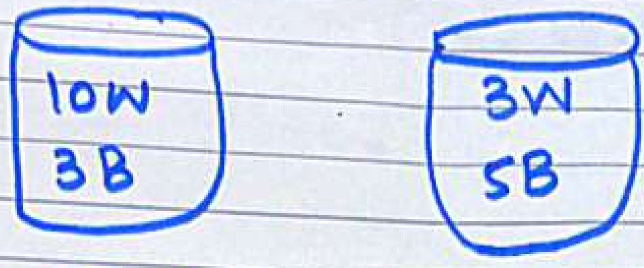
page 156 **Lecture 21 example 1** Masters

Solution:



bag 1 bag 2

Let

E_1 = 2 white balls are transfered from bag 1 to bag 2

E_2 = 2 black balls are transfered from bag 1 to bag 2

E_3 = 1 white ball or 1 black ball are transfered from bg 1 to bag 2.

$$P(E_1) = \frac{^{10}C_2}{^{13}C_2} = \frac{45}{78} = \frac{15}{26}$$

$$P(E_2) = \frac{^{3}C_2}{^{13}C_2} = \frac{3}{78} = \frac{1}{26}$$

$$P(E_3) = \frac{^{10}C_1 \times {^{3}C_1}}{^{13}C_2} = \frac{30}{78} = \frac{5}{13}$$

→ Let A be the event a ball (white ball) is drawn from bag 2.

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5W
5B

→ 2 white balls

bag 2

$$P(A/E_1) = \frac{5}{10} = \frac{1}{2}.$$

3W
7B

→ 2 black balls

$$P(A/E_2) = \frac{3}{10}$$

4W
6B

1 white or 1 black

$$P(A/E_3) = \frac{4}{10} = \frac{2}{5}$$

→ we have to find P(A)

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)$$

$$= \frac{15}{26}\cdot\frac{1}{2} + \frac{1}{26}\cdot\frac{3}{10} + \frac{5}{13}\cdot\frac{2}{5}$$

$$= \frac{15}{52} + \frac{3}{260} + \frac{10}{65}$$

$$= \frac{75+3+40}{260} = \frac{118}{260} = \frac{59}{130} \boxed{= 0.45}$$

Solution:-

Let's define the events

A : The bolt is produced by machine A

B = The bolt is produced by machine B.

C : The bolt is produced by machine C.

D : The bolt is defective

probabilities

$P(A) = 0.25$

$P(B) = 0.35$

$P(C) = 0.40$

Defected bolts

$P(D/A) = 0.02$

$P(D/B) = 0.04$

$P(D/C) = 0.05$

we have to find the Probability that a defected bolt came from machine A.

$$P(A_1/D) = \frac{P(A)\cdot P(D/A)}{P(A)\cdot P(D/A) + P(B)\cdot P(D/B) + P(C)\cdot P(D/C)}$$

$$= \frac{(0.25)(0.02)}{(0.25)(0.02)+(0.35)(0.04)+(0.40)(0.05)}$$

=

$$\frac{0.005}{0.0415} = 0.0121$$

= 1.21% from machine A

the defective bolts are 1.21%

Lecture # 24

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→ Continous Random Variable

A random variable which takes infinite number of values in an interval is called continous random variable.

Example:

→ The weight of a graup of individuals.

→ Height of graup of individua

→ Price of house

→ Probability Density function (p.d.f)

A function $f(x)$ is p.d.f if

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\, dx = 1$

→ Also known as density function

Example:-

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a) Find the value of k so that the function f(x) defined as

$$f(x) = kx, \quad 0 < x < 2$$

Solution:-

Must satisfy the two condition of pdf.

i. $f(x) > 0$

ii. $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$1 = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx$$

$$\int_{-\infty}^{0} kx\,dx + \int_{0}^{2} kx\,dx + \int_{2}^{\infty} kx\,dx = 1$$

$$0 + \int_{0}^{2} kx\,dx + 0 = 1$$

$$\left[k \frac{x^2}{2} \right]_0^2 = 1$$

$$k \left[\frac{(2)^2}{2} - \frac{(0)^2}{2} \right] = 1$$

$$k \left[\frac{4 \cancel{/}_2}{\cancel{x}} \right] = 1$$

$$k2 = 1$$

$$\boxed{k = 1/2}$$

b. Compute $P(X=1)$

$$(X=1) = 0$$

c. Compute $P(X>1)$

$$P(X>1) = \int_1^\infty f(x)\, dx \Rightarrow k \int_1^\infty x\, dx - 1$$

$$\frac{1}{2}\int_{1}^{2} x \, dx$$

$$\frac{1}{2}\left[\frac{x^2}{2}\right]_1^2$$

$$\frac{1}{2}\left[\frac{(2)^2}{2}-\frac{(1)^2}{2}\right]$$

$$\frac{1}{2}\left[\frac{\cancel{4}^{\,2}}{\cancel{2}}-\frac{1}{2}\right] =$$

$$\frac{1}{2}\left[\overset{\times 2}{2}-\frac{1}{2}\right]$$

$$\frac{1}{2}\left[\frac{3}{2}\right] = \frac{3}{4}$$

d. Compute the distribution function $F(x)$

$$F(x) = P(X < x) = \int_{-\infty}^{x} f(x)\,dx$$

for x $\quad -\infty < x \le 0$

$$F(x) = \int_{-\infty}^{x} 0\,dx = 0$$

$$F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{x} \frac{x^2}{2}\,dx$$

$$\left[\frac{x^2}{2+2}\right]_0^x$$

$$= \frac{x^2}{4} - \frac{(0)^2}{2} = \frac{x^2}{4}$$

Now $x > 2$

$$F(x) = \int_{-\infty}^{0} 0\,dx + \int_{0}^{2} \frac{x}{2}\,dx + \int_{2}^{\infty} 0\,dx = 1$$

$$= \frac{x^2}{4}$$

lec # 27

Example # 1

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$$f(x,y) = \frac{x(1+3y^2)}{4}$$

E(X), E(Y)

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_0^1 x\left(\frac{1+3y^2}{4}\right)dy$$

$$= \frac{1}{4}\int_0^1 (x + 3xy^2)\,dy$$

$$= \frac{1}{4}\left[xy + \frac{\cancel{3}xy^3}{\cancel{3}}\right]_0^1$$

$$= \frac{1}{4}\left[xy + xy^3\right]_0^1$$

$$= \frac{1}{4}\left[x(1) - x(0) + x(1)^3 - x(0)^3\right]$$

$$= \frac{1}{4}\left[x + x\right] = \frac{1}{\cancel{4}}\left[\cancel{2}x\right] = \frac{x}{2}$$

$$h(y) = \int_{-\infty} f(x, y)\, dx$$

$$= \int_0^2 \frac{x(1+3y^2)}{4}\, dx$$

$$= \frac{1}{4}\int_0^2 (x + 3xy^2)\, dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 3xy^2\right]_0^2$$

$$= \frac{1}{4}\left[\frac{(2)^2}{2} - \frac{(0)^2}{2} + 3(1)y^2 + 3(0)y^2\right]$$

$$\frac{1}{\cancel{4}_2}\left[\cancel{2} + 3y^2\right]$$

$$\frac{1}{2}\left[1+3y^2\right]$$

$$E(X+Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+y)\, f(x,y)\, dx\, dy$$

$$= \int_0^2 \int_0^1 (x+y)\, \frac{x(1+3y^2)}{4}\, dy\, dx$$

$$= \int_0^2 \int_0^1 (x+y)\, \frac{(x+3xy^2)}{4}\, dy\, dx$$

$$= \int_0^2 \int_0^1 \frac{(x^2+3x^2y^2+yx+3xy^3)}{4}\, dy\, dx$$

$$= \int_0^2 \int_0^1 \frac{x^2+3x^2y^2}{4}\, dx\, dy + \int_0^2 \int_0^1 \frac{xy+3xy^3}{4}\, dy\, dx$$

$$= \int_0^2 \frac{1}{4}\left[x^2y + x^2y^3\right]_0^1 dx$$

$$= \int_0^2 \frac{1}{4}\left[x^2(1) - x^2(0) + x^2(1)^3 - x^2(0)^3\right] dx$$

$$= \int_0^2 \frac{1}{4}\left[x^2 + x^2\right] dx$$

$$= \int_0^2 \frac{1}{\cancel{4}} \cancel{2}x^2\, dx$$

$$= \int_0^2 \frac{1}{2}x^2\, dx$$

$$= \frac{1}{2}\int_0^2 x^2\, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 \Rightarrow \frac{1}{2}\left[\frac{(2)^3}{3} - \frac{(0)^3}{3}\right]$$

part (1)

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{8}\,4}{3}\right] \Rightarrow \frac{4}{3}$$

$$= \int_0^2 \int_0^1 \frac{xy + 3xy^3}{4} \, dy \, dx$$

$$= \int_0^2 \frac{1}{4} \left[\frac{xy^2}{2} + \frac{3xy^4}{4} \right]_0^1 dx$$

$$= \int_0^2 \frac{1}{4} \left[\frac{x(1)^2}{2} - \frac{x(0)^2}{2} + \frac{3x(1)^4}{4} - \frac{3x(0)^4}{4} \right] dx$$

$$= \int_0^2 \frac{1}{4} \left[\frac{x}{2} + \frac{3x}{4} \right] dx$$

$$= \frac{1}{4} \left[\frac{x^2}{2 \times 2} + \frac{3x^2}{2 \times 4} \right]_0^2$$

$$= \frac{1}{4} \left[\frac{x^2}{4} + \frac{3x^2}{8} \right]_0^2$$

$$= \frac{1}{4} \left[\frac{(2)^2}{4} + \frac{3(2)^2}{8} \right]$$

$$= \frac{1}{4}\left[\frac{\cancel{4}}{\cancel{4}} + \frac{\cancel{12}^{\cancel{6}\,3}}{\cancel{8}\,\cancel{4}_{2}}\right]$$

part (2)

$$= \frac{1}{4}\underbrace{\left[1+\frac{3}{2}\right]}_{LCM} = \frac{5}{8}$$

$$\frac{2+3}{2} = \frac{5}{2}$$

$$= \frac{4}{3} + \frac{5}{8} \text{ (By Taking LCM)}$$

$$= \frac{47}{24}$$

$$E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\, dx\, dy$$

$$= \int_0^2\int_0^1 (xy)\, \underbrace{\frac{x(1+3y^2)}{4}}_{\frac{x+3xy^2}{4}}\, dy\, dx$$

$$= \int_0^2\int_0^1 \frac{x^2y + 3x^2y^3}{4}\, dy\, dx$$

$$= \int_0^2 \frac{1}{4}\left[\frac{x^2y^2}{2} + \frac{3x^2y^4}{4}\right]_0^1 dx$$

$$= \int_0^2 \frac{1}{4}\left[\frac{x^2(1)^2}{2} - \frac{x^2(0)}{2} + \frac{3x^2(1)^4}{4} - \frac{3x^2(0)^2}{4}\right] dx$$

$$= \int_0^2 \frac{1}{4}\left[\frac{x^2}{2} + \frac{3x^2}{4}\right] dx$$

$$= \int_0^2 \frac{1}{4}\left[\frac{2x^2 + 3x^2}{4}\right] dx$$

$$\int_0^2 \frac{1}{4}\left(\frac{5x^2}{4}\right) dx$$

$$= \frac{1}{4}\left[\frac{5x^3}{3 \times 4}\right]_0^2$$

$$\frac{1}{4}\left[\frac{5x^3}{12}\right]_0^2 \Rightarrow \frac{1}{4}\left[\frac{5(2)^3}{12} - \frac{5(0)^3}{12}\right]$$

$$= \frac{1}{4}\left[\frac{\cancel{\cancel{40}}^{\cancel{20}\,10}}{\cancel{12}_{\cancel{6}\,3}}\right]$$

$$= \frac{1}{\cancel{4}_2}\left[\frac{\cancel{10}^{5}}{3}\right] = \frac{5}{6}$$

i $E(X) + E(Y) = \frac{4}{3} + \frac{5}{8}$

$= \frac{47}{24}$

ii $E(X)\,E(Y)$

$$\left(\frac{4}{3}\right)\left(\frac{5}{8}\right) = \frac{\cancel{20}^{\cancel{10}\,5}}{\cancel{24}_{\cancel{12}\,6}} = \frac{5}{6}$$

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lecture # 25

Example # 1 page185

$$f(x) = \frac{x}{2}$$

$$P\left(X \le \frac{1}{2} \Big/ \frac{1}{3} \le X \le \frac{2}{3}\right)$$

$$= \frac{P\left(\frac{1}{3} \le X \le \frac{1}{2}\right)}{P\left(\frac{1}{3} \le X \le \frac{2}{3}\right)}$$

$$= \frac{\int_{1/3}^{1/2} \frac{x}{2}\, dx}{\int_{1/3}^{2/3} \frac{x}{2}\, dx}$$

$$= \frac{\left[\frac{x^2}{2 \times 2}\right]_{1/3}^{1/2}}{\left[\frac{x^2}{2 \times 2}\right]_{1/3}^{2/3}}$$

$$= \frac{\left[\frac{x^2}{4}\right]_{1/3}^{1/2}}{\left[\frac{x^2}{4}\right]_{1/3}^{2/3}}$$

$$= \frac{\frac{\left(\frac{1}{2}\right)^2}{4} - \frac{\left(\frac{1}{3}\right)^2}{4}}{\frac{\left(\frac{2}{3}\right)^2}{4} - \frac{\left(\frac{1}{3}\right)^2}{4}}$$

$$= \frac{\frac{\frac{1}{4}}{4} - \frac{\frac{1}{9}}{4}}{\frac{\frac{4}{9}}{4} - \frac{\frac{1}{9}}{4}}$$

$$= \frac{\frac{1}{4} \times 4 - \frac{1}{9} \times 4}{\frac{4}{9} \times 4 - \frac{1}{9} \times 4}$$

$$= \frac{\frac{\cancel{4}}{\cancel{4}} - \frac{4}{9}}{\frac{16}{9} - \frac{4}{9}}$$

$$= \frac{1 - \frac{4}{9}}{\frac{16-4}{9}}$$

$$= \frac{\frac{9-4}{9}}{\frac{\cancel{12}\,4}{\cancel{9}\,3}}$$

$$= \frac{5}{9} \times \frac{3}{4}$$

$$= \frac{15}{36}$$

Example # 02 page 185-186

A continous random variable x has d.f $F(x)$ as follow:

$$F(x) = 0, \quad \text{for } x < 0$$

$$= \frac{2x^2}{5}, \quad 0 < x \leq 1$$

$$= -\frac{3}{5} + \frac{2}{5}\left(3x - \frac{x^2}{2}\right), \quad 1 < x \leq 2;$$

$$= 1 \quad x < 2$$

find

$f(x) =$

$F(x)$

$F(x)$

$f(x)$

$F(x$

$f(x)$

$f(x)$

$f($

find p.d.f and $P(|X|<1.5)$

$$f(x) = \frac{d}{dx} F(x)$$

$F(x) = 0$, $f(x) = 0$

$$F(x) = \frac{2x^2}{5}$$

$$f(x) = \frac{2(2)x}{5} = \frac{4x}{5}$$

$$F(x) = -\frac{3}{5} + \frac{2}{5}\left(3x - \frac{x^2}{2}\right)$$

$$f(x) = 0 + \frac{2}{5}\left(3 - \frac{\cancel{2}x}{\cancel{2}}\right)$$

$$f(x) = \frac{2}{5}\left(3 - x\right)$$

$f(x) = 1$

$f(x) = 0$