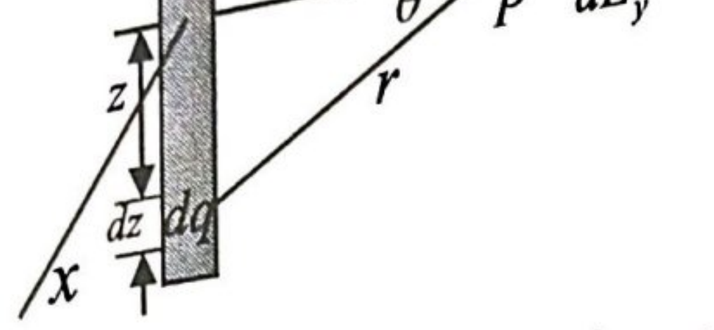
$$E_z = \frac{z\lambda}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\int ds = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}} = \frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}.$$ Answer!!

Note that if you are very far away, the ring looks like a point: $E_z = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{z^2}$, $(z >> R)$.

4. As another example, consider a continuous distribution of charges along a wire that lies along the z-axis, as shown below. We want to know the electric field at a distance x from the wire. By symmetry, the only non-cancelling component lies along the y-axis.

Imp for Final Numerical

See Register.



Applying Coulomb's law to the small amount of charge λdz along the z axis gives,

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dz}{y^2+z^2}$$

the component along the y direction is $dE_y = dE\cos\theta$. Integrating this gives,

$$E_y = \int dE = \int_{z=-\infty}^{z=\infty}\cos\theta dE = 2\int_{z=0}^{z=\infty}\cos\theta dE = \frac{\lambda}{2\pi\varepsilon_0}\int_{z=0}^{z=\infty}\cos\theta\frac{dz}{y^2+z^2}.$$

The rest is just technical: to solve the integral, put $z = y\tan\theta \Rightarrow dz = y\sec^2\theta d\theta$. And so,

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}\int_{\theta=0}^{\theta=\pi/2}\cos\theta d\theta = \frac{\lambda}{2\pi\varepsilon_0 y}.$$ Now, we could have equally well taken the x axis. The only thing that matters is the distance from the wire, and so the answer is better written as

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

5. The **flux** of any vector field is a particularly important concept. It is the measure of the "flow" or penetration of the field vectors through an imaginary fixed surface. So, if there is a uniform electric field that is normal to a surface of area A, the flux is $\Phi = EA$. More generally, for any surface, we divide the surface up into little pieces and take the

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The number of electric field lines passing through the certain Area element is called Flux.

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component of the electric field normal to each little piece, $\Phi_E = \sum \vec{E}_i \cdot \Delta\vec{A}_i$. If the pieces are made small enough, then in this limit, $\Phi = \int \vec{E} \cdot d\vec{A}$.

5. Let us apply the above concept of flux to calculate the flux leaving a sphere which has a charge at its centre. The electric field at any point on the sphere has magnitude equal to $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ and it is directed radially outwards. Let us now divide up the surface of the sphere into small areas. Then $\Phi = \sum E\Delta A = E\sum \Delta A = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\left(4\pi r^2\right)$. So we end up with the important result that the flux leaving this closed surface is $\Phi = \frac{q}{\varepsilon_0}$.

6. **Gauss's Law**: the total electric flux leaving a closed surface is equal to the charge enclosed by the surface divided by ε_0. We can express this directly in terms of the mathematics we have learned, $\Phi \equiv \int \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$. Actually, we have already seen why this law is equivalent to Coulomb's Law in point 5 above, but let's see it again. So, applying Gauss's Law to a sphere containing charge, $\varepsilon_0 \int \vec{E} \cdot d\vec{A} = \varepsilon_0 \int E\, dA = q_{enclosed}$. If the surface is a sphere, then E is constant on the surface and $\varepsilon_0 E \int dA = q$ and from this $\varepsilon_0 E\left(4\pi r^2\right) = q \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$. This is Coulomb's Law again, but the power of Gauss's law is that it holds for any shape of the (closed) surface and for any distribution of charge.

7. Let us apply Gauss's Law to a hollow sphere that has charges only on the surface. At any distance r from the centre, Gauss's Law is $\varepsilon_0 E\left(4\pi r^2\right) = q_{enclosed}$. Now, if we are inside the sphere then $q_{enclosed} = 0$ and there is no electric field. But if we are outside, then the total charge is $q_{enclosed} = q$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, which is as if all the charge was concentrated at the centre.

8. Unfortunately, it will not be possible for me to prove Gauss's Law in the short amount of time and space available but the general method can be outlined as follows: take any volume and divide it up into little cubes. Each little cube may contain some small amount of charge. Then show that for each little cube, Gauss's Law follows from Coulomb's Law. Finally, add up the results. For details, consult any good book on electromagnetism.

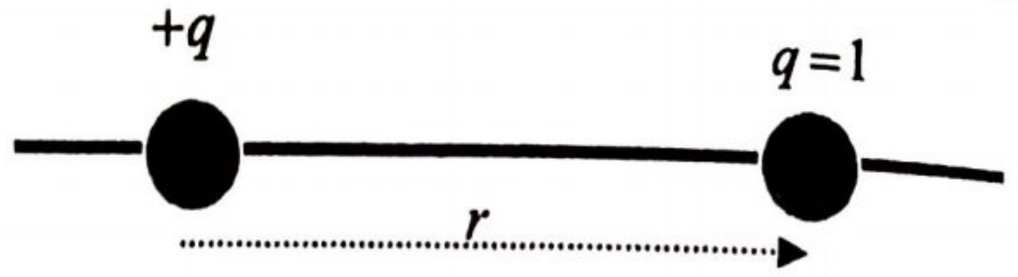
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Summary of Lecture 24 – ELECTRIC POTENTIAL ENERGY

1. You are already familiar with the concept of gravitational potential energy. When lift a weight, you have to do work against the downwards pull of the Earth. That is stored as potential energy. Suppose a force $\vec{F}$ acts on something and displaces Then the work done is $\vec{F} \cdot d\vec{s}$. The work done in going from point a to point b (ca is then got by adding together the little bits of work, $W_{ab} = \int_a^b \vec{F} \cdot d\vec{s}$. The change i potential energy is defined as $\Delta U = U_b - U_a = -W_{ab}$. Remember always that we ca define the potential U at a point if the force is conservative.

2. The electrostatic force is conservative and can be represented by a potential. Let us how to calculate the potential. So consider two charges separated by a distance as



Let us take the point a very far from the fixed charge q, and the unit charge at the point b to be at a distance R from q. Then the work you did in bringing the unit charge from infi to R is, $W_{ab} = \int_{\infty}^{R} (-qE)dr = -\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{R} dr \frac{q}{r^2} = -\frac{1}{4\pi\varepsilon_0} q\left(\frac{1}{R} - \frac{1}{\infty}\right) = -\frac{q}{4\pi\varepsilon_0}\frac{1}{R}$. Since the char repel each other, it is clear that you had to do work in pushing the two charges closer together. So where did the negative sign come from? Answer: the force you exert on the unit charge is directed *towards* the charge q, i.e. is in the negative direction. This is why $\vec{F} \cdot d\vec{s} = (-qE)dr$. Now $\Delta U = U(R) - U(\infty) = \frac{q}{4\pi\varepsilon_0}\frac{1}{R}$. If we take the potential at ∞ to be zero, then the electric potential due to a charge q at the point r is $U(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$.

Remember that we know how to calculate the force given the potential: $F = -\frac{dU}{dr}$. Apply this here and you see that $F = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$, (which also has the correct repulsive sign).

3. From the above, it is quite obvious that the potential energy of two charges q_1, q_2 is, $U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$. Compare this with the formula for gravitational energy, $U(r) = -G\frac{m_1 m}{r}$

What is the difference? From here, you can see that the gravitational force is always negative (which means attractive), whereas the electrostatic force can be both attractive

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So, in our MKS units, the unit of potential is $1\ \frac{\text{Joule}}{\text{Coulomb}} = 1$ Volt. Another useful unit is "electron volt" or eV. The definition is:

One electron - volt = energy gained by moving one electron charge through one Volt

$$= \left(1.6\times10^{-19}C\right)\times 1V = 1.6\times10^{-19}J$$

It is useful to note that 1 Kev $= 10^3 eV$ (kilo-electron-volt)
1 Mev $= 10^6 eV$ (million-electron-volt)
1 Gev $= 10^9 eV$ (giga-electron-volt)
1 Tev $= 10^{12} eV$ (tera-electron-volt)

$\Delta V = -\frac{W}{qV}$

4. Every system seeks to minimize its potential energy (that is why a stone falls down!). So, positive charges accelerate toward regions of lower potential, but negative charges accelerate toward regions of higher potential. Note that only the potential difference matters - even if a charge is placed in a region where there is a high potential, it will not want to move unless there is some other place where the potential is higher/lower.

5. Given a system of charges, we can always compute the force - and hence the potential - that arises from them. Here are some important general statements:

a) Potentials are more positive in regions which have more positive charge.

b) The electric potential is a scalar quantity (a scalar field, actually).

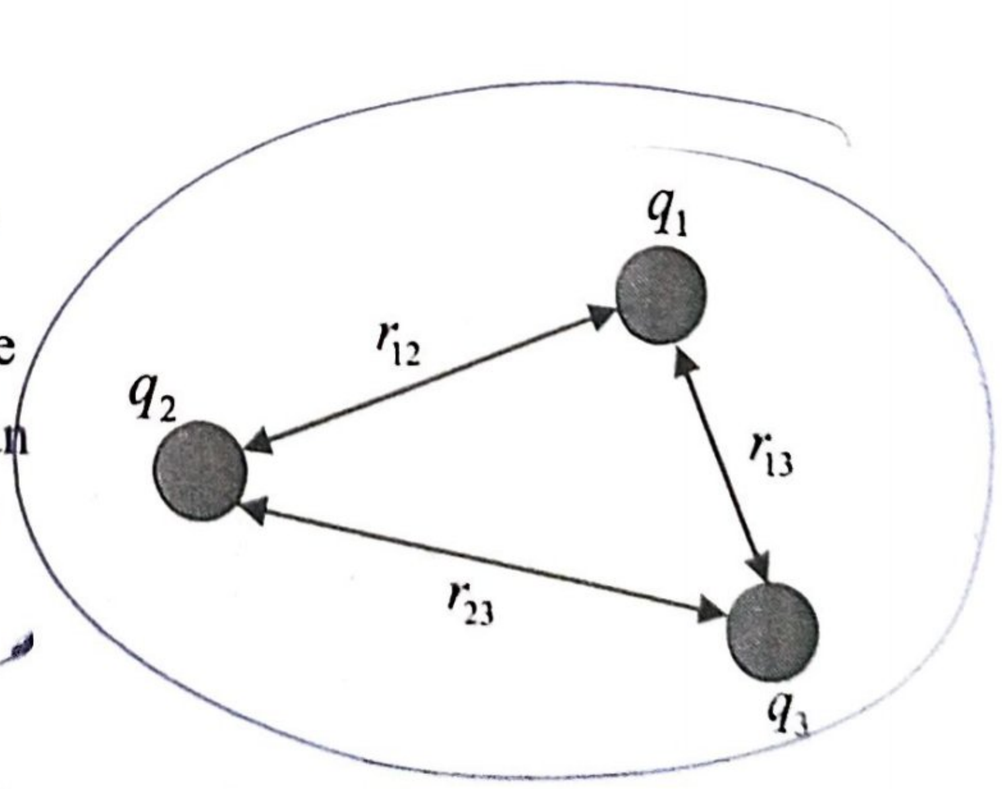
c) The electric potential determines the force through $F = -\frac{dU}{dr}$, and hence the electric field because $F = qE$.

d) The electric potential exists only because the electrostatic force is conservative.

6. To compute the potential at a point, the potentials arising from charges 1, 2, $\cdots$ N must be added up:

$V = V_1 + V_2 + \cdots V_N = \sum_{i=1}^{N} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}$. Here r_i is the distance of the i'th charge from the point where the potential is being calculated or measured. As an example, the potential from the three charges is:

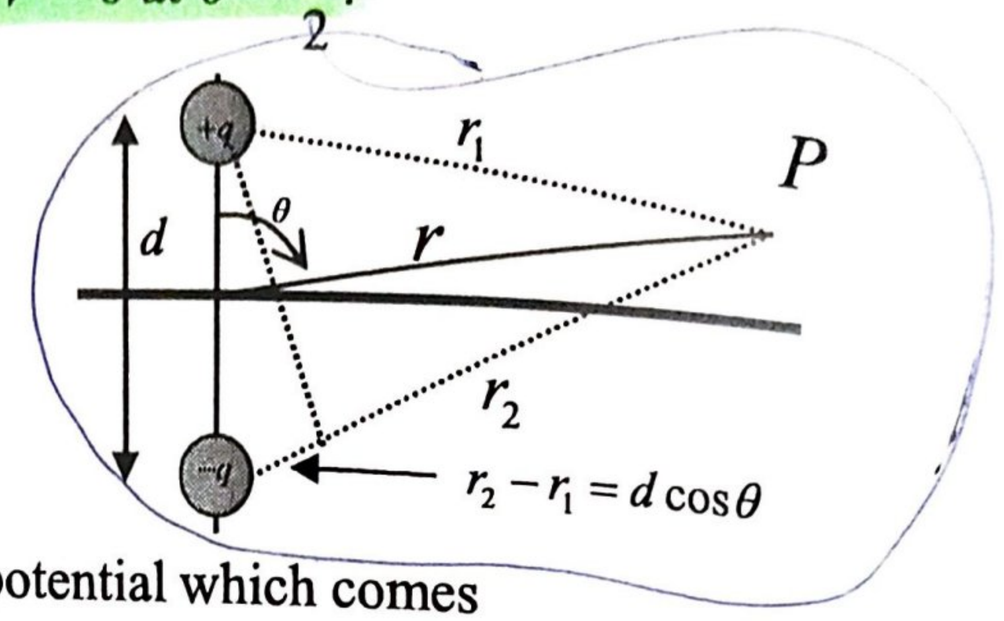
$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_{23}}.$$



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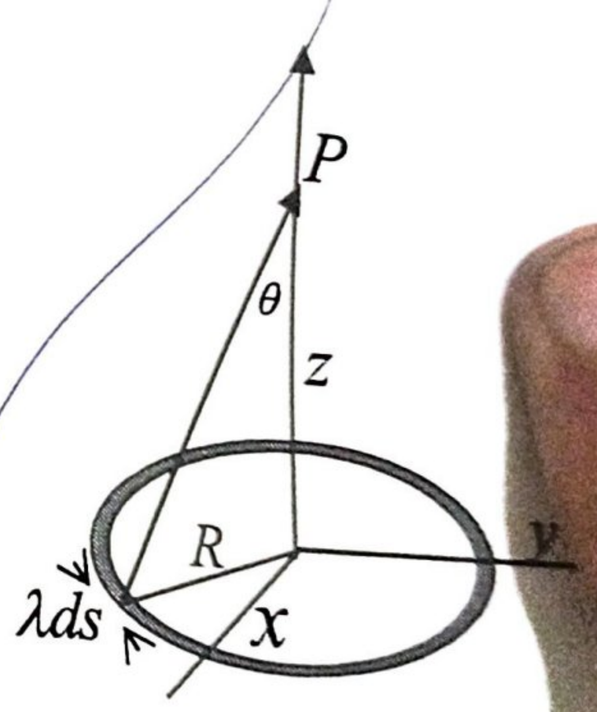
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where $r >> d$. from the diagram you can see that $r_2 - r_1 \approx d\cos\theta$ and that $r_1 r_2 \approx r^2$. Hence $V \approx \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$. So we have calculated the potential at any θ with such little difficulty. Note that $V = 0$ at $\theta = \frac{\pi}{2}$.



8. Now let us calculate the potential which comes from charges that are uniformly spread over a ring. This is the same problem as in the previous lecture, but simpler. Give the small amount of potential coming from the small amount of charge $dq = \lambda ds$ some name, $dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$. Then obviously

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2 + z^2}}.$$



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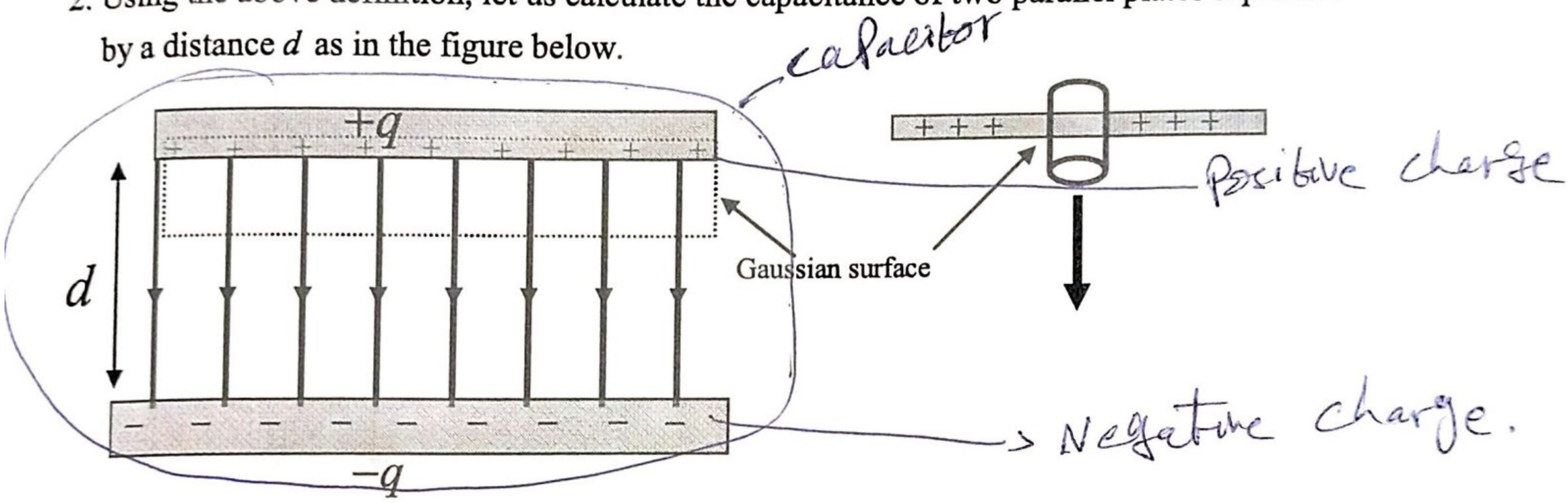
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The device that is used to store energy and charges called capacitor. capacito release 360 Joul/s. energy. capacito is used to give startup to any electric thing.

Summary of Lecture 25 – CAPACITORS AND CURRENTS

1. Two conductors isolated from one another and from their surroundings, form a *capacitor*. These conductors may be of any shape and size, and at any distance from each other. If a potential difference is created between the conductors (say, by connecting the terminals of a battery to them), then there is an electric field in the space between them. The electric field comes from the charges that have been pushed to the plates by the battery. The amount of charge pushed on to the conductors is proportional to the potential difference between the battery terminals (which is the same as between the capacitor plates). ~~Hence, $Q \propto V$. To convert this into an equality, we write $Q = CV$. This provides the definition of~~ capacitance, $C = \frac{Q}{V}$.

2. Using the above definition, let us calculate the capacitance of two parallel plates separated by a distance d as in the figure below.



Recall Gauss's Law: $\Phi \equiv \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$. Draw any Gaussian surface. Since the electric field is zero above the top plate, the flux through the area A of the plate is $\Phi = EA = \frac{Q}{\varepsilon_0}$, where Q is the total charge on the plate. Thus, $E = \frac{Q}{\varepsilon_0 A}$ is the electric field in the gap between the plates. The potential difference is $V = \frac{E}{d}$, and so $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$. You can see that the capacitance will be large if the plates are close to each other, and if the plates have a large area. We have simplified the calculation here by assuming that the electric field is strictly directed downwards. This is only true if the plates are infinitely long. But we can usually neglect the side effects. Note that any arrangement with two plates forms a capacitor: plane, cylindrical, spherical, etc. The capacitance depends upon the geometry, the size of plates and the gap between them.

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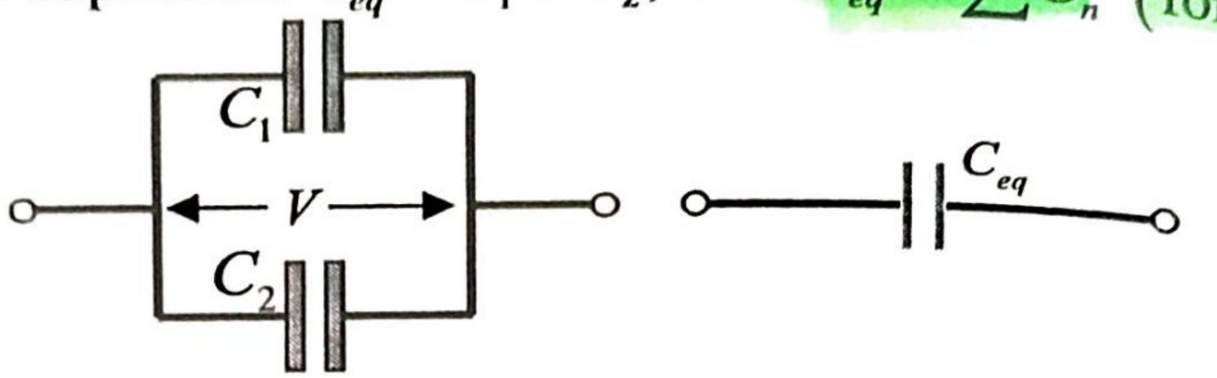
Ability of capacitor to store charge is called capacitance. Farad is Unit

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3. One can take two (or more) capacitors in various ways and thus change the amount of charge they can contain. Consider first two capacitors connected in parallel with each other. The same voltage exists across both. For each capacitor, $q_1 = C_1 V$, $q_2 = C_2 V$ when V is the potential between terminals a and b. The total charge is:

Circuit $$Q = q_1 + q_2 = C_1 V + C_2 V = (C_1 + C_2)V$$

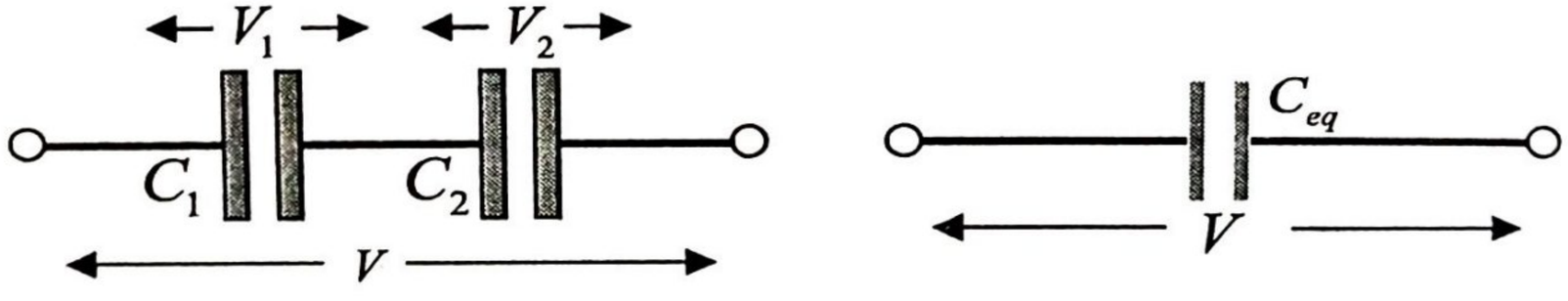
Now, let us define an "effective" or "equivalent" capacitance as $C_{eq} = \frac{Q}{V}$. Then we can immediately see that for 2 capacitors $C_{eq} = C_1 + C_2$, and $C_{eq} = \sum C_n$ (for n capacitors).



4. We can repeat the analysis above when the capacitors are put in series. Here the differer is that now we must start with $V = V_1 + V_2$; where V_1 and V_2 are the voltages across the t Clearly the same charge had to cross both the capacitors. Hence,

Series $$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right).$$

From our definition, $C_{eq} = \frac{Q}{V}$, it follows that $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$. The total capacitance is n less than if they were in parallel. In general, $\frac{1}{C_{eq}} = \sum \frac{1}{C_n}$ (for n capacitors).



Read

5. When a battery is connected to a capacitor, positive and negative charges appear on the opposite plates. Some energy has been transferred from the battery to the capacitor, and

$$dU = \text{v}dq = \frac{q}{C}dq \quad \Rightarrow \quad U = \int dU = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C} = \frac{1}{2}CV^2.$$

But where in the capacitor is the energy stored? Answer, it is present in the electric field in the volume between the two plates. We can calculate the energy density:

$$u = \frac{\text{energy stored in capacitor}}{\text{volume of capacitor}} = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\varepsilon_0}{2}\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2.$$

In the above we have used $C = \frac{\varepsilon_0 A}{d}$, derived earlier. The important result here is that $u \propto E^2$. Turning it around, wherever there is an electric field, there is energy available.

6 **Dielectrics.** Consider a free charge $+Q$. Around it is an electric field, $E = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{r^2}$.

Now suppose this charge is placed among water molecules. These molecules will polarise, i.e. the centre of positive charge and centre of negative charge will be slightly displaced. The negative part of the water molecule will be attracted toward the positive charge $+Q$.

So, in effect, the electric field is weakened by $\frac{1}{\varepsilon_r}$ and becomes, $\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{r^2}$. Here I have

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to high Potential of the battary Poten
called "Electric Current"

Summary of Lecture 26 – ELECTRIC POTENTIAL ENERGY

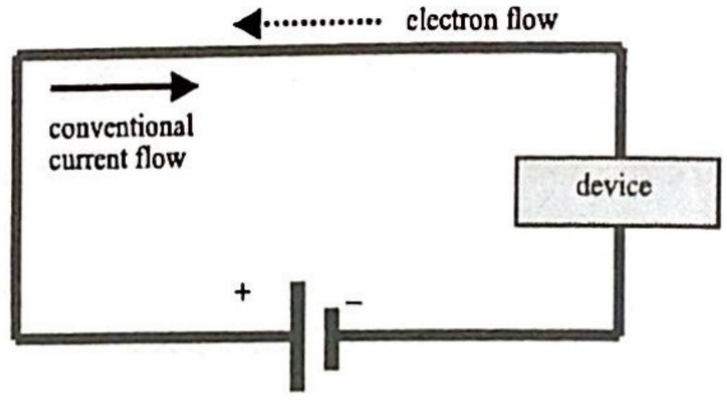
1. Electric current is the flow of electrical charge. If a small amount of charge dq flows in time dt, then the current is $i = \frac{dq}{dt}$. If the current is constant in time, then in time t, the current that flows is $q = i \times t$. The unit of charge is ampere, which is define as:

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{\text{second}}$$

A car's battery supplies upto 50 amperes when starting the car, but often we need to deal with smaller values:

$$1 \text{ milliampere} = 1 \text{ ma} = 10^{-3} A$$
$$1 \text{ microampere} = 1 \mu A = 10^{-6} A$$
$$1 \text{ nanoampere} = 1 \text{ n}A = 10^{-9} A$$
$$1 \text{ picoampere} = 1 \text{ p}A = 10^{-12} A$$

2. The direction of current flow is the direction in which positive charges move. However, in a typical wire, the positive charges are fixed to the atoms and it is really the negative charges (electrons) that move. In that case the direction of current flow is reversed.



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Summary of Lecture 26 – ELECTRIC POTENTIAL ENERGY

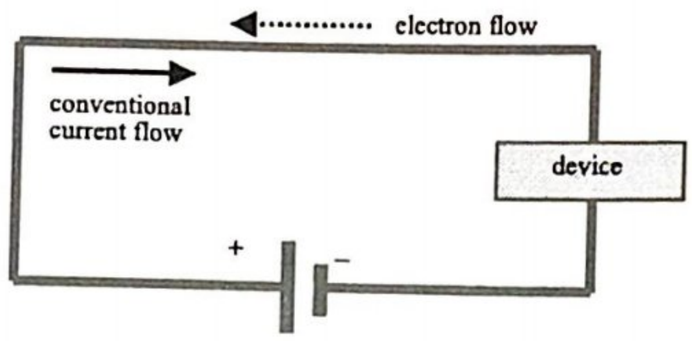
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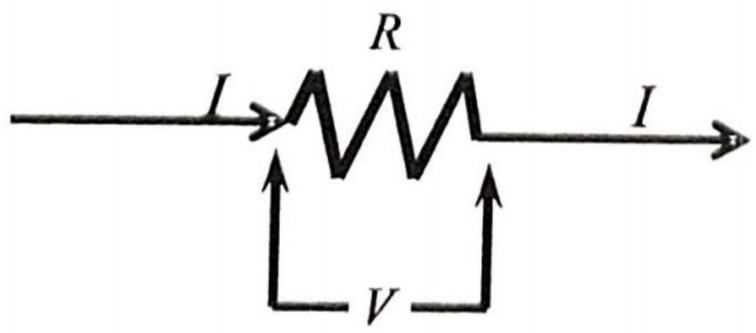
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2. The direction of current flow is the direction in which positive charges move. However, in a typical wire, the positive charges are fixed to the atoms and it is really the negative charges (electrons) that move. In that case the direction of current flow is reversed.



3. Current flows because something forces it around a circuit. That "something" is EMF, electromotive force. But remember that we are using bad terminology and that EMF is not a force - it is actually the difference in electric potentials between two parts of a circuit. So, in the figure below, $V = V_a - V_b$ is the EMF which causes current to flow in the resistor. How much current? Generally, the larger V is , the more current will flow and we expect $I \propto V$. In general this relation will not be completely accurate but when it holds, we say Ohm's Law applies: $I = \frac{V}{R}$. Here, $R = \frac{V}{I}$ is called the resistance.

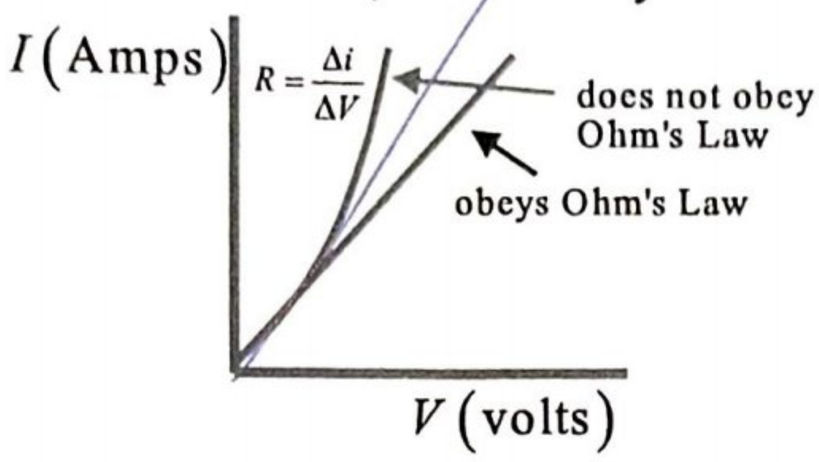


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When current start flow from high Potential to low Potential of the battary called "Convential Current"

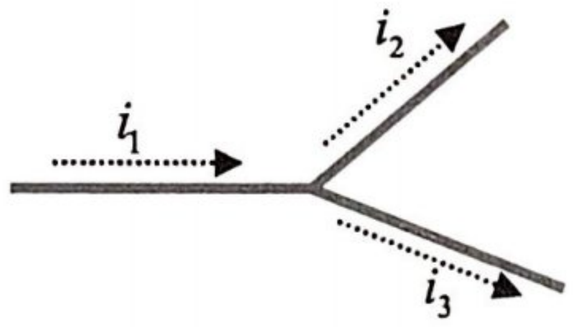
4. Be careful in understanding Ohm's Law. In general the current may depend upon the applied voltage in a complicated way. Another way of saying this is that the resistance may depend upon the current. Example: when current passes through a resistor, it gets hot and its resistance increases. Only when the graph of current versus voltage is a straight line does Ohm's Law hold. Else, we can only define the "incremental resistance".



Ohm law = V ∝ I

V = IR

5. Charge is always conserved, and therefore current is conserved as well. This means that when a current splits into two currents the sum remains constant, $i_1 = i_2 + i_3$.



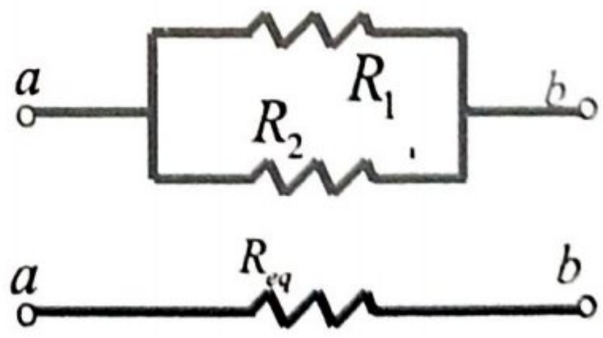
6. When resistors are put in series with each other, the same current flows through both. So, $V_1 = iR_1$ and $V_2 = iR_2$. The total potential drop across the pair is $V = V_1 + V_2 = i(R_1 + R_2)$.
$\Rightarrow R_{eq} = R_1 + R_2$. So resistors in series add up.



7. Resistors can also be put in parallel. This means that the same voltage V is across both. So the currents are $i_1 = \frac{V}{R_1}$, $i_2 = \frac{V}{R_2}$.

Since $i = i_1 + i_2$ it follows that $i = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$

$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ or $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$.



This makes sense: with two possible paths the current will find less resistance than if only one was present.

8. When current flows in a circuit work is done. Suppose a small amount of charge dq is moved through a potential difference V. Then the work done is $dW = Vdq = V\,idt$. Hence

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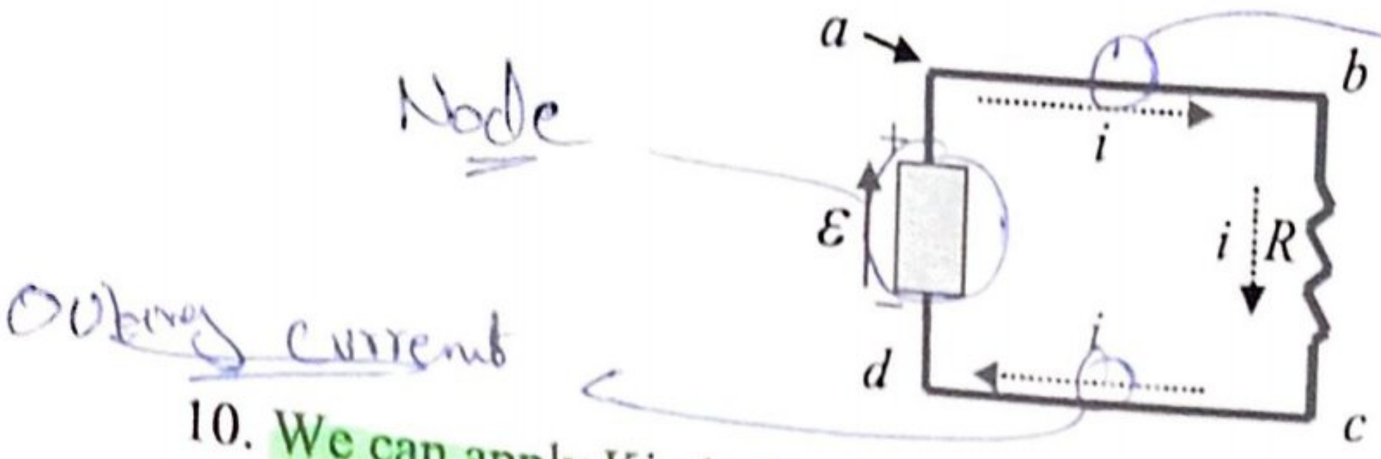
The work done per unit time known as "Power"

$P = \frac{W}{t}$, Unit = Watts

$Vidt = i^2 Rdt$ (because $i = \frac{V}{R}$). The rate of doing work, i.e power, is $P = \frac{dW}{dt} = i^2 R$.

This is an important formula. It can also be written as $P = \frac{V^2}{R}$, or as $P = iV$. The unit of power is: $1 \text{ volt-ampere} = 1 \frac{\text{joule}}{\text{coulomb}} \cdot \frac{\text{coulomb}}{\text{second}} = 1 \frac{\text{joule}}{\text{second}} = 1 \text{ watt}.$

9. Kirchoff's Law: The sum of the potential differences encountered in moving around a closed circuit is zero. This law is easy to prove: since the electric field is conservative, therefore no work is done in taking a charge all around a circuit and putting it back where it was. However, it is very useful in solving problems. As a trivial example, consider the circuit below. The statement that, starting from any point a we get back to the same potential after going around is: $V_a - iR + \mathcal{E} = V_a$. This says $-iR + \mathcal{E} = 0$.



Node

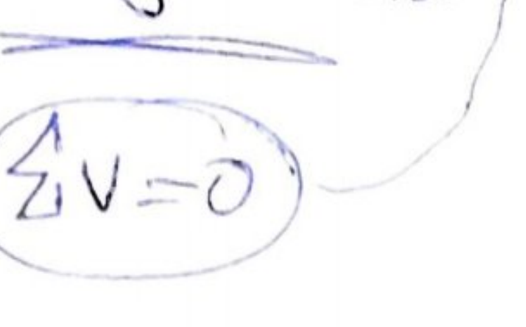
Entering current

$\Sigma V = 0$

Outgoing current

10. We can apply Kirchoff's Law to a circuit that consists of a resistor and capacitor in order to see how current flows through it. Since $q = VC$, we can see that $\frac{q}{C} + iR = 0$. Now differentiate with respect to time to find $\frac{1}{C}\frac{dq}{dt} + \frac{di}{dt}R = 0$,

or $\frac{di}{dt} = -\frac{1}{RC}i$. This equation has solution: $i = i_0 e^{-\frac{t}{RC}}$. The product RC is called the time constant τ, and it gives the time by which the current has fallen to $1/e \approx 1/2.7$ of the initial value.



A reminder about the exponential function, $e^x \equiv 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

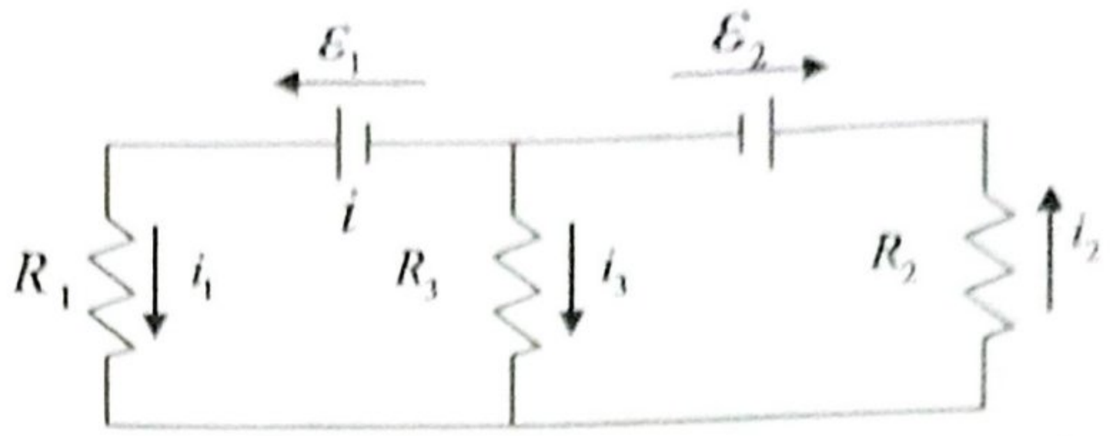
From this, $\frac{\mathrm{d}}{\mathrm{dx}} e^x = 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \cdots = e^x$ Similarly, $\frac{\mathrm{d}}{\mathrm{dx}} e^{-x} = -e^{-x}$

11. Circuits often have two or more loops. To find the voltages and currents in such situations, it is best to apply Kirchoff's Law. In the figure below, you see that there are 3 loops and you can see that:

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$$i_1 + i_3 = i_2$$
$$\mathcal{E}_1 - i_1R_1 + i_3R_3 = 0$$
$$-i_3R_3 - i_2R_2 + \mathcal{E}_2 = 0$$

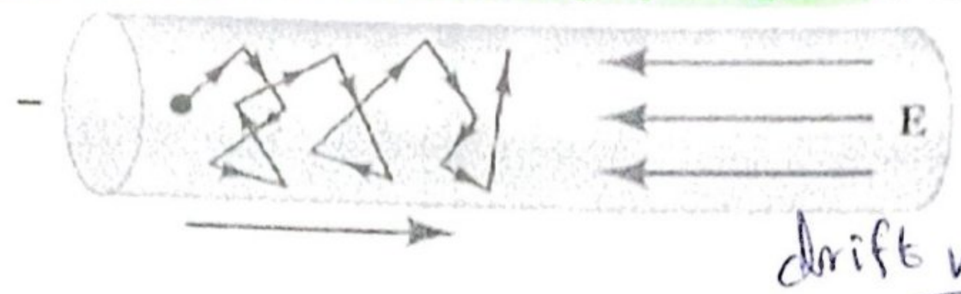


Make sure that you understand each of these, and then check that the solution is:

$$i_1 = \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2R_3}{R_1R_2 + R_2R_3 + R_1R_3} \qquad i_2 = \frac{\mathcal{E}_1R_3 - \mathcal{E}_2(R_1 + R_3)}{R_1R_2 + R_2R_3 + R_1R_3} \qquad i_3 = \frac{-\mathcal{E}_1R_2 - \mathcal{E}_2R_1}{R_1R_2 + R_2R_3 + R_1R_3}$$

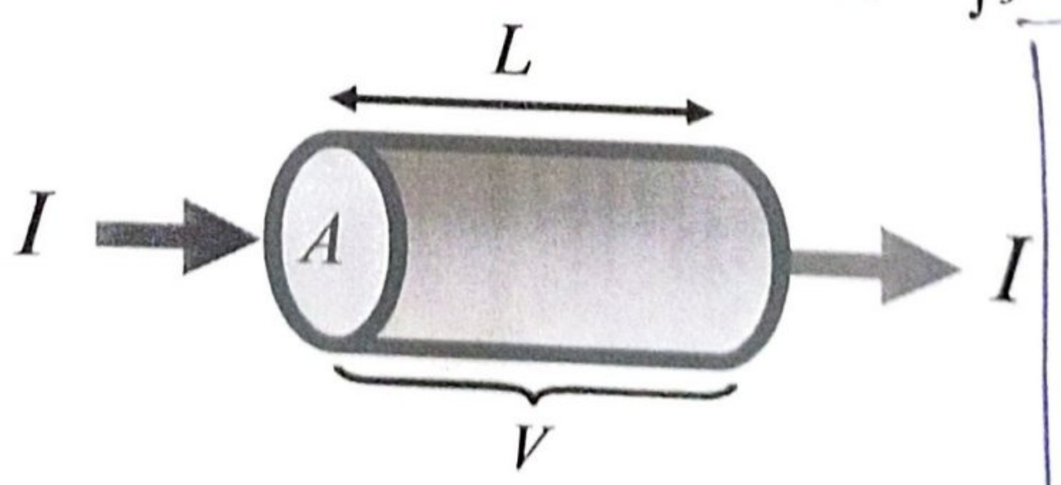
Drift velocity:

12. A charge inside a wire moves under the influence of the applied electric field and suffers many collisions that cause it to move on a highly irregular, jagged path as shown below.



Nevertheless, it moves on the average to the right at the "drift velocity" (or speed).

13. Consider a wire through which charge is flowing. Suppose that the number of charges per unit volume is n. If we multiply n by the crossectional area of the wire A and the length L, then the charge in this section of the wire is $q = (nAL)e$. If the drift velocity of the charges is v_d, then the time taken for the charge to move through the wire is $t = \frac{L}{v_d}$. hence the current is $i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$. From this we can calculate the drift velocity of the charges in terms of the measured current, $v_d = \frac{i}{nAe}$. The current density, which is the current per unit crossectional area is defined as $j = \frac{i}{A} = nev_d$. If j varies inside a volume, then we can easily generalize and write, $i = \int \vec{j} \cdot d\vec{A}$.



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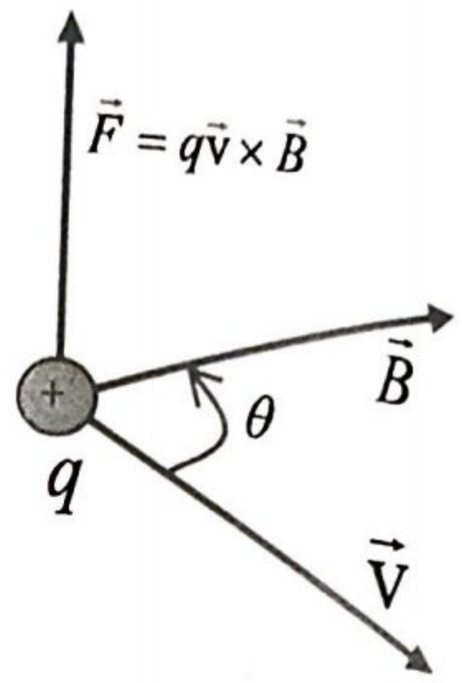
The Force on the Charge Particle move in the magnetic field called magnetic Force.

$F = qVB\sin\theta$

$\theta = 0$	$\theta = 90°$
$F = 0$	$F = Max$

Summary of Lecture 27 – THE MAGNETIC FIELD

1. The magnetic field exerts a force upon any charge that moves in the field. The greater the size of the charge, and the faster it moves, the larger the force. The direction of the force is perpendicular to both the direction of motion and the magnetic field. If θ is the angle between $\vec{v}$ and $\vec{B}$, then $F = qvB\sin\theta$ is the magnitude of the force. This vanishes when $\vec{v}$ and $\vec{B}$ are parallel ($\theta = 0$), and is maximum when they are perpendicular.



2. The unit of magnetic field that is used most commonly is the *tesla*. A charge of one coulomb moving at 1 metre per second perpendicularly to a field of one tesla experiences a force of 1 newton. Equivalently,

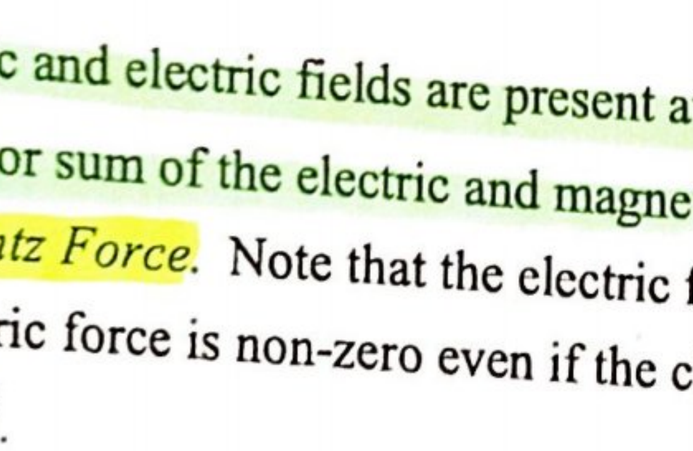
$$1 \text{ tesla} = 1 \frac{\text{newton}}{\text{coulomb} \cdot \text{meter/second}} = 1 \frac{\text{newton}}{\text{ampere} \cdot \text{meter}} = 10^4 \text{ gauss (CGS unit)}$$

In order to have an appreciation for how much a tesla is, here are some typical values of the magnetic field in these units:

Earth's surface	10^{-4} T
Bar magnet	10^{-2} T
Powerful electromagnet	1 T
Superconducting magnet	5 T

3. When both magnetic and electric fields are present at a point, the total force acting upon a charge is the vector sum of the electric and magnetic forces, $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. This is known as the *Lorentz Force*. Note that the electric force and magnetic force are very different. The electric force is non-zero even if the charge is stationary, and it is in the same direction as $\vec{E}$.

4. The Lorentz Force can be used to select charged particles of whichever velocity we want. In the diagram below, particles enter from the left with velocity v. They experience a force due to the perpendicular magnetic field, as well as force downwards because of an electric field. Only particles with speed $v = E/B$ are undeflected and keep going straight.



$qE = qvB \Rightarrow v = \frac{E}{B}$ velocity selector

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Mathematically, $\vec{B}$ is describe as a vector field

It is set of many vectors.

5. A magnetic field can be strong enough to lift an elephant, but it can never increase or decrease the energy of a particle. Proof: suppose the magnetic force $\vec{F}$ moves a particle through a displacement $d\vec{r}$. Then the small amount of work done is,

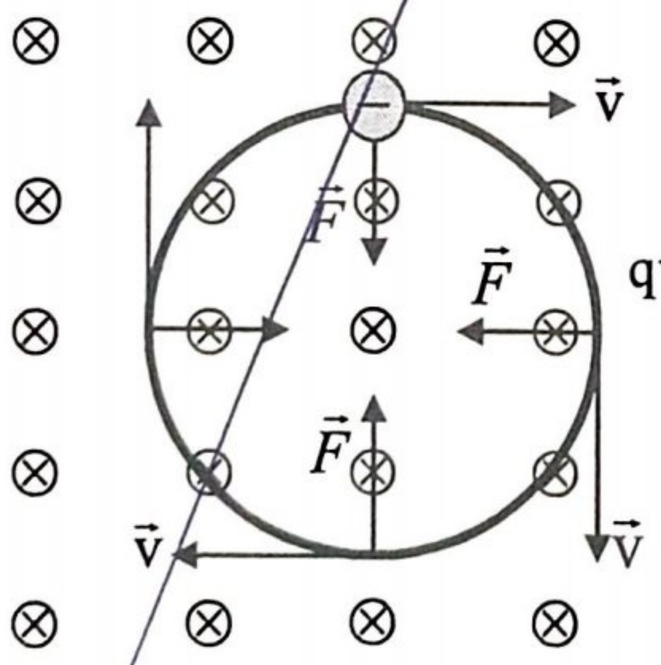
$$dW = \vec{F} \cdot d\vec{r} = q(\vec{v} \times \vec{B}) \cdot d\vec{r}$$

$$= q(\vec{v} \times \vec{B}) \cdot \frac{d\vec{r}}{dt} dt$$

$$= q(\vec{v} \times \vec{B}) \cdot \vec{v}\, dt = 0.$$

Basically the force and direction of force are orthogonal, and hence there can be no work done on the particle or an increase in its energy.

6. A magnetic field bends a charged particle into a circular orbit because the particle feels a force that is directed perpendicular to the magnetic field. As we saw above, the particle cannot change its speed, but it certainly does change direction! So it keeps bending and bending until it makes a full circle. The radius of orbit can be easily calculated: the magnetic and centrifugal forces must balance each other for equilibrium. So, $qvB = m\frac{v^2}{r}$ and we find that $r = \frac{mv}{qB}$. A strong B forces the particle into a tighter orbit, as you can see. We can also calculate the angular frequency, $\omega = \frac{v}{r} = \frac{qB}{m}$. This shows that a strong B makes the particle go around many times in unit time. There are a very large number of applications of these facts.



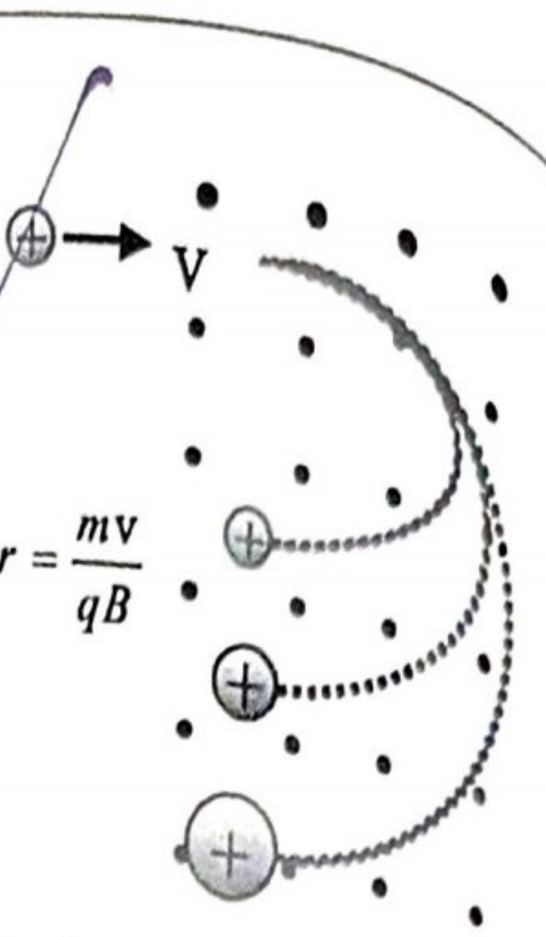
$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}.$$

7. The fact that a magnetic field bends charged particles is responsible for shielding the earth from harmful effects of the "solar wind". A large number of charged particles are released from the sun and reach the earth. These can destroy life. Fortunately the earth's magnetic field deflects these particles, which are then trapped in the "Van Allen" belt around the earth.

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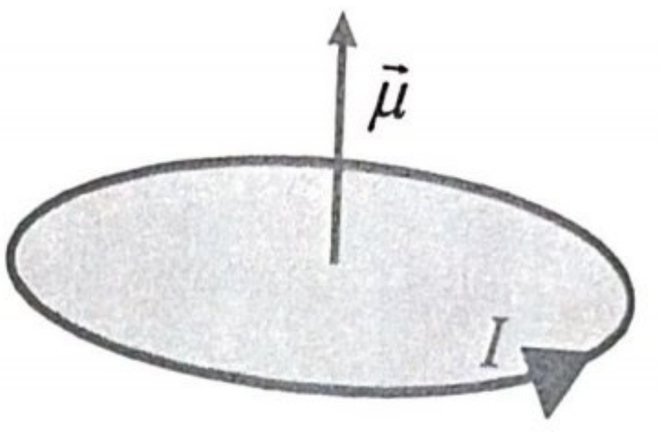
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8. The mass spectrometer is an extremely important equipment that works on the above principle. Ions are made from atoms by stripping away one electron. Then they pass through a velocity selector so that they all have the same speed. In a beam of many different ions, the heavier ones bend less, and lighter ones more, when they are passed through a B field.

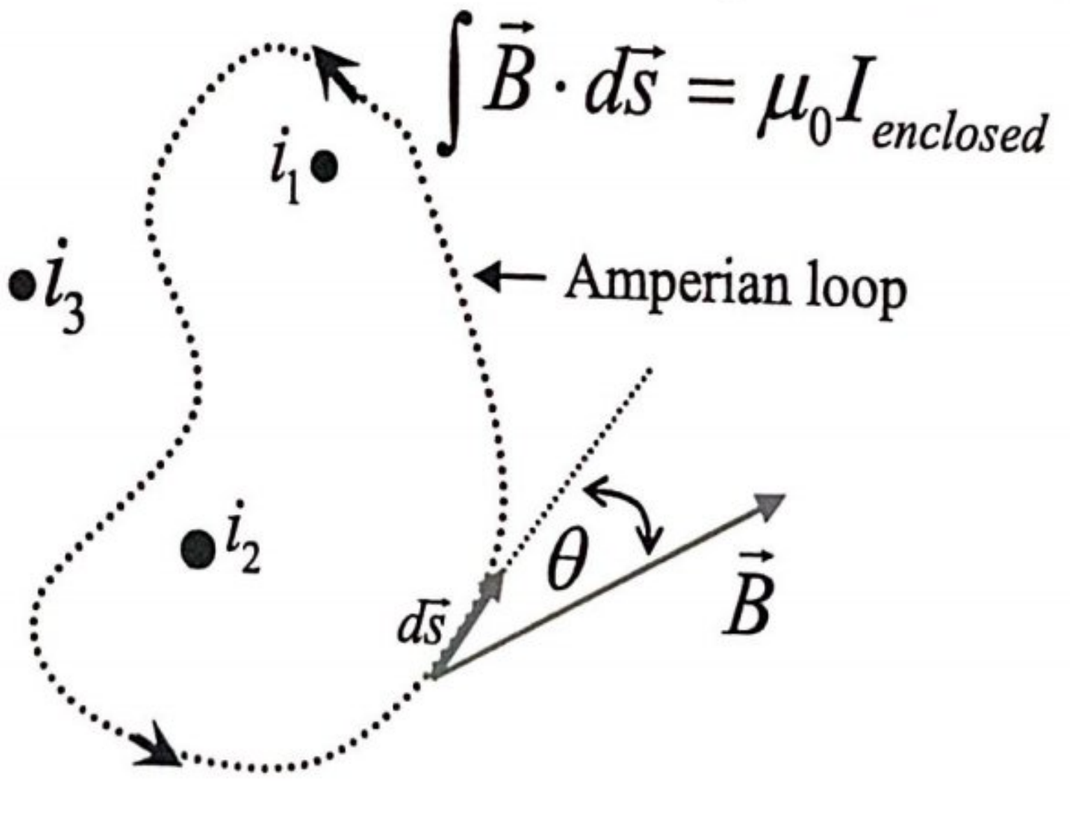


9. A wire carries current, and current is flowing charges. Since each charge experiences a force when placed in a magnetic field, you might expect the same for the current. Indeed, that is exactly the case, and we can easily calculate the force on a wire from the force on individual charges. Suppose N is the total number of charges and they are moving at the average (or drift) velocity $\vec{v}_d$. Then the total force is $\vec{F} = Ne\vec{v}_d \times \vec{B}$. Now suppose that the wire has length L, crossectional area A, and it has n charges per unit volume. Then clearly $N = nAL$, and so $\vec{F} = nALe\vec{v}_d \times \vec{B}$. Remember that the current is the charge that flows through the wire per unit time, and so $nAe\vec{v}_d = \vec{I}$. We get the important result that the force per unit length on the wire is $\vec{F} = \vec{I} \times \vec{B}$.

10. A current that goes around a loop (any shape) produces a magnetic field. We define the *magnetic moment* as the product of current and area, $\vec{\mu} = IA\hat{z}$. Here A is the area of the loop and I the current flowing around it. The direction is perpendicular to the plane of the loop, as shown.



11. Magnetic fields are produced by currents. Every small bit of current produces a small amount of the B field. Ampere's Law, illustrated below, says that if one goes around a loop (of any shape or size) then the integral of the B field around the loop is equal to the enclosed current. In the loop below $I = I_1 + I_2$. Here I_3 is excluded as it lies outside.



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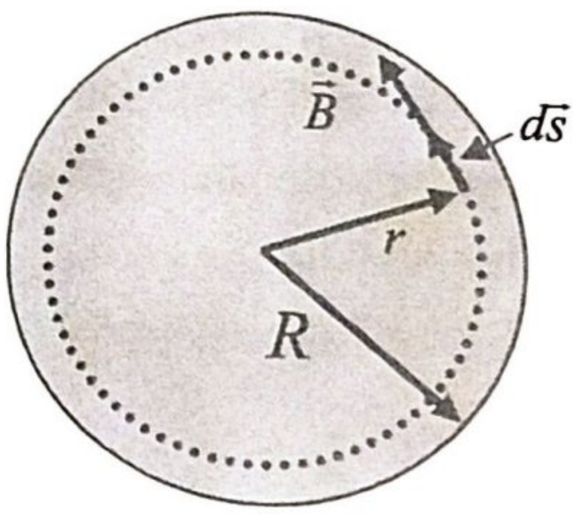
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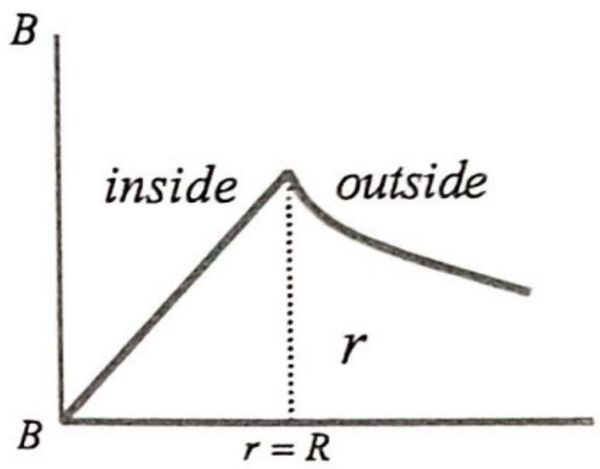
12. Let us apply Ampere's Law to a circular loop of radius r outside an infinitely long wire carrying current I through it. The magnetic field goes around in circles, and so $\vec{B}$ and $d\vec{s}$ are both in the same direction. Hence, $\int \vec{B} \cdot d\vec{s} = B\int ds = B(2\pi r) = \mu_0 I$. We get the important result that $B = \dfrac{\mu_0 I}{2\pi r}$.

13. Assuming that the current flows uniformly over the crossection, we can use Ampere's Law to calculate the magnetic field at distance r, where r now lies inside the wire.

$$B(2\pi r) = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right) \Rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

Here is a sketch of the B field inside and outside the wire as a function of distance r.





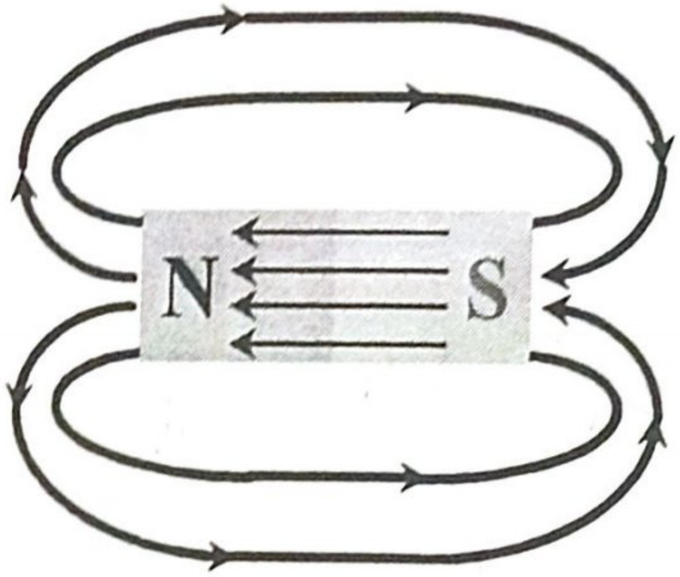
Properties of Magnetic field lines.

i) Magnetic field lines are imaginary.
ii) Never cross each other.
iii) Density indicated strength of field lines.
iv) They always make close loops.
v) They always starts from north to South Pole.

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zero, $\Phi_B = \int \vec{B} \cdot d\vec{A} = 0$. Note that this is very different from what you learned earlier in electrostatics where the flux is essentially the electric charge. There is no such thing as a magnetic charge! What we call the magnetic north (or south) pole of a magnet are actually due to the particular electronic currents, not magnetic charges. In the bar magnet below, no matter which closed surface you draw, the amount of flux leaving the surface is equal to that entering it.



Imp→ **Example:** A sphere of radius R is placed near a long, straight wire that carries a steady current I. The magnetic field generated by the current is B. Find the total magnetic flux passing through the sphere.



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Induced Voltage $(\mathcal{E}) = -(N)\dfrac{(\Delta\Phi)}{(\Delta t)}$ → magnetic flux; → change in time.

Number of loops

Answer: zero, of course!

3. **Faraday's Law for Induced EMF:** when the magnetic flux changes in a circuit, an electro-motive force is induced which is proportional to the rate of change of flux. Mathematically, $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ where $\mathcal{E}$ is the induced emf. If the coil consists of N turns, then $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$.

How does the flux through a coil change? Consider a coil and magnet. We can:

a) move the magnet,

b) change the size and shape of the coil by squeezing it,

c) move the coil.

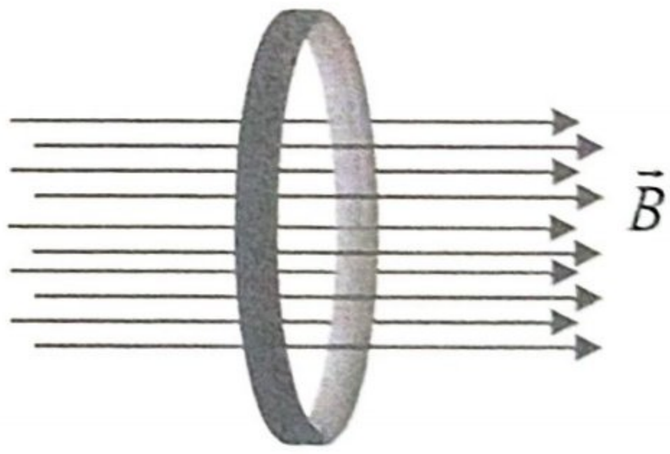
In all cases, the flux through the coil changes and $\dfrac{d\Phi_B}{dt}$ is non-zero leading to an induced emf.

Example: A flexible loop has a radius of 12cm and is in a magnetic field of strength 0.15T. The loop is grasped at points A and B and stretched until it closes. If it takes 0.20s to close the loop, find the magnitude of the average induced emf in it during this time.

Solution: Here the loop area changes, hence the flux. So the induced emf is:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \approx -\left[\frac{\text{final flux} - \text{initial flux}}{\text{time taken}}\right] = -\left[\frac{0 - \pi(0.12)^2 \times 0.15}{0.2}\right] = 0.034 \text{ Volts.}$$

Example : A wire loop of radius 0.30m lies so that an external magnetic field of +0.30T is perpendicular to the loop. The field changes to -0.20T in 1.5s. Find the magnitude of the average induced EMF in the loop during this time.



Solution: Again, we will find the initial and final fluxes first and then divide by the time taken for the change.

Use $\Phi = BA = B\pi r^2$ to calculate the flux.

$$\Phi_i = 0.30 \times \pi \times (0.30)^2 = 0.085 \text{ Tm}^2$$

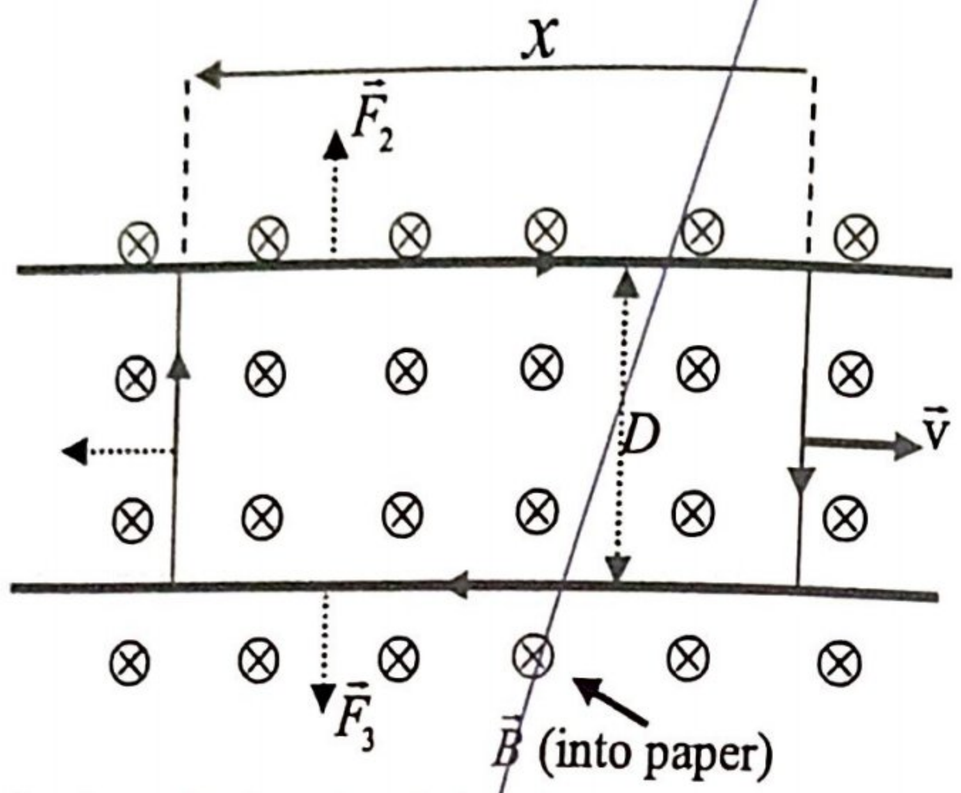
$$\Phi_f = -0.20 \times \pi \times (0.30)^2 = -0.057 \text{ Tm}^2$$

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = \frac{\Phi_f - \Phi_i}{\Delta t} = \frac{0.085 - 0.057}{1.5} = 0.095 \text{ V}$$

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4. Remember that the electromotive force is not really a force but the difference in electric potentials between two points. In going around a circular wire, where the electric field is constant as a function of angle, the emf is $\varepsilon = E(2\pi r)$. More generally, for any size or shape of a closed circuit, $\varepsilon = \int \vec{E} \cdot d\vec{s}$. So Faraday's Law reads: $\int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$.

Example : A conducting wire rests upon two parallel rails and is pulled towards the right with speed v. A magnetic field B is perpendicular to the plain of the rails as shown below. Find the current that flows in the circuit.



Solution: As the wire is pulled to the right, the area of the circuit increases and so the flux increases. Measure x as above so that $x = vt$, i.e. x keeps increasing as we pull. The flux at any value of x is $\Phi_B = BDx$, and so,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BDx)}{dt} = -BD\frac{dx}{dt} = BDv.$$

To calculate the current, we simply use Ohm's Law: $I = \frac{\varepsilon}{R} = \frac{BDv}{R}$. Here R is the resistance of the circuit.

Example : Find the power dissipated in the above circuit using I^2R, and then by directly calculating the work you do by pulling the wire.

Solution: Clearly $I^2R = \frac{B^2D^2v^2}{R}$ is the power dissipated, as per usual formula. Now let us calculate the force acting upon the piece of wire (of length D) that you are pulling. From the formula for the force on a wire, $\vec{F} = I\vec{L} \times \vec{B}$, the magnitude is $F = IBD = \frac{B^2D^2v}{R}$. So, the power is $P = Fv = \frac{B^2D^2v^2}{R}$. This is exactly the value calculated above!

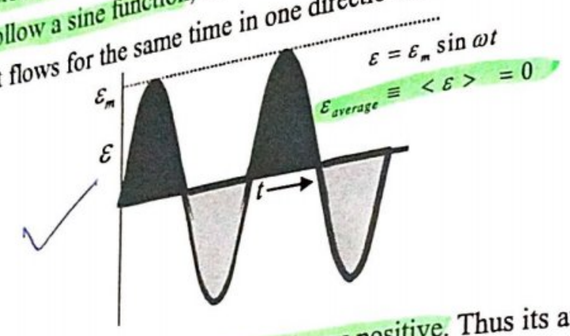
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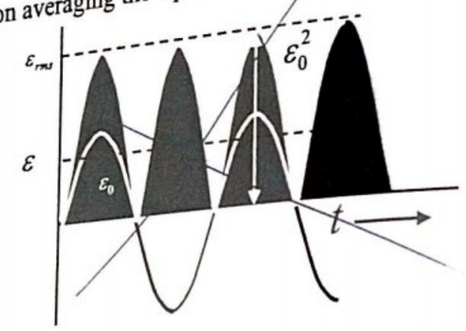
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Summary of Lecture 29 – ALTERNATING CURRENT

1. Alternating current (AC) is current that flows first in one direction along a wire, and then in the reverse direction. The most common AC is sinuisoidal in which the current (and voltage) follow a sine function, as in the graph below. The average value is zero because the current flows for the same time in one direction as in the other.



However, the square of any AC wave is always positive. Thus its average is not zero. As we shall see, upon averaging the square we get half the square of the peak value.



The calculation follows: if there is a sine wave of amplitude (height) equal to one and frequency ω, ($\omega = 2\pi/T$, T=time period), then the squared amplitude is $\sin^2 \omega t$ and its average is:

$$\langle \sin^2 \omega t \rangle \equiv \frac{1}{T}\int_0^T dt \sin^2 \omega t = \frac{1}{T}\int_0^T dt\left(\frac{1-\cos 2\omega t}{2}\right) = \frac{1}{T}\int_0^T dt \frac{1}{2} = \frac{1}{2}$$

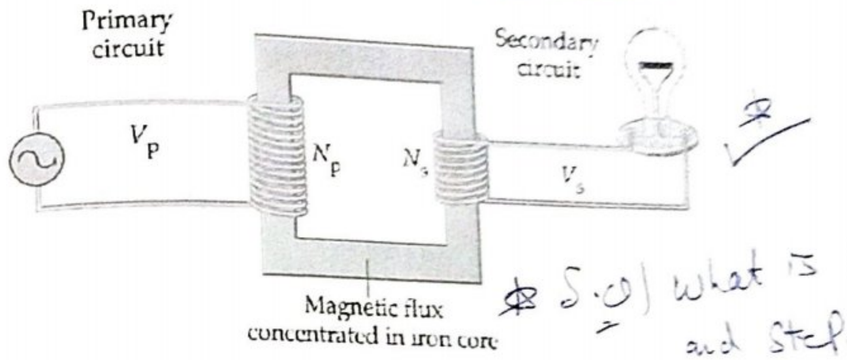
Taking the square root gives the root mean square value as $1/\sqrt{2}$ of the maximum value. Of course, it does not matter whether this is of the voltage or current:

$$\varepsilon_{rms} = \sqrt{\frac{\varepsilon_m^2}{2}} = \frac{\varepsilon_m}{\sqrt{2}} = 0.707\varepsilon_m, \text{ and } I_{rms} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m.$$

Exactly the same results are obtained for cosine waves. This is what one expects since the difference between sine and cosine is only that one starts earlier than the other.

2. AC is generated by a coil rotating in a magnetic field. We know from Faraday's Law, $\varepsilon = -\frac{d\Phi_B}{dt}$, that a changing magnetic flux gives rise to an emf. Imagine a magnetic field and a coil of area A, rotating with frequency ω so that the flux through the coil at any instant of time is $\Phi_B = BA\cos\omega t$. Then the induced emf is $\varepsilon = BA\omega \sin \omega t$.

3. AC is particular useful because *transformers* make it possible to step up or step down voltages. The basic transformer consists of two coils - the primary and secondary - both wrapped around a core (typically iron) that enhances the magnetic field.



Suppose that the flux in the core is Φ and that its rate of change is $\frac{\partial\Phi}{\partial t}$. Call the number of turns in the primary and secondary N_p and N_s respectively. Then, from Faraday's law, the primary emf is $\varepsilon_p = -N_p\frac{\partial\Phi}{\partial t}$ and the secondary emf is $\varepsilon_s = -N_s\frac{\partial\Phi}{\partial t}$. The ratio is, $\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s}$. So, the secondary emf is $\varepsilon_s = \frac{N_s}{N_p}\varepsilon_p$. If $\frac{N_s}{N_p}$ is less than one, then it is called a step-down transformer because the secondary voltage is less than the input voltage. Else, it is a step-up transformer. Both types are used.

4. If this is a lossless transformer (and good transformers are 99% lossless), then the input power must equal the output power, $\varepsilon_p I_p = \varepsilon_s I_s$. From above, this shows that the ratio of currents is $I_s = \frac{N_p}{N_s} I_p$.

5. Whatever the shape or size of a current carrying loop, the magnetic flux that passes through it is proportional to the current, $\Phi \propto I$. The inductance L (called the self-inductance if

inductance is ...

capacitance, is purely geometr...

It does not depend on the current.

6. Let us calculate the inductance of a long coil wound with n turns per unit length. As calculated earlier, the B field is $B = \mu_0 nI$, and the flux passing through N turns of the coil of length l and area A is $N\Phi_B = (nl)(BA) = \mu_0 n^2 lAI$. From the definition, we find:

$L = \frac{N\Phi_B}{I} = \frac{\mu_0 n^2 lIA}{I} = \mu_0 n^2 lA$ (inductance of long solenoid).

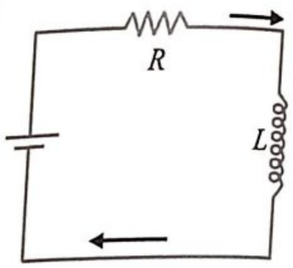
Imp->Example: Find the inductance of a coil with 3500 turns, length 10 cm, and radius 5cm.

Solution: $L = 4\pi \cdot 10^{-7} \frac{T \cdot m}{A} \cdot 3500^2 \cdot \frac{\pi (0.05m)^2}{0.10m} = 1.21 \frac{T \cdot m^2}{A} = 1.21H.$

Using $\Rightarrow \mu_0 n^2 \frac{\pi r^2}{l}$

7. When the current changes through an inductor, by Faraday's Law it induces an emf equal to $-\frac{d\Phi_B}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$.

Let us use this to calculate the current through the circuit shown here, where a resistor is present. Then, by using Kirchoff's Law, $\varepsilon = IR + L\frac{dI}{dt}$ or $\varepsilon I = I^2 R + LI\frac{dI}{dt}$. In words: the power expended by the battery equals energy dissipated in the resistor + work done on inductor.



8. Before we go on to the AC case, suppose that we suddenly connect a battery to an inductor so that the voltage suddenly increases from zero to ε across the circuit. Then, $L\frac{dI}{dt} + IR = \varepsilon$ has solution: $I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$ where $\tau = \frac{L}{R}$. You can easily see that this true using $\frac{dI}{dt} = \frac{\varepsilon}{R}\frac{1}{\tau}e^{-t/\tau}$. The solution shows that the current increases from zero to a maximum of $\frac{\varepsilon}{R}$, i.e. that given by Ohm's Law. We need to pay a little attention to the "time constant" τ, which is the time after which the constant approaches 63% of its final value. Units: $[\tau] = \frac{[L]}{[R]} = \frac{\text{henry}}{\text{ohm}} = \frac{\text{volt.second/ampere}}{\text{ohm}} = \left(\frac{\text{volt}}{\text{ampere.ohm}}\right)\text{second} = \text{second}.$

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by Faraday's ...

current through. How much work? The power, or rate of doing work, is emf × current.

Let U_B be the work done in passing current I. Then, $\frac{dU_B}{dt} = \left(L\frac{dI}{dt}\right)I = LI\frac{dI}{dt}$. Let us integrate $dU_B = LI\,dI$. Then, $\int_0^{U_B} dU_B = \int_0^I LI dI \Rightarrow U_B = \frac{1}{2}LI^2$. This is an important result. It tells us that an inductor L carrying current I requires work $\frac{1}{2}LI^2$. By conservation of energy, this is also the energy stored in the inductor. Compare this result with the result $U_E = \frac{1}{2}CV^2$ for a capacitor. Notice that $C \leftrightarrow L$ and $V \leftrightarrow I$.

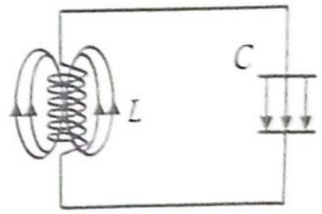
10. Let us use the result derived earlier for the inductance of a solenoid and the magnetic field in it, $L = \mu_0 n^2 lA$ and $B = \mu_0 nI$. Putting this into $U_B = \frac{1}{2}LI^2$ gives $U_B = \frac{1}{2}(\mu_0 n^2 lA)(B/\mu_0 n)^2$.

Divide the energy by the volume of the solenoid, $\frac{U_B}{\text{volume}} = \frac{B^2}{2\mu_0}$. This directly gives the energy density (energy per unit volume) contained in a magnetic field.

$\frac{U_B}{\text{volume}}$

11. **Electromagnetic oscillations in an LC circuit.**

We know that energy can be stored in a capacitor as well as in an inductor. What happens when we connect them up together and put some charge on the capacitor? As it discharges, it creates a current that transfers energy to the inductor. The total energy remains constant, of course.



This means that the sum $U = U_B + U_E = \frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}$ is constant. Differentiate this:

$0 = \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}\right)$. Since $I = \frac{dq}{dt}$, and $\frac{dI}{dt} = \frac{d^2q}{dt^2}$, $\therefore \quad \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$

To make this look nicer, put $\omega^2 \equiv \frac{1}{LC} \Rightarrow \frac{d^2q}{dt^2} + \omega^2 q = 0$. We have seen this equation many times earlier. The solution is: $q = q_m \cos\omega t$. The important result here is that the

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charge, current, and voltage will oscillate with frequency $\omega = \sqrt{\frac{1}{LC}}$. This oscillation will go on for forever if there is no resistance in the circuit.

Electromagnetic oscillations in LC circuits

Total energy = $U = U_B + U_E$

Before flow of current = U_E Maximum

U_B minimum.

After flow of current =

$U_B < U_E$

$U_E < U_B$.

Energy will be conserved.

$$U = \left(\frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}\right)$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}\right)$$

$$= \frac{2(LI^2 + q^2)}{2C}$$

$$= \frac{LI^2 + q^2}{C}$$

$$= \frac{LI^2}{L} + \frac{q^2}{LC}$$

$$= I^2 + \frac{q^2}{LC}$$

$$\frac{d^2q}{dt^2} + \omega = \sqrt{\frac{1}{LC}}$$

Summary of Lecture 30 – ELECTROMAGNETIC WAVES

1. Before the investigations of James Clerk Maxwell around 1865, the known laws of electromagnetism were:

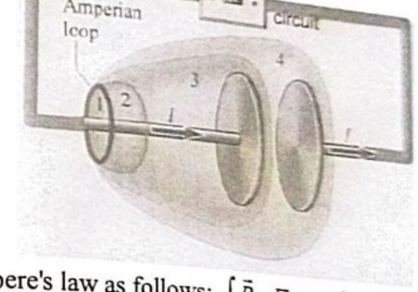
   a) Gauss' law of electricity: $\int \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$ (integral is over any closed surface)

   b) Gauss' law of magnetism: $\int \vec{B}\cdot d\vec{A} = 0$ (integral is over any closed surface)

   c) Faraday's law of induction: $\int \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ (integral is over any closed loop)

   d) Ampere's law: $\int \vec{B}\cdot d\vec{s} = \mu_0 I$ (integral is over any closed loop)

2. But Maxwell realized that the above 4 laws were not consistent with the conservation of charge, which is a fundamental principle. He argued that if you take the space between two capacitors (see below) and take different surfaces 1,2,3,4 then applying Ampere's Law gives an inconsistency: $\left[\int \vec{B}\cdot d\vec{s}\right]_{(1,2,4)} \neq \left[\int \vec{B}\cdot d\vec{s}\right]_{(3)}$ because obviously charge cannot flow in the gap between plates. So Ampere's Law gives different results depending upon which surface is bounded by the loop shown!



Maxwell modified Ampere's law as follows: $\int \vec{B}\cdot d\vec{s} = \mu_0(I + I_d)$ where the "displacement current" is $I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$. Let's look at the reasoning that led to Maxwell's discovery of the displacement current. The current that flows in the circuit is $I = \frac{dQ}{dt}$. But the charge on the capacitor plate is $Q = \varepsilon_0 EA$. Hence, $I = \frac{d}{dt}(\varepsilon_0 EA) = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_D$. In words, the changing electric field in the gap acts as source of the magnetic field in just the same way as the current in the outside wires. This is really the most important point - a magnetic field may have two separate reasons for existence - flowing charges or changing electric fields.

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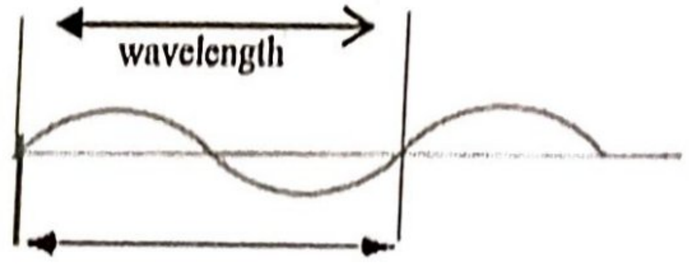
years later by Hertz. Note that for these waves:

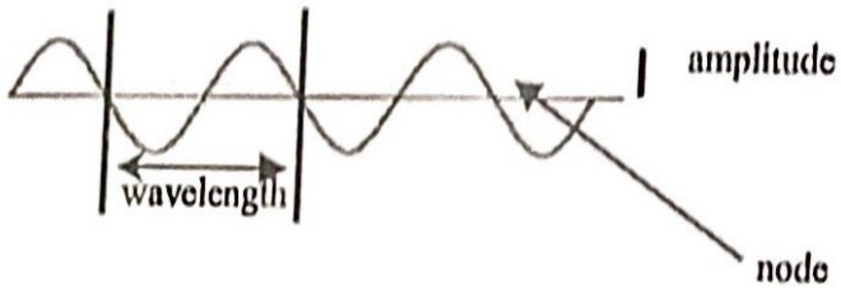
a) Absolutely no medium is required - they travel through vacuum.

b) The speed of propagation is c for all waves in the vacuum.

c) There is no limit to the amplitude or frequency.

A wave is characterized by the amplitude and frequency, as illustrated below.

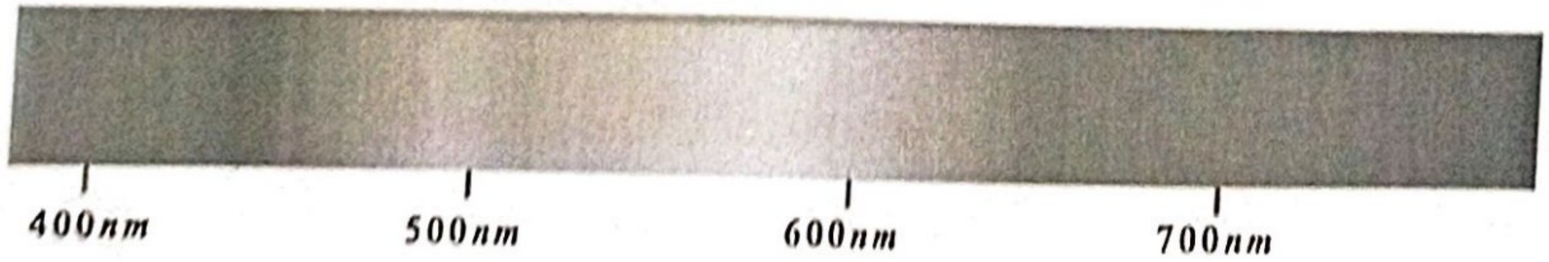




Example: Red light has $\lambda = 700$ nm. The frequency ν is calculated as follows:

$$\nu = \frac{3.0\times10^{8}\, m/\sec}{7\times10^{-7}\, m} = 4.29\times10^{14}\, Hertz$$

By comparison, the electromagnetic waves inside a microwave oven have wavelength of 6 cm, radio waves are a few metres long. For visible light, see below. On the other hand, X-rays and gamma-rays have wavelengths of the size of atoms and even much smaller.

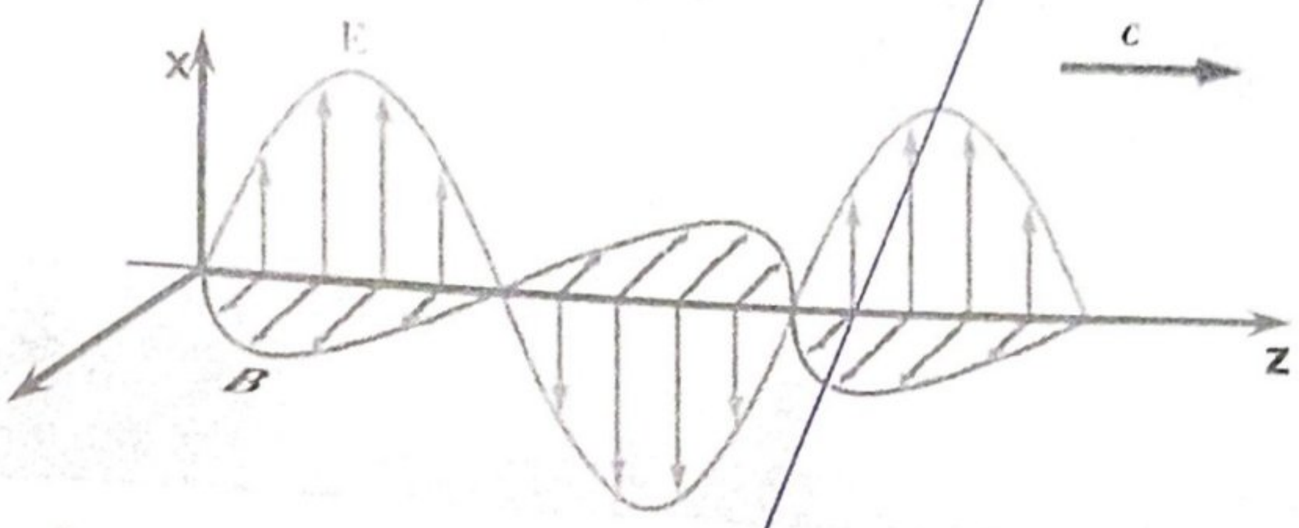


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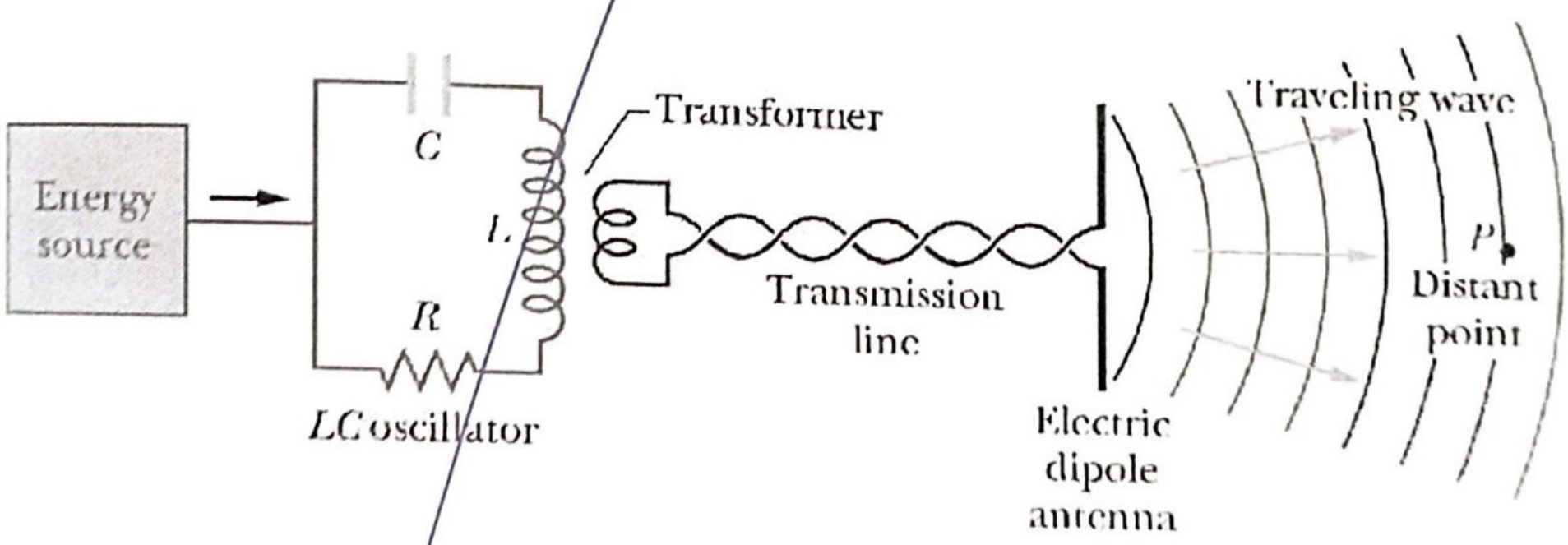
If light waves vibrate in more than one plane called Unpolarised light. To convert Unpolarised to Polarised we used Polarizer.

5. We now consider how electromagnetic waves can travel through empty space. Suppose an electric field (due to distant charges in an antenna) has been created. If this changes then this creates a changing electric flux $\frac{d\Phi_E}{dt}$ which, through $\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$, creates a changing magnetic flux $\frac{d\Phi_B}{dt}$. This, through $\int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$, creates a changing electric field. This chain of events in free space then allows a wave to propagate.



In the diagram above, an electromagnetic wave is moving in the z direction. The electric field is in the x direction, $E_x = E_0 \sin(kz - \omega t)$, and the magnetic field is perpendicular to it, $B_y = B_0 \sin(k z - \omega t)$. Here $\omega = kc$. From Maxwell's equations the amplitudes of the two fields are related by $E_0 = cB_0$. Note that the two fields are in phase with each other.

6. The production of electromagnetic waves is done by forcing current to vary rapidly in a small piece of wire. Consider your mobile phone, for example. Using the power from the battery, the circuits inside produce a high frequency current that goes into a "dipole antenna" made of two small pieces of conductor. The electric field between the two oppositely charged pieces is rapidly changing and so creates a magnetic field. Both fields propagate outwards, the amplitude falling as $1/r$.

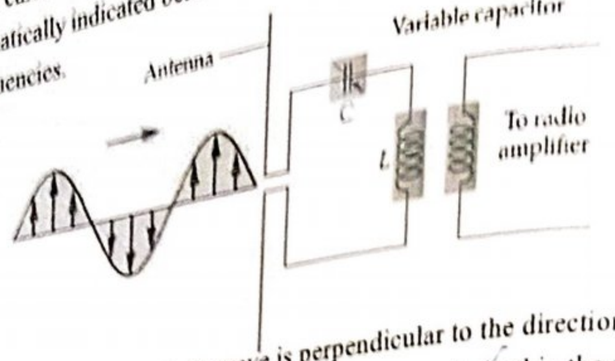


The power, which is the square of the amplitude, falls off as $I(\theta) \propto \frac{\sin^2\theta}{r^2}$. The $\sin^2\theta$ dependence shows that the power is radiated unequally as a function of direction. The maximum power is at $\theta = \pi/2$ and the least at $\theta = 0$.

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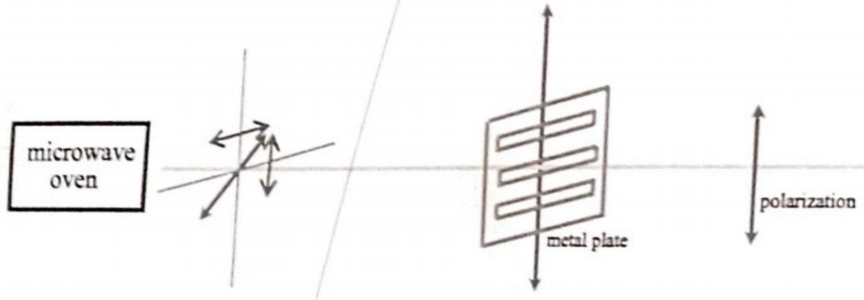
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7. The reception of electromagnetic
electric field that forces the electrons to run up
a tiny electric current. This current is then amplified (increased in amplitude) electronically
This is schematically indicated below. Here the variable capacitor is used to tune to
different frequencies.



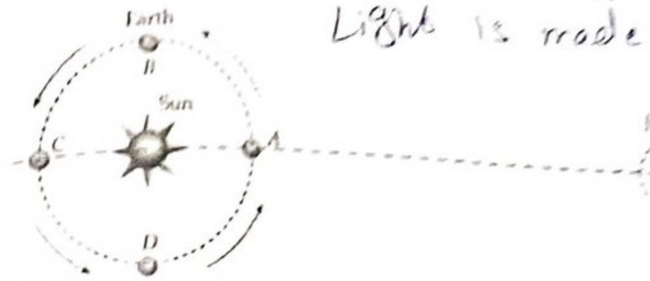
8. As we have seen, the electric field of a wave is perpendicular to the direction of its motion. If this is a fixed direction (say, $\hat{x}$), then we say that wave is polarized in the x direction. Most sources - a candle, the sun, any light bulb - produce light that is unpolarized. In this case, there is no definite direction of the electric field, no definite phase between the orthogonal components, and the atomic or molecular dipoles that emit the light are randomly oriented in the source. But for a typical linearly polarized plane electromagnetic wave polarized along $\hat{x}$, $E_x = E_0 \sin(kz - \omega t)$, $B_y = \frac{E_0}{c}\sin(kz - \omega t)$ with all other components zero. Of course, it may be that the wave is polarized at an angle θ relative to $\hat{x}$, in which case $E_x = E_0 \cos\theta \cdot \sin(kz - \omega t)$, $E_y = E_0 \sin\theta \cdot \sin(kz - \omega t)$, $E_z = 0$.

9. Electromagnetic waves from an unpolarized source (e.g. a burning candle or microwave oven) can be polarized by passing them through a simple polarizer of the kind below. A metal plate with slits cut into will allow only the electric field component perpendicular to the slits. Thus, it will produce linearly polarized waves from unpolarized ones.



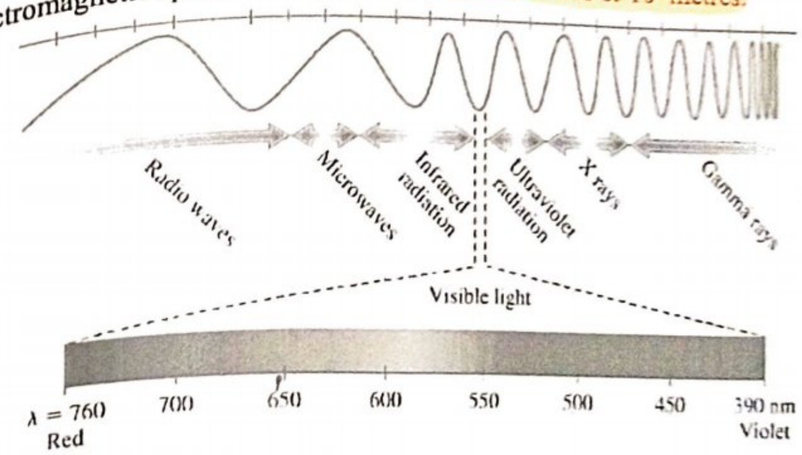
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using ca
of the eclipse of one of Jupiter's moon called Io. In the diagram below Io is observed with
Earth at A and then at C. The eclipse is 16.6 minutes late, which is the time taken for light
to travel AC. Roemer estimated that $c = 3\times10^8$ metres/sec, a value that is remarkably
close to the best modern measurement, $c = 299792458.6$ metres/sec.



Light is made up of Particles called Photons.

2. Light is electromagnetic waves. Different frequencies ν correspond to different colours. Equivalently, different wavelength λ correspond to different colours. Recall that the product $\nu \times \lambda = c$. In the diagram below you see that visible light is only one small part of the total electromagnetic spectrum. Here nm means nanometres or 10^{-9} metres.



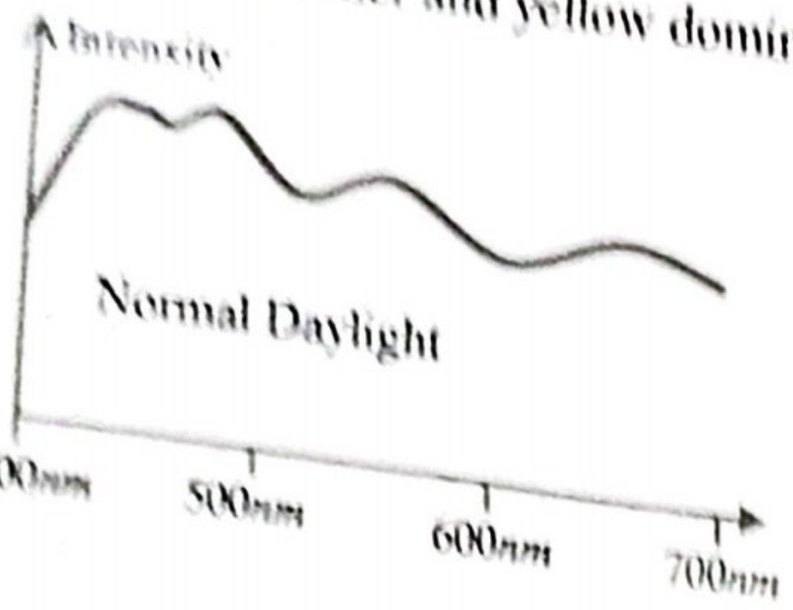
ν = diff frequency
λ = wavelength
c = Speed of light

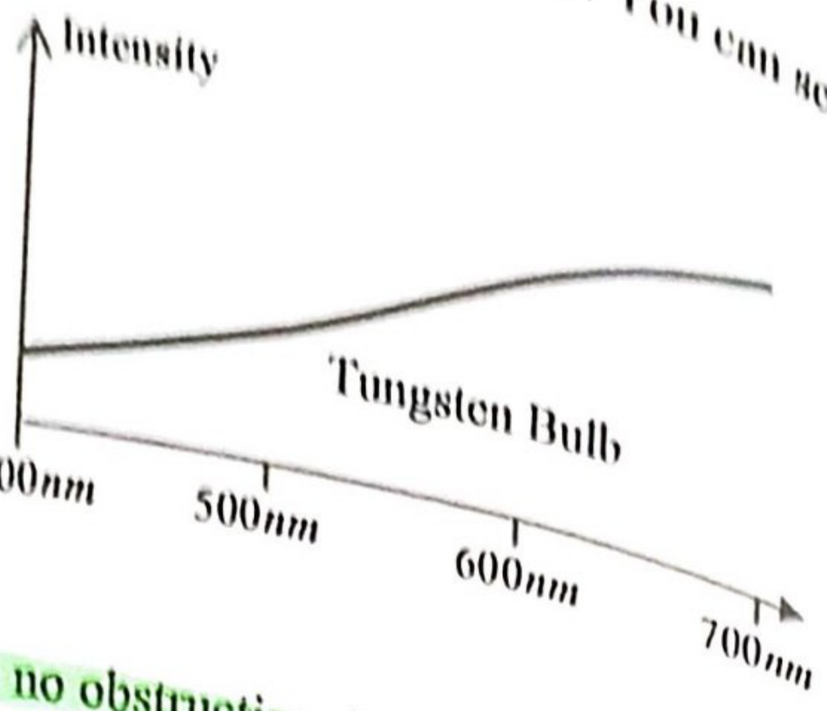
3. If light contained all frequencies with equal strength, it would appear as white to us. course, most things around us appear coloured. That is because they radiate more strongly in one range of frequency than in others. If there is more intensity in the yellow range than the green range, we will see mostly yellow. The sky appears blue to us on a clear day because tiny dust particles high above in the atmosphere reflect a lot of the blue light coming from the sun. In the figure below you can see the hump at smaller wavelengths.

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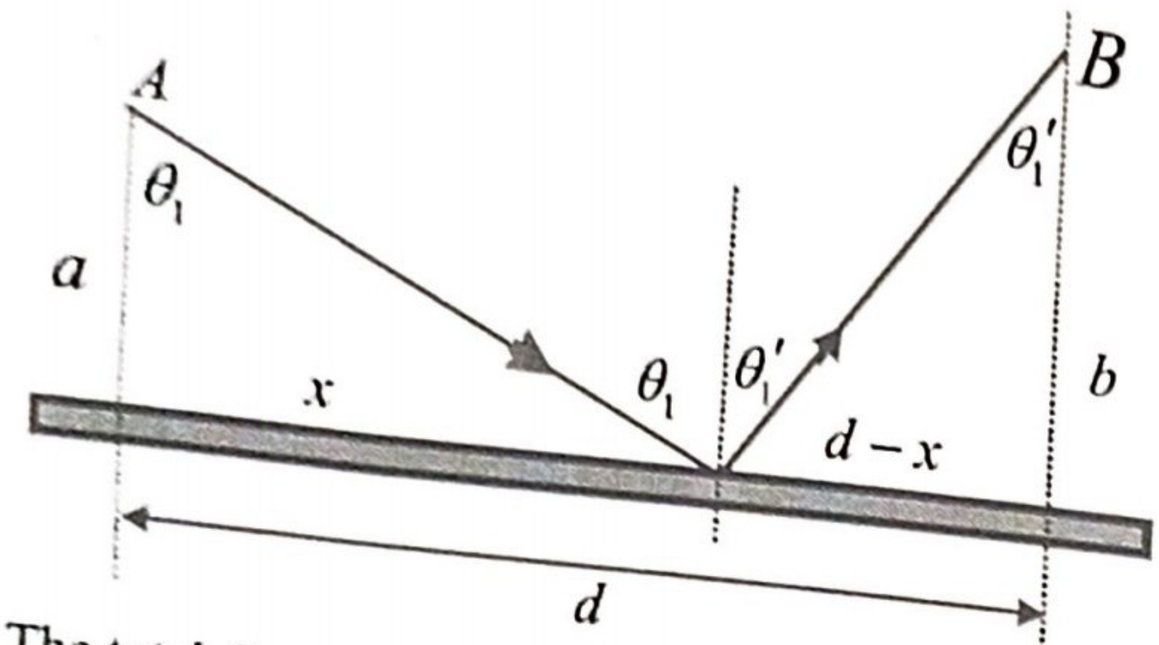
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Compare this with the spectrum of light emitted from a tungsten bulb. You can see that this is smoother and yellow dominates.





4. What path does light travel upon? If there is no obstruction, it obviously likes to travel on straight line which is the shortest path between any two points, say A and B. Fermat's Principle states that in all situations, light will always take that path for which it takes the least time. As an example, let us apply Fermat's Principle to the case of light reflected from a mirror, as below.



The total distance travelled by the ray is $L = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d-x)^2}$. The time taken is $t = L/c$. To find the smallest time, we must differentiate and then set the derivative to zero,

$$0 = \frac{dt}{dx} = \frac{1}{c}\frac{dL}{dx}$$

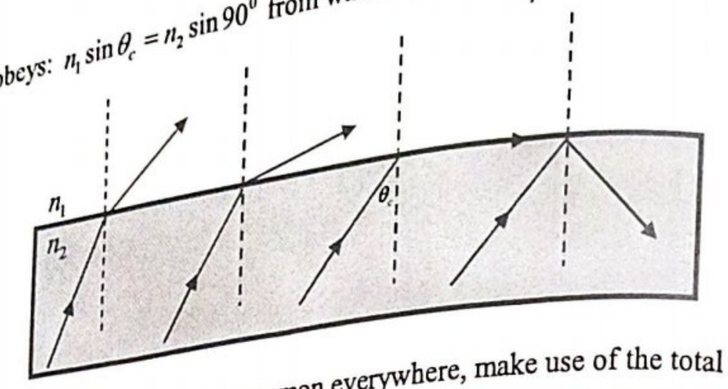
$$= \frac{1}{2c}\left(a^2 + x^2\right)^{-1/2}(2x) + \frac{1}{2c}\left[b^2 + (d-x)^2\right]^{1/2}(2)(d-x)(-1)$$

From here we immediately see that $\frac{x}{\sqrt{a^2 + x^2}} = \frac{d-x}{\sqrt{b^2 + (d-x)^2}}$ From the above diagram, $\sin\theta_1 = \sin\theta_1'$. Of course, it is no surprise that the angle of reflection equals the angle of incidence. You knew this from before, and this seems like a very complicated derivation of a simple fact. But it is still nice to see that there is a deeper principle behind it.

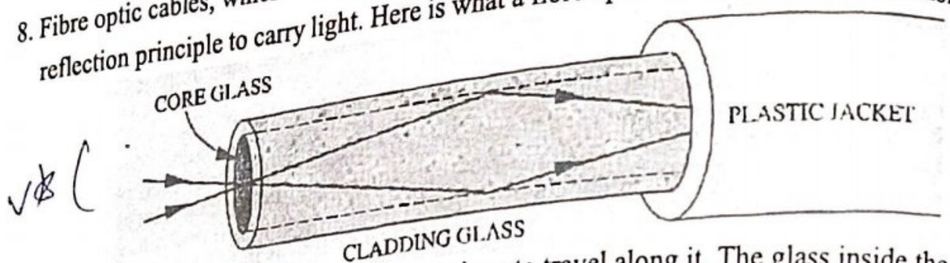
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angle. It obeys: $n_1 \sin\theta_c = n_2 \sin 90^0$ from which $\theta_c = \sin^{-1}\frac{n_2}{n_1}$.



8. Fibre optic cables, which are now common everywhere, make use of the total internal reflection principle to carry light. Here is what a fibre optic cable looks like from inside:



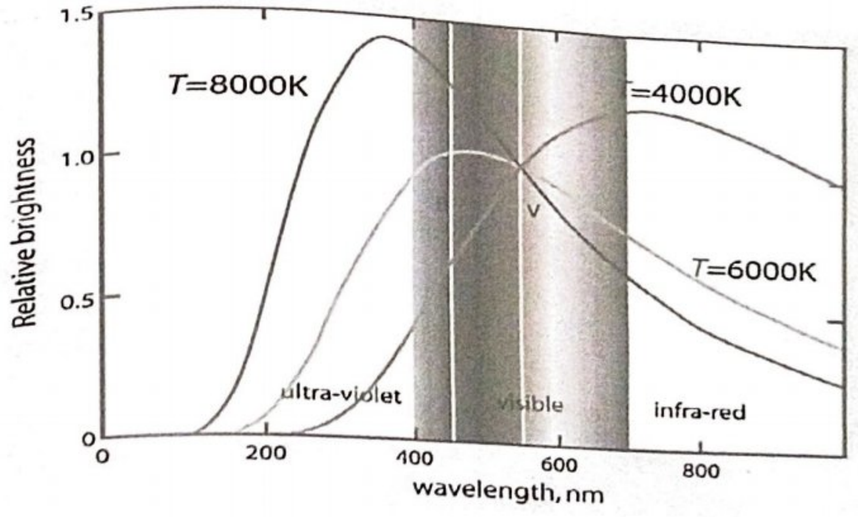
Even if the cable is bent, the light will continue to travel along it. The glass inside the cable must have exceedingly good consistency - if it thicker or thinner in any part, the refractive index will become non-uniform and a lot of light will get lost. Optical fibres now carry thousands of telephone calls in a cable whose diameter is only a little bigger than a human hair!

LCD - Liquid crystal display.

Summary of Lecture 32 – INTERACTION OF LIGHT WITH MATTER

1. In this lecture I shall deal with the 4 basic ways in which light interacts with matter:
   a) Emission - matter releases energy as light.
   b) Absorption - matter takes energy from light.
   c) Transmission - matter allows light to pass through it.
   d) Reflection - matter repels light in another direction.

2. When an object (for example, an iron rod or the filament of a tungsten bulb) is heated, it emits light. When the temperature is around 800°C, it is red hot. Around 2500°C it is yellowish-white. At temperatures lower than 800°C, infrared (IR) light is emitted but our eyes cannot see this. This kind of emission is called blackbody radiation. Blackbody radiation is continuous - all wavelengths are emitted. However most of the energy is radiated close to the peak. As you can see in the graph, the position of the peak goes to smaller wavelengths (or higher frequencies) as the object becomes hotter. The scale of temperature is shown in degrees Kelvin (°K). To convert from °C to °K, simply add 273. We shall have more to say about the Kelvin scale later.



Where exactly does the peak occur? Wien's Law states that $\lambda_{max} T = 2.90\times10^{-3}$ m K. We can derive this in an advanced physics course, but for now you must take this as given.

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3. In the lecture on electromagnetic waves you had learnt that these waves are emitted when charges accelerate. Blackbody radiation occurs for exactly this reason as well. When a body is heated up, the electrons, atoms, and molecules which it contains undergo violent random motion. Light may emitted by electrons in one atom and absorbed in another. Even an empty box will be filled with blackbody radiation because the sides of the box are made up of material that has charged constituents that radiate energy when they undergo acceleration during their random motion.

4. When can you use Wien's Law? ~~More generally~~ Or, when can you expect a body to emit blackbody radiation? Answer: only for objects that emit light, not for those that merely reflect light (e.g. flowers). The Sun and other stars obey Wien's Law since the gases they are composed of emit radiation that is in equilibrium with the other materials. Wien's law allows astronomers to determine the temperature of a star because the wavelength at which a star is brightest is related to its temperature.

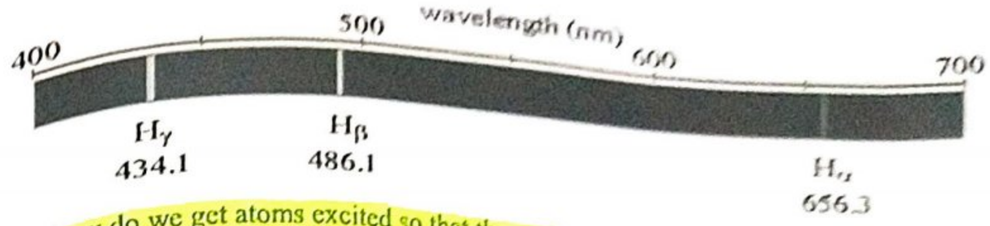
5. All heated matter radiates energy, and hotter objects radiate more energy. The famous Stefan-Boltzman Law, which we unfortunately cannot derive in this introductory course, states that the power radiated per unit area of a hot body is $P = \sigma T^4$, where the Stefan-Boltzman constant is $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$.

6. Let us apply $P = \sigma T^4$ for finding the temperature of a planet that is at distance R from the sun. The sun has temperature T_{sun} and radius R_{sun}. In equilibrium, the energy received from the sun is exactly equal to the energy radiated by the planet. Now, the total energy radiated by the sun is $\sigma T_0^4 \times 4\pi R_{sun}^2$. But on a unit area of the planet, only $\frac{1}{4\pi R^2}$ of this is received.

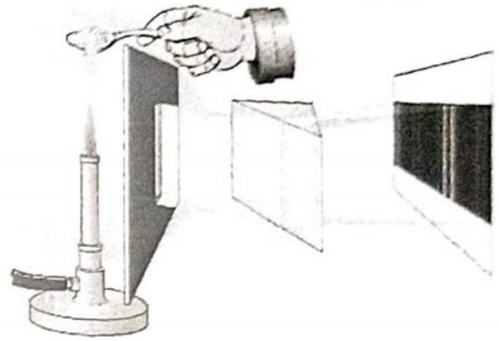
So the energy received per unit area on the planet is $\sigma T_0^4 \times 4\pi R_{sun}^2 \times \frac{1}{4\pi R^2}$. This must be equal to $\sigma T^4 \Rightarrow T = T_0\sqrt{\frac{R_{sun}}{R}}$.

7. The above was for blackbody radiation where the emitted light has a continuous spectrum. But if a gas of identical atoms is excited by some mechanism, then only a few discrete wavelengths are emitted. Each chemical element produces a very distinct pattern of colors called an emission spectrum. So, for example, laboratory hydrogen gas lamps emit 3 lines in the visible region, as you can see below. Whenever we see 3 lines spaced apart in this way, we immediately know that hydrogen gas is present. It is as good as the thumbprint of a man!

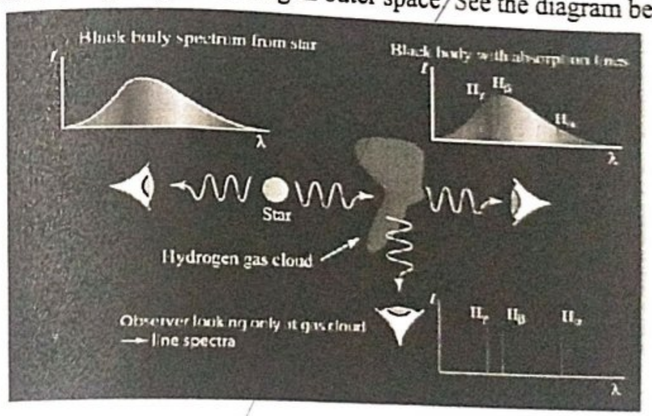
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8. But how do we get atoms excited so that they can start revealing their identity? One way is to simply heat material containing those atoms. You saw in the lecture how different colours come from sprinkling different materials on a flame.



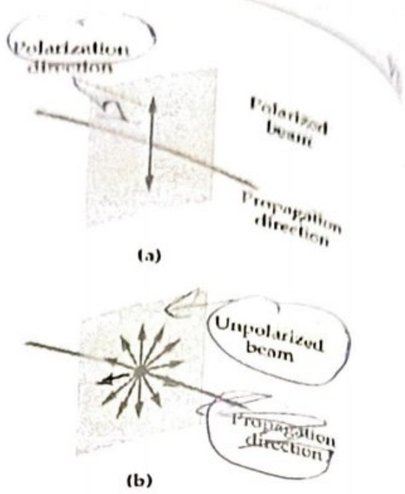
9. Everything that I have said about the emission of light applies exactly to the *absorption* of light as well. So, for example, when white light (which has all different frequencies within it) passes through hydrogen gas, you will see that all wavelengths survive except the three on the previous page. So the absorption spectrum looks exactly the same as the emission spectrum - the same lines are emitted and absorbed. This how we know that there are huge clouds of hydrogen floating in outer space. See the diagram below.



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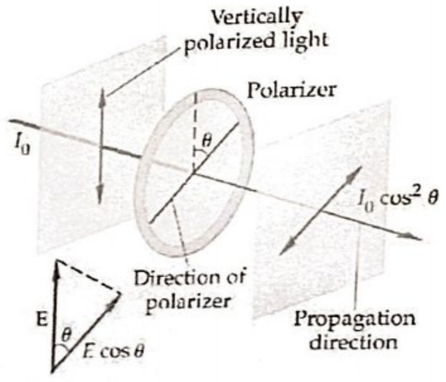
11. As you learned earlier, light is an electromagnetic wave that has an electric field vector. This vector is always perpendicular to the direction of travel, but it can be pointing anywhere in the plane. If it is pointing in a definite direction, we say that the wave is polarized, else it is unpolarized. In general, the light emitted from a source, such as a flame, will be unpolarized and it is equally possible to find any direction of the electric field in the wave. Of course, the magnetic field is perpendicular to both the direction of travel and the electric field.



10. The atmosphere contains various gases which absorb light at many different wavelengths. Molecules of oxygen, nitrogen, ozone, and water have their own absorption spectra, just as atoms have their own.

11. All the beauty of colours we see is due to the selective absorption by molecules of certain frequencies. So, for example, *carotene* is a long, complicated molecule that makes carrots orange, tomatoes red, sarson yellow, and which absorbs blue light. Similarly *chlorophyll* makes leaves green and which absorbs red and blue light.

12. From unpolarized light we can make polarized light by passing it through a polarizer as shown below.



Each wave is reduced in amplitude by $\cos\theta$, and in intensity by $\cos^2\theta$. The wave that emerges is now polarized in the θ direction.

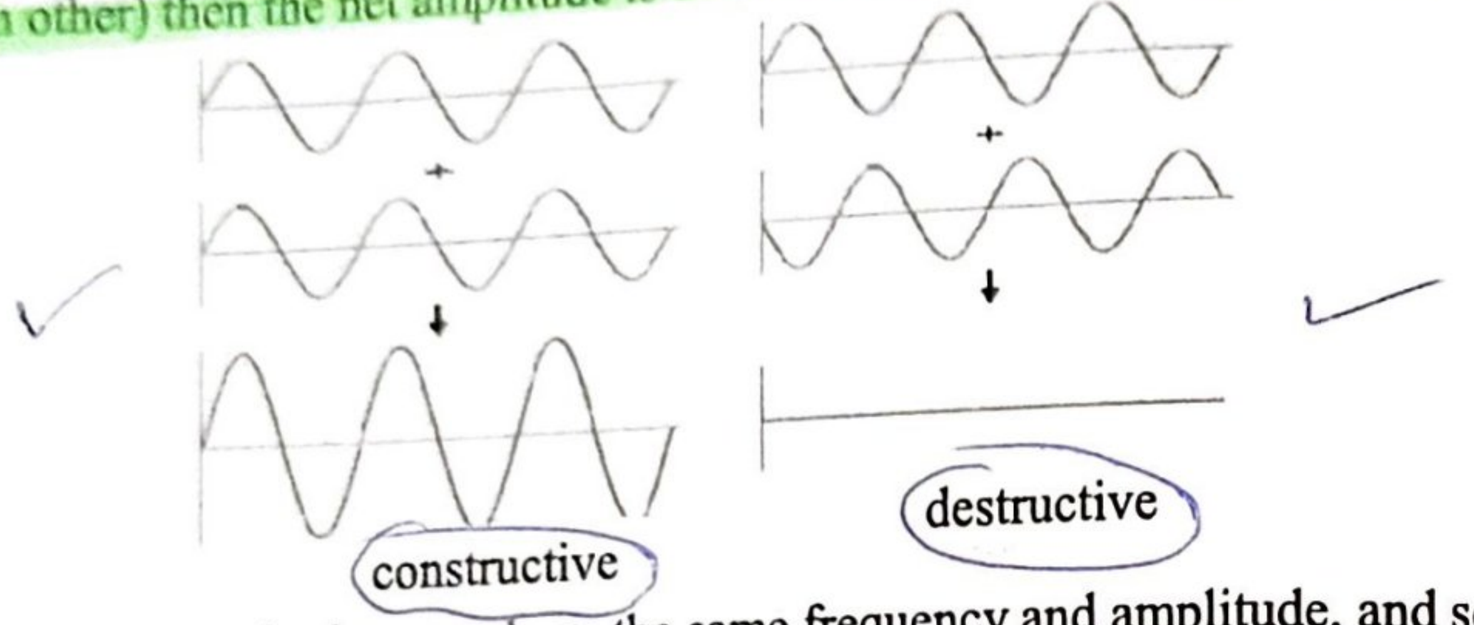
13. We can design materials (crystals or stressed plastics) so that they have different optical properties in the two transverse directions. These are called birefringent materials. They are used to make commonly used liquid crystal displays (LCD) in watches and mobile phones. Birefringence can occur in any material that possesses some asymmetry in its structure where the material is more springy in one direction than another.

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Summary of Lecture 33 – INTERFERENCE AND DIFFRACTION

Interference:

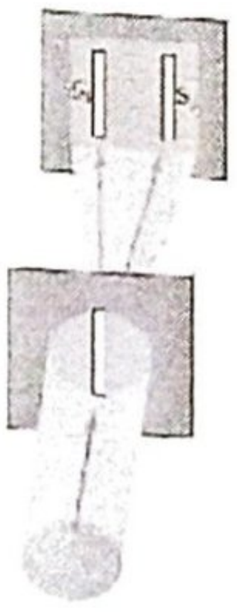
1. Two waves (of any kind) add up together, with the net result being the simple sum of the two waves. Consider two waves, both of the same frequency, shown below. If they start together (i.e. are *in phase* with each other) then the net amplitude is increased. This is called *constructive interference*. But if they start at different times (i.e. are *out of phase* with each other) then the net amplitude is decreased. This is called *destructive interference*.



In the example above, both waves have the same frequency and amplitude, and so the resulting amplitude is doubled (constructive) or zero (destructive). But interference occurs for any two waves even when their amplitudes and frequencies are different.

2. Although any waves from different sources interfere, if one wants to observe the interference of light then it is necessary to have a *coherent* source of light. Coherent means that both waves should have a fixed phase relative to each other. Even with lasers, it is very difficult to produce coherent light from two separate sources. Observing interference usually requires taking two waves from a single source, with each going along a different path. In the figure, an incoherent light source illuminates the first slit. This creates a uniform and coherent illumination of the second screen. Then waves from the slits S_1 and S_2 meet on the third screen and create a pattern of alternating light and dark fringes.



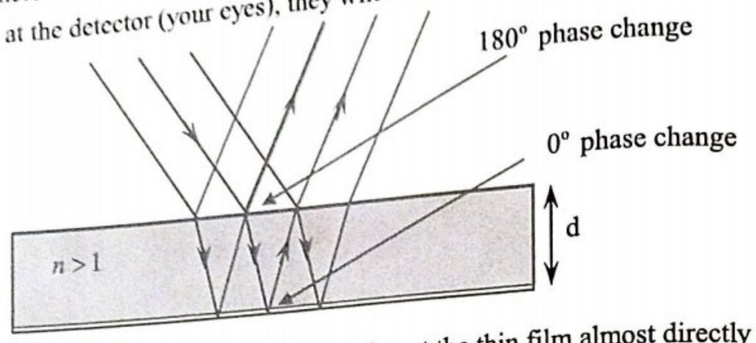


3. Wherever there is a bright fringe, constructive interference has occured, and wherever there is a dark fringe, destructive interference has occured. We shall now calculate where on the third screen the interference is constructive. Take any point on the third screen. Light reaches this point from both S_1 and S_2, but it will take different amounts of time to get there. Hence there will be a phase difference that we can calculate. Look at the diagram below. You can see that light from one of the slits has to travel an extra

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5. When light falls upon a thin film of soapy water, oil, etc. it is reflected from two surfaces. On the top surface, the reflection is with change of phase by π whereas at the lower surface there is no change of phase. This means that when waves from the two surfaces combine at the detector (your eyes), they will interfere.



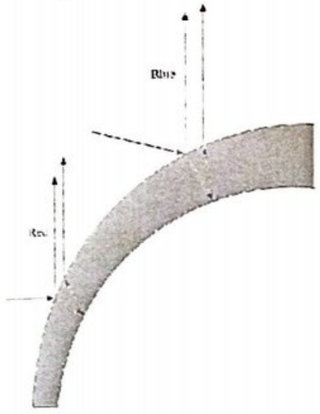
To simplify matters, suppose that you are looking at the thin film almost directly from above. Here n is the index of refraction for the medium. Then,

The condition for destructive interference is: $2nd = m\lambda \quad (m = 0,1,2,...)$

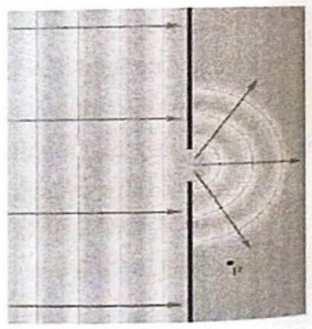
The condition for constructive interference is: $2nd = (m+\frac{1}{2})\lambda \quad (m = 0,1,2,...)$.

Prove it!

6. Interference is why thin films give rise to colours. A drop of oil floating on water spreads out until it is just a few microns thick. It will have thick and thin portions. Thick portions of any non-uniform thin film appear blue because the long-wavelength red light experiences destructive interference. Thinner portions appear red because the short-wavelength blue light interferes destructively.



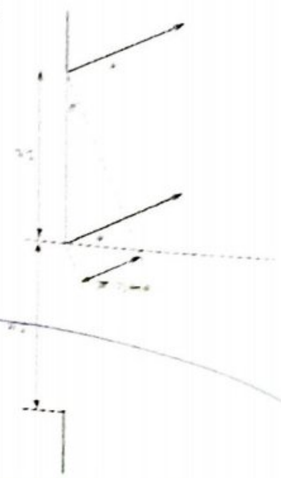
7. **Diffraction :** The bending of light around objects ~~(into what would otherwise be a shadowed region)~~ is known as diffraction. Diffraction occurs when light passes through very small apertures or near sharp edges. Diffraction is actually just interference, with the difference being only in the source of the interfering waves. Interference from a single slit, as in the figure here, is called diffraction.



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We can get an interference pattern with a single slit provided its size is approximately equal to the wavelength of the light (neither too small nor too large).

8. Let's work out the condition necessary for diffraction of light from a single slit. With reference to the figure, imagine that a wave is incident from the left. It will cause secondary waves to be radiated from the edges of the slit. If one looks at angle θ, the extra distance that the wave emitted from the lower slit must travel is $W\sin\theta$. If this is a multiple of the wavelength λ, then constructive interference will occur. So the condition becomes $W\sin\theta = m\lambda \quad \text{with} \quad m = \pm1, \pm2, \pm3,$. So, even from a single slit one will see a pattern of light and dark fringes when observed from the other side.



9. Light with wavelength of 511 nm forms a diffraction pattern after passing through a single slit of width 2.2×10^{-6}m. Find the angle associated with (a) the first and (b) the second bright fringe above the central bright fringe.

SOLUTION: For $m = 1$, $\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left(\frac{(1)(511\times10^{-9}\,m}{2.20\times10^{-6}\,m}\right) = 13.4°$

For $m = 2$, $\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left(\frac{(2)(511\times10^{-9}\,m}{2.20\times10^{-6}\,m}\right) = 27.7°$

10. Diffraction puts fundamental limits on the capacity of telescopes and microscopes to separate the objects being observed because light from the sides of a circular aperture interferes. One can calculate that the first dark fringe is at $\theta_{min} = 1.22\frac{\lambda}{D}$, where D is diameter of the aperture. Two objects can be barely resolved if the diffraction maximum of one object lies in the diffraction minimum of the second object. Clearly, the larger D is, the smaller the angular diameter separation. We say that larger apertures lead to better resolution.

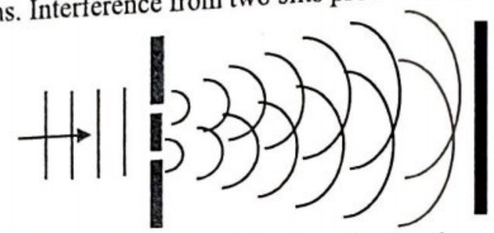


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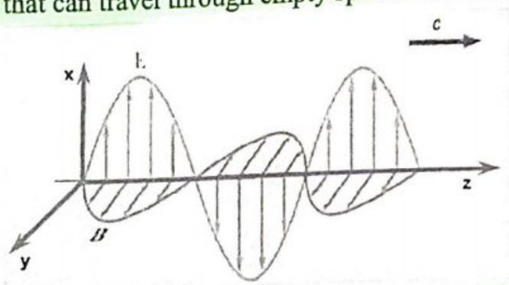
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Summary of Lecture 34 – THE PARTICLE NATURE OF LIGHT

1. In the previous lecture I gave you some very strong reasons to believe that light is waves. Else, it is impossible to explain the interference and diffraction phenomena that we see in innumerable situations. Interference from two slits produces the characteristic pattern.

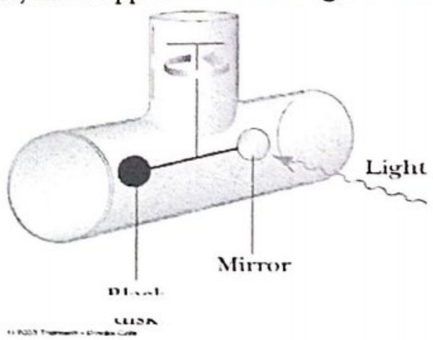


2. Light is waves, but waves in what? of what? The thought that there is some invisible medium (given the name *aether*) turned out to be wrong. Light is actually electric and magnetic waves that can travel through empty space. The electric and magnetic waves



are perpendicular to each other and to the direction of travel (here the z direction).

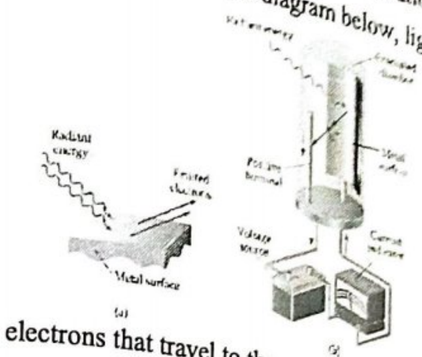
3. Electromagnetic waves transport linear momentum and energy. If the energy per unit volume in a wave is U then it is carrying momentum p, where $p = U/c$. Waves with large amplitude carry more energy and momentum. For the sun's light on earth the momentum is rather small (although it is very large close to or inside the sun). Nevertheless, it is easily measureable as, for example, in the apparatus below. Light strikes a mirror and rebounds.



Thus the momentum of the light changes and this creates a force that rotates the mirror. The force is quite small - just 5×10^{-6} Newtons per unit area (in metre^2) of the mirror.

1-Human can see flash about 30 Photons
The radiant reached retina is 2×10^3 Photons

4. So strong was the evidence of light as waves that observation of the photoelectric effect came as a big shock to everybody. In the diagram below, light hits a metal surface and

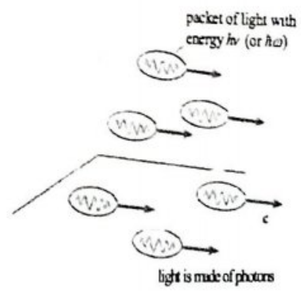


and knocks out electrons that travel to the anode. A current flows only as long as the light is shining. Above the threshold frequency, the number of electrons ejected depends on the intensity of the light. This was called the photoelectric effect. The following was observed:

a) The photoelectric effect does not occur for all frequencies ν; it does not occur at all when ν is below a certain value. *But classically (meaning according to the Maxwell nature of light as an electromagnetic wave) electrons should be ejected at any ν. If an electron is shaken violently enough by the wave, it should surely be ejected!*

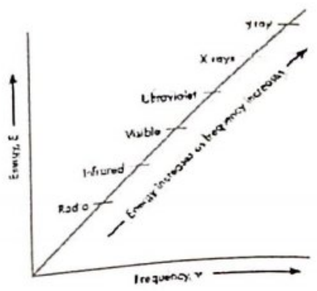
b) It is observed that the first photoelectrons are emitted instantaneously. *But classically the first photoelectrons should be emitted some time after the light first strikes the surface and excites its atoms enough to cause ionization of their electrons.*

5. Explanation of the photoelectric puzzle came from Einstein, for which he got the Nobel Prize in 1905. Einstein proposed that the light striking the surface was actually made of little packets (called quanta in plural, quantum in singular). Each quantum has an energy $\varepsilon = h\nu$ ~~(or $\varepsilon = \hbar\omega$ where $\hbar = h/2\pi$ and $\omega = 2\pi\nu$)~~ where h is the famous Planck's constant, 6.626×10^{-34} Joule-seconds. An electron is kicked out of the metal only when a quantum has energy (and frequency) big enough to do the job. It doesn't matter how many quanta of light - called photons - are fired at the metal. No photoelectrons will be released unless ν is large enough. Furthermore the photoelectrons are released immediately when the photon hits an electron.



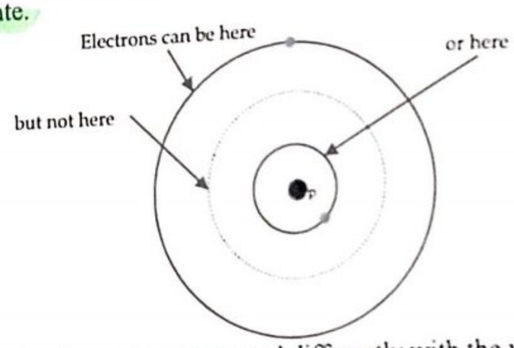
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6. Microwaves, radio and TV waves, X-rays, γ-rays, etc. are all photons but of very different frequencies. Because Planck's constant h is an extremely small number, the energy of each photon is very small.

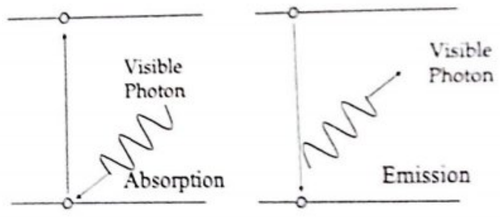


7. How many photons do we see? Here is a table that gives us some interesting numbers:
   a) Sunny day (outdoors): 10^{15} photons per second enter eye (2 mm pupil).
   b) Moonlit night (outdoors): 5×10^{10} photons/sec (6 mm pupil).
   c) Moonless night (clear, starry sky): 10^{8} photons/sec (6 mm pupil).
   d) Light from dimmest naked eye star (mag 6.5): 1000 photons/sec entering eye.

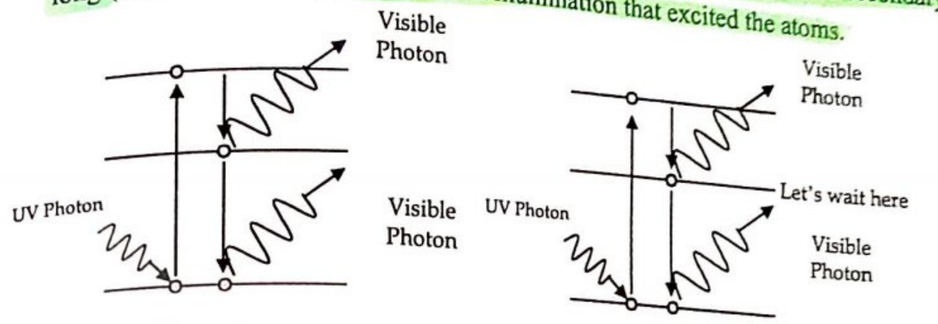
8. Where do photons come from? For this it is necessary to first understand that electrons inside an atom can only be in certain definite energy states. When an electron drops from a state with higher energy to one with lower energy, a photon is released whose energy is exactly equal to the difference of energies. Similarly a photon is absorbed when a photon of just the right energy hits an electron in the lower state and knocks it into a higher state.



The upper and lower levels can be represented differently with the vertical direction representing energy. The emission and absorption of photons is shown below.



9. Fluorescence and phosphorence are two phenomena observed in some materials. When they are exposed to a source of light of a particular colour, they continue to emit light of a different colour even after the source has been turned off. So these materials can be seen to glow even in the dark. In phosphorescence, a high-frequency photon raises an electron to an excited state. The electron jumps to an intermediate states, and then drops after a little while to the ground state. This is fluorescence. Phosphorescence is similar to fluorescence, except that phosphorescence materials continue to give off a secondary glow long (seconds to hours) after the initial illumination that excited the atoms.



10. One of the most important inventions of the 20th century is the laser which is short for:

    LASER ≡ Light Amplification by the Stimulated Emission of Radiation

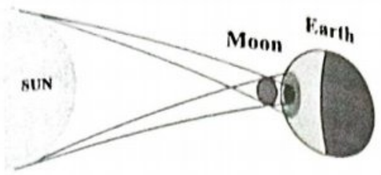
Lasers are important because they emit a very large number of photons all with one single frequency. How is this done? By some means - called optical pumping - atoms are excited to a high energy level. When one atom starts decaying to the lower state, it encourages all the others to decay as well. This is called spontaneous emission of radiation.

or How Lasers work?

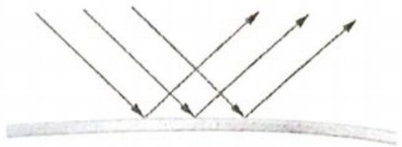
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Summary of Lecture 35 – GEOMETRICAL OPTICS

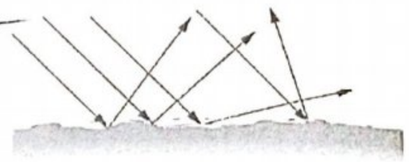
1. In the previous lecture we learned that light is waves, and that waves spread out from every point. But in many circumstances we can ignore the spreading (diffraction and interference), and light can then be assumed to travel along straight lines as rays. This is hown by the existence of sharp shadows, as for case of the eclipse illustrated here.



2. When light falls on a flat surface, the angle of incidence equals the angle of reflection. You can verify this by using a torch and a mirror, or just by sticking pins on a piece of paper in front of a mirror. But what if the surface is not perfectly flat? In that case, ~~as shown in fig. (b),~~ the angle of incidence and reflection are equal at every point, but the normal direction differs from point to point. This is called "diffuse reflection". Polishing a surface reduces the diffusiveness.

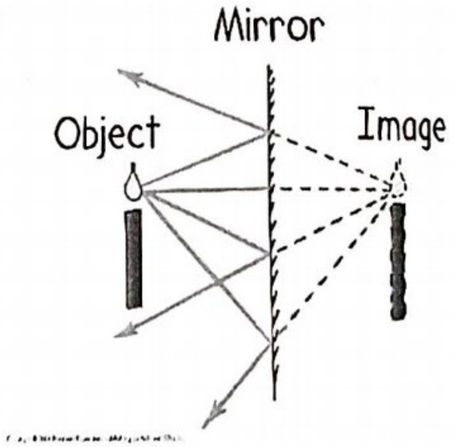


(a) Specular reflection

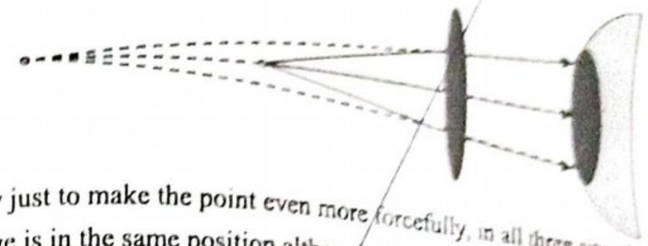


(b) Diffuse reflection

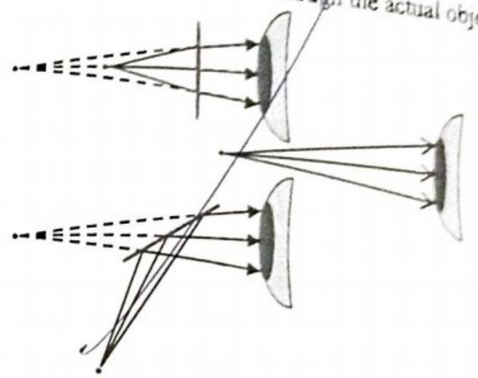
3. If you look at an object in the mirror, you will see its image. It is not the real thing, and that is why it is called a "virtual" image. You can see how the virtual image of a candle is formed in this diagram. At each point on the surface, there is an incident and reflected ray. If we extend each reflected ray backwards, it appears as if they are all coming from the same point. This point is the image of the tip of the flame. If we take other points on the candle, we will get their images in just the same way. This way we will have the image of the whole candle. The candle and its image are at equal distance from the mirror.



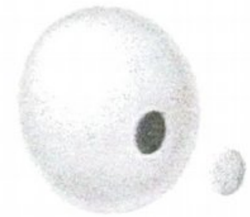
4. Here is another example of image formation. A source of light is placed in front of a bi-convex lens which bends the light as shown. The eye receives rays of light which seem to originate from a positon that is further away than the actual source.



Now just to make the point even more forcefully, in all three situations below, the virtual image is in the same position although the actual object is in 3 different places.



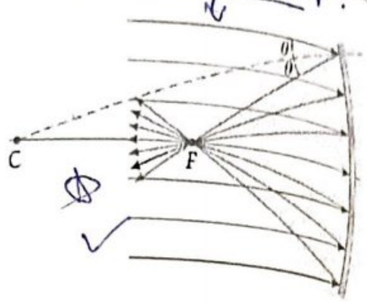
5. Imagine that you have a sphere of radius R and that you can cut out any piece you want. The outside or inside surface can be silvered, as you want. You can make spherical mirrors in this way. These can be of two kinds. In the first case, the silvering can be on the inside surface of the sphere, in which case this is is called a convex spherical mirror. The normal directed from the shiny surface to the centre of the sphere (from which it was cut out from) is called the principal axis, and the radius of curvature is *R*. The other situation is that in which the outside surface is shiny. Again, the principal axis the same, but now the radius of curvature (by definition) is - *R*. What does a negative curvature mean? It means precisely what has been illustrated - a convex surface has a positive and a concave surface has a negative curvature.



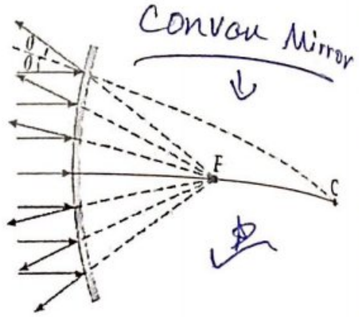


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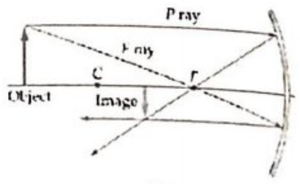
6. Now we come to the important point: a beam of parallel rays will reflect off a convex mirror and all get together (or converge) at one single point, called the focus. Now look at the figure. Since the incident and reflected angles are equal for every ray, you can see that the ray will have to pass through the focus at F, at distance $R/2$. We say that the focal length of such a mirror is $f = R/2$.



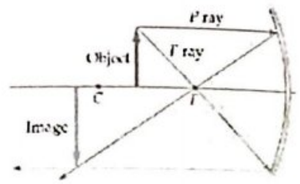
7. The concave mirror is different. A parallel beam incident from the left is reflected and the rays now spread out (diverge). However, if we were to look at any outgoing ray and extend it backward, all the rays would appear to be coming from one single point. This is the *virtual focus* because there is, in fact, no real source of light there. From the diagram you can see the virtual focus F is at distance $R/2$ *behind* the mirror. We say that the focal length of such a mirror is $f = -R/2$.



8. Now let's see how images are formed by concave mirrors. Take two rays that are emitted from the top of an object placed in front of the mirror. Call them the P and F ray, and see what happens to them after reflection. Where the two cross, that is the image point. As you can see the image is inverted and smaller than the actual object if the object is far away (i.e. lies to the left of C) and is larger if close to the mirror. The magnification of size makes this useful as a shaving mirror, among other things.

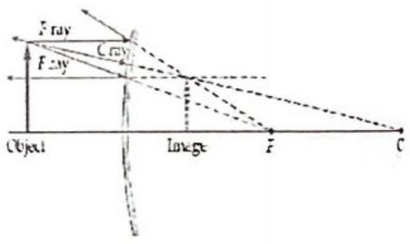


(a)



(b)

9. Now repeat for a concave mirror. Here the rays will never actually meet so we can have only a virtual object. Take 3 rays - call them P,F, and C - and see how they are reflected from the mirror's surface. Where they meet is the position of the image. The image is always smaller than the actual size of the object. This is obviously useful for driving a car because you can see a wide area.

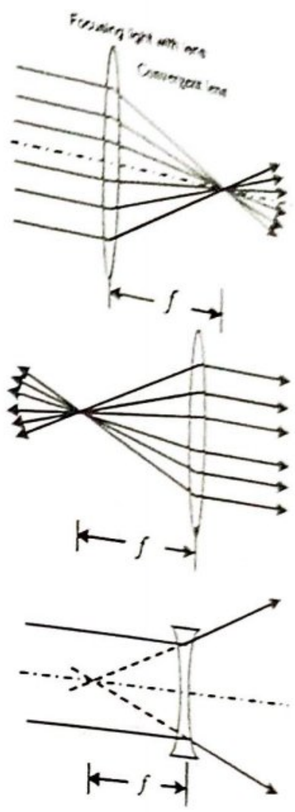


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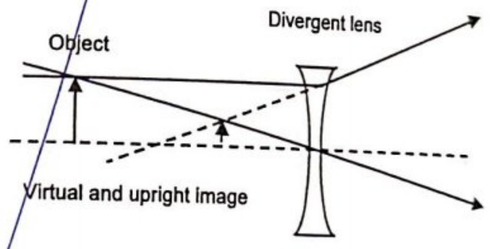
10. A lens is a piece of glass curved in a definite way. Because the refractive index of glass is bigger than one, every ray bends towards the normal. Here you see a double convex lens that focuses a beam of parallel rays. For a perfect lens, all the rays will converge to one single point that is (again) called the focus. The distance f is called the focal length.

Of course, light can equally travel the other way, so if a point source is placed at the focus of a convex lens then a parallel beam of light will emerge from the other side. This is how some film projectors produce a parallel beam.

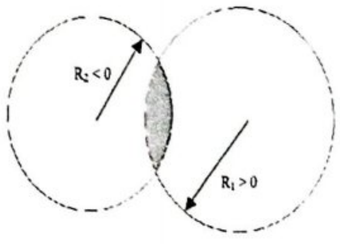
A concave lens does not cause a parallel beam to converge. On the contrary, it makes it diverge, as shown. Note, however, that if the rays are continued backwards, then they appear to come from one single point, which is here the virtual focus. The distance f is the focal length.



12. Here is how a concave (or divergent) lens forms an image. An observer on the side opposite to the object will see the image upright and smaller in size than the object.



13. A lens can be imagined as cut out of two spheres of glass as shown, with the spheres having radii R_1 and R_2. Note that they tend to bend light in opposite ways. By convention, R_2 is negative. It is possible to show that the focal length of the lens is , $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

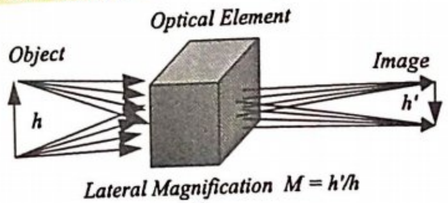


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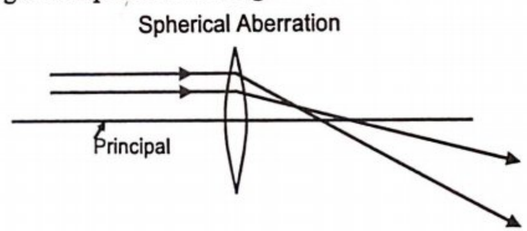
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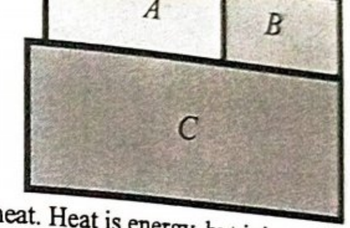
15. The "strength" of a lens is measured by its focal length. If the focal length is measured in metres, the diopter of the lens is defined as $D = 1/f$. If the refractive index of the glass in a lens is n, then the diopters due to the first interface D_1 and the second interface D_2 are, $D_1 = (n-1)/R_1$ and $D_2 = (1-n)/R_2$. The total diopter of the lens is $D = D_1 + D_2$.

16. For any optical system - meaning a collection of lenses and mirrors - we can define a magnification factor as a ratio of sizes – see the diagram below.



17. The perfect lens will focus a parallel beam of rays all to exactly the same focus. But no lens is perfect, and every lens suffers from aberration although this can be made quite small by following one lens with another. Below you see an example of "spherical aberration". Rays crossing different parts of the lens do not reach exactly the same focus. This distorts the image. Computers can design lens surfaces to minimize this aberration.





3. Now let us understand heat. Heat is energy, but it is a very special kind of energy: it is that energy which flows from a system at high temprature to a system at low temperature.

high T → heat → low T

Stated in a slightly different way: heat is the flow of internal energy due to a temperature difference. (Note that we do not have to know about atoms, molecules, and the internal composition of a body to be able to define heat - all that will come later).

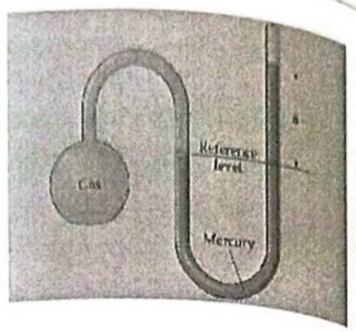
4. The term "thermal equilibrium" is extremely important to our understanding of heat. If some objects (~~say, a glass of water placed in the open atmosphere~~) are put in thermal contact but there is no heat exchange, then we say that the objects are in thermal equilibrium. So, if the glass of water is hot or cold initially, after sufficient time passes it will be in thermal equilibrium and will neither receive nor lose heat to the atmosphere.

5. We have an intuitive understanding of temperature, but how do we measure it? Answer: by looking at some physical property that changes when the temperature changes. So, for example, when the temperature rises most things expand, the electrical resistance changes, some things change colour, etc. These are called "thermometric properties".

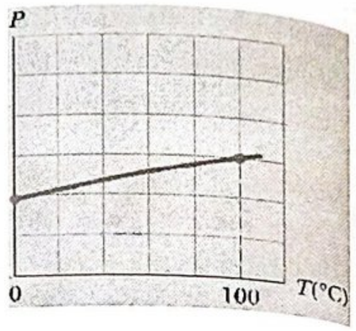
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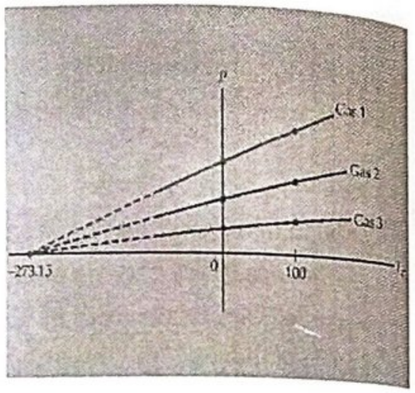
6. Let's take a practical example: the constant volume gas thermometer. Here the reference level is kept fixed by raising or lowering the tube on the right side (the tube below is made of rubber or some flexible material). So the gas volume is fixed. The gas pressure is $\rho \times g \times h$, and so the pressure is known from measuring h.



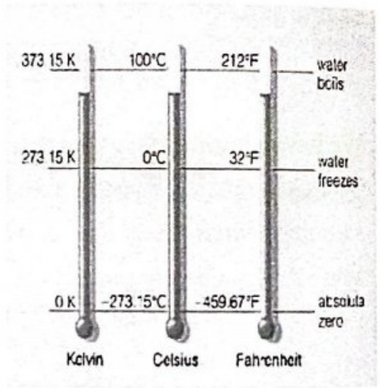
To find the temperature of a substance, the gas flask is placed in thermal contact with the substance. When the temperature is high, the pressure is large. From the graph of pressure versus temperature, you can easily read off the temperature. Note that we are using two fixed points, which we call 0^0C and 100^0C. This is called the Centigrade scale, and the two fixed points correspond to the freezing and boiling of water.



7. There is a temperature below which it is not possible to go. In other words, if you cool and cool there comes a point after which you cannot cool any more! How do we know this? Take different gases and plot how their pressure changes as you cool them down. You can see from the graph that all the lines, when drawn backwards, meet exactly at one point. This point is the absolute zero of temperature and lies at about -273.15^0C. This is also called 0^0K, or zero degrees Kelvin.



8. Temperature Scales. If you had been born 300 years ago, and if you had discovered a reliable way to tell different levels of "hotness", then maybe today there would be a temperature scale named after you! The relation between Centigrade, Fahrenheit, and Kelvin scales is illustrated to your right. You can see that absolute zero corresponds to -460^0K, -273.15^0C, and 0^0K. The Kelvin scale is the most suited for scientific purposes, and you should be careful to use this in all heat related calculations.

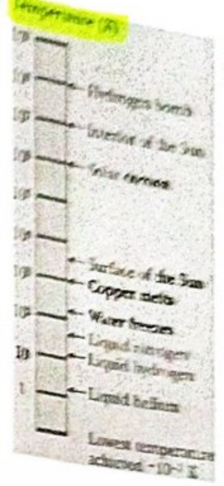


Conversion between degrees Celsius and degrees Fahrenheit: $T_F = (\frac{9}{5}°F/°C)T_C + 32°F$

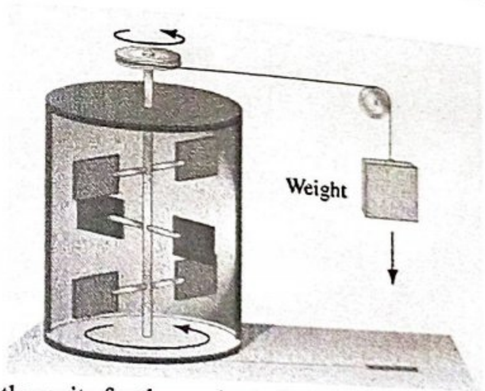
Conversion between degrees Fahrenheit and degrees Celsius: $T_C = (\frac{5}{9}°C/°F)(T_F - 32°F)$

Conversion between Celsius and Kelvin temperatures: $T = T_C + 273.15$

9. How hot is hot, and how cold is cold? Whenever a scientist says something is large or small, it is always relative to something. The hottest thing ever was the universe when it just came into existence (more about this in the last lecture). After this comes the hydrogen bomb (10^8K). The surface of the sun is not so hot, only 5500K. Copper melts around 1000K, water turns to steam at 373K. If you cool further, then all the gases start to solidify. The lowest temperature that has ever been achieved is $10^{-7}K$, which is one tenth of one millionth of one degree! Why not still lower? We shall later why this is not possible.



10. When you rub your hands, they get hot. Mechanical work has been converted into heat. The first person to investigate this scientifically was Joule. In the experiment below, he allowed a weight to drop. This turned a paddle that stirred up the water and caused the temperature to rise. The water got hotter if the weight was released from a greater height.



Joule established the units for the mechanical equivalent of heat. The units we use today are:

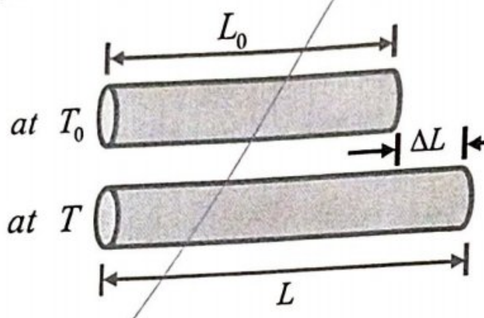
1 calorie (1 cal) raises the temperature of 1 g of water by 1 °C

1 cal = 4.186 Joule, 1 kilocalorie (1 kcal) = 1000 cal

Remember also that joule is the unit of work: when a force of one newton acts through a distance of one metre, the work done is one joule.

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11. Let's now consider one important effect of heat - most things expand when heated. Of course, our world is 3-dimensional but if there is a thin long rod then the most visible effect of heating it is that the rod increases in length. By how much?



Look at the above diagram. Call the length of the rod at T_0 as L_0. When the rod is heated to T, then the length increases to $L = L_0\left[1+\alpha\left(T-T_0\right)\right]$. The difference of the lengths is, $L - L_0 = \alpha L_0\left(T - T_0\right)$, or $\Delta L = \alpha L_0 \Delta T$. Here α is just a dimensionless number that tells you how a particular material expands. It is called the coefficient of linear expansion. If α was zero, then the material would not expand at all. You can also write it as $\alpha = \frac{\Delta L / L_0}{\Delta T}$.

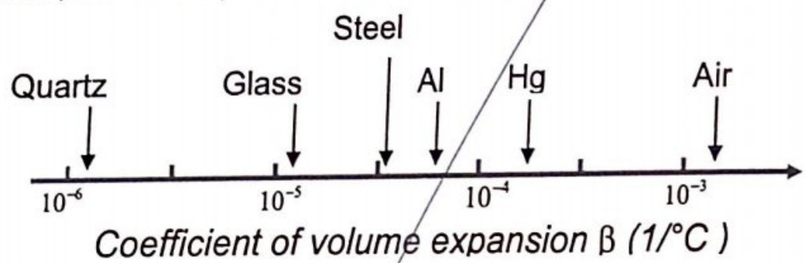
Similarly, define a coefficient of volume expansion – call it β – as $\beta \equiv \frac{\Delta V / V_0}{\Delta T}$.

12. There is a relation between α and β, the linear and volume coefficients. Let's look at the change in volume due to expansion: $V' = (L + \Delta L)^3 = (L + \alpha L \Delta T)^3$

$$= L^3 + 3\alpha L^3 \Delta T + 3\alpha^2 L^3 \Delta T^2 + \alpha^3 L^3 \Delta T^3$$

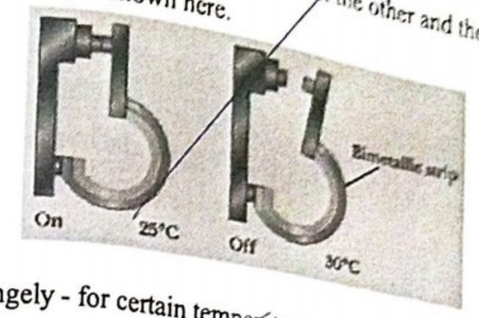
$$\approx L^3 + 3\alpha L^3 \Delta T = V + 3\alpha V \Delta T$$

We are only looking for small changes, so the higher terms in ΔT can be safely dropped. Hence $\Delta V = 3\alpha V \Delta T$. From the definition, this we immediately see that $\boxed{\beta = 3\alpha}$. Just to get an idea, here is what β looks like for various different materials:

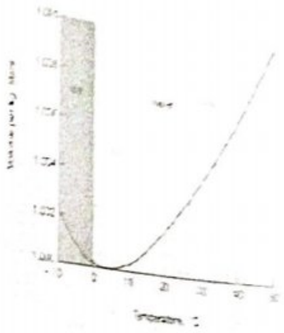


13. We can use the fact that different metals expand at different rates to make thermostats. For example, you need a thermostat to prevent an electric iron from getting too hot or a

refrigerator from getting too cold. In the diagram below, one metal is bonded to another. When they expand together, one expands less than the other and the shape is distorted. This breaks off a circuit, as shown here.



14. Water behaves strangely - for certain temperatures, it contracts when heated (instead of expanding)! Look at the graph: between 0^0C and 4^0C, it exhibits strange or *anomalous* behaviour. The reason is complicated and has to do with the molecular structure of water. If water behaved normally, it would be very bad for fish in the winter because they rely upon the bottom of the lake or sea to remain liquid even though there the surface is frozen.



15. We define the "heat capacity" of a body as $C = \frac{Q}{\Delta T}$, where ΔT is the increase in temperature when an amount of heat Q is added to the body. Heat capacity is always positive; Q and ΔT have the same sign. The larger the heat capacity, the smaller is the change in the body's temperature when a fixed amount of heat is added. In general, $Q = mc\Delta T$, where Q = heat added, m = mass, c = specific heat, and ΔT = change in temperature. Water has a very large specific heat c, $c = 1.0\ cal/(°C\ g)$; this means it takes one calorie to raise the temperature of 1 gm of water by 1 degree Celsius. In joules per kilogram this is the same as 4186. In the table are the specific heats of various common materials. You can see that metals have small c, which means that it is relatively easy to raise or lower their temperatures. The opposite is true of water. Note also that steam and ice have smaller c's than water. This shows that knowing the chemical composition is not enough.

ΔT = increase in temp
Q = Amount of heat

Specific Heats at Atmospheric Pressure

Substance	Specific heat, c [J / (kg · K)]
Water	4186
Ice	2092
Steam	2010
Beryllium	1820
Air	1004
Aluminum	900
Glass	837
Silicon	703
Iron (steel)	448
Copper	387
Silver	234
Gold	129
Lead	128

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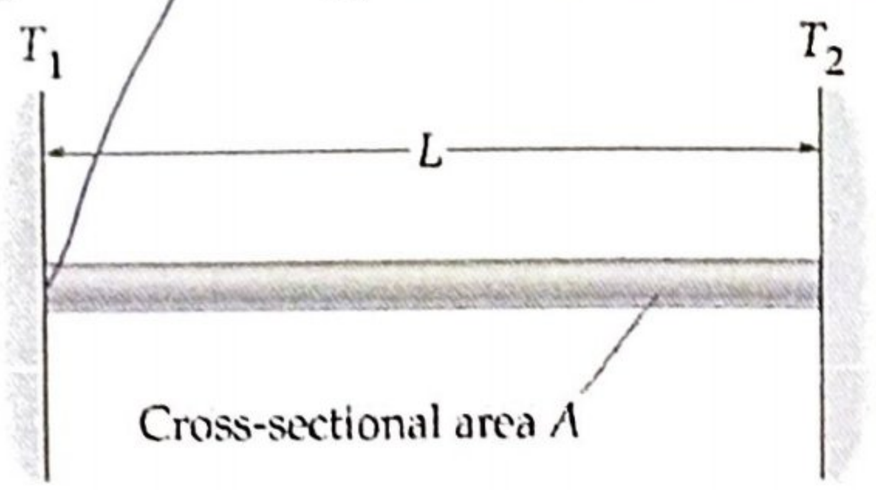
16. A 0.5-kg block of metal with an initial temperature of $30.0^{\circ}C$ is dropped into a container holding 1.12 kg of water at $20.0^{\circ}C$. If the final temperature of the block-water system is $20.4^{\circ}C$, what is the specific heat of the metal?

SOLUTION: Write an expression for the heat flow out of the block $Q_{block} = m_b c_b (T_b - T)$. Do the same for water, $Q_{water} = m_w c_w (T - T_w)$. Now use the fact that all the energy that is lost by the block is gained by the water: $Q_{block} = Q_{water} \Rightarrow Tm_b c_b (T_b - T) - m_w c_w (T - T_w) = 0$

From this, $c_b = \dfrac{m_w c_w (T - T_w)}{m_b (T_b - T)} = \dfrac{(1.12kg)[4186J/(kg \cdot K)](20.4^{\circ}C - 20.0^{\circ}C)}{(0.500kg)(30.0^{\circ}C - 20.4^{\circ}C)}$

$= 391J/(kg \cdot K)$

17. When lifting a "daigchee" from a stove, you would be wise to use a cloth. Why? Because metals transfer, or conduct, heat easily whereas cloth does not. Scientifically we define conductivity using experiments and apparatus similar to the following:



Heat flows from the hotter to the colder plate. Let us use the following symbols:

k = thermal conductivity
Q = heat transferred
A = cross sectional area
t = duration of heat transfer
L = length
ΔT = temperature difference

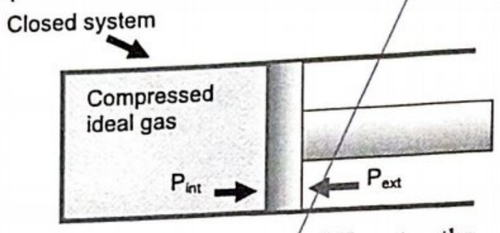
Then the heat transferred in time t is, $Q = kA\left(\dfrac{\Delta T}{L}\right)t$. This formula allows us to measure k if all the other quantities in it are measured.

18. Conduction is one possible way by which heat is transferred from one portion of a system to another. It does not involve physical transport of particles. However, there is another way by which heat can be transferred - convection. In convection, heat is carried by a moving fluid. So when you heat a pot of water, molecules at the bottom move up, and the ones at the top come down - the water has currents inside it that transfer heat. Another mechanism for transferring heat is through radiation. We have already talked about this while discussing blackbody radiation and the Stefan-Boltzman Law.

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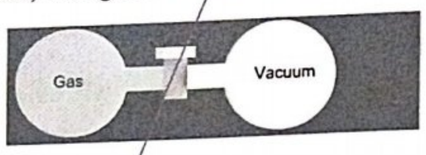
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4. Let us calculate the work done by an expanding gas. In the diagram below, suppose the piston moves out by distance dx, then the work done is $dw = Fdx$ where F is the force exerted on the gas. But $F = P \times A = -P_{ext} \times A$. (The minus sign is important to understand; remember that the force exerted by the gas on the piston is the negative of the force that the piston exerts on the gas). So $dw = Fdx = -P_{ext} \times A \times dx = -P_{ext} dV$. The work done by the expanding gas is $w = -\int_{V_1}^{V_2} P_{ext} dV$ where V_1 and V_2 are the initial and final volumes.

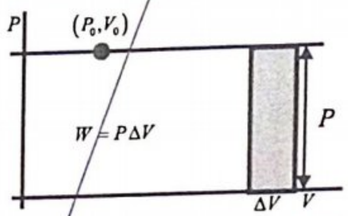


Note that w depends on the path. Let's consider 3 different paths.

a)Suppose that the gas expands into the vacuum, $P_{ext} = 0$. Then $w = 0$. So, if in the figure below, the valve is opened, then gas will flow into the vacuum without doing work.



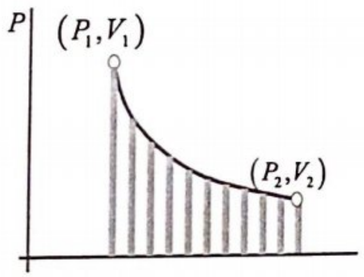
b)Suppose the pressure outside has some constant value. Then, $w = P_{ext} \times (V_2 - V_1)$ or, $w = P_{ext} \Delta V$. You can see that this is just the area of the rectangle below.



c)We can also let the gas expand so that the internal and external pressures are almost the same (this is called reversible expansion). Then the work done *on* the system is: $dw = -P_{ext} dV$. Using $PV = Nk_B T$ gives,

$$w = -\int_{V_1}^{V_2} \frac{Nk_B T}{V} dV = -Nk_B T(\ln V_2 - \ln V_1) = -Nk_B T \ln \frac{V_2}{V_1}$$

Of course, the work done *by* the gas as it expands is positive since V_2 is larger than V_1. Remember that the log function is positive if its argument is bigger than 1.



5. The internal energy of a gas depends only on the number of molecules it contains and on the temperature, $E = \frac{3}{2} Nk_B T$. In a free expansion of gas (it doesn't do any work in this case), the initial and final internal energies are equal, $E_i = E_f$.

6. Imagine that a substance is heated while keeping its volume constant. Obviously, you have to supply more heat as you raise the temperature higher, $\Delta Q_V = C_V \Delta T$, where C_V is given the name "specific heat at constant volume". No work is done since there is no expansion, $\Delta W = 0$. Hence, using the First Law, $\Delta Q_V = \Delta E + \Delta W = \Delta E$, and so $\Delta E = C_V \Delta T$.

7. Now take the same system as above, but allow it to expand at constant pressure as it is heated. Then you will have to supply an amount of heat, $\Delta Q_P = C_P \Delta T$. Here, C_P is called the specific heat at constant pressure, $\Delta Q_P = \Delta E + \Delta w = \Delta E + P\Delta V$. Hence, $C_P \Delta T = \Delta U + P\Delta V$. If we supply only a small amount of heat then, $C_P dT = dU + PdV$. This gives, $C_P dT = C_V dT + PdV$. If we consider the special case of an ideal gas, then $PV = Nk_B T$. Hence, $PdV = Nk_B dT$ and $C_P dT = C_V dT + Nk_B dT$. Hence, $C_P = C_V + Nk_B$. Since the internal energy is $E = \frac{3}{2} Nk_B T$, it follows that $C_V = \frac{dU}{dT} = \frac{3}{2} Nk_B$. From this, the specific heat at constant pressure is: $C_P = \frac{3}{2} Nk_B + Nk_B = \frac{5}{2} Nk_B$. From the equation $C_P = C_V + Nk_B$, it follows that $\frac{Nk_B}{C_V} = \frac{C_P - C_V}{C_V}$. Let us define a new symbol, $\frac{C_P}{C_V} \equiv \gamma$.

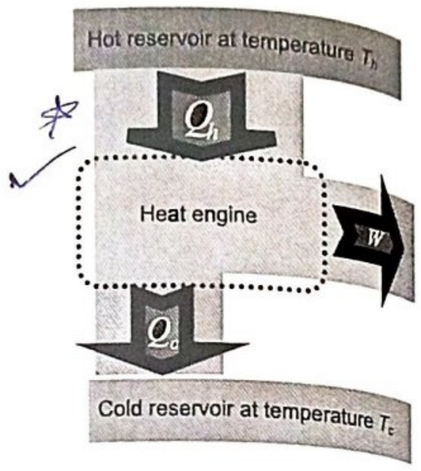
8. We can find the EOS of an ideal gas when it expands without losing any heat (this is called adiabatic expansion). In this case, $dQ = dE + dW = 0$. Hence, $C_V dT + PdV = 0$, or $C_V dT + Nk_B T \frac{dV}{V} = 0$. Dividing by dT gives, $\frac{dT}{T} + \frac{Nk_B}{C_V} \frac{dV}{V} = 0$ or $\frac{dT}{T} + (\gamma - 1)\frac{dV}{V} = 0$. Integrating this gives, $\ln T + (\gamma - 1) \ln V = \text{Constant}$. Equivalently, $\ln(TV^{\gamma - 1}) = \text{Constant}$. A more convenient form is: $TV^{\gamma - 1} = \text{Constant}$. This holds for the entire adiabatic expansion.

9. Almost nothing beats the importance of the Second Law Of Thermodynamics. There are many equivalent ways of stating it. The one I like is: "There can be no process whose only final result is to transfer thermal energy from a cooler object to a hotter object". This seems extremely simple (and almost useless), but in fact it tells you, among other things, that no perpetual motion machine (such as that which generates electricity without any fuel input) can ever be built. If someone could build such a machine, then it could also be used to transfer heat from a cold object to a hot object - a contradiction with the Second Law.

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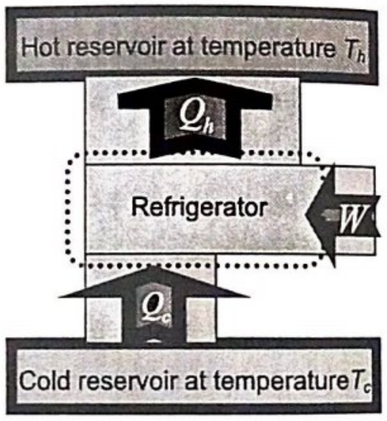
10. Before we consider some implications of the Second Law, I want to introduce the concept of a *thermal reservoir*. Suppose you put a thermometer in your mouth. You will transfer a small amount of heat to the thermometer, but because your body is so big this will make no measureable difference to your body temperature. Your body therefore is a thermal reservoir as far as the thermometer is concerned. More generally, any large mass of material will act as a thermal reservoir if it is at constant temperature. A *heat engine* (e.g. a car engine, refrigerator, etc) operates between reservoirs at two different temperatures.

11. A heat engine works between a high temperature T_H and a low temperature T_c. It aborbs heat $Q_{in,h}$ from the hot reservoir and rejects heat $Q_{out,c}$ into the cold reservoir. Since the internal energy of the engine does not change, $\Delta U = 0$ and the First Law gives simply, $Q = \Delta U + W = W$. The work done by the engine is $W = Q_{in,h} - Q_{out,c}$. Now define the efficiency of the machine as $\varepsilon = \dfrac{\text{work done by engine}}{\text{heat put in}} = \dfrac{W}{Q_{in,h}}$. Then



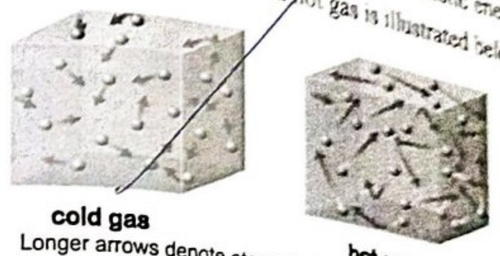
$\varepsilon = \dfrac{Q_{in,h} - Q_{out,c}}{Q_{in,h}} = 1 - \dfrac{Q_{out,c}}{Q_{in,h}}$. Now, the heat that is transferred out of or into a reservoir is proportional to its absolute temperature. Hence we arrive at the important result that $\varepsilon = 1 - \dfrac{T_C}{T_H}$. This number is always less than one, showing that no machine can convert all the input heat into useful work. As an example, a nuclear reactor has temperature $300^0 C$ at the core and rejects heat into a river at $30^0 C$. The maximum efficiency it can have is $\varepsilon = 1 - \dfrac{30+273}{300+273} = 0.471$

12. A refrigerator is a heat engine working in reverse. Work is done (by an electric motor) to pump heat from a cold reservoir (inside of refrigerator) to the hot exterior. The Second Law can be shown to imply the following: it is impossible for a refrigerator to produce no other effect than the transfer of thermal energy from a cold object to a hot object. Again, the efficiency of a refrigerator is always less than one.



Summary of Lecture 38 – THERMAL PHYSICS III

1. In the two previous lectures, I concentrated exclusively on heat as a form of energy that flows from a hot to a cold body. I made no mention of the fact that all matter is made of atoms and molecules; the notion of heat would exist even if we did not know this. But, the modern understanding of heat is that it is the random kinetic energy of atoms. So, for example, the difference between a cold and hot gas is illustrated below



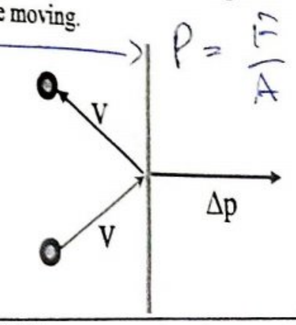
cold gas **hot gas**

Longer arrows denote atoms that are moving faster
The hotter gas has, on the average, faster moving atoms.

2. The study of heat, considered as arising from the random motion of the basic constituents of matter, is an area of physics called statistical mechanics. Its goal is the understanding and prediction of macroscopic phenomena, and the calculation of macroscopic properties from the properties of individual molecules. Temperature is the average energy related to the speed of atoms in an object, and heat is the amount of energy transferred from one object to another.

3. Imagine a gas so dilute that atoms rarely collide with each other (this is also called an ideal gas). We can readily understand why the pressure is directly proportional to the temperature for a gas confined to a box: increasing the temperature T makes gas molecules move faster, striking the walls of the container harder and more often, thus giving an increase in pressure P, i.e. $P \propto T$. On the other hand, for an ideal gas at constant pressure, the volume is directly proportional to the temperature : $V \propto T$. So, as the air in a balloon is heated up, its volume will increase in direct proportion because of the impact of the atoms on the balloon walls. Note that this is where the "absolute" or Kelvin scale comes from: at 0° K an ideal gas would have zero volume because the atoms would not be moving.

4. Pressure is related to the outward force per unit area exerted by the gas on the container wall. Newton's 2nd Law is: $F = \dfrac{\Delta p}{\Delta t}$. Here Δp is to be understood as the momentum destroyed in time Δt. Using this, we can do a little calculation to find the pressure in an ideal gas.



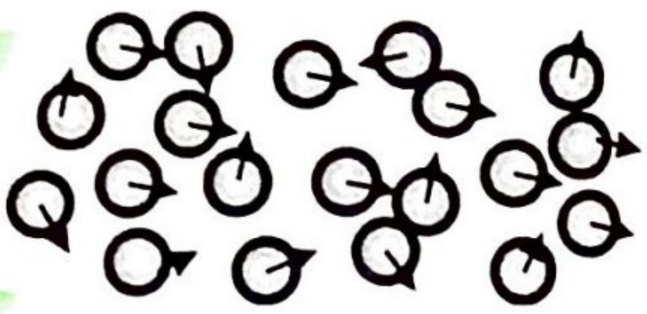
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Suppose there are N atoms in a box of volume, and the average speed of an atom in the x direction is v_x. By symmetry, half are moving in the $+x$ and half in the $-x$ directions. So the number of atoms that hit the wall in time Δt is $\frac{N}{2}\frac{1}{V}v_x\Delta t A$. Hence the change in momentum is $\Delta p = (2mv_x)\times\left(\frac{1}{2}\frac{N}{V}v_x\Delta t A\right) = \frac{N}{V}mv_x^2 A\Delta t$. From this, we can calculate the pressure $P = \frac{F}{A} = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{N}{V}mv_x^2$. Hence $PV = Nmv_x^2 = 2N\left(\frac{1}{2}mv_x^2\right)_{av}$.

5. Let us return to our intuitive understanding of heat as the random energy of small particles. Now, $\left(\frac{1}{2}mv_x^2\right)_{av}$ is the average kinetic energy of a particle on account of its motion in the x direction. This will be greater for higher temperatures, so $\left(\frac{1}{2}mv_x^2\right)_{av} \propto T$, where T is the absolute temperature. So we write $\left(\frac{1}{2}mv_x^2\right)_{av} = k_B T$, which you can think of as the definition of the Boltzmann constant, k_B. So we immediately get $PV = Nk_B T$, the equation obeyed by an ideal gas (dilute, no collisions). Now, there is nothing special about any particular direction, so $\left(v_x^2\right)_{av} = \left(v_y^2\right)_{av} = \left(v_z^2\right)_{av}$ and therefore the average total velocity is $\left(v^2\right)_{av} = \left(v_x^2\right)_{av} + \left(v_y^2\right)_{av} + \left(v_z^2\right)_{av} = 3\left(v_x^2\right)_{av}$. So the total average kinetic energy of one atom is, $K_{av} = \left(\frac{1}{2}mv^2\right)_{av} = \frac{3}{2}k_B T$, and the energy of the entire gas is $K = N\left(\frac{1}{2}mv^2\right)_{av} = \frac{3}{2}Nk_B T$.

From this, the mean squared speed of atoms in a gas at temperature is $\left(v^2\right)_{av} = \frac{3k_B T}{m}$.

6. Heat, which is random kinetic energy, causes changes of phase in matter:
   a) Water molecules attract each other, but if water is heated then they can escape and water becomes steam.
   b) If steam is heated, the water molecules break up and water becomes separated hydrogen and oxygen atoms.
   c) If a gas of atoms gets hot enough, then the atoms collide so violently that they lose electrons (i.e. get ionized).
   d) More heat (such as inside the core of a star) will break up atomic nuclei into protons and neutrons.



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9. A very important concept is that of *entropy*. It was first introduced in the context of thermodynamics. Then, when the statistical nature of heat became clear a century later, it was understood in very different terms. Let us begin with thermodynamics: suppose a system is at temperature T and a small amount of heat dQ is added to it. Then, the small increase in the entropy of the system is $dS = \frac{dQ}{T}$. What if we keep adding little bits of heat? Then the temperature of the system will change by a finite amount, and the change in entropy is got by adding up all the small changes: $\Delta S = \int_{T_1}^{T_2} \frac{dQ}{T}$.

Now let's work out how much the entropy changes when we add heat dQ to an ideal gas. From the First Law, $dQ = dU + dW = dU + PdV = C_V dT + Nk_B T\frac{dV}{V}$. The entropy change is $dS = \frac{dQ}{T} = C_V \frac{dT}{T} + Nk_B \frac{dV}{V}$, and for a finite change we simply integrate,

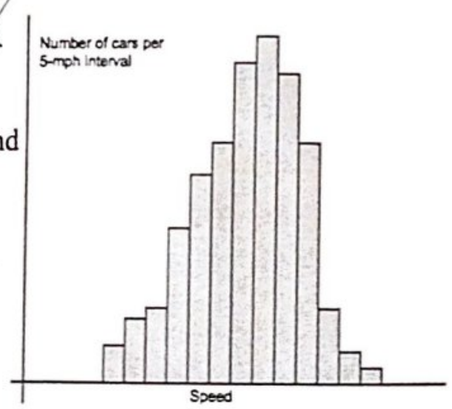
$\Delta S = \int \frac{dQ}{T} = C_V \ln\frac{T_2}{T_1} + Nk_B \ln\frac{V_2}{V_1} = \frac{3}{2}Nk_B \ln\frac{T_2}{T_1} + Nk_B \ln\frac{V_2}{V_1}$. Note that the following:

a) The entropy increases if we heat a gas ($T_2 > T_1$).

b) The entropy increases if the volume increases ($V_2 > V_1$).

10. In statistical mechanics, we interpret entropy as the degree of disorder. A gas with all atoms at rest is considered ordered, while a hot gas having atoms buzzing around in all directions is more disordered and has greater entropy. Similarly, as in the above example, when a gas expands and occupies greater volume, it becomes even more disordered and the entropy increases.

11. We always talk about averages, but how do you define them mathematically? Take the example of cars moving along a road at different speeds. Call $n(v_i)$ the number of cars with speed v_i, then $N = \sum_i n(v_i)$ is the total number of cars and the average speed is defined as $\bar{v} = \frac{\sum_i v_i n(v_i)}{\sum_i n(v_i)}$.



We define the probability of finding a car with speed v_i as $P(v_i) \equiv \frac{n(v_i)}{\sum_i n(v_i)} = \frac{n(v_i)}{N}$. We can also write the average speed as, $\bar{v} = \sum_i v_i P(v_i)$.

Summary of Lecture 39 – SPECIAL RELATIVITY I



1. The Special Theory of Relativity was created by Albert Einstein, when he was a very young man in 1905. It is one of the most solid pillars of physics and has been tested thousands of times. Special Relativity was a big revolution in understanding the nature of space and time, as well as mass and energy. It is absolutely necessary for a proper understanding of fast-moving particles (electrons, photons, neutrinos..). Einstein's General Theory of Relativity - which we shall not even touch here - goes beyond this and also deals with gravity.

2. Relativity deals with time and space. So let us first get some understanding of how we measure these two fundamental quantities:

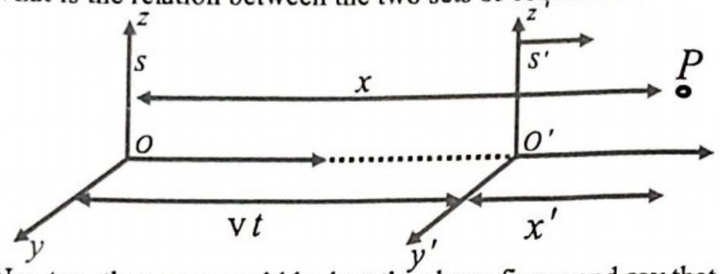
a) *Time*: we measure time by looking at some phenomenon that repeats itself. There are endless examples: your heart beat, a pendulum, a vibrating quartz crystal, rotation of the earth around its axis, the revolution of the earth about the sun,... These can all be used as clocks. Of course, an atomic clock is far more accurate than using your heart beat and is accurate to one part in a trillion. Although different systems of measurement have different units it is fortunate that time is always measured in seconds.

b) *Distance*: intuitively we know the difference between short and long. But to do a measurement, we first have to agree on what should be the unit of length. If you use a metre as the unit, then you can use a metre rod and measure any length you want. Of course, sometimes we may use more sophisticated means (such as finding how high a satellite is) but the basic idea is the same: the distance between point A and point B is the number of metre rods (or fractions thereof) that can be made to fit in between the two points.

3. Newton had believed that there was one single time for the entire universe. In other words, time was absolute and could be measured by one clock held somewhere in the centre of the universe. Similarly, he believed that space was absolute and that the true laws of physics could be seen in that particular frame which was fixed to the centre of the universe. (As we shall see, Einstein shocked the world by showing that time and space are not absolute quantities, but depend on the speed of your reference frame. Even more shocking was his proof that time and space are not entirely separate quantities!)

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4. An *event* is something that happens at some point in space at some time. With respect to the frame S below, the event P happened at (x,y,z,t). Now imagine a girl running to the right at fixed speed v in frame S′. According to her, the same event P happened at (x′,y′,z′,t′). What is the relation between the two sets of coordinates?



If you were Newton, then you would look at the above figure and say that obviously it is the following: $x' = x - vt,\ y' = y,\ z' = z,\ t' = t$. These are called *Galilean coordinate transformations*. Note that it is assumed here that the time is the same in both frames because of the Newtonian belief that there is only one true time in the world.

5. There are certain obvious consequences of using the Galilean transformations. So, for example a rod is at rest in S-frame. The length in S-frame $= x_B - x_A$, while the length in the S′-frame $= x'_B - x'_A = x_B - x_A - v(t_B - t_A)$. Since $t_B = t_A$, the length is the same in both frames: $x'_B - x'_A = x_B - x_A$. (As we shall see later this will not be true in Einstein's Special Relativity).



6. Let us now see what the Galilean transformation of coordinates implies for transformations of velocities. Start with $x' = x - vt$ and differentiate both sides. Then $\frac{dx'}{dt} = \frac{dx}{dt} - v$. Since $t = t'$, it follows that $\frac{dx'}{dt} = \frac{dx'}{dt'}$ and so $\frac{dx'}{dt'} = \frac{dx}{dt} - v$. Similarly, $\frac{dy'}{dt'} = \frac{dy}{dt}$ and $\frac{dz'}{dt'} = \frac{dz}{dt}$.

Now, $\frac{dx'}{dt'} = u'_x$ is the x-component of the velocity measured in S′-frame, and similarly $\frac{dx}{dt} = u_x$ is the x-component of the velocity measured in S-frame. So we have found that:

$$u'_x = u_x - v,\quad u'_y = u_y,\quad u'_z = u_z \text{ (or, in vector form, } \vec{u}' = \vec{u} - \vec{v})$$

Taking one further derivative, $\frac{du'_x}{dt'} = \frac{d}{dt}(u_x - v) = \frac{du_x}{dt}$ (remember that v = constant), we find that the components of acceleration are the same in S and S′:

$$\frac{du'_x}{dt'} = \frac{du_x}{dt},\quad \frac{du'_y}{dt'} = \frac{du_y}{dt},\quad \frac{du'_z}{dt'} = \frac{du_z}{dt}.$$

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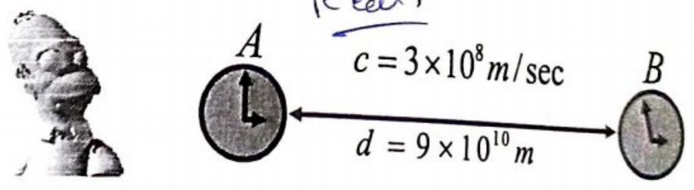
7. We had learned in a previous chapter that light is electromagnetic wave that travels at a speed measured to be $c = 2.907925 \times 10^8\ m/sec$. Einstein, when he was 16 years old, asked himself the question: in which frame does this light travel at such a speed? If I run holding a torch, will the light coming from the torch also move faster? Yes, if we use the formula for addition of velocities derived in the previous section! So if light actually travels at $2.907925 \times 10^8\ m/sec$ then it is with respect to the frame in which the "aether" (a massless fluid which we cannot feel) is at rest. But there is no evidence for the aether!

8. Einstein made the two following postulates (or assumptions).
   1) The laws of physics have the same form in all inertial frames. (In other words, there is no constant-velocity frame which is preferred, or better, than any other).
   2) The speed of light in vacuum has the same value in all inertial systems, independent of the relative motion of source and observer.

[Note: as stressed in the lecture, no postulate of physics can ever be mathematically proved. You have to work out the consequences that follow from the postulates to know whether the postulates are good ones or not.]

9. Let's first get one thing clear: in any one inertial frame, we can imagine that there are rulers and clocks to measure distances and times. We can synchonize all the clocks to read one time, which will be called the time in that frame S. But how do we do this? If are two clocks, then we can set them to read the same time (i.e. synchronize them) by taking account of the time light takes to travel between them.

Example: The observer with clock A sees the time on clock B as 2:55pm. But he knows that light took 5 minutes to travel from B to A, and therefore A and B are actually reading exactly the same time.



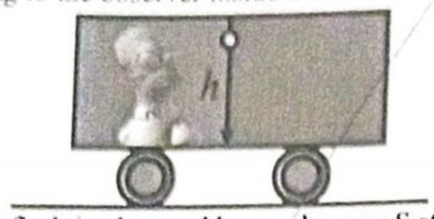
(Time taken by light in going from B to A is equal to $\frac{d}{c} = \frac{9 \times 10^{10}\ m}{3 \times 10^8\ m/sec} = 300\,sec = 5\,min.$)

10. Now I shall derive the famous formula which shows that a moving clock runs slow. This will be something completely different from the older Newtonian conception of time. Einstein derived this formula using a "gedanken" experiment, meaning an experiment which can imagine but not necessarily do. So imagine the following: a rail carriage has a

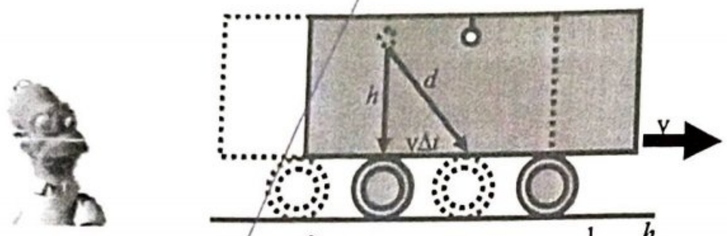
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a bulb that is fixed to the ceiling. The bulb suddenly flashes, and the light reaches the floor. Time taken according to the observer inside the train is $\Delta t' = h/c$.



Now suppose that the same flash is observed by an observer S standing on the ground. According to S, the train is moving with speed v. Let's consider the same light ray. Clearly, the train has moved forward between the time when the light left the ceiling and when it reached the floor. According to S, the time taken is $\Delta t = \frac{d}{c} = \frac{\sqrt{h^2 + (v\Delta t)^2}}{c}$. Now

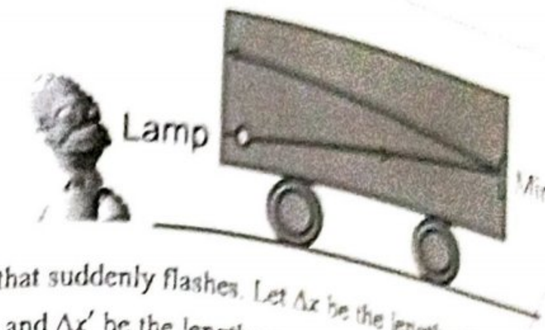


square both sides: $(c\Delta t)^2 = h^2 + (v\Delta t)^2$, which gives $\Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{h}{c} = \gamma\Delta t'$. Here γ is the relativistic factor, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and is a number that is always bigger than one. As v gets closer and closer to c, the value of γ gets larger and larger. For v=4c/5, γ=5/3. So, if 1 sec elapses between the ticks of a clock in S′ (i.e. $\Delta t' = 1$), the observer in S will see 5/3 seconds between the ticks. In other words, he will think that the moving clock is slow!

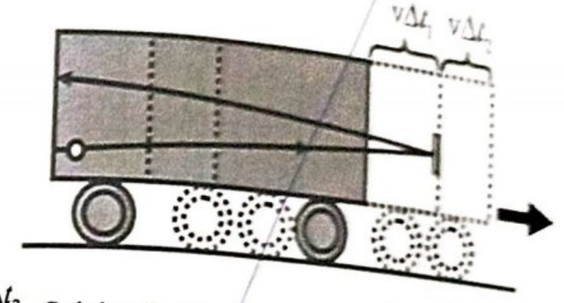
11. The muon is an unstable particle. If at rest, it decays in just 10^{-18} seconds. But if traveling at 3/5 the speed of the light, it will last 25% longer because $\gamma = \frac{1}{\sqrt{1-(3/5)^2}} = \frac{5}{4}$. If it is traveling at v=0.999999c, it will last 707 times longer. We can observe these shifts due to time dilation quite easily, and they are an important confirmation of Relativity.

12. Another amazing prediction of Relativity is that objects are shortened (or contracted) along the direction of their motion. Einstein reached this astonishing conclusion on the basis of yet another gedanken experiment. Again, consider a moving railway carriage with a bulb

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at one end that suddenly flashes. Let Δx be the length of the carriage according to ground observer S, and $\Delta x'$ be the length according to the observer S′ inside the carriage. So, $\Delta t' = \frac{2\Delta x'}{c}$ is the time taken for the light to go from one end to the other, and then return after being reflected by a mirror. Now let's look at this from the point of view of S, who is fixed to the ground. Let Δt_1 be the time for the signal to reach the front end. Then, because the mirror is moving forward, $\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c}$. Call Δt_2 the return time. Then,



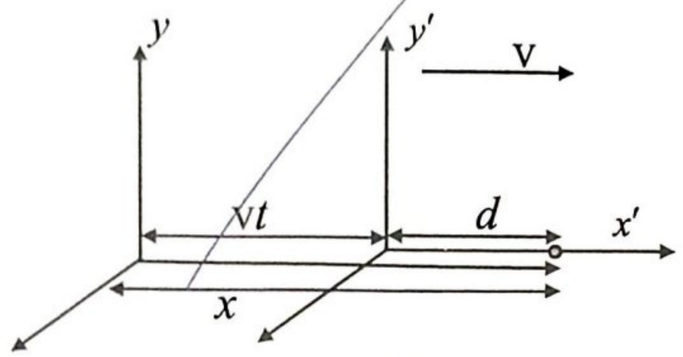
$\Delta t_2 = \frac{\Delta x - v\Delta t_2}{c}$. Solving for Δt_1 and Δt_2: $\Delta t_1 = \frac{\Delta x}{c-v}$ and $\Delta t_2 = \frac{\Delta x}{c+v}$. The total time is therefore $\Delta t = \Delta t_1 + \Delta t_2 = \frac{2\Delta x/c}{(1-v^2/c^2)}$. Now, from the time dilation result derived earlier, $\Delta t' = \sqrt{1-v^2/c^2}\,\Delta t$, and so $\Delta t' = \sqrt{1-v^2/c^2}\left(\frac{2\Delta x/c}{(1-v^2/c^2)}\right) = \frac{2\Delta x/c}{\sqrt{1-v^2/c^2}}$. This gives, $\Delta x' = \frac{\Delta x}{\sqrt{1-v^2/c^2}}$, or $\Delta x = \Delta x'/\gamma$. This an astonishing result! Suppose that there is a metre rod. Then the observer riding with it has $\Delta x' = 1$. But according to someone who sees the metre rod moving towards/away from him, the length is $1/\gamma$. This is less than 1 metre!

13. Although an object shrinks in the direction of motion (both while approaching and receding), the dimensions perpendicular to the velocity are not contracted. It is easy to conceive of a gedanken experiment that will demonstrate this. One of the exercises will guide you in this direction.

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14. We shall now derive the "Lorentz Transformation", which is the relativistic version of the Galilean transformation discussed earlier. Consider an event that occurs at position x (as measured in S) at a distance d (again, as seen in S) from the origin of S′. Then, $x = d + vt$. From the Lorentz contraction formula derived earlier, $d = \frac{x'}{\gamma}$ where x' is the distance measured in S′. This gives $x' = \gamma(x - vt)$. Now, by the same logic, $x' = d' - vt'$.



Here $d' = \frac{x}{\gamma}$. This gives $x = \gamma(x' + vt') = \gamma\left(\gamma(x - vt) + vt'\right)$. We find that the time in S′ is related to the time in S by $t' = \gamma\left(t - \frac{v}{c^2}x\right)$. Note that if we make c very large, then $t' = t$.

To summarize:

LORENTZ TRANSFORMATION

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right).$$

(Note: in various books you will find slightly different derivations of the above Lorentz transformation. You should look at one of your choice and understand that as well.)

Summary of Lecture 40 – SPECIAL RELATIVITY II

1. Recall the Lorentz Transformation: $x' = \gamma(x - vt)$ and $t' = \gamma\left(t - \frac{v}{c^2}x\right)$. Suppose we take the space interval between two events $\Delta x = x_1 - x_2$, and the time interval $\Delta t = t_1 - t_2$. Then, these intervals will be seen in S′ as $\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$ and $\Delta x' = \gamma(\Delta x - v\Delta t)$.

Now consider two particular cases:

a) Suppose the two events occur at the same place (so $\Delta x = 0$) but at different times (so $\Delta t \neq 0$). Note that in S′ they do not occur at the same point: $\Delta x' = \gamma(0 - v\Delta t)$!

b) Suppose the two events occur at the same time (so $\Delta t = 0$) but at different places (so $\Delta x \neq 0$). Note that in S′ they are not simultaneous: $\Delta t' = \gamma\left(0 - \frac{v}{c^2}\Delta x\right)$.

2. As seen in the frame S, suppose a particle moves a distance dx in time dt. Its velocity u is then $u = \frac{dx}{dt}$ (in S-frame). As seen in the S′-frame, meanwhile, it has moved a distance dx' where $dx' = \gamma(dx - vdt)$ and the time that has elapsed is $dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$. The velocity in S′-frame is therefore $u' = \frac{dx'}{dt'} = \frac{\gamma(dx - vdt)}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{dx/dt - v}{1 - \frac{v}{c^2}dx/dt} = \frac{u - v}{1 - \frac{uv}{c^2}}$. This is the Einstein velocity addition rule. It is an easy exercise to solve this for u in terms of u′,

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}.$$

3. Note one very interesting result of the above: suppose that a car is moving at speed v and it turns on its headlight. What will the speed of the light be according to the observer on the ground? If we use the Galilean transformation result, the answer is v+c (wrong!). But using the relativistic result we have $u' = c$ and $u = \frac{c + v}{1 + \frac{cv}{c^2}} = \frac{c + v}{1 + \frac{v}{c}} = c\frac{c + v}{c + v} = c$. In other words, the speed of the source makes no difference to the speed of light in your frame. Note that if either u or v is much less than c, then u' reduces to the familiar result:

$u' = \frac{u - v}{1 - \frac{uv}{c^2}} \rightarrow u - v$, which is the Galilean velocity addition rule.

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4. The Lorentz transformations have an interesting property that we shall now explore. Take the time and space intervals between two events as observed in frame S, and the corresponding quantities as observed in S′. We will now prove that the quantities defined respectively as $I = (c\Delta t)^2 - (\Delta x)^2$ and $I' = (c\Delta t')^2 - (\Delta x')^2$ are equal. Let's start with I':

$$I' = (c\Delta t')^2 - (\Delta x')^2 = \gamma^2\left((c\Delta t)^2 + (\Delta x)^2 v^2/c^2 - 2v\Delta t\Delta x - (\Delta x)^2 - (v\Delta t)^2 + 2v\Delta t\Delta x\right)$$

$$= \frac{1}{(1 - v^2/c^2)}\left((c^2 - v^2)(\Delta t)^2 - (\Delta x)^2(1 - v^2/c^2)\right)$$

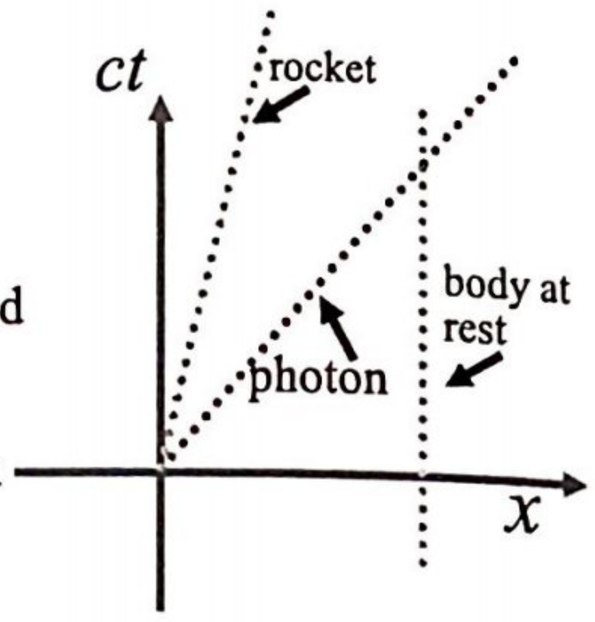
$$= (c\Delta t)^2 - (\Delta x)^2 = I$$

This guarantees that all inertial observers measure the same speed of light !!

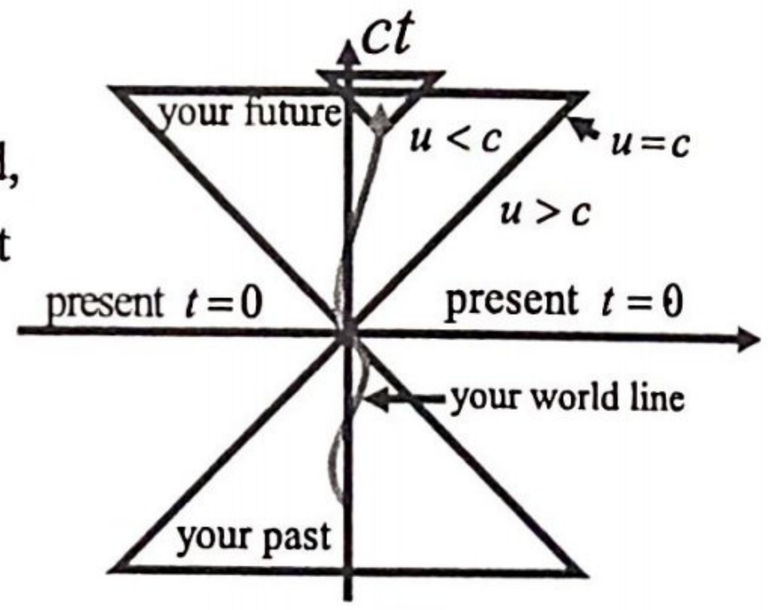
5. If the time separation is large, then $I > 0$ and we call the interval *timelike*.
If the space separation is large, then $I < 0$ and we call the interval *spacelike*.
If $I = 0$ and we call the interval *lightlike*.

Note : a) If an interval is timelike in one frame, it is timelike in all other frames as well.
b) If interval between two events is timelike, their time ordering is absolute.
c) If the interval is spacelike the ordering of two events depends on the frame from which they are observed.

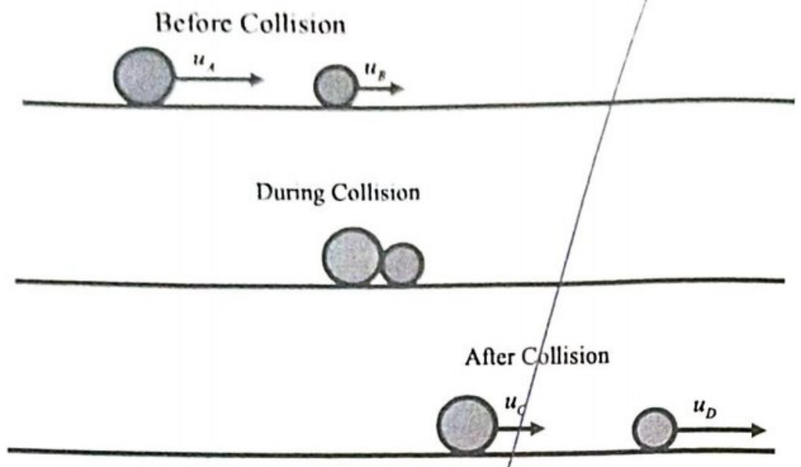
6. It is sometimes nice to look at things graphically. Here is a graph of position versus time for objects that move with different speeds along a fixed direction. First look at an object at rest. Its position is fixed even though time (plotted on the vertical axis) keeps increasing. Then look at the rocket moving at constant speed (which has to be less than c), and finally a photon (which can only move at c).



7. The trajectory of a body as it moves through space-time is called its world-line. Let's take a rocket that is at $x = 0$ at $t = 0$. It moves with non-constant speed, and that is why its world-line is wavy. A photon that moves to the right will have a world line with slope equal to +1, and that to the right with slope -1. The upper triangle (with t positive) is called the future light cone (don't forget we also have y and x). The lower light cone consists of past events.



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In the frame fixed to the lab (S-frame) conservation of momentum implies: $m_A u_A + m_B u_B = m_C u_C + m_D u_D$. Mass is also conserved: $m_A + m_B = m_C + m_D$. Now suppose we wish to observed the collision from a frame S′ moving at speed v. Then, from the relativistic addition of velocities formula, $u' = \dfrac{u - \mathrm{v}}{1 - \dfrac{u\mathrm{v}}{c^2}}$ and, from it, $u = \dfrac{u' + \mathrm{v}}{1 + \dfrac{u'\mathrm{v}}{c^2}}$. Insert this into the equation of conservation of momentum (using $p = m\mathrm{v}$ as the definition):

$$m_A\left(\frac{u'_A + \mathrm{v}}{1 + u'_A\mathrm{v}/c^2}\right) + m_B\left(\frac{u'_B + \mathrm{v}}{1 + u'_B\mathrm{v}/c^2}\right) = m_C\left(\frac{u'_C + \mathrm{v}}{1 + u'_C\mathrm{v}/c^2}\right) + m_D\left(\frac{u'_D + \mathrm{v}}{1 + u'_D\mathrm{v}/c^2}\right)$$

This is clearly not the equation $m_A u'_A + m_B u'_B = m_C u'_C + m_D u'_D$ So momentum will not be conserved relativistically if we insist on using the old definition!!

10. Can we save the situation and make the conservation of momentum hold by finding some suitable new definition of momentum? The new definition must have two properties:
    1) At low speeds it must reduce to the old one.
    2) At all speeds momentum must be conserved.

    Let's see if the definition $\vec{p} = \dfrac{m\vec{u}}{\sqrt{1 - u^2/c^2}} = \gamma m\vec{u}$ will do the job. Obviously if u<<c we get $\vec{p} = m\vec{u}$, so requirement 1 is clearly satisfied. . Let's now see if the conservation of momentum equation will hold if the new definition of momentum is used:

    $$m_A\gamma_A u_A + m_B\gamma_B u_B = m_C\gamma_C u_C + m_D\gamma_D u_D.$$

    After doing some algebra you find,

    $$m_A\left(\frac{1}{\gamma}\gamma'_A u'_A + \gamma_A\mathrm{v}\right) + m_B\left(\frac{1}{\gamma}\gamma'_B u'_B + \gamma_B\mathrm{v}\right) = m_C\left(\frac{1}{\gamma}\gamma'_C u'_C + \gamma_C\mathrm{v}\right) + m_D\left(\frac{1}{\gamma}\gamma'_D u'_D + \gamma_D\mathrm{v}\right)$$

    This gives $m_A\gamma'_A u'_A + m_B\gamma'_B u'_B = m_C\gamma'_C u'_C + m_D\gamma'_D u'_D$, which is just what we want.

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11. We shall now consider how energy must be redefined relativistically. The usual expression for the kinetic energy $K = \dfrac{1}{2}mu^2$ is not consistent with relativistic mechanics (it does not satisfy the law of conservation of energy in relativity). To discover a new definition, let us start from the basics: the work done by a force F when it moves through distance dx adds up to an increase in knetic energy, $K = \int F dx = \int \dfrac{dp}{dt}dx = \int \dfrac{dx}{dt}dp = \int u dp$. Now use:

$$dp = md\left(\frac{u}{\sqrt{1 - u^2/c^2}}\right) = \frac{mdu}{\sqrt{1 - u^2/c^2}} + \frac{m\left(u^2/c^2\right)du}{\left(1 - u^2/c^2\right)^{3/2}} = \frac{mdu}{\left(1 - u^2/c^2\right)^{3/2}}$$

This give $K = \int_0^u u dp = m\int_0^u \dfrac{udu}{\left(1 - u^2/c^2\right)^{3/2}} = \dfrac{mc^2}{\sqrt{1 - u^2/c^2}} - mc^2$

Note that K is zero if there is no motion, or $K = E - E_0$ where $E = \dfrac{mc^2}{\sqrt{1 - u^2/c^2}}$ and $E_0 = mc^2$. Now expand $\dfrac{1}{\sqrt{1 - u^2/c^2}} = \left(1 - u^2/c^2\right)^{-1/2} = 1 - \dfrac{u^2}{2c^2} - \cdots$

Hence, $K = mc^2\left(1 + \dfrac{u^2}{2c^2} + \cdots\right) - mc^2 \to \dfrac{1}{2}mu^2$ as $u/c \to 0$.

12. Now that we have done all the real work, let us derive some alternative expressions using our two main formulae: $\vec{p} = \dfrac{m\vec{u}}{\sqrt{1 - u^2/c^2}} = \gamma m\vec{u}$ and $E = \dfrac{mc^2}{\sqrt{1 - u^2/c^2}} = \gamma mc^2$.

    a) $\vec{u} = \dfrac{\vec{p}}{\gamma m} = \dfrac{\vec{p}c^2}{E}$ or $pc = E\left(\dfrac{u}{c}\right)$

    b) Clearly $(pc)^2 = E^2\left(\dfrac{u}{c}\right)^2 = \gamma^2 m^2 c^4\left(1 - 1/\gamma^2\right) = \gamma^2 m^2 c^4 - m^2 c^4 = E^2 - m^2 c^4$. Hence, $E^2 = p^2 c^2 + m^2 c^4$.

    c) For a massless particle ($m = 0$), $E = pc$.

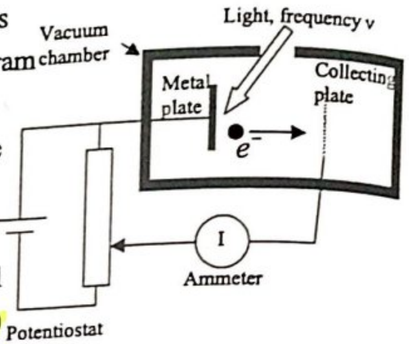
13. A particle with mass m has energy mc^2 even though it is at rest. This is called its rest energy. Since c is a very large quantity, even a small m corresponds to a very large energy. We interpret this as follows: suppose all the mass could somehow be converted into energy. Then an amount of energy equal to mc^2 would be released.

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Summary of Lecture 41 – WAVES AND PARTICLES

1. We think of particles as matter highly concentrated in some volume of space, and of waves as being highly spread out. Think of a cricket ball, and of waves in the ocean. The two are completely different! And yet today we are convinced that matter takes the form of waves in some situations and behaves as particles in other situations. This is called wave-particle duality. But do not be afraid - there is no logical contradiction here! In this lecture we shall first consider the evidence that shows the particle nature of light.
2. The photoelectric effect, noted nearly 100 years ago, was crucial for understanding the nature of light. In the diagram shown, when light falls upon a metal plate connected to the cathode of a battery, electrons are knocked out of the plate. They reach a collecting plate that is connected to the battery's anode, and a current is observed. A vacuum is created in the apparatus so that the electrons can travel without hindrance. According to classical (meaning old!) physics we expect the following:

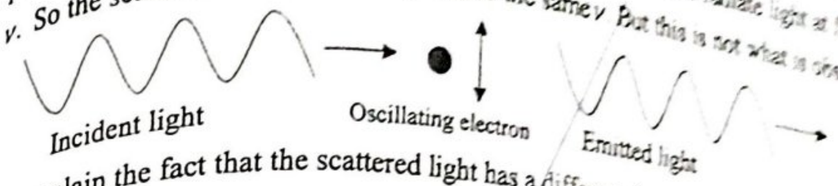


   a) As intensity of light increases, the kinetic energy of the ejected electrons should increase.
   b) Electrons should be emitted for any frequency of light ν, so long as the intensity of the light is sufficiently large.

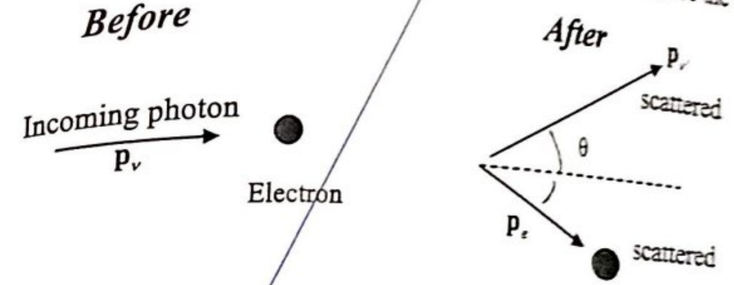
   But the actual observation was completely different and showed the following:
   a) The maximum kinetic energy of the emitted electrons was completely independent of the light intensity, but it did depend on ν.
   b) For $\nu<\nu_0$ (i.e. for frequencies below a cut-off frequency) no electrons are emitted no matter how large the light intensity was.

3. In 1905, Einstein realized that the photoelectric effect could be explained if light actually comes in little packets (or quanta) of energy with the energy of each quantum $E = h\nu$. Here h is a universal constant of nature with value $h = 6.63\times10^{-34}$ Joule-seconds, and is

4. That light is made of photons was confirmed by yet another experiment, carried out by Arthur Compton in 1922. Suppose an electron is placed in the path of a light beam. What will happen? Because light is electromagnetic waves, we expect the electron to oscillate with the same frequency as the frequency of the incident light ν. But because a charged particle radiates em waves, we expect that the electron will also radiate light at frequency ν. So the scattered and incident light have the same ν. But this is not what is observed!



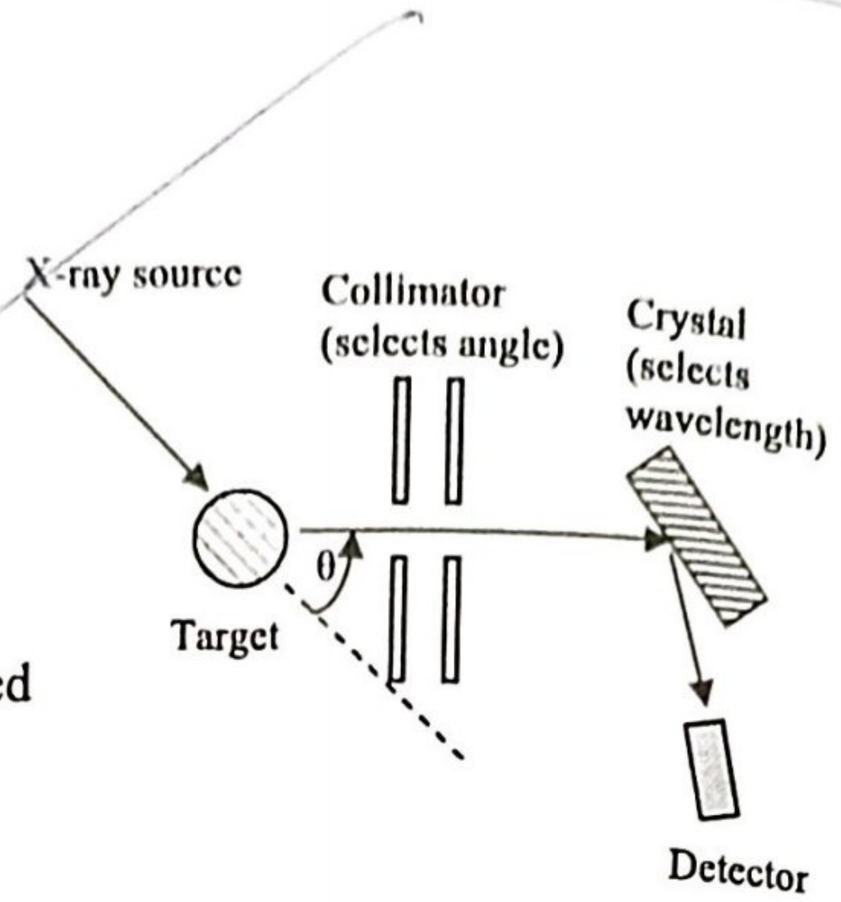
To explain the fact that the scattered light has a different frequency (or wavelength), Compton said that the scattering is a collision between particles of light and electrons. But we know that momentum and energy is conserved in scattering between particles. Specifically, from conservation of energy $h\nu + m_e c^2 = h\nu' + \left(p_e^2c^2 + m_e^2c^4\right)^{1/2}$. The last term is the energy of the scattered electron with mass m_e. Next, use the conservation



of momentum. The initial momentum of the photon is entirely along the $\hat{z}$ direction. $\mathbf{p}_\nu = \frac{h}{\lambda}\hat{z} = \mathbf{p}_{\nu'} + \mathbf{p}_e$. By resolving the components and doing a bit of algebra, you can get the change in wavelength $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda_c(1-\cos\theta)$, where the Compton wavelength $\lambda_c = \frac{h}{m_e c} = 2.4\times10^{-12}$ m. Note that $\lambda' - \lambda$ is always positive because $\cos\theta$ has magnitude less than 1. In other words, the frequency of the scattered photon is

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5. In the apparatus shown, X-rays are incident upon a target which contains electrons. Those X-rays which are scattered in a particular angle θ are then selected by the collimator and are incident upon a crystal. The crystal diffracts the X-rays and, as you will recall from the lecture on diffraction, $2d\sin\theta_D = n\lambda$ is the necessary condition. It was thus determined that the changed wavelength of the scattered follows $\lambda' - \lambda = \lambda_c(1-\cos\theta)$.



6. Here is a summary of photon facts:

a) The relation between the energy and frequency of a photon is $E = h\nu$.

b) The relativistic formula relating energy and momentum for photons is $E = pc$. Note that in general $E^2 = p^2c^2 + m^2c^4$ for a massive particle. The photon has $m = 0$.

c) The relation between frequency, wavelength, and photon speed is $\lambda\nu = c$.

d) From the abbove, the momentum-wavelength relation for photons is $p = \frac{h\nu}{c} = \frac{h}{\lambda}$.

e) An alternative way of expressing the above is: $E = \hbar\omega$ and $p = \hbar k$. Here $\omega = 2\pi\nu$ and $k = \frac{2\pi}{\lambda}$, $\hbar \equiv \frac{h}{2\pi}$ ($\hbar$ is pronounced h-bar).

f) Light is always detected as packets (photons); if we look, we never observe half a photon. The number of photons is proportional to the energy density (i.e. to square of the electromagnetic field strength).

7. So light behaves as if made of particles. But do all particles of matter behave as if they are waves? In 1923 a Frenchman, Louis de Broglie, postulated that ordinary matter can have wave-like properties with the wavelength λ related to the particle's momentum p in the same way as for light, $\lambda = \frac{h}{p}$. We shall call λ the de Broglie (pronounced as Deebrolee!) wavelength.

$(E^2 = p^2c^2 + m^2c^4)$

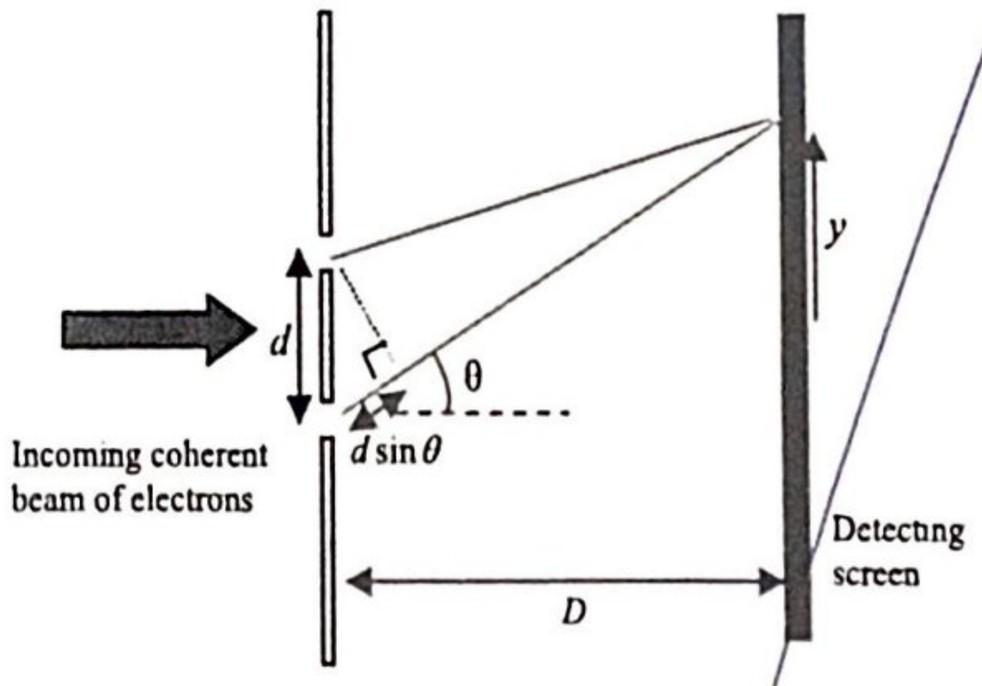
8. Let us estimate some typical De Broglie wavelengths: Photon has $m = 0$

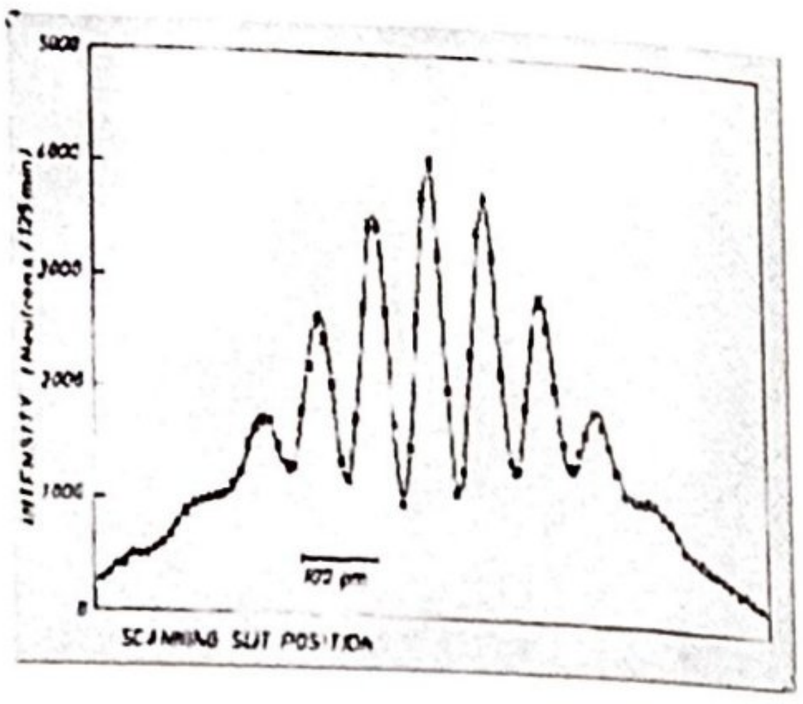
a) The wavelength of 0.5 kg cricket ball moving at 2 m/sec is:

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{0.5\times2} = 6.63\times10^{-34}\ \text{m}$$

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11. Let's take a still simpler situation: electrons are incident upon a metal plate with two tiny holes punched into it. The holes - spearated by distance d - are very close together. A screen behind, at distance D, is made of material that flashes whenever it is hit by an electron. A clear interference pattern with peaks and valleys is observed. Let us analyze: there will be a maximum when $d\sin\theta = n\lambda$. If the screen is very far away, i.e. $D >> d$, then θ will be very small and $\sin\theta \approx \theta$. So we then have $\theta \approx \frac{n\lambda}{d}$, and the angular separation between two adjacent minima is $\Delta\theta \approx \frac{\lambda}{d}$. The position on the screen is y, and $y = D\tan\theta \approx D\theta$. So the separation between adjacent maxima is $\Delta y \approx D\Delta\theta$ and hence $\Delta y = \frac{\lambda D}{d}$. This is the separation between two adjacent bright spots. You can see from the experimental data that this is exactly what is observed.





12. The double-slit experiment is so important that we need to discuss it further. Note the following:

a) It doesn't matter whether we use light, electrons, or atoms - they all behave as waves. in this experiment. The wavelength of a matter wave is unconnected to any internal size of particle. Instead it is determined by the momentum, $\lambda = \frac{h}{p}$.

b) If one slit is closed, the interference disappears. So, in fact, each particle goes through both slits at once.

c) The flux of particles arriving at the slits can be reduced so that only one particle arrives at a time. Interference fringes are still observed !Wave-behaviour can be shown by a single atom. In other words, a matter wave can interfere with itself.

d) If we try to find out which slit the particle goes through the interference pattern vanishes! We cannot see the wave/particle nature at the same time.

All this is so mysterious and against all our expectations. But that's how Nature is!

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Thus we have found that $\Delta p_y \Delta y \approx h$. The important point here is that by localizing the y position of the electron to the width of the slit, we have forced the electron to acquire a momentum in the y direction whose uncertainty is Δp_y.

15. In a proper course in quantum mechanics, one can give a definite mathematical meaning to Δx and Δp_x etc, and derive the uncertainty relations:

$$\Delta x \Delta p_x \geq \hbar/2, \quad \Delta y \Delta p_y \geq \hbar/2, \quad \Delta z \Delta p_z \geq \hbar/2$$

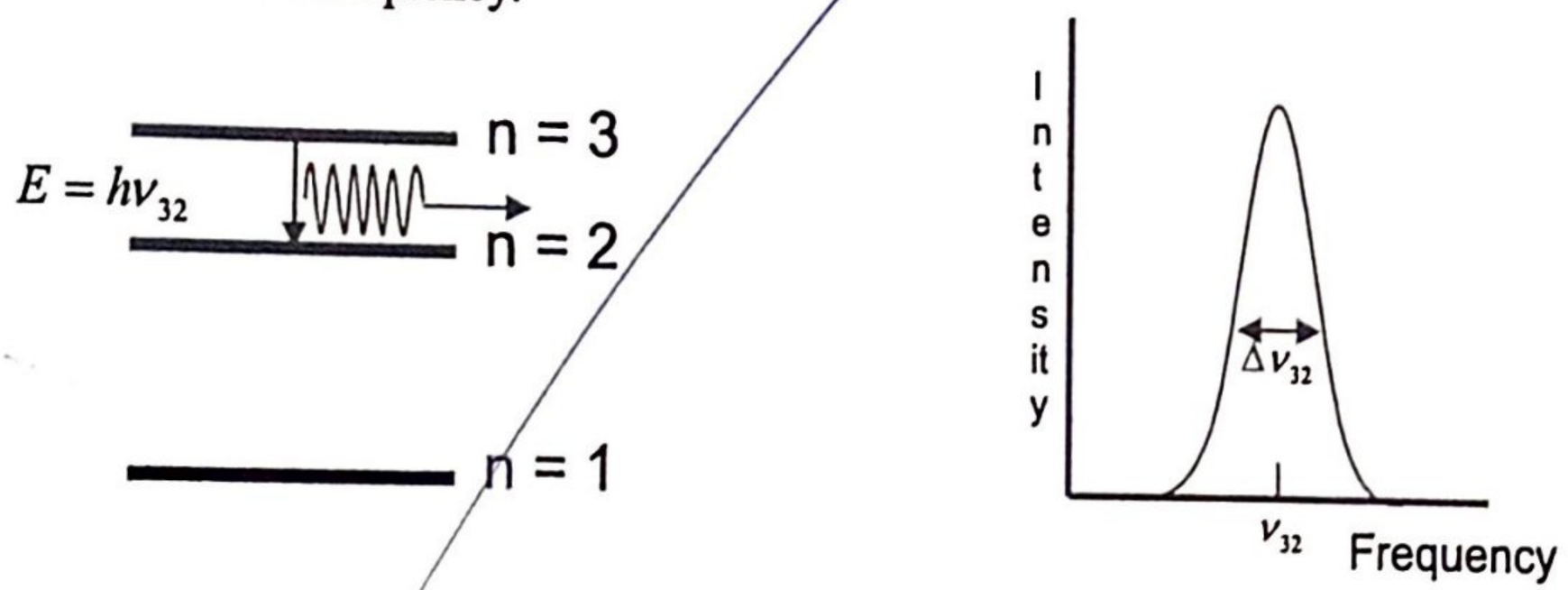
Note the following:

a) There no uncertainty principle for the product $\Delta x \Delta p_y$. In other words, we can know in principle the position in one direction precisely together with the momentum in another direction.

b) The thought experiment I discussed seems to imply that, while prior to experiment we have well defined values, it is the act of measurement which introduces the uncertainty by disturbing the particle's position and momentum. Nowadays it is more widely accepted that quantum uncertainty (lack of determinism) is intrinsic to the theory and does not come about just because of the act of measurement.

16. There is also an Energy-Time Uncertainty Principle which states that $\Delta E \Delta t \geq \hbar/2$. This says that the principle of energy conservation can be violated by amount ΔE, but only for a short time given by Δt. The quantity ΔE is called the uncertainty in the energy of a system.

17. One consequence of $\Delta E \Delta t \geq \hbar/2$ is that the level of an atom does not have an exact value. So, transitions between energy levels of atoms are never perfectly sharp in frequency. So, for example, as shown below an electron in the $n = 3$ state will decay to a lower level after a lifetime of order $t \approx 10^{-8}$s. There is a corresponding "spread" in the emitted frequency.



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4. What brought about the Quantum Revolution? By the end of the 19th century a number of serious discrepancies had been found between experimental results and classical theory. The most serious ones were:

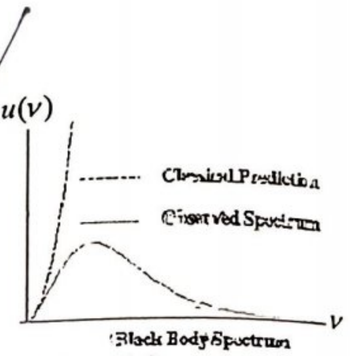
A) The blackbody radiation law

B) The photo-electric effect

C) The stability of the atom, and puzzles of atomic spectra

In the following, we shall briefly consider these discrepancies and the manner in which quantum mechanics resolved them.

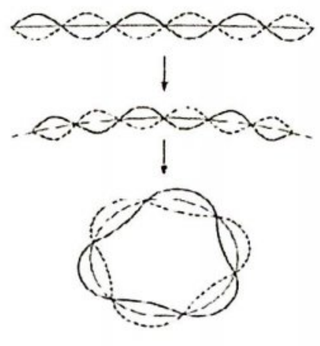
A)Classical physics gives the wrong behaviour for radiation emitted from a hot body. Although in this lecture it is not possible to do the classical calculation, it is not difficult to show that the electromagnetic energy $u(\nu)$ radiated at frequency ν increases as ν^3 (see graph). So the energy radiated over all frequencies is infinite. This is clearly wrong. The correct calculation was done by Max Planck.



Planck's result is shown above, and it leads to the sensible result that u(ν) goes to zero at large ν. He assumed that radiation of a given frequency ν could only be emitted and absorbed in quanta of energy $\varepsilon = h\nu$. If the electromagnetic field is thought of as harmonic oscillators, Planck assumed that the total energy of this large number of oscillators is made of finite energy elements $h\nu$. With this assumption, he came up with a formula that fitted well with the data. But he called his theory "an act of desperation" because he did not understand the deeper reasons.

B) I have already discussed the photoelectric effect in the previous lecture. Briefly, Einstein (1905) postulated a quantum of light called photon, which had particle properties like energy and momentum. The photon is responsible for knocking electrons out of the metal - but only if it has enough energy.

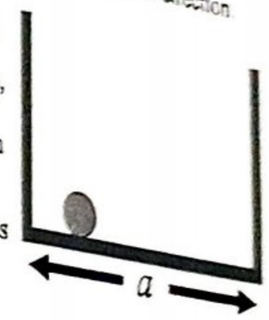
C) Classical physics cannot explain the fact that atoms are stable. An accelerating charge always radiates energy if classical electromagnetism is correct. So why does the hydrogen atom not collapse? In 1921 Niels Bohr, a great Danish physicist, made the following hypothesis: if an electron moves around a nucleus so that its angular momentum is $\hbar, 2\hbar, 3\hbar, \cdots$ then it will not radiate energy. In the next lecture we explore the consequences of this hypothesis. Bohr's hypothesis called for the quantization



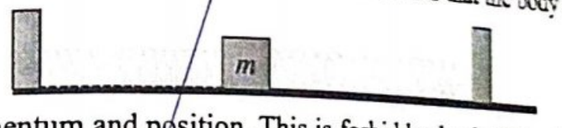
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of angular momentum. If the electron is regarded as a De Broglie wave, then there must be an integer number of waves that go around the centre. Of course, this is not proper quantum mechanics and cannot be taken too seriously, but it definitely was a major step in the ultimate development of the subject. Even if one's understanding was imperfect it was now possible to understand why atoms had certain energies only, and why the light radiated by atoms was of discrete frequencies only. In contrast, classical physics predicted that atoms could radiate at any frequency.

5. One consequence of quantum mechanics is that it explains through the uncertainty principle the stability of the atom. But before talking of that, let us consider a particle moving between two walls. Each time it hits a wall, a force pushes it in the opposite direction. There is no friction, so classically the particle just keeps moving forever between the two potential walls. In QM the uncertainty of the particle's position is $\Delta x = a$ and so, from $\Delta x \Delta p \geq \hbar/2$, we have $\Delta p\, a \approx \frac{\hbar}{2}$. From this we learn that the kinetic energy $\frac{(\Delta p)^2}{2m} \approx \frac{\hbar^2}{8ma^2}$. This is telling us that as we squeeze the particle into a tighter and tighter space, the kinetic energy goes up and up!



6. The story repeats for a harmonic oscillator. So imagine that a mass moves in a potential of the type shown below, and that its frequency of oscillation is ω. Classically, the lowest energy would be that in which the mass is at rest (no kinetic energy) and it is at the position where the potential is minimum. But this means that the body has both a



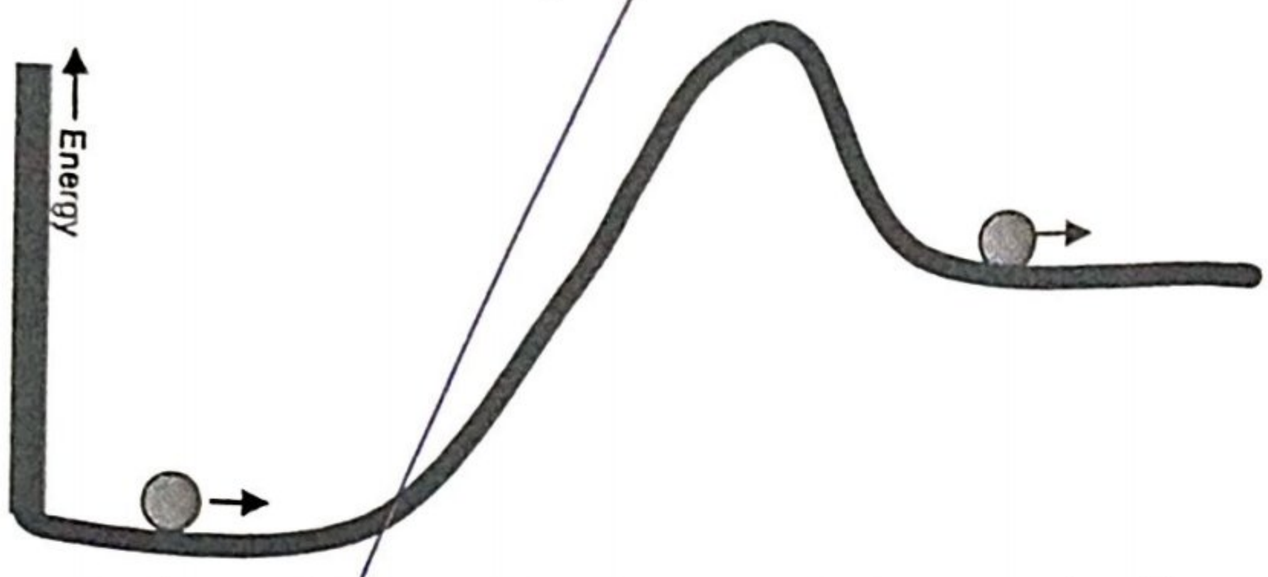
well-defined momentum and position. This is forbidden by the Heisenberg uncertainty principle. A proper quantum mechanical calculation shows that the minimum energy is actually $\frac{1}{2}\hbar\omega$. This is called the zero-point energy and comes about because it is not possible for the mass to be a rest. The oscillator's other energy states have energies $\frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega, \frac{5}{2}\hbar\omega, \frac{7}{2}\hbar\omega, \cdots$ We say that the energies are quantized. This is completely impossible to understand from classical mechanics where we know that we can excite an oscillator to have any energy we want. Quantized energy levels are indeed what we observe in so many physical systems: atoms, rotating or vibrating molecules, nuclei, etc.

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7. We can understand from both the previous examples why the hydrogen atom does not collapse even if the electron does not have any orbital angular momentum around the proton. The uncertainty principle essentially forces the electron to stay away from the proton - if it tried to get too close, the kinetic energy would rise enormously because $\Delta x \Delta p \geq \hbar/2$ says that if Δx becomes small then Δp must become big to compensate.

8. Quantum mechanics predicts what is called "tunneling" of particles through a potential barrier. Again, we shall use the uncertainty principle but this time the energy-time one. Imagine a particle that moves in a potential of the shape shown below. Suppose it does not have enough energy to go over the peak and on to the other side. In that case, we



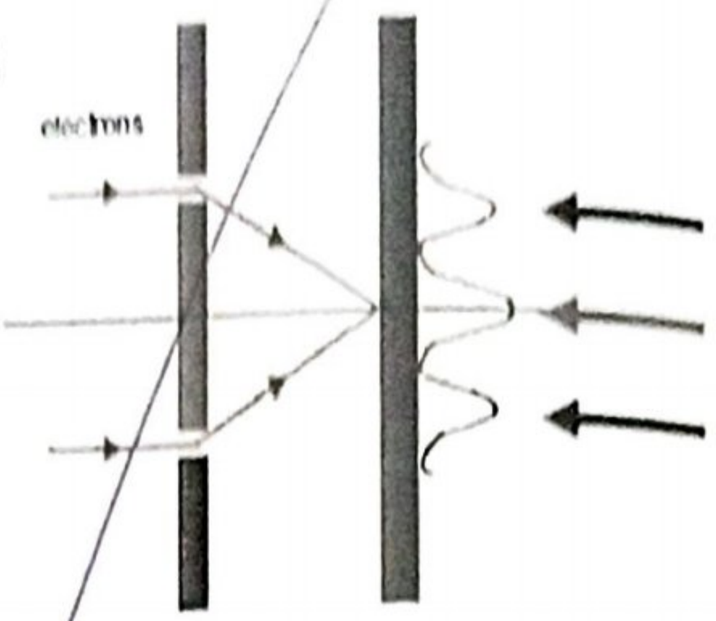
know from our experience that it will just keep oscillating - moving first towards the hill and then down again, etc. But quantum mechanically, it can "steal" energy ΔE for a time Δt and this may be enough to surmount the hill. Of course, the particle must respect $\Delta E \Delta t \geq \hbar/2$ so the time is small if it needs a large amount of energy to cross over. Again, I have given only a rough argument here, but in quantum mechanics we can do proper calculations to find tunneling probabilities.

9. Without tunneling our sun would go cold. As you may know, it is powered by hydrogen fusion. The protons must somehow overcome electrostatic repulsion to get close enough so that they can feel the nuclear force and fuse with each other. But the thermal energy at the core of the sun is not high enough. It is only because the tunneling effect allows protons to sometimes get close enough that fusion happens.

10. Quantum mechanics is all about probabilities. What is probability? Probability is a measure of the likelihood that an event will occur. Probability values are assigned on a scale of zero (the event can never occur) to one (the event definitely occurs). More precisely, suppose we do an experiment (like rolling dice or flipping a coin) N times where N is large. Then if a certain outcome occurs n times, the probability is $P = n/N$.

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14. Let us return to the double slit experiment discussed earlier. Here the amplitude for an electron wave coming from one slit interferes with the amplitude for an electron wave coming from the other slit. This is what causes a pattern to emerge in which electron are completely absent in certain places (destructive interference) and are present in large numbers where there is constructive interference. So what we must deal with are matter waves. But how to treat this mathematically?



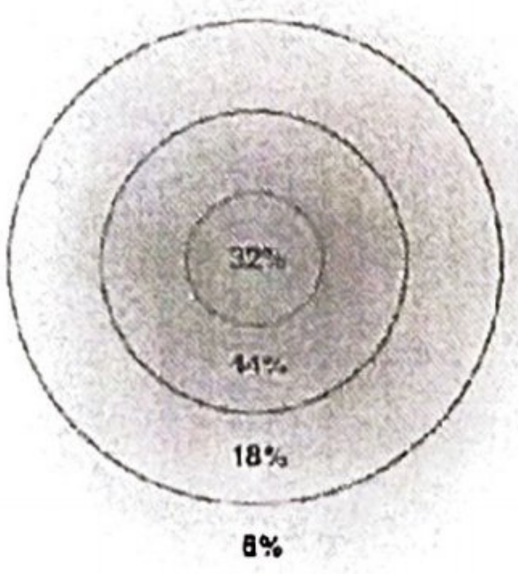
15. The above brings us to the concept of a "wave function". In 1926 Schrödinger proposed a quantity that would describe electron waves (or, more generally, matter waves).

- The wavefunction $\Psi(x,t)$ of a particle is the amplitude to be at position x at time t.

- The probability of finding the particle at position x between x and $x+dx$ (at time t) is $|\Psi(x,t)|^2\,dx$. Since the particle has to be somewhere, if we add up all possibilities then we must get one, i.e. $\int_{-\infty}^{+\infty} |\Psi(x,t)|^2\,dx = 1$.

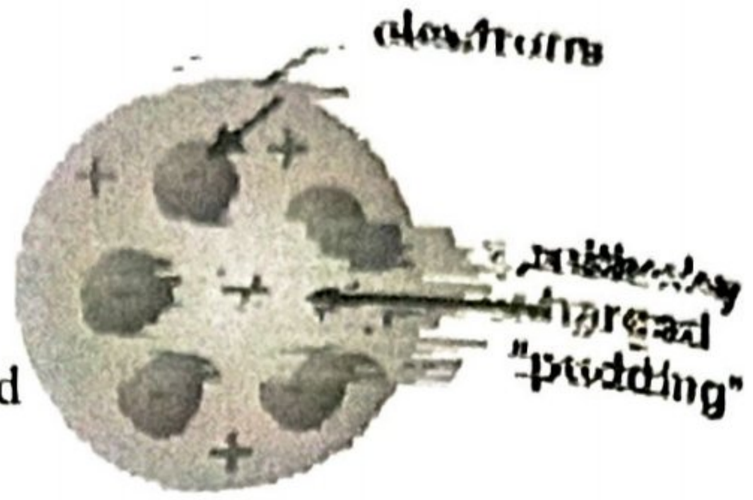
- $\Psi(x,t)$ is determined by solving the "Schrodinger equation" which, unfortunately, I shall not be able to discuss here. This is one of the most important equations of physics. If it is solved for the atom then it tells you all that is possible to know: energies, the probability of finding an electron here or there, the momenta with which they move, etc. Of course, one usually cannot solve this equation in complicated situations (like a large molecule, for example) and this is what makes the subject both difficult and interesting.

16. For an electron moving around a nucleus, one can easily solve the Schrodinger equation and thus find the wavefunction $\Psi(x,t)$. From this we compute $|\Psi(x,t)|^2$, which is large where the electron is more likely to be found. In this picture, the probability of finding the electron inside the first circle is 32%, between the second and first is 44%, etc.

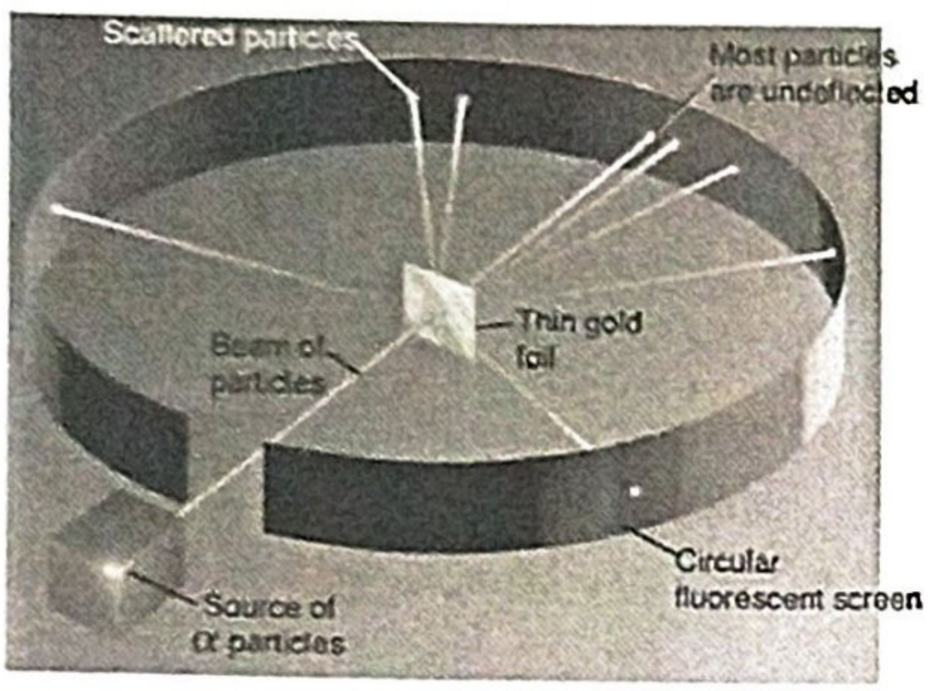


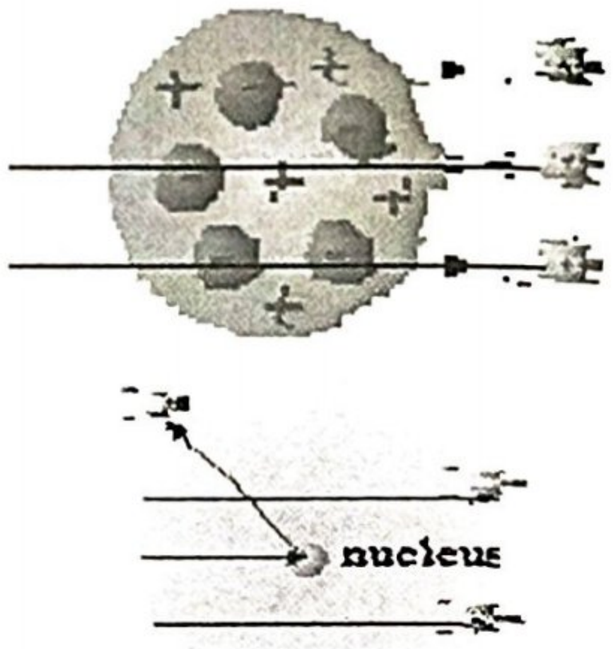
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4. Even if you know how big an atom is, this does not mean that its internal structure is known. In 1895 J.J. Thomson proposed the "plum pudding" model of an atom. Here the atom is considered as made of a positively charged material with the negatively charged electrons scattered through it.



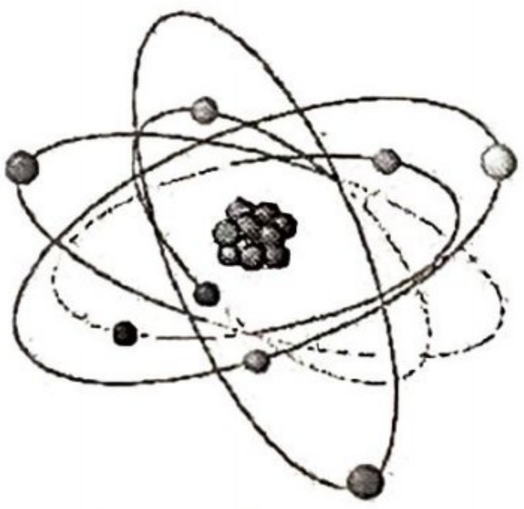
5. But the plum-pudding model was wrong. In 1911, Rutherford carried out his famous experiment that showed the existence of a small but very heavy core of the atom. He arranged for a beam of α particles to strike gold atoms in a thin foil of gold.





If the positive and negative charges in the atom were randomly distributed, all $\alpha's$ would go through without any deflection. But a lot of backscattering was seen, and some α's were even deflected back in the direction of the incident beam. This was possible only if they were colliding with a very heavy object inside the atom. Rutherford had discovered the atomic nucleus.

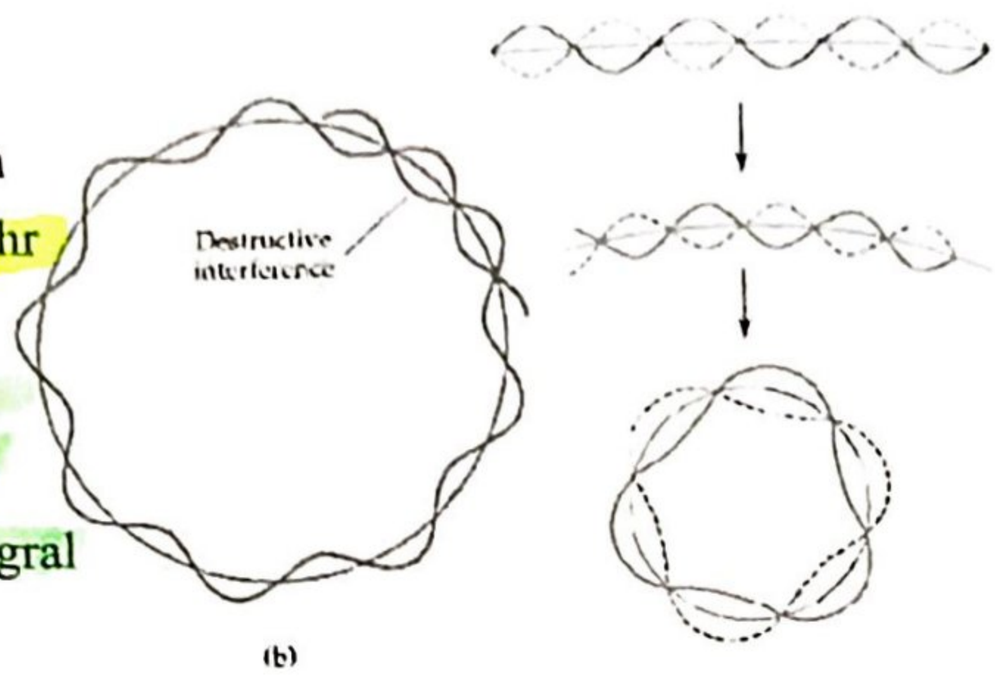
6. The picture that emerged after Rutherford's discovery was like that of the solar system - the atom was now thought of as mostly empty space with a small, positive nucleus that contained protons. Negative electrons moved around the outside in orbits hat resembled those of planets, attracted towards the centre by a coulomb force.



7. This sounds fine, but there is a serious problem: we know that a charge that accelerates radiates energy. In fact the power radiated is $P \propto e^2 a^2$, where e is the charge and a is the acceleration. Now, a particle moving in a circular orbit has an acceleration even if it is moving at constant speed because it is changing its direction all the time. So this means

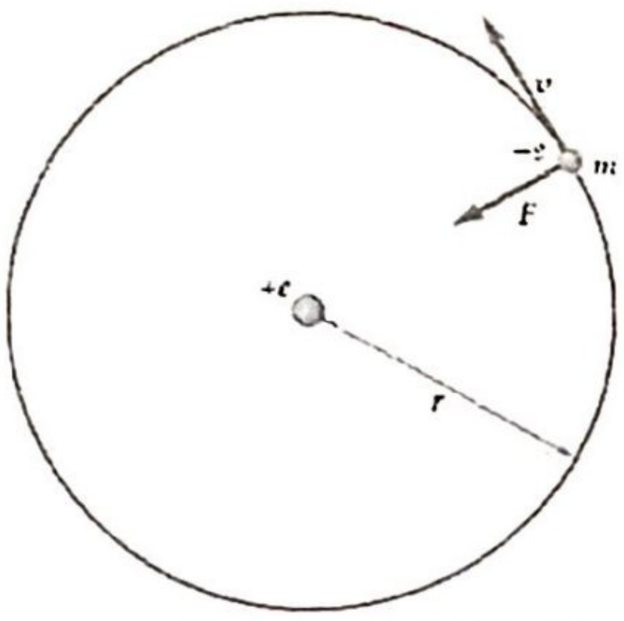
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10. Then came Niels Bohr. By this time it was known that electrons had a dual character as waves (De Broglie relation and Davisson-Germer experiment). Bohr said: suppose I bend a standing wave into a circle. If the wavelength is not exactly correct, wave interference will make the wave disappear. So only integral numbers of wavelengths can interfere constructively.



11. Let us pursue this idea further. The electron has a wavelength and forms standing waves in its orbit around the nucleus. An integral number of electron wavelengths must fit into the circumference of the circular orbit. Hence $n\lambda = 2\pi r$ with $n = 1,2,3\cdots$ The momentum is $p = mv = \frac{h}{\lambda} = \frac{h}{(2\pi r/n)} = \frac{n\hbar}{r}$. The angular momentum $L = rmv = n\hbar$ is therefore quantized in units of $\hbar$.

12. Now let us suppose that the electron moves in an orbit of radius r when it has $L_n = n\hbar$. Equilibrium demands that the centrifugal force be equal to the coulomb attraction: $\frac{mv_n^2}{r} = k\frac{e^2}{r^2}$. From $v_n = \frac{n\hbar}{mr_n}$ we find that the radius $r_n = \left(\frac{\hbar^2}{mke^2}\right)n^2$.

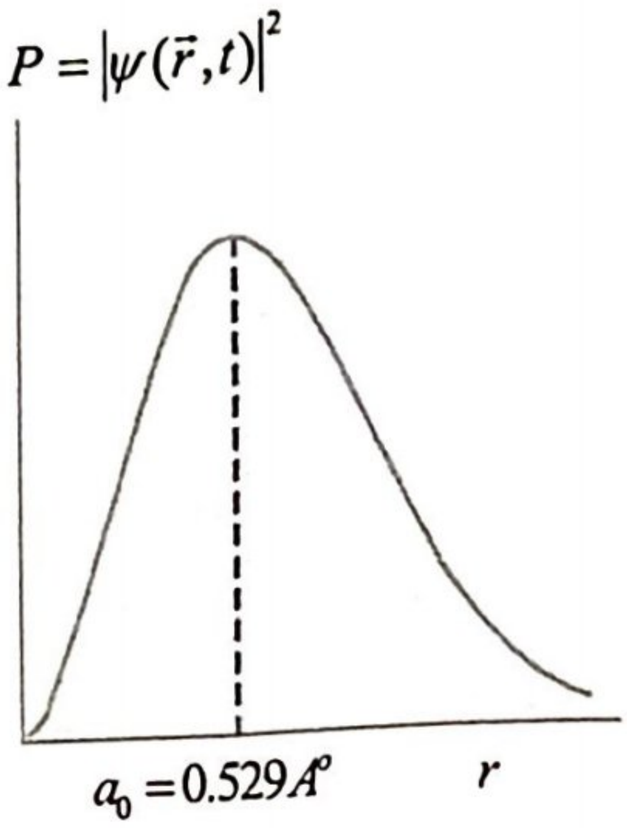


- For $n = 1$ the electron orbit which is closest to the nucleus, $r_n = a_0 \equiv 0.53\times10^{-8}\,cm$ (this is called the Bohr radius).
- For higher n, $r_n = a_0 n^2$. The atom becomes huge for n ≈ 100, the so-called Rydberg atom. Such atoms are of experimental interest these days.
- Note that the speed of the electron is smaller in orbits farther from the nucleus, $v_n = \frac{ke^2}{n\hbar}$. As n becomes very large, the electron is very far out and very slow.
- In the above $n = 0$ is strictly not allowed. As you can see, none of the formulae make any sense for this case. The minimum angular momentum that the electron can have is $\hbar$. (In proper quantum mechanics the minimum is $0\hbar$ and this is a big difference with the Bohr model of the atom).

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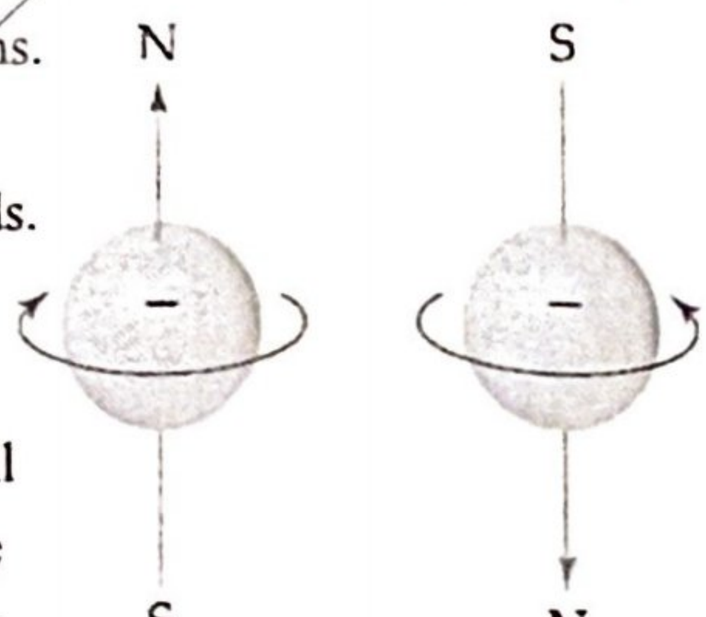
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16. Quantum mechanics gives a picture that is quite different from the solar-system model. We solve the "Schrodinger Equation" to find the wavefunction $\psi(\vec{r},t)$, whose square gives the probability of finding the electron at the point $\vec{r}$. In the lowest energy state, the electron can be viewed as a spherical cloud surrounding the nucleus. The densest regions of the cloud represent the highest probability for finding the electron.



- The principal quantum number $n = 1,2,3\cdots$ determines the allowed energy levels. An electron can only have energy $E_n = -\frac{13.6}{n^2}$ eV. Miraculously this is the same result as in the Bohr model.
- The orbital angular momentum is determined by the number l, and $L = \sqrt{l(l+1)}\hbar$. Allowed values of l are, $l = 0,1,2,3\cdots, n-1$.
- The magnetic quantum number m_l determines the projection (or component) of the vector angular momentum $\vec{L}$ on to any fixed axis, $L_z = m_l\hbar$. Allowed values are, $m_l = -l,\cdots -2,-1,0,1,2,\cdots,l$.

17. Electrons can be thought of as little spinning balls of charge. All electrons spin at the same speed (more accurately, their spin angular momentum is the same and equals $\hbar/2$). An electron can spin in only one of two possible directions. When charges move around in a circle, that constitutes a current. As you know, currents give rise to magnetic fields. This is why electrons are also little magnets that interact with other magnets. Now go back to the Stern-Gerlach experiment described in the previous lecture, and you will understand better why the electrons were deflected by the applied magnetic field. Now that we have learned that the electron has spin, we can describe the two spin states by giving the "magnetic" quantum number m_s where $m_s = -\frac{1}{2},\frac{1}{2}$. These two states have the same energy except when there is some magnetic field present.



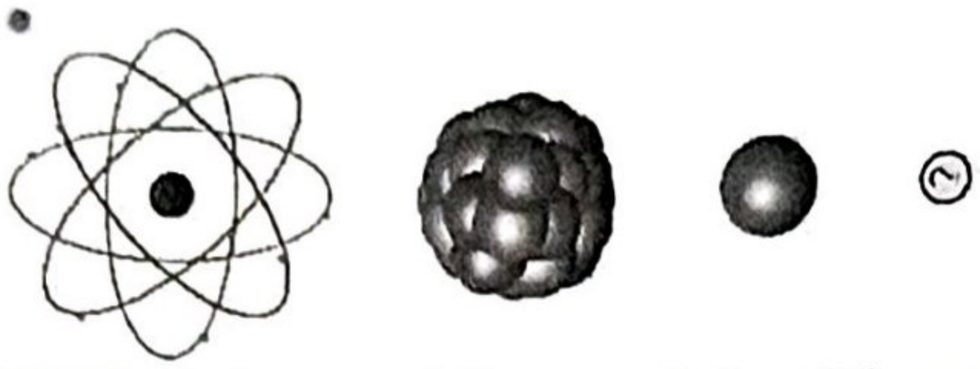
18. States (or orbitals) having $l = 0,1,2,\cdots$ are called s,p,d,$\cdots$. The s-states are spherical. As n increases, the s-orbitals get larger and the wavefunction is larger away from the nucleus.

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Summary of Lecture 44 – INTRODUCTION TO NUCLEAR PHYSICS

Q.1 In the previous lecture you learned how it was discovered that the atom is mostly empty space with a cloud of electrons. At the centre is a small but very heavy nucleus that has protons and neutrons. The word "nucleon" refers to both of these. So you can think of



the neutron or proton as being two different varieties of the nucleon. The masses of the two are very similar, and they are roughly 2000 times heavier than the electron.

Imp. (proton mass $= M_p = 1.672 \times 10^{-27} kg$

neutron mass $= M_n = 1.675 \times 10^{-27} kg$

electron mass $= M_e = 9.109 \times 10^{-31} kg$)

The neutron is neutral, of course, but the charge on the proton is $1.6 \times 10^{-19} C$ while the charge on the electron is the negative of this, $-1.6 \times 10^{-19} C$.

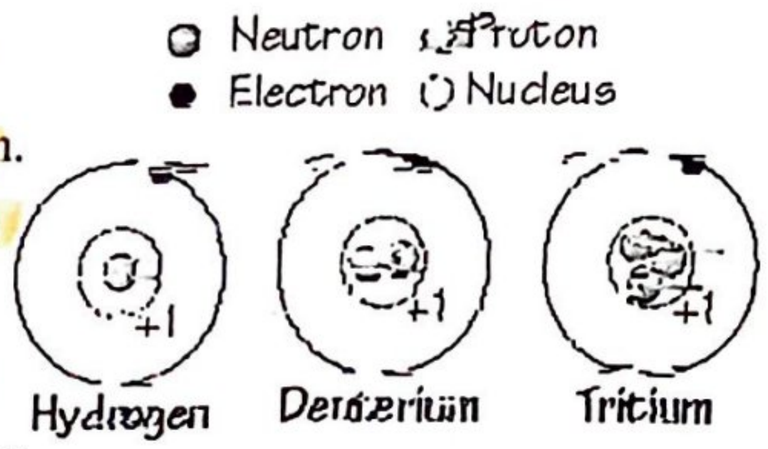
2. Using kilograms is very awkward if you are dealing with such small particles. Instead we we use $E = mc^2$ to write the mass of a particle in terms of its rest energy, $m = E/c^2$. So mass is measured in units of MeV/c^2.

proton mass $= M_p = 938\ MeV/c^2$

neutron mass $= M_n = 940\ MeV/c^2$

electron mass $= M_e = 0.5\ MeV/c^2$

3. a) A hydrogen nucleus is just one proton.
   b) A deuteron has a proton plus one neutron.
   c) A triton (or tritium nucleus) has a proton plus two neutrons.



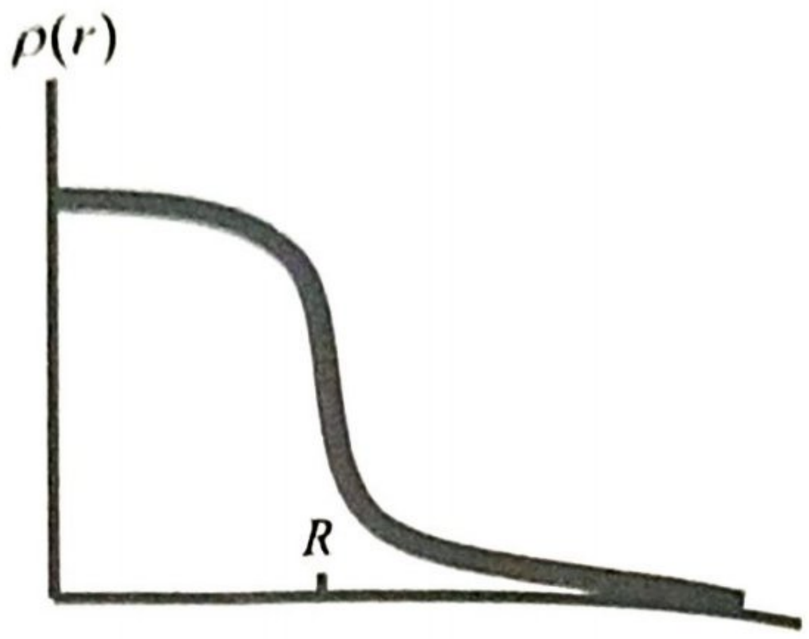
All three atoms have one electron only, and thus completely identical chemical properties.

4. Hydrogen, deuterium, and tritium are called isotopes. If a nucleus with Z protons has N neutrons then its isotopes will have fewer, or more, neutrons. Since the number of electrons is also Z, the chemical properties of all isotopes are exactly the same. But for any given element, at most there is only one stable isotope.

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8. From electron scattering we see that the proton density is almost constant throughout the nucleus and falls sharply at the surface. So nuclei should be thought of as rather fuzzy balls. A typical plot of density versus distance from the centre looks like this. Here the distance $R \approx r_0 A^{1/3}$ can be called the nuclear radius. The distribution of neutrons is very similar to this plot, but requires other techniques.



9. Let us consider the implication of the approximate formula for the nuclear radius, $r \approx r_0 A^{1/3}$ where $r_0 = 1.2\ fm$. The volume of the nucleus is: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_0^{\,3} A$. From this $\frac{A}{V} = \frac{3}{4\pi r_0^{\,3}} \approx 0.14$ nucleons/fm^3. This is the number of nucleons per cubic fermi, and is independent of the nucleus considered. This is the density you would find at the centre of any nucleus. Of course, this is approximately true only but it is quite remarkable.

10. Protons repel protons through the electrostatic force. So why does the nucleus not blow apart. Obviously there must be some attractive force that is stronger than this repulsion. It is, in fact, called the strong force. From what we have learned so far, we can guess some of its important features:
    a) Since neutron and proton distributions are almost the same, the N-P force cannot be very different from the N-N or P-P force.
    b) Since the density of nucleons in large nuclei is the same as in lighter nuclei, this means that a given nucleon feels only the force due to its immediate neighbours, and does not interact much with nucleons on the other side of the nucleus. In other words, the range of the nucleon-nucleon force is very short and of the order of 1-2 fm only.

11. The force between two charges is always of one sign - repulsive if the signs are the same, and attractive if they are opposite. In the early years of quantum theory, people realized that this force comes about because of the exchange of photons between charges. The nucleon-nucleon force is different. It has to be attractive to keep the nucleus together, and has to short range (as discussed above). But, to prevent nucleons from sticking to each other, it must be repulsive at short distances. Now here, "short" and long means distances on the scale of fermis. Typically the distances between nucleons is on this scale as well.

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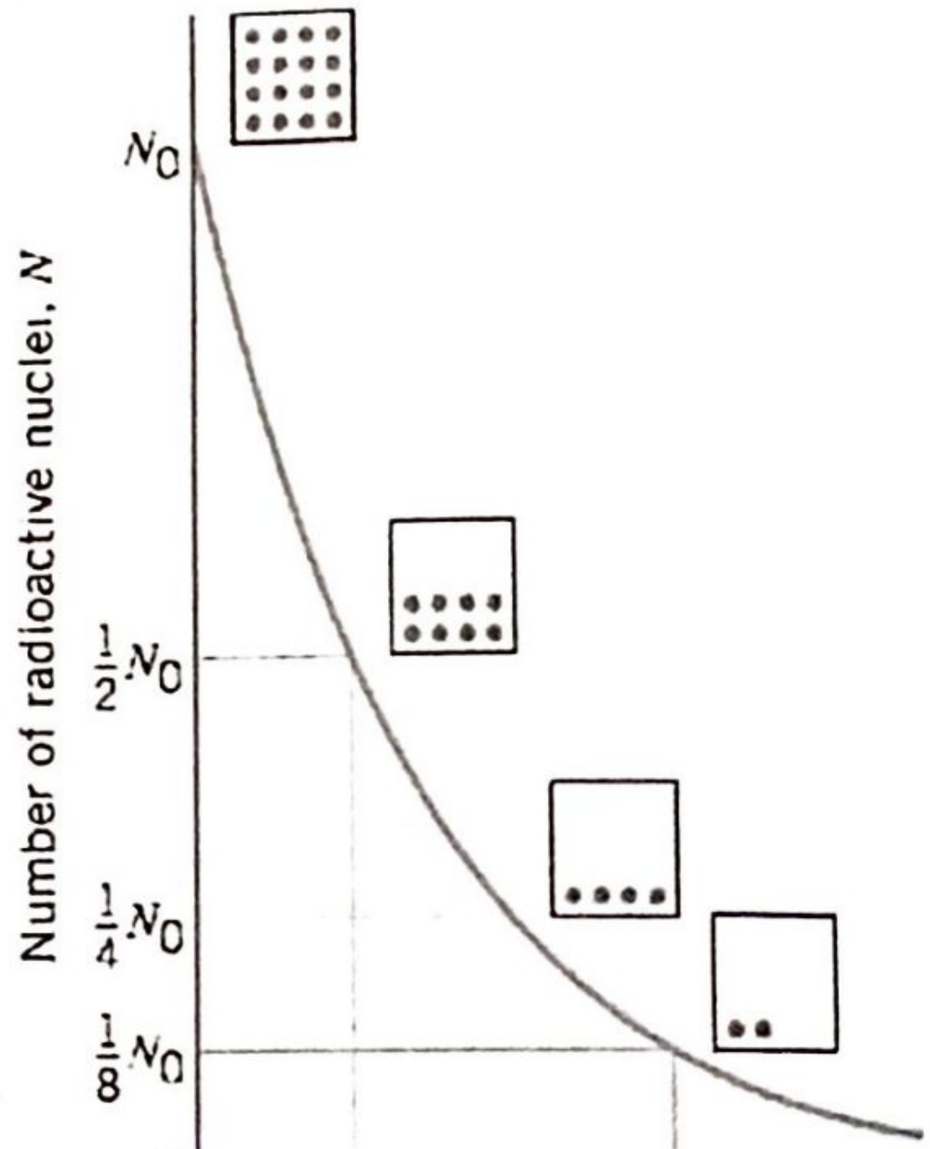
13. A few nuclei are stable, most decay. The decay law is simply derived: if the number of nuclei decreases by dn in time dt, then dn must be proportional to both the the number of nuclei n that are decaying and dt, so $dn \propto -ndt$ (minus sign for decrease). with the proportionality constant λ, we have $dn = -\lambda n dt$, or $\frac{dn}{dt} = -\lambda n$. We have encountered the solution of this type of equation before, $n = n_0 e^{-\lambda t}$. You can see that at $t = 0$, $n = n_0$. Taking the log, we have $\ln \frac{n}{n_0} = -\lambda t$. We define the half-life $T_{\frac{1}{2}}$ as the time it takes for half the original sample to decay. If $n = n_0/2$ then $\log \frac{n_0}{2n_0} = -\lambda t$, from which the half life is related to λ by, $T_{\frac{1}{2}} = \frac{\log 2}{\lambda} = \frac{0.693}{\lambda}$. The larger λ, the more radioactively unstable a nucleus is. Some typical half-lives are:

Polonium	$^{214}_{84}P$	$1.64 \times 10^{-4} s$
Krypton	$^{89}_{36}$K	3.16 minutes
Strontium	$^{90}_{38}$Sr	28.5 years
Radium	$^{226}_{88}$Ra	1600 years
Carbon	$^{14}_{6}$C	5730 years
Uranium	$^{238}_{92}$U	4.5×109 years

Recall

You can see how hugely different the lifetimes of different nuclei are!

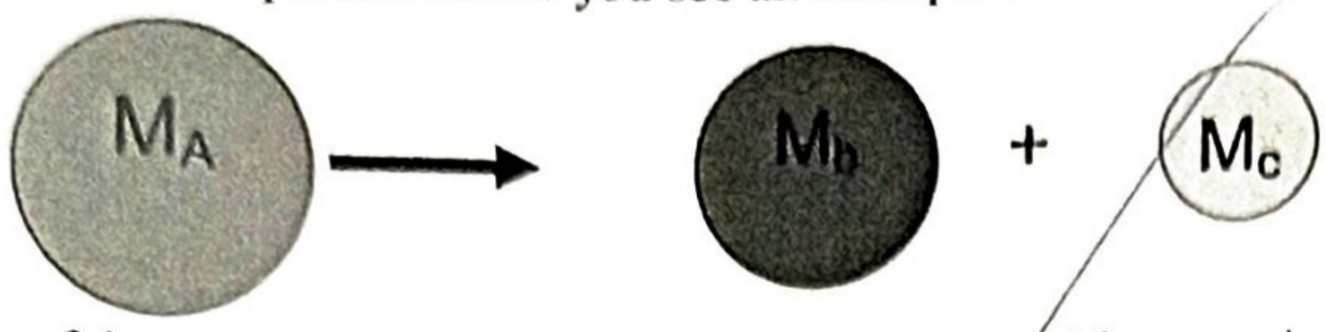
14. Here is a plot of the number of unstable nuclei left as a function of time. After each half-life, the number of nuclei decreases in number by half of the previous. Eventually there is only one nucleus left, and that too will eventually decay. So how can the derivation for the decay law be correct? Strictly speaking, we are not allowed to write down, or solve, a differential equation like $\frac{dn}{dt} = -\lambda n$ because this assumes that $n(t)$ is a continuous function. But this is almost true because in real life we deal with very large numbers of nuclei and so it makes a lot of sense to think of $n(t)$ as continuous.



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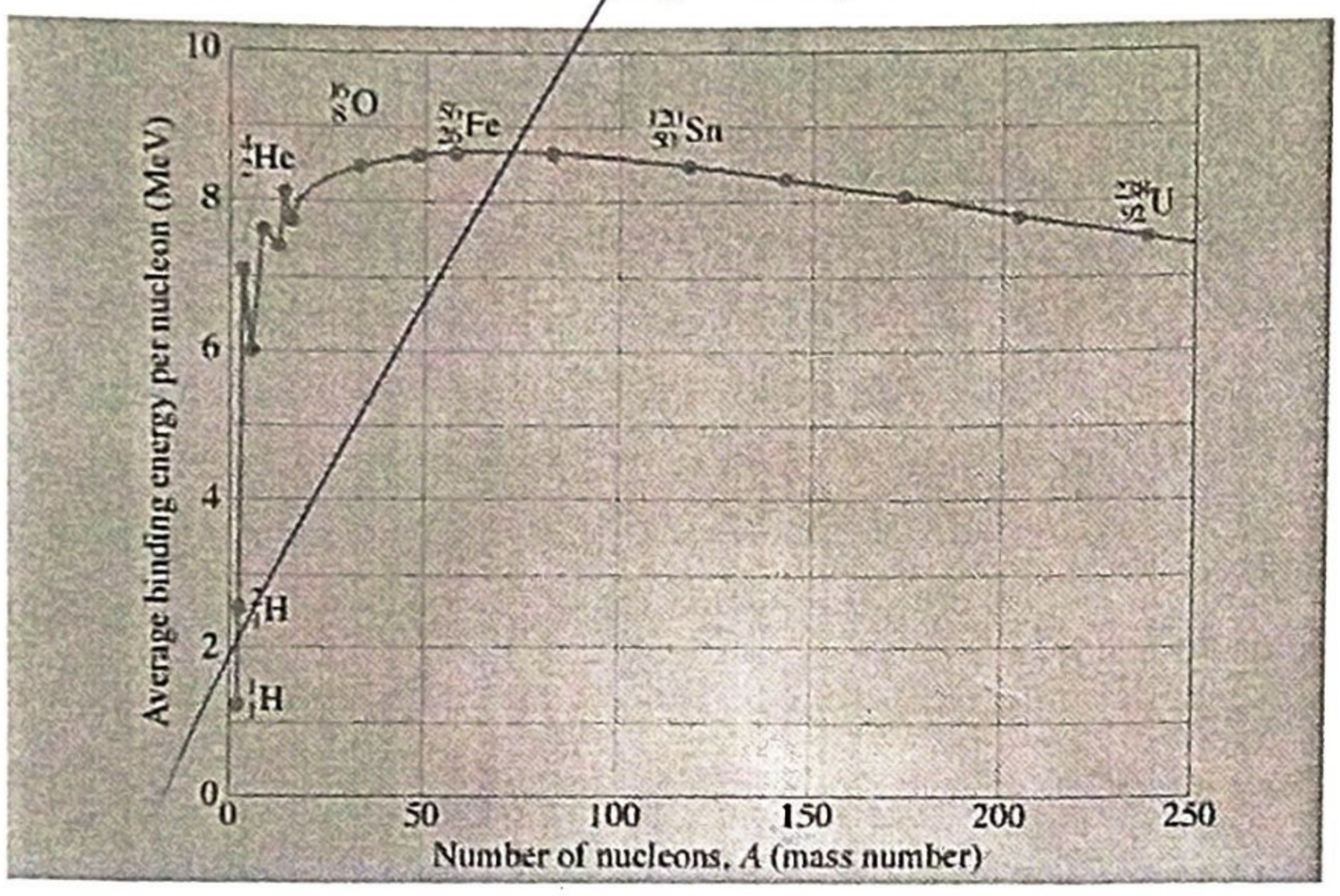
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usual situation. In the picture below you see an example of fission.



The masses of the two nuclei add up to less than the mass of the parent nucleus, and the energy released is $Q = (M_A - M_b - M_c)c^2$. This goes into kinetic energy and sends the two daughter nuclei flying apart at a large velocity. There happen to be NO completely stable nuclei above $Z = 82$, and no naturally occurring nuclei above $Z = 92$. Above these limits the nuclei decay or fall apart in some fashion to get below these limits.

19. A very useful concept is "binding energy". Suppose you want to take a nucleon out of a nucleus. The binding energy is the amount of energy that you would have to provide to pull it on the average. Nuclei with the largest BE per nucleon are the most stable. As you



can see from the graph below, the most stable element is iron, ^{56}Fe with a BE per nucleon of about 8.6 MeV. This is why iron is the heavy element found in the largest quantity on earth and inside stars. The curve is not smmoth and you see that a ^{4}He nucleus (i.e. an α parrticle, has a relatively high binding energy and so is relatively stable. In contrast, Uranium ^{235}U or deuterium ^{2}H are much less bound and they decay.

20. Nuclei can be unstable in different ways. A nucleus can emit α, β, and γ radiations. Usually a nucleus will emit one of these three, but it is possible to emit two, or even all three of these. (In addition, as we have discussed above, a nucleus can break up into

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Summary of Lecture 45 – PHYSICS OF THE SUN

1. In this lecture I shall pull together the different parts of physics that you have learned in this course and apply them to understanding the source of all life on earth - our sun. We will learn that the sun operates according to principles that we can understand, and on the basis of this we can even predict the manner in which it will eventually die.

2. Basic solar facts.

   a) Mass of sun $= 2\times10^{30}$ kg = 333,000 Earth's

   b) Diameter of sun = 1,392,000 km = 10° Earth's

   c) Age of sun = 4.6 billion years

   d) Rotation Period = 25 days at equator, 36 at poles (surface)

   e) Temperature = 15 million °K at core, 5770 °K at surface

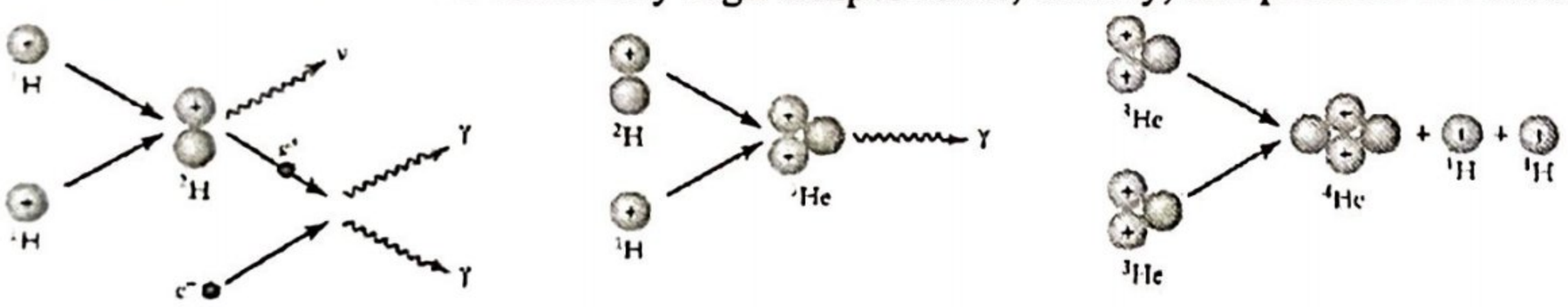
   f) Density = 8 × gold at the core, average is ~ 1.5 water

   g) Composition: 72% H, 25% He, rest is metals



3. The sun puts out a huge amount of energy. In quantitative terms we measure this in by its luminosity, 3.83×10^{27} joules per second. The power output is 3.83×10^{24} kilowatts. This is equal to 8×10^{16} of the largest power plants on Earth, meaning those which produce ~5000 MW of power. Another way of expressing this: every second the sun puts out as much energy as 2.5×10^{9} (2 billion) such power plants would put out every year.

4. What powers the sun? The earth is very old (billions of years). If there was a chemical fuel (say, coal or oil) at most that would last a few million years. But the sun is many thousands of tons older than that. Only after the discovery of $E = mc^2$ did we know the real secret. The sun gets its energy from the fusion (the coming together and combining of atomic nuclei. For this extremely high temperatures, density, and pressure is needed.



a Step 1:

- Two protons (hydrogen nuclei, ^1H) collide.
- One of the protons changes into a neutron (shown in blue), a neutral, nearly massless neutrino (ν), and a positron (e^+), a positively charged electron.
- The proton and neutron form a hydrogen isotope (^2H).
- The positron encounters an ordinary electron (e^-), annihilating both particles and converting them into gamma-ray photons (γ).

b Step 2:

- The ^2H nucleus from the first step collides with a third proton.
- A helium isotope (^3He) is formed and another gamma-ray photon is released.

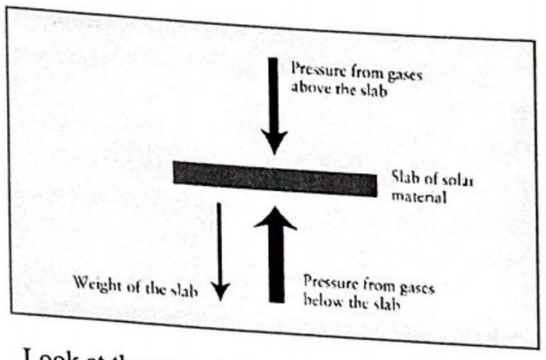
c Step 3:

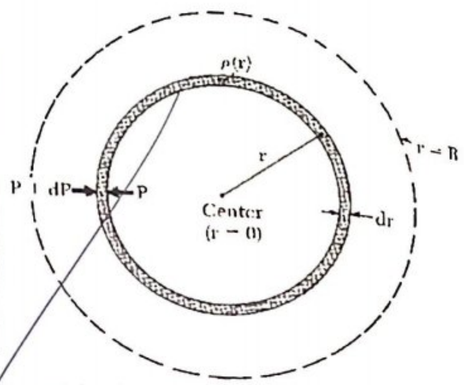
- Two ^3He nuclei collide.
- A different helium isotope with two protons and two neutrons (^4He) is formed and two protons are released.

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7. For over 4 billion years the sun has been nearly steady in maintaining its size. It is in a state of equilibrium between two big forces acting oppositely to each other:
   a) The hot hydrogen gas seeks to expand outwards because the thermal velocity of H atoms leads to a pressure directed outwards.
   b) Gravity tries to squeeze the star inwards because every piece of matter attracts every other piece. So the gravitational pressure is inward, increasing toward the core.

As in the diagrams below, imagine a piece of solar material at some distance from the centre of the sun. The two forces acting upon it must exactly balance in equilibrium. If for some reason the sun cools down, the pressure of gas will decrease and the sun will contract to a new equilibrium point. Conversely, the sun will expand if the rate of fusion were to increase.



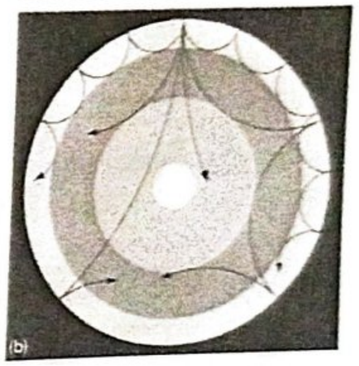


Look at the second diagram. Let $M(r)$ be the mass contained within radius r. We will not assume that the density is constant in r. First find the inward directed gravitational force on a shell of matter at radius r and thickness dr, $dF = -\dfrac{GM(r)\times\rho(r)4\pi r^2 dr}{r^2}$. There is a net pressure as shown, and we will call dP the difference of pressures. Then obviously $dF = dP\times 4\pi r^2$. Hence, $dP = -\dfrac{GM(r)\rho(r)dr}{r^2}$ or, $\dfrac{dP}{dr} = -\dfrac{GM(r)\rho(r)}{r^2}$. The total mass upto radius r is, $M(r) = \int_0^r \rho(r')4\pi r'^2 dr'$. If we knew $\rho(r)$ then $M(r)$ would also be known. By solving the differential equation, we would also then know $P(r)$. To give us a better understanding, suppose for simplicity that the sun is approximately uniform. Then the density is, $\rho \approx \dfrac{M}{(4\pi/3)R^3} \approx 1.4\text{gm cm}^{-3}$. Hence the pressure at the centre of the sun can be computed, $P_{centre} \approx \dfrac{GM\rho}{R} \approx 3\times10^9$ atmospheres.

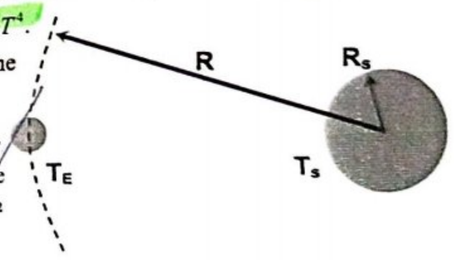
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8. The equilibrium of forces is a nearly perfect one, but there are small disturbances and these cause "earthquakes" (actually sun-quakes) to occur on the surface. The study of the surface of the sun and sun-quakes is an area known as "helioseismology" (helio=sun, seismology is the study of vibrations and earthquakes). The pulsating motions of the sun can be seen by measuring the Doppler shifts of hydrogen lines across the face of the sun. Some parts are expanding towards the earth while adjacent regions contract away. This is like the modes of a ringing bell. Vibrations propagate inside the sun, and the waves travel through a hot dense gas. They experience reflection and refraction since the speed at which a wave travels changes when it goes from a region of high to low density or vice-versa. All this is being studied by astrophysicists today as they map out the sun's interior.





9. Let us calculate the surface temperature of a planet circulating the sun. We shall use the Stefan Boltzman law that you studied in the lecture of heat: the power radiated by a black body per unit surface area at temperature T is σT^4. For thermal equilibrium, we must have that all the power absorbed from the sun is re-radiated by the planet. Let P_S be the sun's flux at its surface, Then, $P_S = \sigma T_S^4$. Since radiation decreases by the distance squared, P_R = flux at earth $= \sigma T_S^4 \times \left(\frac{R_s}{R}\right)^2$

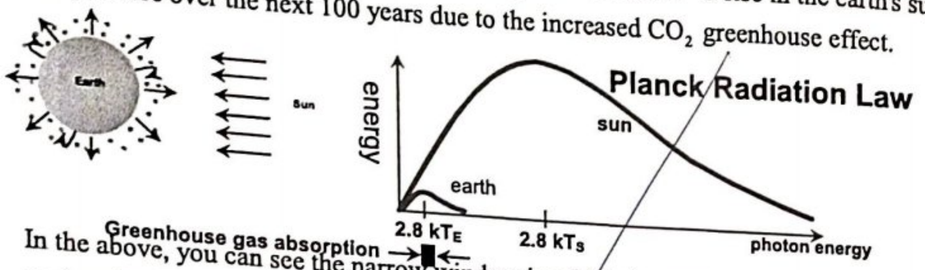


This must be multiplied by πR_E^2, which is the crossectional area of the planet. On the other hand, for emission from the planet, P_E = flux at its surface $= \sigma T_E^4$. We must multiply this by $4\pi R_E^2$, the area of planet, to get the total radiated power. Now impose the equilibrium condition: $P_R \times \pi R_E^2 = P_E \times 4\pi R_E^2$. This gives $\sigma T_S^4 \times \left(\frac{R_s}{R}\right)^2 = 4\sigma T_E^4$, and hence $T_E = \left(\frac{R_s}{2R}\right)^{1/2} T_S$. This is true for any planet, so let's see what this gives for the earth using $R_S = 7\times10^8$ metres, $R = 1.5\times10^{11}$ metres, and $T_S = 5800^\circ K$. This gives $T_E = 280K$, which is very sensible! Of course, we have assumed that the earth absorbs and emits as a black body. True?

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10. Actually, the black body assumption is only approximately true. About 30% of the sun's radiation reflects off our atmosphere! In fact the average surface temperature of the earth is about 290K, or about 13 ^{0}C. The extra warming is mainly due to the "greenhouse effect". Certain gases trap the sun's radiation and it is not able to get out, thus leading to greater absorption. The main greenhouse gases are CO_2, CH_4, H_2O. (N_2, O_2 and Ar are transparent to solar and earth radiation.) CO_2 is now 360 ppm (parts per million) of the atmosphere, up from 227 ppm in 1750, before the industrial revolution. Plants turn H_2O + CO_2 into O_2 plus organics. Some estimates predict a 3 to 10 ^{0}C rise in the earth's surface temperature over the next 100 years due to the increased CO_2 greenhouse effect.

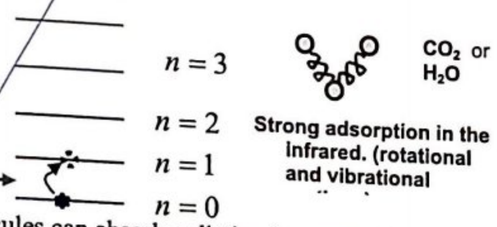


In the above, you can see the narrow window in which the greenhouse gases absorb the sun's radiation. But why are only some gases responsible, and not others?

11. The answer lies in quantum mechanics. In a previous lecture you learned that molecules can exist only in certain states that have very specific values of energy. For example, a molecule of CO_2 can oscillate and have equally spaced energy levels as shown below:

Energy levels of an oscillator:

$$\varepsilon_n = (n + \frac{1}{2})\hbar\omega$$

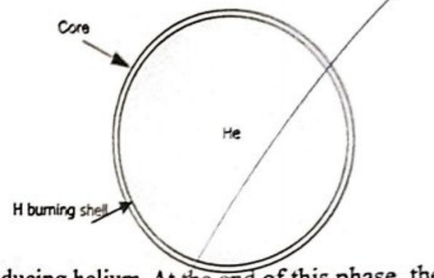


The energy at which these molecules can absorb radiation happen to lie in the spectrum of the re-radiated energy from the earth's surface. Thus, they effectively trap the out-going radiation, leading to enhanced earth temperatures. For any molecule, we can both calculate (using quantum mechanics) the frequencies at which radiation is absorbed or emitted. We can also experimentally measure the frequencies. Both of these are in very good agreement, and this is one of the reasons why we have such confidence in the correctness of quantum mechanics.

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12. Finally, I have some bad news: the sun is going to die because the hydrogen supply is eventually going to run out. There is, of course, no immediate danger - the Sun can last another 5 billion years on core hydrogen fusion. During this time it will be consuming

The Sun in Five Billion Years



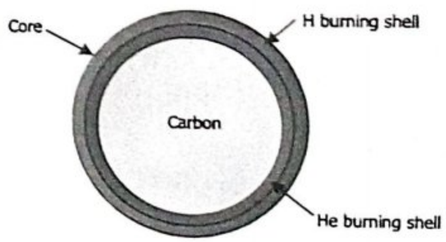
hydrogen and producing helium. At the end of this phase, the core of the sun will have mostly helium with the little bit of hydrogen left almost entirely outside in a thin layer. The reaction rate will fall, and the core will no longer be able to balance the pull due to gravity. This will cause a shrinkage of the core. As a consequence the temperature will increase. A new phase of the sun is about to start.

13. At this point a fusion reaction in next core zone begins. This lifts the envelopes and the sun will brighten up significantly. It is now a Red Giant star, and as you can see below, it is huge! The earth will be swallowed up by the expanded sun.



14. As remarked earlier, helium can also "burn" to produce carbon, but it needs a much higher temperature. Eventually even the He will be depleted, and the reaction rate will fall. Again the small reaction rate means that the core will shrink, and heat up to the point that it crosses the carbon fusion threshold of 600 million K. Low mass stars cannot reach this temperature. Envelope expands to a supergiant, many times larger than even a Red Giant.

Helium Depletion in Core



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