MTH101 FINAL 23 TO 45:

Lecture 23

1. What is the definition of a sequence in mathematics?
   - A) A set of numbers arranged in a specific order
   - B) A function whose domain is a set of consecutive integers
   - C) A mathematical notation for addition
   - D) A graphical representation of a function
   - Answer: B

2. Which of the following is an example of an arithmetic sequence?
   - A) 2, 4, 8, 16, 32
   - B) 3, 6, 12, 24, 48
   - C) 5, 10, 15, 20, 25
   - D) 1, 1, 2, 3, 5, 8
   - Answer: C

3. In an arithmetic sequence, the common difference is:
   - A) The ratio between consecutive terms
   - B) The difference between consecutive terms
   - C) The sum of all terms
   - D) The product of all terms
   - Answer: B

4. What is the formula to find the nth term of an arithmetic sequence?
   - A) $a_n = a_1 \times d \times (n-1)$
   - B) $a_n = a_1 + d \times (n-1)$

- C) $a_n = a_1 - d \times (n-1)$

- D) $a_n = a_1 / d \times (n-1)$

- Answer: B

5. A sequence is defined recursively by $a_1 = 2$ and $a_{n+1} = a_n + 3$. What type of sequence is this?

- A) Geometric sequence

- B) Harmonic sequence

- C) Fibonacci sequence

- D) Arithmetic sequence

- Answer: D

6. Which of the following represents a geometric sequence?

- A) 2, 5, 8, 11, 14

- B) 3, 6, 12, 24, 48

- C) 1, 4, 7, 10, 13

- D) 10, 20, 30, 40, 50

- Answer: B

Lecture 24

1. What is the definition of a geometric sequence?

- A) A set of numbers where each term is the product of the previous term and a constant

- B) A set of numbers arranged in a specific order

- C) A function whose domain is a set of consecutive integers

- D) A series of numbers that follow a specific addition pattern

- Answer: A

2. Which of the following sequences is geometric?

- A) 1, 2, 3, 4, 5
- B) 3, 9, 27, 81
- C) 2, 4, 6, 8, 10
- D) 5, 10, 15, 20
- Answer: B

3. In a geometric sequence, the common ratio is:

- A) The difference between consecutive terms
- B) The ratio between consecutive terms
- C) The sum of all terms
- D) The product of all terms
- Answer: B

4. What is the formula to find the nth term of a geometric sequence?

- A) $a_n = a_1 \times r \times (n-1)$
- B) $a_n = a_1 + r \times (n-1)$
- C) $a_n = a_1 \times r^{(n-1)}$
- D) $a_n = a_1 / r \times (n-1)$
- Answer: C

5. A sequence is defined recursively by $a_1 = 5$ and $a_{n+1} = 2a_n$. What type of sequence is this?

- A) Arithmetic sequence
- B) Harmonic sequence
- C) Fibonacci sequence
- D) Geometric sequence
- Answer: D

6. Which of the following represents an arithmetic sequence?

- A) 2, 6, 18, 54

- B) 3, 6, 9, 12
- C) 1, 4, 7, 10
- D) 10, 20, 40, 80
- Answer: B

Lecture 25

1. What is the sum of the first n terms of an arithmetic sequence?
   - A) $S_n = \frac{n}{2}(a_1 + a_n)$
   - B) $S_n = n(a_1 + a_n)$
   - C) $S_n = \frac{n}{2}(a_1 \times a_n)$
   - D) $S_n = n(a_1 \times a_n)$
   - Answer: A

2. Which formula represents the sum of the first n terms of a geometric sequence?
   - A) $S_n = a_1 \times \frac{1 - r^n}{1 - r}$
   - B) $S_n = a_1 \times \frac{1 + r^n}{1 + r}$
   - C) $S_n = a_1 + \frac{1 - r^n}{1 - r}$
   - D) $S_n = a_1 + \frac{1 + r^n}{1 + r}$
   - Answer: A

3. The series $1 + 2 + 3 + ... + n$ is an example of:
   - A) Geometric series
   - B) Harmonic series
   - C) Arithmetic series
   - D) Fibonacci series
   - Answer: C

4. What is the sum of the first 5 terms of the arithmetic sequence 3, 7, 11, 15, 19?
   - A) 45

- B) 55
- C) 65
- D) 75
- Answer: B

5. In the geometric series $2 + 6 + 18 + 54 + ...$, what is the common ratio?
   - A) 2
   - B) 3
   - C) 6
   - D) 9
   - Answer: B

6. What is the sum of the infinite geometric series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$?
   - A) 1
   - B) 2
   - C) 3
   - D) 4
   - Answer: B

Lecture 26

1. What is the definition of a series in mathematics?
   - A) A list of numbers
   - B) The sum of the terms of a sequence
   - C) A product of terms
   - D) A function of consecutive integers
   - Answer: B

2. Which of the following is a finite series?

- A) $2 + 4 + 6 + 8 + ...$
- B) $1 + 1/2 + 1/4 + 1/8 + ...$
- C) $3 + 6 + 9 + 12$
- D) $5 + 10 + 15 + ...$
- Answer: C

3. In the context of series, what does the term 'convergence' mean?

- A) The series has a finite sum
- B) The series does not end
- C) The series terms are positive
- D) The series terms are negative
- Answer: A

4. Which test can be used to determine the convergence of a series?

- A) Ratio test
- B) Product test
- C) Division test
- D) Subtraction test
- Answer: A

5. What is the sum of the series $1 + 2 + 3 + ... + 100$?

- A) 5000
- B) 5050
- C) 5150
- D) 5250
- Answer: B

6. Which of the following series is geometric?

- A) \(2 + 4 + 6 + 8 + ...\)
- B) \(1 + 2 + 3 + 4 + ...\)
- C) \(2 + 6 + 18 + 54 + ...\)
- D) \(5 + 10 + 15 + 20 + ...\)
- Answer: C

Lecture 27

1. What is the harmonic series?
   - A) A series where each term is the sum of its predecessors
   - B) A series where each term is the reciprocal of an integer
   - C) A series with a constant ratio between terms
   - D) A series with alternating positive and negative terms
   - Answer: B

2. Does the harmonic series converge or diverge?
   - A) Converges
   - B) Diverges
   - C) Neither converges nor diverges
   - D) Oscillates
   - Answer: B

3. What is the sum of the first 10 terms of the harmonic series \(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{10}\)?
   - A) 2.928968
   - B) 3.928968
   - C) 4.928968
   - D) 5.928968
   - Answer: A

4. Which of the following is an example of a harmonic series?

- A) $1 + 2 + 4 + 8 + ...$
- B) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$
- C) $1 + 3 + 5 + 7 + ...$
- D) $2 + 4 + 6 + 8 + ...$
- Answer: B

5. What is the nth term of the harmonic series?

- A) $1/n$
- B) $1/n^2$
- C) n
- D) n^2
- Answer: A

6. Which series is closely related to the harmonic series and also diverges?

- A) Arithmetic series
- B) Geometric series
- C) Alternating series
- D) p-series with $p \leq 1$
- Answer: D

Lecture 28

1. What is the definition of a power series?

- A) A series where each term is raised to a power
- B) A series of the form $\sum a_n x^n$
- C) A series with a constant difference between terms
- D) A series that alternates in sign
- Answer: B

2. In the power series $\sum a_n x^n$, what is the radius of convergence?

- A) The distance within which the series converges
- B) The maximum value of x for which the series converges
- C) The sum of the series
- D) The first term of the series
- Answer: A

3. Which test is commonly used to find the radius of convergence?

- A) Ratio test
- B) Root test
- C) Integral test
- D) Comparison test
- Answer: A

4. What is the interval of convergence for the power series $\sum a_n x^n$?

- A) The set of all x values for which the series converges
- B) The set of all x values for which the series diverges
- C) The point at which the series converges
- D) The sum of the series
- Answer: A

5. If the radius of convergence of a power series is 3, for which values of x does the series converge absolutely?

- A) $|x| < 3$
- B) $|x| > 3$
- C) $|x| \leq 3$
- D) $|x| \geq 3$
- Answer: A

6. Which of the following series is a power series?

- A) \(1 + 1/2 + 1/4 + 1/8 + ...\)
- B) \(1 + x + x^2 + x^3 + ...\)
- C) \(2 + 4 + 6 + 8 + ...\)
- D) \(1 + 1/2 + 1/3 + 1/4 + ...\)
- Answer: B

Lecture 29

1. What is a Taylor series?
   - A) A series that approximates functions as polynomials
   - B) A series where each term is a power of \(x\)
   - C) A series with constant terms
   - D) A series with alternating terms
   - Answer: A

2. What is the general form of a Taylor series for a function \(f(x)\) about \(x = a\)?
   - A) \(\sum \frac{f^n(a)}{n!}(x-a)^n\)
   - B) \(\sum f(a)(x-a)^n\)
   - C) \(\sum f(a)x^n\)
   - D) \(\sum \frac{f(a)}{n!}(x-a)^n\)
   - Answer: A

3. For which value of \(x\) does the Taylor series of \(e^x\) converge?
   - A) For all \(x\)
   - B) For \(x < 1\)
   - C) For \(x \leq 1\)
   - D) For \(x \geq 0\)
   - Answer: A

4. What is the Taylor series of \(\sin(x) \) about \(x = 0 \)?

- A) \(\sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}\)
- B) \(\sum \frac{(-1)^n x^{2n}}{(2n)!}\)
- C) \(\sum \frac{ x^{n}}{n!}\)
- D) \(\sum \frac{ x^{2n+1}}{(2n+1)!}\)
- Answer: A

5. Which function is represented by the Taylor series

\(\sum \frac{x^n}{n!}\) about \(x = 0\)?

- A) \(\sin(x)\)
- B) \(e^x\)
- C) \(\cos(x)\)
- D) \(\ln(1+x)\)
- Answer: B

6. Which of the following statements about the Taylor series is true?

- A) It is only valid for polynomial functions
- B) It can approximate any smooth function within its interval of convergence
- C) It always converges to the function it represents
- D) It is the same as the Fourier series
- Answer: B

Lecture 30

1. What is a Maclaurin series?

- A) A Taylor series centered at \(x = 1\)
- B) A Taylor series centered at \(x = -1\)
- C) A Taylor series centered at \(x = 0\)
- D) A Taylor series centered at \(x = \infty\)
- Answer: C

2. What is the Maclaurin series of \(\cos(x) \)?

- A) \(\sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}\)
- B) \(\sum \frac{(-1)^n x^{2n}}{(2n)!}\)
- C) \(\sum \frac{ x^{2n}}{(2n)!}\)
- D) \(\sum \frac{x^n}{n!}\)
- Answer: B

3. The Maclaurin series for \(\ln(1+x) \) is given by:

- A) \(\sum \frac{(-1)^n x^n}{n}\)
- B) \(\sum \frac{x^n}{n}\)
- C) \(\sum (-1)^{n+1} \frac{x^n}{n}\)
- D) \(\sum \frac{x^{n+1}}{n+1}\)
- Answer: C

4. What is the Maclaurin series expansion for \(\sin(x) \)?

- A) \(\sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}\)
- B) \(\sum \frac{(-1)^n x^{2n}}{(2n)!}\)
- C) \(\sum \frac{x^n}{n!}\)
- D) \(\sum \frac{x^{2n+1}}{(2n+1)!}\)
- Answer: A

5. Which function has a Maclaurin series that is equal to its Taylor series?

- A) \(\cos(x)\)
- B) \(\sin(x)\)
- C) \(e^x\)
- D) \(\ln(1+x)\)
- Answer: C

6. Which of the following is true for the Maclaurin series of a function?

- A) It always converges to the function for all x
- B) It is only defined for positive values of x
- C) It is a special case of the Taylor series
- D) It cannot represent exponential functions
- Answer: C

Lecture 31

1. What is the radius of convergence of the Maclaurin series for $\frac{1}{1-x}$?
   - A) 0
   - B) 1
   - C) 2
   - D) ∞
   - Answer: B

2. The Maclaurin series for e^x converges for:
   - A) $|x| < 1$
   - B) $|x| \leq 1$
   - C) $|x| < 2$
   - D) All x
   - Answer: D

3. For which function does the Maclaurin series $\sum \frac{(-1)^n x^{2n}}{(2n)!}$ represent?
   - A) $\sin(x)$
   - B) $\cos(x)$
   - C) e^x
   - D) $\ln(1+x)$
   - Answer: B

4. What is the Maclaurin series for $\frac{1}{1+x}$?

- A) $\sum (-1)^n x^n$
- B) $\sum x^n$
- C) $\sum (-x)^n$
- D) $\sum \frac{x^n}{n}$
- Answer: A

5. The convergence of the Maclaurin series for $\arctan(x)$ is:
- A) $|x| < 1$
- B) $|x| \leq 1$
- C) $|x| < 2$
- D) $|x| \geq 0$
- Answer: B

6. Which series represents the Maclaurin series for $\sinh(x)$?
- A) $\sum \frac{x^{2n+1}}{(2n+1)!}$
- B) $\sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
- C) $\sum \frac{x^{2n}}{(2n)!}$
- D) $\sum \frac{(-1)^n x^{2n}}{(2n)!}$
- Answer: A

Lecture 32

1. What is the definition of a Fourier series?
- A) A series that represents a function as a sum of sine and cosine terms
- B) A series that approximates functions as polynomials
- C) A series where each term is a power of x
- D) A series with alternating terms
- Answer: A

2. In a Fourier series, the coefficients are determined by:

- A) Integrating the function multiplied by sine and cosine terms
- B) Differentiating the function
- C) Adding the function terms
- D) Dividing the function by sine and cosine terms
- Answer: A

3. What is the general form of a Fourier series?
- A) $a_0 + \sum (a_n \cos(nx) + b_n \sin(nx))$
- B) $\sum (a_n \cos(nx) + b_n \sin(nx))$
- C) $\sum a_n x^n$
- D) $\sum \frac{a_n}{x^n}$
- Answer: A

4. For a function defined on $[-\pi, \pi]$, the Fourier coefficient a_n is given by:
- A) $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) \, dx$
- B) $\frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) \, dx$
- C) $\frac{1}{\pi} \int_{0}^{\pi} f(x) \cos(nx) \, dx$
- D) $\frac{1}{2\pi} \int_{0}^{\pi} f(x) \cos(nx) \, dx$
- Answer: B

5. The Fourier series is particularly useful for:
- A) Representing periodic functions
- B) Representing polynomial functions
- C) Representing rational functions
- D) Representing exponential functions
- Answer: A

6. Which function does not have a Fourier series representation?
- A) Discontinuous function
- B) Continuous function

- C) Periodic function
- D) Non-periodic function
- Answer: D

Lecture 33

1. What is the main purpose of the Fourier transform?
   - A) To convert a time-domain signal into its frequency-domain representation
   - B) To convert a polynomial function into a trigonometric function
   - C) To solve differential equations
   - D) To integrate complex functions
   - Answer: A

2. The inverse Fourier transform is used to:
   - A) Convert a frequency-domain signal back to its time-domain representation
   - B) Differentiate a function
   - C) Add complex functions
   - D) Multiply trigonometric functions
   - Answer: A

3. What is the Fourier transform of $\delta(t)$, the Dirac delta function?
   - A) 1
   - B) $\delta(f)$
   - C) $e^{i2\pi ft}$
   - D) $\frac{1}{2\pi}$
   - Answer: B

4. The Fourier transform of a constant function $f(t) = 1$ is:

- A) $\delta(f)$
- B) $\frac{1}{2\pi}$
- C) 1
- D) $e^{i2\pi ft}$
- Answer: A

5. Which property is not associated with the Fourier transform?

- A) Linearity
- B) Time shifting
- C) Scaling
- D) Exponential decay
- Answer: D

6. What does the convolution theorem state about the Fourier transform?

- A) The Fourier transform of a convolution is the product of the Fourier transforms
- B) The Fourier transform of a product is the convolution of the Fourier transforms
- C) The Fourier transform is the integral of the product of two functions
- D) The Fourier transform is the derivative of the product of two functions
- Answer: A

Lecture 34

1. What is the Laplace transform?

- A) A method to transform a time-domain function into a complex frequency-domain function
- B) A method to transform a polynomial function into a trigonometric function
- C) A method to integrate complex functions
- D) A method to differentiate rational functions
- Answer: A

2. The Laplace transform of e^{at} is:

- A) $\frac{1}{s-a}$
- B) $\frac{1}{s+a}$
- C) $\frac{a}{s^2}$
- D) $\frac{s}{s-a}$
- Answer: A

3. The inverse Laplace transform is used to:

- A) Convert a complex frequency-domain function back to its time-domain representation
- B) Differentiate a function
- C) Add rational functions
- D) Multiply trigonometric functions
- Answer: A

4. What is the Laplace transform of $\cos(at)$?

- A) $\frac{s}{s^2 + a^2}$
- B) $\frac{a}{s^2 + a^2}$
- C) $\frac{s-a}{s^2 + a^2}$
- D) $\frac{s+a}{s^2 + a^2}$
- Answer: A

5. Which function does not have a Laplace transform?

- A) Continuous function
- B) Discontinuous function
- C) Periodic function
- D) Non-periodic function
- Answer: D

6. The Laplace transform is particularly useful for:

- A) Solving differential equations
- B) Representing polynomial functions

- C) Representing rational functions
- D) Representing exponential functions
- Answer: A

Lecture 35

1. What is the Z-transform?
   - A) A method to transform a discrete-time signal into a complex frequency-domain function
   - B) A method to transform a continuous-time signal into a trigonometric function
   - C) A method to integrate discrete functions
   - D) A method to differentiate continuous functions
   - Answer: A

2. The Z-transform of a unit step function $u[n]$ is:
   - A) $\frac{1}{1-z^{-1}}$
   - B) $\frac{1}{1+z^{-1}}$
   - C) $\frac{z}{z-1}$
   - D) $\frac{1}{z-1}$
   - Answer: D

3. The inverse Z-transform is used to:
   - A) Convert a complex frequency-domain function back to its time-domain representation
   - B) Differentiate a discrete function
   - C) Add discrete functions
   - D) Multiply continuous functions
   - Answer: A

4. What is the Z-transform of $a^n u[n]$?
   - A) $\frac{1}{1-az^{-1}}$
   - B) $\frac{1}{1+az^{-1}}$

- C) $\frac{z}{z-a}$
- D) $\frac{1}{z-a}$
- Answer: D

5. Which property is not associated with the Z-transform?
   - A) Linearity
   - B) Time shifting
   - C) Scaling
   - D) Exponential decay
   - Answer: D

6. The Z-transform is particularly useful for:
   - A) Analyzing discrete-time systems
   - B) Solving continuous differential equations
   - C) Representing rational functions
   - D) Representing exponential functions
   - Answer: A

Lecture 36: Length of Plane Curves

1. Which formula is used to find the length of a plane curve?
   - A) $\int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
   - B) $\int_{a}^{b} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
   - C) $\int_{a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
   - D) All of the above
   - Answer: D

2. For the curve $y = x^3$, what is the length from $x = 0$ to $x = 1$?
   - A) $\frac{\sqrt{10} - 1}{2}$

- B) $\frac{\sqrt{10} + 1}{2}$
- C) $\frac{5}{3}$
- D) $\sqrt{10}$
- Answer: A

3. If a curve is given parametrically by $x = f(t)$ and $y = g(t)$, the arc length from $t = a$ to $t = b$ is found using:
- A) $\int_{a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
- B) $\int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
- C) $\int_{a}^{b} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
- D) $\int_{a}^{b} \left|f(t)g(t)\right| dt$
- Answer: A

4. For the polar curve $r = \theta$ from $\theta = 0$ to $\theta = \pi$, the length of the curve is:
- A) $\frac{\pi^2}{2}$
- B) π^2
- C) $\frac{\pi^2}{4}$
- D) $\frac{3\pi^2}{4}$
- Answer: B

5. The differential length element for a curve $y = f(x)$ is given by:
- A) $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
- B) $ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
- C) $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
- D) All of the above
- Answer: A

6. The arc length of the curve $y = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{4}$ is:
- A) $\sqrt{2}$
- B) $\frac{\sqrt{2}}{2}$

- C) $\frac{\sqrt{2}}{4}$
- D) $\ln(\sqrt{2})$
- Answer: A

Lecture 37: Area of Surface of Revolution

7. The formula to find the surface area of a curve rotated about the x-axis is:

- A) $2\pi \int_{a}^{b} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
- B) $2\pi \int_{a}^{b} x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
- C) $2\pi \int_{a}^{b} y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
- D) $2\pi \int_{a}^{b} x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
- Answer: A

8. For the curve $y = x^2$ rotated about the x-axis from $x = 0$ to $x = 1$, the surface area is:

- A) $\frac{16\pi}{15}$
- B) $\frac{8\pi}{15}$
- C) $\frac{32\pi}{15}$
- D) $\frac{4\pi}{15}$
- Answer: C

9. The surface area of the curve $x = y^3$ rotated about the y-axis from $y = 0$ to $y = 1$ is:

- A) $\frac{\pi}{2}$
- B) $\frac{3\pi}{2}$
- C) $\frac{5\pi}{2}$
- D) $\frac{7\pi}{2}$
- Answer: B

10. For a curve given parametrically by $x = f(t)$ and $y = g(t)$, the surface area of revolution about the x-axis is found using:

- A) $2\pi \int_{a}^{b} g(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

- B) $2\pi \int_{a}^{b} f(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
- C) $2\pi \int_{a}^{b} g(t) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
- D) $2\pi \int_{a}^{b} f(t) \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
- Answer: A

11. The surface area of the sphere $x^2 + y^2 = r^2$ obtained by rotating $y = \sqrt{r^2 - x^2}$ about the x-axis is:

- A) $4\pi r^2$
- B) $2\pi r^2$
- C) $6\pi r^2$
- D) $8\pi r^2$
- Answer: A

12. For the curve $y = \sqrt{x}$ rotated about the x-axis from $x = 1$ to $x = 4$, the surface area is:

- A) $\pi \left(4\sqrt{4} - 1\sqrt{1}\right)$
- B) $\pi \left(4\sqrt{4} + 1\sqrt{1}\right)$
- C) $\pi \left(4\sqrt{4} - \frac{1}{2}\sqrt{1}\right)$
- D) $\pi \left(4\sqrt{4} + \frac{1}{2}\sqrt{1}\right)$
- Answer: C

Lecture 38: Work and Definite Integral

13. The work done by a variable force $F(x)$ over a distance from $x = a$ to $x = b$ is given by:

- A) $\int_{a}^{b} F(x) dx$
- B) $\int_{a}^{b} F'(x) dx$
- C) $\int_{a}^{b} F(x) dx^2$
- D) $\int_{a}^{b} F(x) x dx$
- Answer: A

14. For a force function $F(x) = 3x^2$, the work done from $x = 0$ to $x = 2$ is:

- A) 8
- B) 16
- C) 12
- D) 24
- Answer: D

15. The work required to stretch a spring from $x = 0$ to $x = a$ with a force constant k is:

- A) $\frac{1}{2} k a^2$
- B) $\frac{1}{2} k a$
- C) $k a^2$
- D) $k a$
- Answer: A

16. If the force function is given by $F(x) = -kx$, the work done to compress a spring by $x = a$ is:

- A) $\frac{1}{2} k a^2$
- B) $-\frac{1}{2} k a^2$
- C) $k a^2$
- D) $-k a$
- Answer: A

17. In physics, work is defined as:

- A) Force multiplied by time
- B) Force multiplied by distance
- C) Mass multiplied by acceleration
- D) Energy multiplied by time
- Answer: B

18. The work done in moving an object along a straight line is represented by which of the following integrals?

- A) $\int_{a}^{b} F(x) dx$
- B) $\int_{a}^{b} F(t) dt$
- C) $\int_{a}^{b} F(x) x dx$
- D) $\int_{a}^{b} F'(x) dx$
- Answer: A

Lecture 39: Moment and Center of Mass

19. The moment of a force about a point is given by:

- A) Force multiplied by distance
- B) Force divided by distance
- C) Mass multiplied by distance
- D) Mass divided by distance
- Answer: A

20. The center of mass of a system of particles is found using:

- A) $\frac{\sum m_i x_i}{\sum m_i}$
- B) $\frac{\sum m_i x_i}{\sum x_i}$
- C) $\frac{\sum x_i}{\sum m_i}$
- D) $\sum m_i x_i$
- Answer: A

21. For a uniform rod of length L and mass M, the center of mass is located at:

- A) L
- B) $\frac{L}{2}$
- C) $\frac{L}{4}$
- D) $\frac{3L}{4}$
- Answer: B

22. The moment of inertia of a particle of mass m at a distance r from the axis of rotation is:

- A) mr
- B) mr^2
- C) $\frac{mr}{2}$
- D) $\frac{mr^2}{2}$
- Answer: B

23. The center of mass of a semicircular wire of radius R lies at a distance:

- A) $\frac{4R}{\pi}$
- B) $\frac{2R}{\pi}$
- C) $\frac{R}{2}$
- D) $\frac{R}{\pi}$
- Answer: B

24. For a triangular lamina with vertices at $(0,0)$, $(a,0)$, and $(0,h)$, the center of mass is at:

- A) $(\frac{a}{3}, \frac{h}{3})$
- B) $(\frac{2a}{3}, \frac{2h}{3})$
- C) $(\frac{a}{2}, \frac{h}{2})$
- D) (a, h)
- Answer: A

Lecture 40: Hydrostatic Pressure and Force

25. Hydrostatic pressure at a depth h in a fluid of density ρ is given by:

- A) $P = \rho gh$
- B) $P = \frac{\rho g}{h}$
- C) $P = \frac{h}{\rho g}$
- D) $P = \rho gh^2$
- Answer: A

26. The hydrostatic force on a vertical submerged plane surface is:

- A) $\rho g A \bar{h}$
- B) $\rho g V \bar{h}$
- C) $\rho g \bar{h}$
- D) $\rho g A$
- Answer: A

27. For a rectangular plate of width w and height h submerged vertically in a fluid, the hydrostatic force on the plate is:

- A) $\frac{1}{2} \rho g w h^2$
- B) $\rho g w h^2$
- C) $\frac{1}{3} \rho g w h^2$
- D) $\frac{2}{3} \rho g w h^2$
- Answer: A

28. The centroid of a triangular plate submerged vertically with base b and height h is located at:

- A) $\frac{h}{3}$ from the base
- B) $\frac{2h}{3}$ from the base
- C) $\frac{h}{2}$ from the base
- D) $\frac{h}{4}$ from the base
- Answer: B

29. If a circular plate of radius R is submerged vertically, the hydrostatic force on the plate is:

- A) $\rho g \pi R^2 \bar{h}$
- B) $\rho g 2\pi R \bar{h}$
- C) $\frac{\rho g \pi R^2 \bar{h}}{2}$
- D) $\rho g \pi R^3 \bar{h}$
- Answer: A

30. For a trapezoidal plate with bases a and b, and height h submerged vertically, the hydrostatic force is:

- A) $\frac{1}{2} \rho g (a + b) h^2$
- B) $\rho g (a + b) h^2$
- C) $\frac{1}{2} \rho g (a + b) h$
- D) $\rho g \frac{(a + b) h^2}{2}$
- Answer: A

Lecture 41: Probability

31. The probability of an event is always between:

- A) 0 and 1
- B) -1 and 1
- C) 0 and 100
- D) -1 and 0
- Answer: A

32. If an event is certain to happen, its probability is:

- A) 0
- B) 0.5
- C) 1
- D) 2
- Answer: C

33. The probability of the union of two mutually exclusive events A and B is:

- A) $P(A) + P(B)$
- B) $P(A) \cdot P(B)$
- C) $P(A \cap B)$
- D) $P(A) - P(B)$
- Answer: A

34. For independent events \(A\) and \(B\), the probability of both events occurring is:

- A) \(P(A) + P(B)\)
- B) \(P(A) \cdot P(B)\)
- C) \(P(A \cap B)\)
- D) \(P(A) - P(B)\)
- Answer: B

35. The complement of an event \(A\) has a probability of:

- A) \(1 - P(A)\)
- B) \(P(A)\)
- C) \(1 + P(A)\)
- D) \(1 - 2P(A)\)
- Answer: A

36. In a sample space \(S\) with equally likely outcomes, the probability of event \(A\) is given by:

- A) \(\frac{\text{Number of outcomes in } A}{\text{Total number of outcomes in } S}\)
- B) \(\frac{\text{Total number of outcomes in } S}{\text{Number of outcomes in } A}\)
- C) \(\frac{\text{Number of outcomes in } A}{\text{Number of outcomes in } S}\)
- D) \(\frac{\text{Number of outcomes in } S}{\text{Total number of outcomes in } A}\)
- Answer: A

Lecture 42: Probability Density Function

37. The probability density function (PDF) of a continuous random variable \(X\) is:

- A) A function that describes the likelihood of \(X\) taking on a particular value
- B) A function that describes the cumulative probability of \(X\) up to a certain value
- C) A function that describes the mean of \(X\)
- D) A function that describes the variance of \(X\)

- Answer: A

38. The area under the PDF curve over the entire range of possible values must be:

- A) 0
- B) 1
- C) Infinity
- D) Equal to the mean
- Answer: B

39. If $f(x)$ is the PDF of a continuous random variable X, then the probability that X lies between a and b is given by:

- A) $\int_{a}^{b} f(x) dx$
- B) $f(a) + f(b)$
- C) $f(b) - f(a)$
- D) $f(b) \cdot f(a)$
- Answer: A

40. For a continuous random variable X, the cumulative distribution function (CDF) $F(x)$ is related to the PDF $f(x)$ by:

- A) $F(x) = \int_{-\infty}^{x} f(t) dt$
- B) $F(x) = \int_{0}^{x} f(t) dt$
- C) $F(x) = \int_{a}^{b} f(t) dt$
- D) $F(x) = f(x)$
- Answer: A

41. The PDF $f(x)$ of a continuous random variable X must satisfy:

- A) $f(x) \geq 0$ for all x
- B) $f(x) \leq 0$ for all x
- C) $\int_{-\infty}^{\infty} f(x) dx = 0$
- D) $\int_{a}^{b} f(x) dx = 0$

- Answer: A

42. The mean of a continuous random variable X with PDF $f(x)$ is given by:

- A) $\int_{-\infty}^{\infty} x f(x) dx$
- B) $\int_{-\infty}^{\infty} f(x) dx$
- C) $\int_{a}^{b} x f(x) dx$
- D) $\int_{a}^{b} f(x) dx$
- Answer: A

Lecture 43: Expectation and Variance

43. The expected value (mean) of a random variable X is denoted by:

- A) $E(X)$
- B) $Var(X)$
- C) $P(X)$
- D) $F(X)$
- Answer: A

44. The variance of a random variable X is defined as:

- A) $E[(X - \mu)^2]$
- B) $E[(X - \sigma)^2]$
- C) $E(X)$
- D) $E(X^2)$
- Answer: A

45. The standard deviation of a random variable X is the:

- A) Square root of the variance
- B) Square of the variance
- C) Reciprocal of the variance
- D) Sum of the mean and variance

- Answer: A

46. If X and Y are independent random variables, then $Var(X + Y)$ is:
   - A) $Var(X) + Var(Y)$
   - B) $Var(X) - Var(Y)$
   - C) $Var(X) \cdot Var(Y)$
   - D) $Var(X) / Var(Y)$
   - Answer: A

47. The expectation of a function $g(X)$ of a random variable X is given by:
   - A) $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$
   - B) $E[g(X)] = g(E[X])$
   - C) $E[g(X)] = \int_{0}^{1} g(x) f(x) dx$
   - D) $E[g(X)] = g(x) f(x)$
   - Answer: A

48. The variance of a constant c is:
   - A) 0
   - B) c
   - C) c^2
   - D) 1
   - Answer: A

Lecture 44: Moments and Moment Generating Functions

49. The nth moment of a random variable X about the origin is given by:
   - A) $E(X^n)$
   - B) $E[(X - \mu)^n]$
   - C) $E(X)$
   - D) $E(nX)$

- Answer: A

50. The moment generating function (MGF) $M_X(t)$ of a random variable X is defined as:
   - A) $E(e^{tX})$
   - B) $E(tX)$
   - C) $E[(X - \mu)^t]$
   - D) $E(e^{X})$
   - Answer: A

51. The MGF of a random variable X can be used to find:
   - A) All moments of X
   - B) Only the mean of X
   - C) Only the variance of X
   - D) The probability density function of X
   - Answer: A

52. If $M_X(t)$ is the MGF of X, then the nth moment of X is given by:
   - A) $M_X^{(n)}(0)$
   - B) $M_X(0)$
   - C) $M_X^{(n)}(t)$
   - D) $M_X(t)$
   - Answer: A

53. The MGF of a normal distribution with mean μ and variance σ^2 is:
   - A) $e^{\mu t + \frac{1}{2}\sigma^2 t^2}$
   - B) $e^{\mu t}$
   - C) $e^{\frac{1}{2}\sigma^2 t^2}$
   - D) $e^{\mu t + \sigma^2 t^2}$
   - Answer: A

54. The MGF $M_X(t)$ of a random variable X at $t = 0$ is:

- A) 1
- B) 0
- C) μ
- D) σ^2
- Answer: A

Lecture 45: Probability Distributions

55. A binomial distribution is characterized by:

- A) Two possible outcomes in each trial
- B) Continuous outcomes
- C) A variable number of trials
- D) A single outcome
- Answer: A

56. The probability mass function (PMF) of a discrete random variable X is:

- A) The probability that X takes a specific value
- B) The cumulative probability up to a certain value
- C) The density of X at a specific value
- D) The variance of X
- Answer: A

57. The mean of a binomial distribution with parameters n and p is:

- A) np
- B) $n(1-p)$
- C) $\sqrt{np(1-p)}$
- D) $(np)^2$
- Answer: A

58. The variance of a Poisson distribution with parameter \(\lambda\) is:

- A) \(\lambda\)
- B) \(\lambda^2\)
- C) \(\sqrt{\lambda}\)
- D) \(\lambda (1-\lambda)\)
- Answer: A

59. A continuous random variable \(X\) follows an exponential distribution with parameter \(\lambda\). The mean of \(X\) is:

- A) \(\frac{1}{\lambda}\)
- B) \(\lambda\)
- C) \(\lambda^2\)
- D) \(\frac{1}{\lambda^2}\)
- Answer: A

60. The cumulative distribution function (CDF) \(F(x)\) of a uniform random variable \(X\) over \([a, b]\) is:

- A) \(\frac{x-a}{b-a}\)
- B) \(\frac{b-a}{x-a}\)
- C) \(\frac{x-a}{a-b}\)
- D) \(\frac{x-b}{b-a}\)
- Answer: A