MTH304 Notes

Lecture # 01

Mechanics:-

Mechanics is a branch of Physical Sciences which studies the action of force on bodies. under the action of these forces, bodies are in state of rest or of motion relative to some frame of reference.

Mechanics is further divided into two areas:- (i) Statis (ii) dynamics.

- **Statics:-**

Statics is that branch of mechanics which deals with forces acting on a body which is at rest.

- **Dynamics-**

Dynamics is that branch of Mechanics which accelerated motion

Inertial and non-Intertial frame of Reference:-

An Intertial frame of reference is a frame of reference in which the law of Inertia and other physics law are valid "Any frame moving at a Constant velocity relative to another frame is also an Intertial frame of Reference."

* A non-Interial frame of reference is a reference frame in which the law of Inertia does not hold.

Lec#02 Lecture#02.

Types of forces

- **Contact force:-**

forces that act through direct Contact between two objects.

- **Applied forces:-**

Friction

- **Long Range forces:-**

forces that can act over distance.

- **Gravity:-** Electromagnetic force (EMF)

Net force.

When Mutiple forces are acting on an object. The Net force is the amount of force that is left after adding all the forces on the object.

Net force (F_{NET}) = Resultant force (F_R)

Blanced forces:-

Balanced forces are forces that are equal and opposite So that they cancel out.

10N East →   10N West ←

Net force = $+\cancel{10}N + -\cancel{10}N = 0$

unblagnaced forces:-

do not cancel out and cause a change in the motion of object.

Inertia

Inertia measures the tendency of an object to resist changes in motion.

* Galileo came up with the idea of Inertia.

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* object do not want their motion to change
* Mass measures how much inertia an object has.-
(More Mass = More Inertia)

Mass & Weight

* Mass is amount of matter that an object possess. Mass does not change with location.
* Weight is the gravitational force that a large body exerts on other object.
* weight is a force! It is measured in Newtons.
* weight = Mass x Acceleration due to Gravity.
* $W = mg$.
* weight does change with location! ("g" will change with location)

Vector:-

The term vector is used by Scientists to Indicate a quantity that has both magnitude and direction.

Presenting a vector:-

A vector is often represented by an arrow or a directed line Segment.

* The length of the arrows represents the magnitude of the vector.

* The arrow points in the direction of the vector.

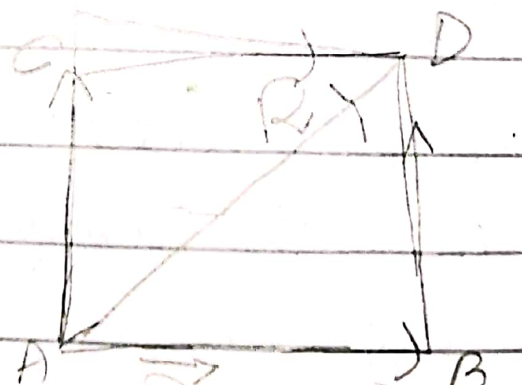
* Two non zero vectors are parallel if they are scalar Mutiples of one-another.

* The vector $-v = (-1).v$ has the same length as v but points in the opposite direction.

Lecture # 03

*** Parallelogram law:-**

If two forces act on a body at a point A along the adjacent sides of a parallelogram, then their resultant is the diagonal of the parallelogram with A as initial point.



$$\vec{R} = \vec{AB} + \vec{BD}$$

$$\overline{R} = \overline{AC} + \overline{BD}$$

Explaination of parallelogram law:-

* Vectors and the Geometry of Space:-

Vectors :-

The term vector is used by scientists to indicate a quantity (such as displacement or velocity or force) that has both Magnitude and direction.

* A vector is often represented by an arrow or a directed line segment.

* The length of the arrow represent the magnitude of the vector.

* The arrow Points in the direction of the vector.

• Zero vector:-

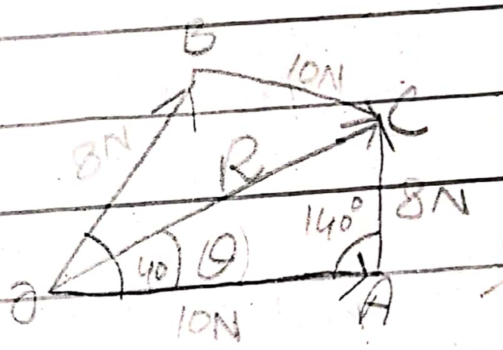
The zero vector, denoted by O, has length O.

* It is the only vector with no specific direction.

Lecture # 04

Parallelogram law Example #01:-

Two forces of Magnitude 8N and 10N are acting on a particle as shown in the figure. Find the resultant of these forces by applying parallelogram law. Find also the angle made by the resultant with 10N force.



ΔOAC

law of Cosines.

$R^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos(140)^\circ$

$= 64 + 100 - 160 \cos 140^\circ$

$= 164 + 122.57$

$= 286.57$

$R = \sqrt{286.57}$

$R = 17N.$

law of sines:-

$$\frac{R}{\sin 140} = \frac{8}{\sin\theta}$$

$$\frac{17}{\sin 140^\circ} = \frac{8}{\sin\theta}$$

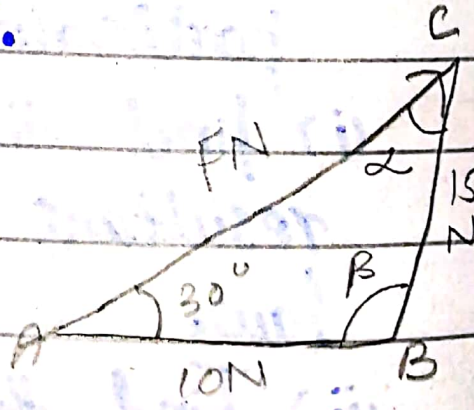
$$\sin\theta = \frac{8}{17} \times \sin 140^\circ$$

$$\theta = \sin^{-1}(0.3)$$

$$\theta = 17.6^\circ$$

Example #02:-

Two forces of Magnitude 10N and 15N are the resolved parts of force of Magnitude FN as shown in the figure. Find the Magnitude of force F.



using law of sines:-

$$\frac{15}{\sin 30^\circ} = \frac{10}{\sin\alpha}$$

$$\sin\alpha = \frac{10}{15} \times \sin 30^\circ$$

$$\alpha = \sin^{-1}\left(\frac{1}{3}\right) = 19.5^\circ$$

$\beta = 180 - 30^\circ - 19.5^\circ$
$= 130.5^\circ$

law of Cosines:-

$F^2 = 10^2 + 15^2 - 2(10)(15)\cos(130.5^\circ)$
$= 325 - 300\cos 130.5$
$= 325 + 194.83$
$= 519.83$
$F = \sqrt{519.83}$

Example #03:-

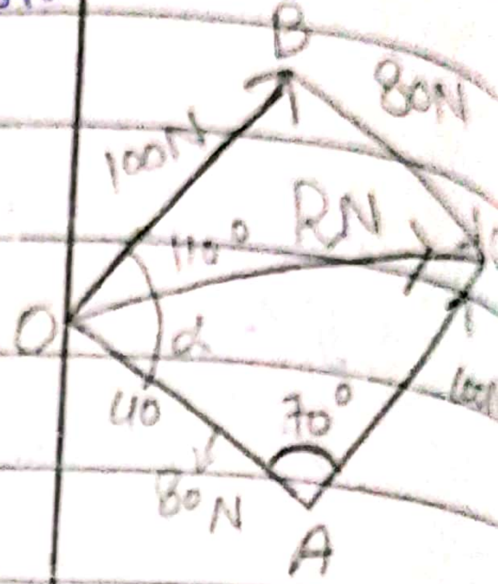
Two forces of Magnitude 100N and 80N are acting on a particale as shown in fig.
(i) find the Magnitude of the resultant by applying parallelogram law.
(ii) Find the Angle that the Resultant Makes With

using law of Cosines

$R^2 = 100^2 + 80^2 - 2 \times 100 \times 80 \times \cos 70^\circ$

$R = \sqrt{16,400 - 1600\cos 70^\circ}$

$R = 104.54$



using law of sines:-

$$\frac{\sin\alpha}{100} = \frac{\sin 70^\circ}{104.54}$$

$$\sin\alpha = \frac{100 \times \sin 70^\circ}{104.54}$$

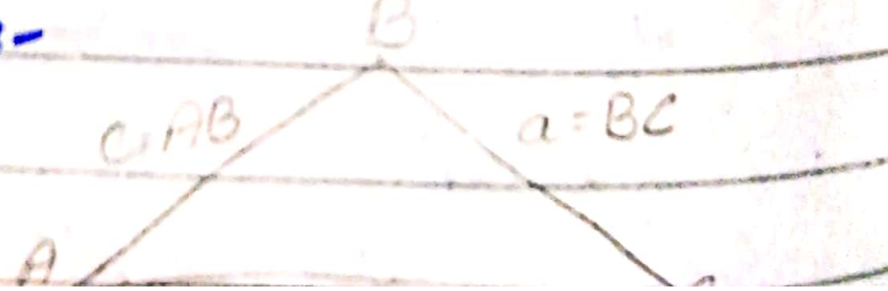
$\alpha = 64^\circ$

Angle with verticle $= 64^\circ + 40^\circ$

$= 104^\circ$

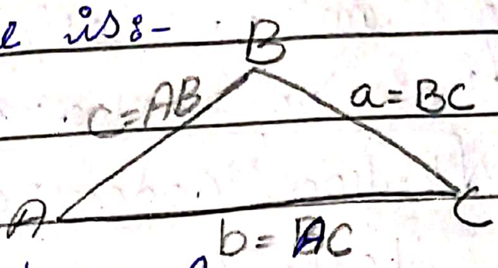
The Sine And Cosine Rule:-

(i) The Sine rule:-



2- The Cosine rule:-

The Cosins rule is:-



$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\frac{d}{dx}$ cosec:

Exercise:-

1- Solve the triangle ABC in which AC = 105cm AB = 76cm and A = 29°. 0-1

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = (105)^2 + (76)^2 - 2(105)(76)\cos(29°)$

$\cancel{= 841 \times 0.874}$

$\cancel{a^2 = 735.5°}$

$\sqrt{a^2} = \sqrt{2851.96}$

$a = 53.40$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$2ac\cos B = a^2 + c^2 - b^2$$

$$\cos B = \frac{a^2 + c^2 - b^2}{}$$

$$= \frac{(53.40)^2 + (76)^2 - (105)^2}{2(53.40)(76)}$$

$$\cos B = -0.2937$$

$$B = \cos^{-1}(-0.2937)$$

$$B = 107.08^\circ$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{(53.40)^2 + (105)^2 - \cancel{2(53.40)(105)}\ (76)^2}{2(53.40)(105)}$$

$$= \frac{8100.56}{11214}$$

$$= \cos^{-1}(0.722)$$

$$C = 43.72^\circ$$

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lecture # 05

Resolution of forces.

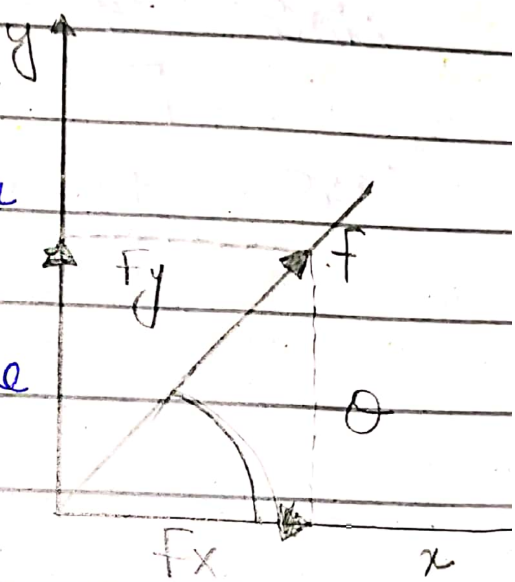
* The process of replacing a force system by its Components is called Resolution.

* To resolve a force into x-y Componets means to express the force as the sum of two forces, one in the x-direction and one in the y-direction.
When resolving a force into x-y components, we must have information on the direction of the force and the magnitude of the force.

L

* When the angle of the force relative to the x- or y-axis is known, we can use trigonometry to find the components.

Let (θ) be the angle that the forces makes with the positive x-axis. Using trigonometry, we find the components (f_x and f_y) as follows:-



$$\sin\theta = \frac{f_y}{F} \Rightarrow f_y = F \cdot \sin\theta$$

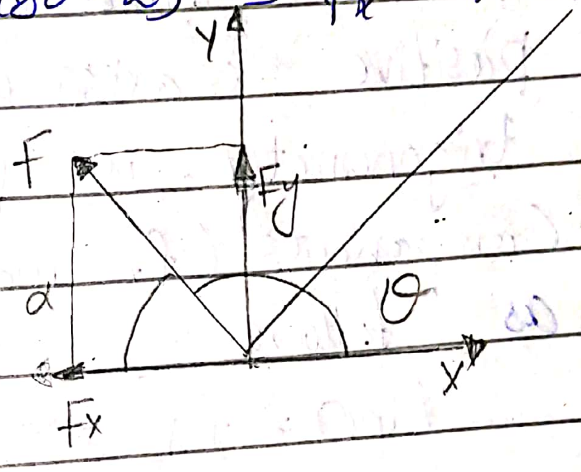
$$\cos\theta = \frac{f_x}{F} \Rightarrow f_x = F \cdot \cos\theta.$$

* It is usually easiest to find the magnitudes of the components from the acute angle of the triangle defined by the force and the axes. Note that the components may have positive or negative signs. You need

to recognize the signs of the Components so they agree with their Senses.

$$\sin\theta = \frac{F_y}{F} = \sin(180-\alpha) \Rightarrow f_y = f \cdot \sin\alpha$$

$$\cos\theta = \frac{F_x}{F} = \cos(180-\alpha) \Rightarrow f_x = -f\cos\alpha$$

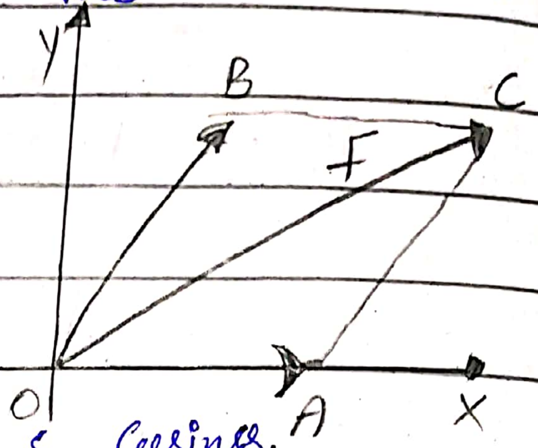


Oblique Components:-

* If non rectangular Components of a force are needed several methods are available for determing them.

* The Component of the force (F) shown as (OA) and (OB) can be determine graphically

by drawing the parallelogram to any Conveniant scale. The Magnitudes of the Components can be determined ~~graphically~~ ~~by draw~~ Algebracially from the law of sines & cosines. with sides of (a, b, c,).



Example

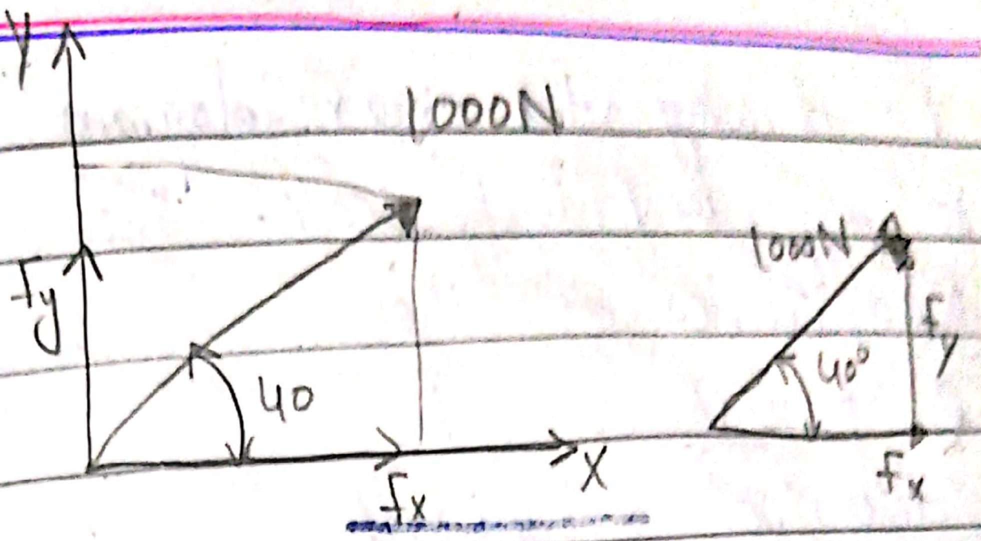
: Resolve the (1000N) force acting on the pipe, into Components in the (a) X and Y direction and (b) X' and Y direction.

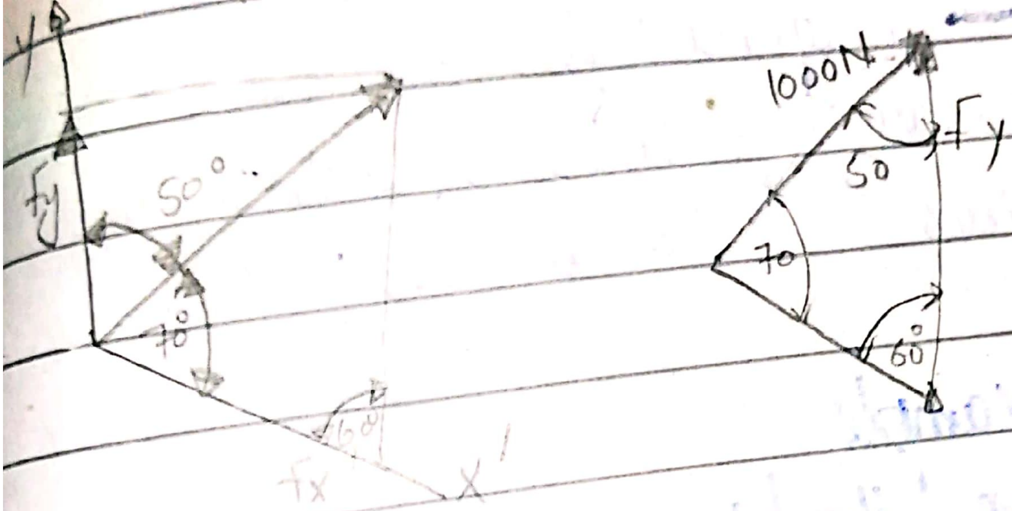
(a) $F_x = 1000 \cos 40° = 766N$

$F_y = 1000 \sin 40° = 643N.$

(b) $\frac{F_{x'}}{\sin 50°} = \frac{1000}{\sin 60°} \Rightarrow F_{x'} = 1000 \cdot \frac{\sin 50}{\sin 60}$

$= 884.6 N.$





$$\frac{F_y}{\sin 70} = \frac{1000}{\sin 60^\circ} \rightarrow F_y = 1000 \frac{\sin 70^\circ}{\sin 60^\circ}$$

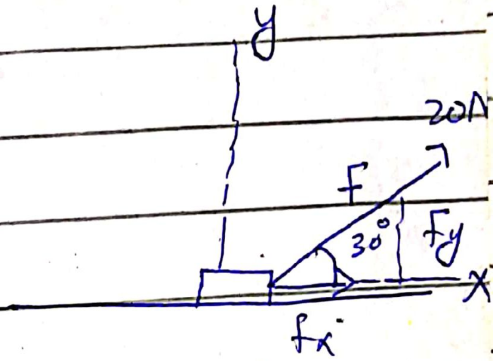
$\therefore$

$$= 1085\,N$$

lecture # 06 Application of Resolution of forces :

Example # 01

A force of 20N is acting on a block. Find its resolved parts along the Coordinate axes.



$$F_x = F\cos\theta$$

$$= 20\cos(30^\circ)$$

$$= 20\left(\frac{\sqrt{3}}{2}\right)$$

$$= 10\sqrt{3}\ N$$

$$F_y = F\sin\theta$$

$$= 20\sin(30^\circ)$$

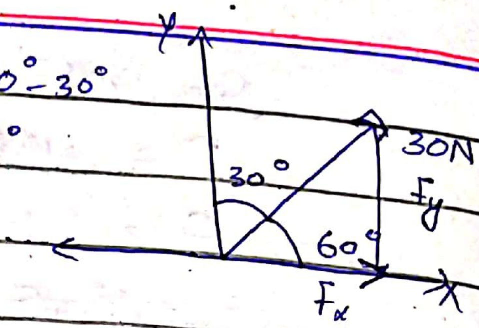
$$F_y = 20\frac{1}{2} \Rightarrow 10N$$

Example # 02 :-

A force of 30N is acting on the particle at an angle of 30 degree with y-axis. Find the resolved parts along x-axis and y-axis.

Angle with x-axis = $90^\circ - 30^\circ$

$= 60^\circ$



$F_x = F\cos\theta$

$F_x = 30\cos(60^\circ)$

$F_x = 30\left(\frac{1}{2}\right)$

$F_x = 15N$

$F_y = F\sin\theta$

$F_y = 30\sin(60^\circ)$

$= 30\,\sqrt{3}/2$

$F_y = 15\sqrt{3}$ F

Example # 03

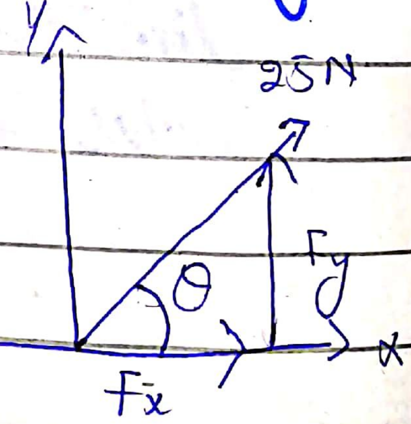
A force of 25N is acting on the particle. Resolved part of force along x- axis is 15N. Find.

(i) The angle that the force makes with x-axis.

(ii) Resolved part of the force along y- axis.

$F_x = 15N$, $F = 25N$.

(i) $\theta = ?$, (ii) $F_y = ?$



(i) $F_x = F\cos\theta$

$\cos\theta = F_x/F$

$\theta = \cos^{-1}\left(\frac{15}{25}\right)$

$\theta = \cos^{-1}(0.6)$

$\theta = 53^\circ.13^\circ$

~~$F_y = F\sin\theta$~~

$F^2 = F_x^2 + F_y^2$

$25^2 = 15^2 + F_y^2$

$625 - 225 = F_y^2$

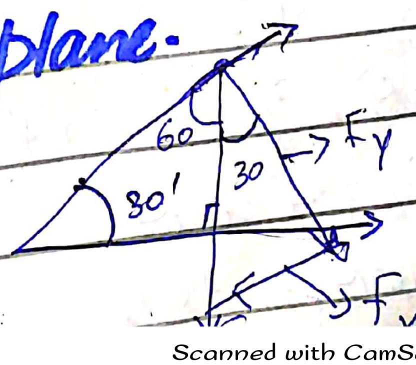
$F_y^2 = 400$

$\sqrt{F_y^2} = \sqrt{400}$

$F_y = 20N$

Example # 04

A particle of Mass 5kg is placed on an inclined plane ~~at~~. Find the resolved parts of the weight of the particle in the direction parallel and perpendicular to the plane.



885 823 302

Fnf 88nji

$m = 5kg$

$W = mg$

$W = 5(10)$ $\because g = 9.81$ —

$W = 50N$

$f_x = mg \sin 30^\circ$

$= 5(10)\sin(30^\circ)$

$= 50 \sin 30^\circ$

$= 50 \frac{1}{2}$

$f_x = 25 N$

$F_y = F\cos\theta$ $= mg \cos 30^\circ$

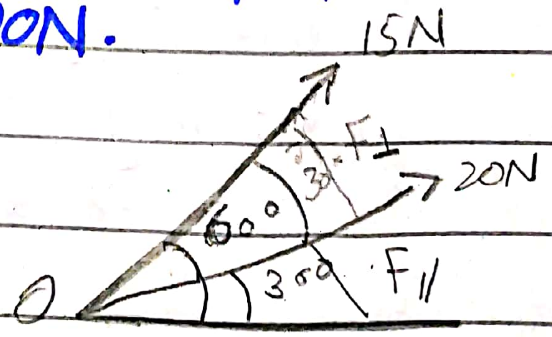
$= 50\cos(30)$

$= 50(\sqrt{3}/2)$

$F_y = 25\sqrt{3} N$

Example #05

A force of 15N and 20N are acting on the point. Find the resolved component of the force of 15N in the direction of parallel and perpendicular to the force of 20N.



$F_{\parallel} = F\cos\theta$

$= 15\cos 30$

$= 15 \times \sqrt{3}/2$

$= 7.5\sqrt{3}N \Rightarrow \frac{15}{2}\sqrt{3}\,N$

$F_{\perp} = F\sin\theta$

$= 15\sin(30^\circ)$

$= 15\left(\frac{1}{2}\right)$

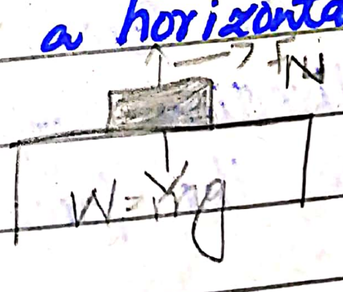
$F_{\perp} = 7.5\,N$

Lecture # 07.

Contact Forces on a plane and inclined plane.

Example # 01:-

Calculate the normal contact force when a book of mass 3kg is placed on a horizontal Table.



$m = 3kg$

$W = mg$

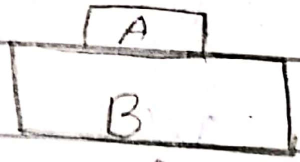
$= 3(10) = 30$

$f_N = W$

$= 30N$

Example #02

Two smooth boxes A and B of masses 20kg and 30kg respectively are placed. Find the magnitude of the forces acting on each box.



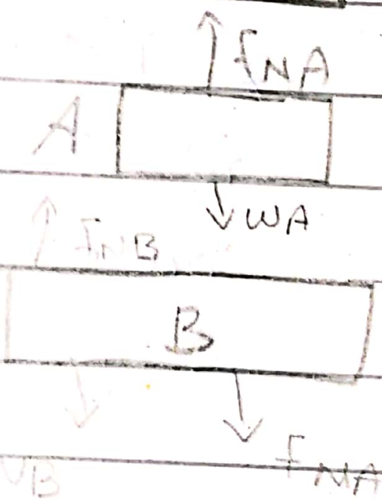
Consider Box A.

$m_A = 20kg$

$W_A = mg$

$= 20(10) = 200N$

$f_{NA} = W_A = 200N.$



Consider Box B:-

$m_B = 30kg$

$W_B = mg = 30(10) = 300N$

$f_{NB} = W_B + f_{nA} = 300 + 200$

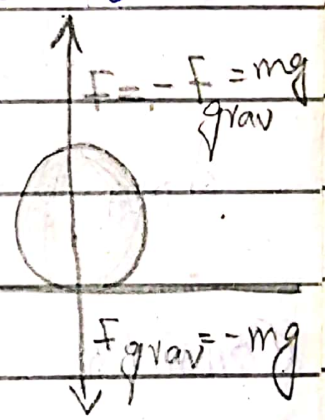
$f_{NB} = 500N$

~~Example # 03:-~~

Contact force:-

Gravity at rest:-

* The force of gravity acts on all objects all the time.
* if an object is at rest, the law of interia says that the net force is zero.
* There must be a force opposite to gravity that cancels it out.



NORMAL Force:-

* The force that opposes gravity for objects on the ground is called the normal force.

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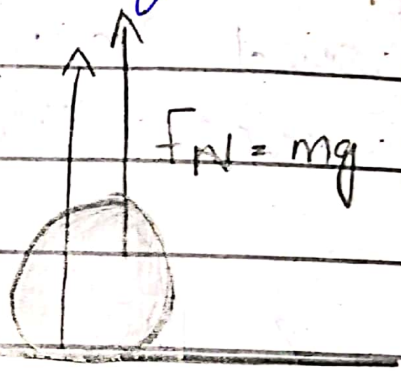
* It is perpendicular to the plane of the ground.

* The force is the result of the law of reaction.

$F_{sb} = -F_{bs} = mg$



Normal force and weight:-

* The normal force pushing up against gravity can be measured.

* we measures weight with a scale that measure normal force.

* weight is a force, not a mass.
* pounds measure weight, so force can be measured in pounds.

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Tension force:-

* A taut rope has a force exerted on it.
* If the rope is lightweight and flexible the force is uniform over the entire length.

* This force is called tension and points along the rope.
* Tension acts on both sides.

Tension or Normal force:-

* Tension and normal forces are different.
  * A pull on an object - tension
  * A push from a surface - Normal force.

* Either one or both may be present.

Equilibrium in one Dimension:-

* Two weights are hung supported by strings.
  - on the lower block the two forces balance: $F_{T_2} = m_2 g$
* on the upper block there are three $F_{T_1} = m_1 g + F_{T_2}$
  - $F_{T_1} = (m_1 + m_2) g$
* The upper string has more tension than the lower string.

Action - Reaction.

Newton's Third law of Motion:

* Newton's first two laws of motion explain how the motion of a single objects changes.

* Newton's third law describes something else that happens when one object exerts a force on another object.

* According to Newton's third law of motion, forces always act in equal

but opposite pairs.

** The forces exerted by two objects on each other are often called an action and reaction force pair.

* Either force can be considered the action force or the reaction force.

* Action and reaction forces pairs don't cancel because they act on different objects.

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lecture # 08

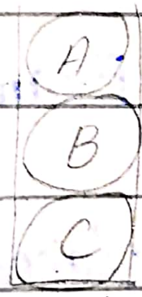
Example:-

Three identical Smooth balls each of mass m kg are labelled as A, B and they rest in Contact with each other in a smooth cylinderical Container. write down, in term of m and g:

(i) The Magnitude of the force exerted on ball A by ball B.

(ii) The Magnitude of the force exerted on ball B by ball C.

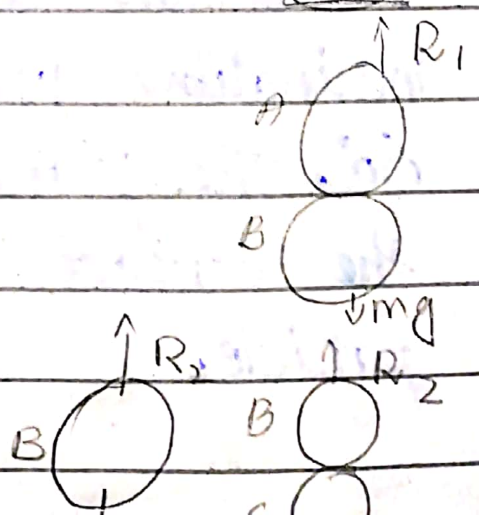
(iii) The Magnitude and direction of the resultant force exerted by ball A and C on ball B.



(i) $R_1 = mg$

(ii) $R_2 = mg + R_1$

$R_2 = mg + mg = 2mg$



(iii) $R = R_2 - mg$

$= 2mg - mg$

$= mg$ act upward.

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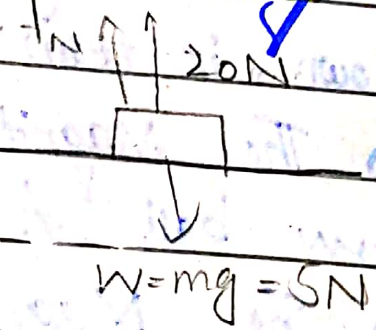
Example:-

A box of mass 5kg is placed on a smooth table. A force of 20N is acting on the box. Find the contact force acting on the box.



$m = 5kg$

$W = mg$

$W = 5(10)$

$W = 50N$

$F_N + 20 = 50$

$F_N = 50 - 20$

$F_N = 30$

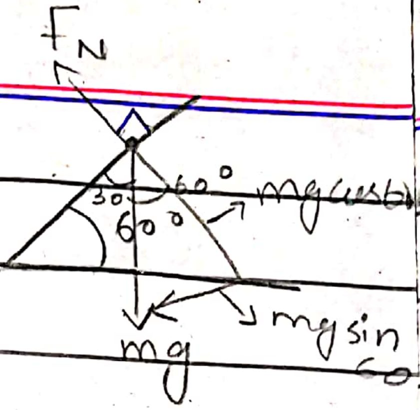
Example:-

A particle of mass 4kg is placed on an inclined plane whose inclination with the horizontal is 60°. The plane is smooth. Find the contact force acting on the particle.

$F_N = mg\cos 60^\circ$

$= 4(10)\,1/2$

$= 20N$



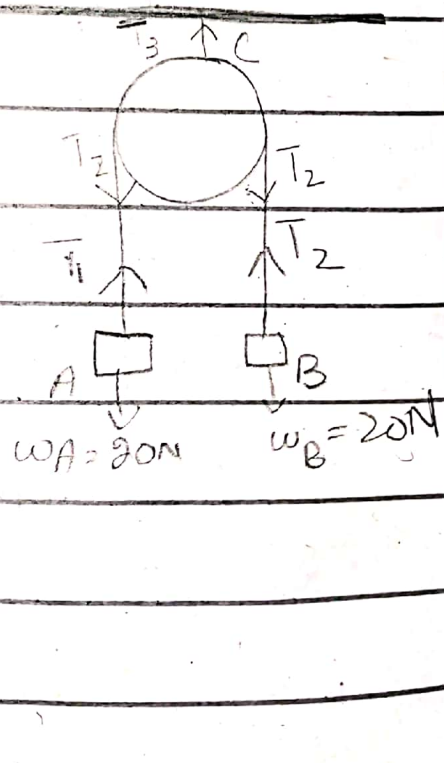
Example:-

Two particles of mass 2kg each are connected by a light inextensible string passing over a smooth fixed pulley, which is attached to a string C, the particles are at rest. Find the tension in the string C.

$m_A = 2kg$

$W_A = mg = 2(10) = 20N$

$W_B = mg = 2(10) = 20N$



At particle A:-

$T_1 = 20N$

At particle B:-

$T_2 = 20N$

At Pully:-

$T_3 = T_1 + T_2$

$T_3 = 20 + 20 = 40N$

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Lecture # 09

Resultant of two forces acting on the particle

1- Concurrent Forces:-

Two or more forces line of action intersect at one point. Called Concurrent forces. Such forces always have a resultant Through the point of ~~cure~~ Concurrence.

2. Resultant of two forces acting at a point:-

$$\vec{R} = \vec{F_1} + \vec{F_2} \longrightarrow (1)$$

$$\vec{R}\cdot\vec{R} = \vec{R}\cdot(\vec{F_1} + \vec{F_2})$$

$$\vec{R}^2 = (\vec{F_1} + \vec{F_2})(\vec{F_1} + \vec{F_2})$$

$$= \vec{F_1}\cdot\vec{F_1} + \vec{F_2}\cdot\vec{F_1} + \vec{F_1}\cdot\vec{F_2} + \vec{F_2}\cdot\vec{F_2}$$

$$= F_1F_1\cos 0 + F_2F_1\cos\alpha + F_1F_2\cos\alpha + F_2F_2\cos 0$$

$$R^2 = F_1^2 + 2F_1F_2\cos\alpha + F_2^2$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha} \longrightarrow (2)$$

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$\vec{F_1} \cdot \vec{R} = \vec{F_1} \cdot (\vec{F_1} + \vec{F_2})$

$F_1 R \cos\theta = F_1^2 + \vec{F_1} \cdot \vec{F_2}$

$F_1 R \cos\theta = F_1^2 + F_1 F_2 \cos\alpha$

$R\cos\theta = F_1 + F_2 \cos\alpha \rightarrow (3)$

$|\vec{F_1} \times \vec{R}| = |\vec{F_1} \times (\vec{F_1} + \vec{F_2})|$

$F_1 R \sin\theta = |\vec{F_1} \times \vec{F_1} + \vec{F_1} \times \vec{F_2}|$

$F_1 R \sin\theta = |\vec{F_1} \times \vec{F_2}|$

$\cancel{F_1} R \sin\theta = \cancel{F_1} \times F_2 \sin\alpha$

$R\sin\theta = F_2 \sin\alpha \rightarrow (4)$

$$\frac{R\sin\theta}{R\cos\theta} = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}$$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}$$

$$\theta = \tan^{-1} \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha} \rightarrow (5)$$

Case-1

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$

R is Max when $\cos\alpha = 1$

i.e. $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2}$

$= \sqrt{(F_1 + F_2)^2}$

$R = F_1 + F_2$

Case-II:-

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$

$$\cos\alpha = -1 \Rightarrow R_{min}$$

$$\alpha = 180^\circ$$

$$R_{min} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2(-1)}$$

$$= \sqrt{F_1^2 + F_2^2 - 2F_1F_2}$$

$$R_{min} = \sqrt{(F_1 - F_2)^2}$$

$$R_{min} = (F_1 - F_2)$$

Case-III:-

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$

$$\alpha = 90^\circ$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^\circ}$$

$$R = \sqrt{F_1^2 + F_2^2}$$

$$\theta = \tan^{-1}\left(\frac{F_2\sin(90^\circ)}{F_1 + F_2\cos 90^\circ}\right)$$

$$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right)$$

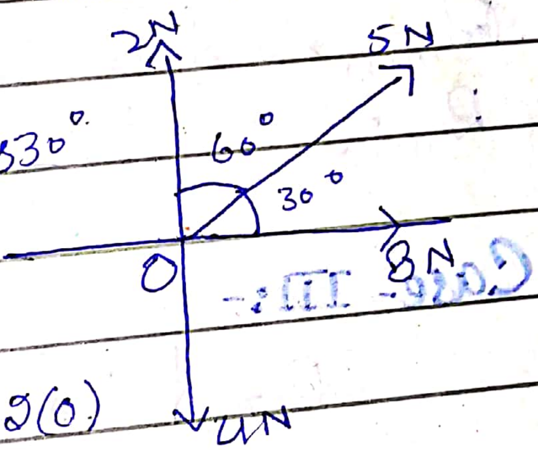
lecture # 10

Application of resultant of two forces acting on the particle

Example #01

Resultant of forces.

find the magniude and direction of the Resultant of the forces.



$X = 8\cos 0^\circ + 5\cos 30^\circ + 2\cos 90^\circ + 4\cos 270^\circ$

$X = 8(1) + 5(0.866) + 2(0) + 4(0)$

$X = 8 + 4.33 = 12.33\,N$

$Y = 8\sin 0^\circ + 5\sin 30^\circ + 2\sin 90^\circ + 4\sin 270^\circ$

$= 8(0) + 5(\frac{1}{2}) + 2(1) + 4(-1)$

$Y = 2.5 + 2 - 4 = 0.5\,N$

$R = \sqrt{x^2 + y^2} = \sqrt{(12.33)^2 + (0.5)^2} = 12.34\,N$

$R = \sqrt{152.27}$

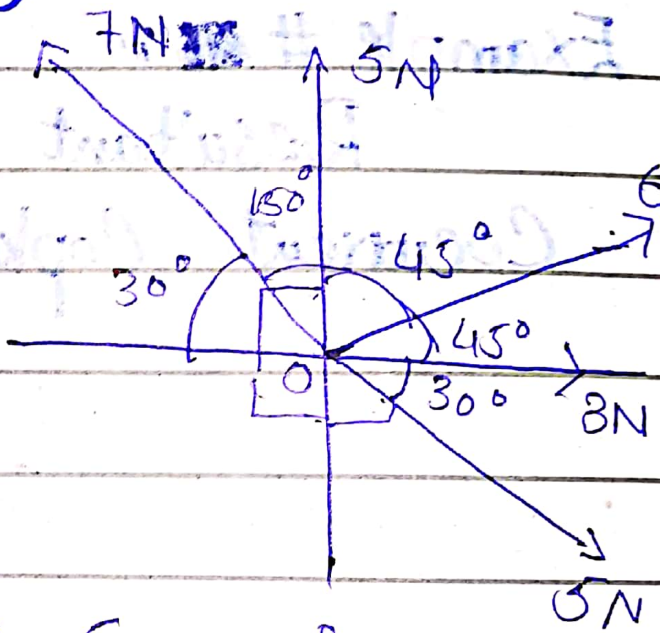
$\theta = \tan^{-1}\left(\frac{Y}{X}\right)$

$\theta = \tan^{-1}\left(\frac{0.5}{12.33}\right) = 2.3^\circ$ with 8N force.

Example #02

Resultant of forces

Find the Magnitude and direction of the resultant of two forces shown in fig.



$X = 3\cos 0^\circ + 6\cos 45^\circ + 5\cos 90^\circ + 7\cos 150^\circ$
$+ 5\cos 330^\circ$

$= 3 + 4.243 + 0 - 6.062 + 4.330$

$X = 5.511\,N$

$Y = 3\sin 0 + 6\sin 45^\circ + 5\sin 90^\circ + 7\sin 150^\circ$
$+ 5\sin 330^\circ$

$= 0 + 4.243 + 5 + 3.5 - 2.5$

$Y = 10.243\,N$

$$R = \sqrt{x^2 + y^2}$$

$$= \sqrt{(5.511)^2 + (10.243)^2}$$

$$R = 11.63\,N$$

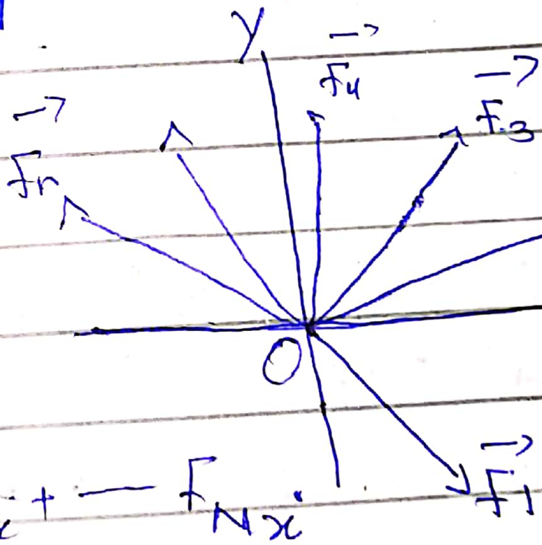
$$\theta = \tan^{-1}\frac{y}{x}$$

$$= \tan^{-1}\frac{(10.243)}{(5.511)}$$

$\theta = 61.7^\circ$ with 3N force

Example # [redacted] :-

Resultant of system of Cocurrent Coplanner forces.



$$R_x = F_{1x} + F_{2x} + F_{3x} + \text{---} F_{Nx}$$

$$R_y = F_{1y} + F_{2y} + F_{3y} + \text{---} F_{Ny}.$$

$$R^2 = R_x^2 + R_y^2$$

$$R = \sqrt{R_x^2 + R_y^2} \longrightarrow (1)$$

66f9vdqst

$$\tan\theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \longrightarrow (2)$$

Lecture # 11

Resultant of Multiple forces acting on the particle.

Example #03:-

Resultant of forces.

Two forces each of Magnitude 8N act at a point in the direction OA and OB. The Angle between the forces is x. The resultant of the two forces has component 9N in the direction OA. Find:-

(i) the Value of x.

(ii) the Magnitude of the resultant of the two forces.

(i)

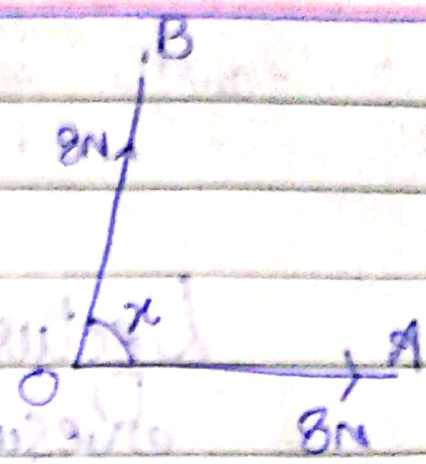
$R_x = 8N + 8\cos x$

$9N = 8N + 8\cos x$

$8\cos x = 1$

$\cos x = \frac{1}{8}$

$x = 82.8°$



(ii) $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos x}$

$= \sqrt{8^2 + 8^2 + 2 \times 8 \times 8 \times \cos 82.8}$

$= \sqrt{64 + 64 + 128\cos 82.8}$

$= 12N$

Example # 04:-

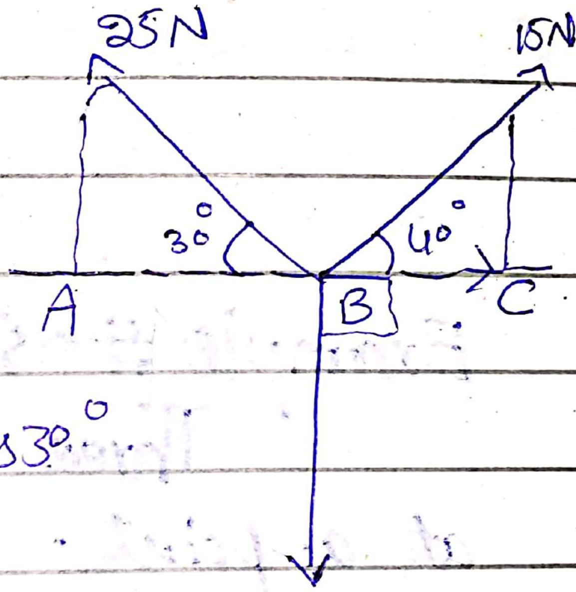
Resultant of forces

Coplaner forces of Magnitudes, 15N, 25N and 30N act at a point B, on the line ABC in the direction.

(i) Find the magnitude and direction of the resultant force.

(ii) The force of magnitude 15N is now replaced by a force of magnitude FN acting in the same direction. The new resultant force has zero component in the direction BC. Find the value of F and also find the magnitude and direction of New resultant force.

Sol:-



$R_x = 15\cos 40° - 25\cos 30°$

$= -10.16$

$R_y = 15\sin 40° + 25\sin 30° - 30$

$= -7.858$

$R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{(-10.16)^2 + (-7.858)^2}$

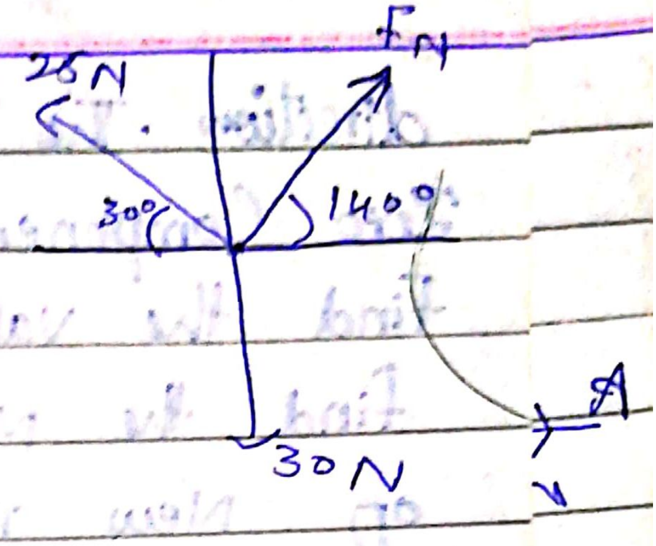
$R = 12.8N$

$\theta = \tan^{-1}(R_y/R_x)$

$= \tan^{-1}\left(\frac{-7.858}{10.16}\right)$

$= 37.7°$ with BA downward.

(ii)



$F\cos 40° = 25\cos 30°$

$F = \dfrac{25\cos 30°}{\cos 40°}$

$= 28.3N$

$R_y = F\sin 40° + 25\sin 30° - 30$

$= 28.3\sin 40° + 25\sin 30° - 30$

$= 0.667N$ upwards.

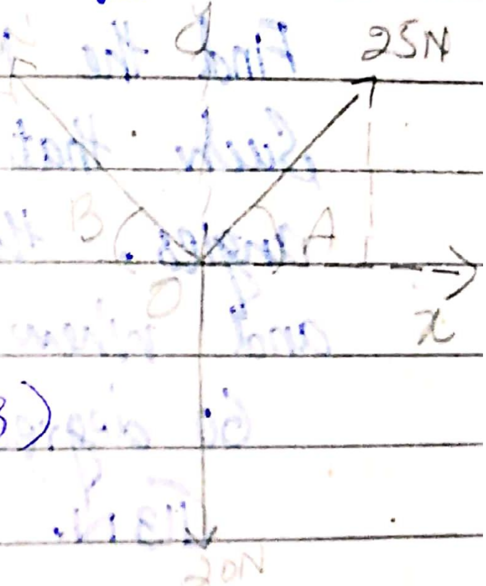
Example #05

Three coplaner forces act at a point. The Magnitude of the forces are 20N, 25N and 30N, and the direction in which forces act, where $\sin A = 0.28$ and $\cos A = 0.96$ and $\sin B = 0.6$ and $\cos B = 0.8$.

i) Show that the resultant of the three forces has a zero component in the x-direction.

(ii) Find the Magnitude and the direction of the resultant of the three forces.

(iii) The force of Magnitude 20N is replaced by another force. The effect is that the resultant force is unchanged is the Magnitude but reverse in direction. State the Magnitude and the direction of the replacement force.



Solution :-

$R_x = \sum F_x$

$= 25\cos A - 30\cos B$

$= 25(0.96) - 30(0.8)$

$= 24 - 24 = 0$

$R_y = \sum F_y$

$= 25\sin A + 30\sin B$

$= 25(0.28) + 30(0.6) - 20$

$= 5N$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0)^2 + (5N)^2}$

$= \sqrt{25} = 5N$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$= \tan^{-1}\left(\frac{5}{0}\right)$

90°

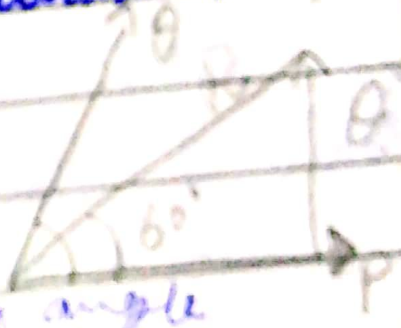
(iii) $X = 30N$ downward

Lecture #12

Example #06

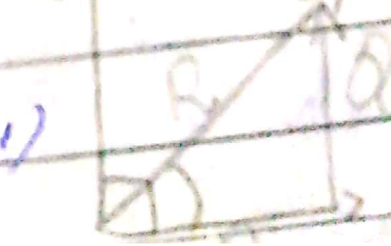
Resultant of forces.

Find the Magnitude of two forces Such that if they act at right angles, their resultant is $\sqrt{10}$ N and when the act at an angle of 60 degrees, their resultant is $\sqrt{13}$ N.



Let the force be P and q.

$R^2 = P^2 + q^2 \rightarrow$ Right angle



$(\sqrt{10})^2 = P^2 + q^2$

$P^2 + q^2 = 10 \longrightarrow (1)$

When angle is not Right angle.

$$R^2 = P^2 + q^2 + 2pq\cos\theta$$

$$(\sqrt{13})^2 = P^2 + q^2 + 2pq\cos 60^\circ$$

$$13 = 10 + 2pq\left(\tfrac{1}{2}\right)$$

$$3 = pq$$

$$Pq = 3 \longrightarrow (2)$$

$$q = 3/p \longrightarrow (3)$$

$$(1) \Rightarrow P^2 + \left(\frac{3}{p}\right)^2 = 10$$

$$p^2 + \frac{9}{p^2} = 10$$

$$P^4 + 9 = 10P^2$$

$$P^4 - 10P^2 + 9 = 0$$

let $t = P^2$

$$t^2 - 10t + 9 = 0$$

$$t^2 - 9t - t + 9 = 0$$

$$t(t-9) - 1(t-9) = 0$$

$$(t-1)(t-9) = 0$$

$$t = 9, \quad t = 1$$

$\Rightarrow P^2 = 1, \quad P = \pm 1$

$\Rightarrow P^2 = 9, \quad P = 3$

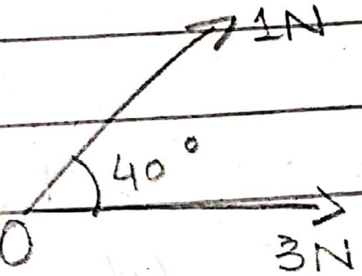
using eq (3), $q = 3/3$

$q = 3/1, \quad q = 3, \quad q = 1$

so the forces are 3N and 1N.

Example # 07

Two forces each of magnitude 1N and 3N act on a particle in the direction. Caculate the resultant Force on the particle and the angle between the resultant and 3N force.



$R = \sqrt{P^2 + q^2 + 2Pq\cos\theta}$

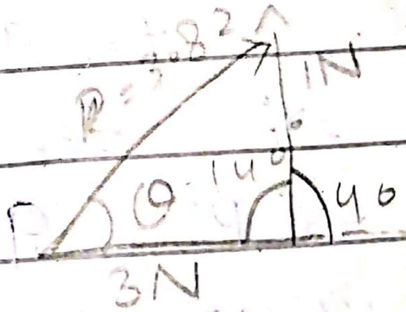
$= \sqrt{(1)^2 + (3)^2 + 2(1)(3)\cos 40^\circ}$

$= \sqrt{1+9+6(0.7660)}$

$= \sqrt{14.5963}$

$= 3.82N$

using sine rule



$\frac{\sin\theta}{1} = \frac{\sin 140^\circ}{3.82}$

$\sin\theta = 0.1682$

$\theta = \sin^{-1}(0.1682) = 9.68^\circ = 9.7^\circ$

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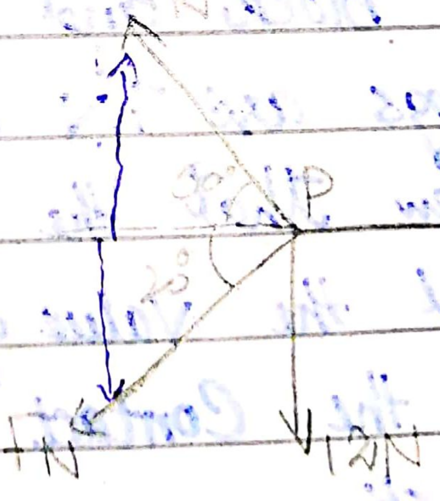
Lecture # 13
Equilibrium and its Conditions

- Equilibrium of a particle

Example 1:-

A particle P is in equilibrium on a smoth Horizontal table under the action of Horizontal forces of the magnitude FN, FN and 12N acting in the direction. find the value of F & G.



$X = 0$ & $Y = 0$

$Y = 0$

$F\sin 50^\circ = F\sin 20^\circ + 12$

$F\sin 50^\circ - F\sin 20^\circ = 12$

$F(\sin 50^\circ - \sin 20^\circ) = 12$

$$F = \frac{12}{\sin 50^\circ - \sin 20^\circ} = 28.3N$$

$X = 0$

$\therefore G = F\cos 50^\circ + F\cos 20^\circ$

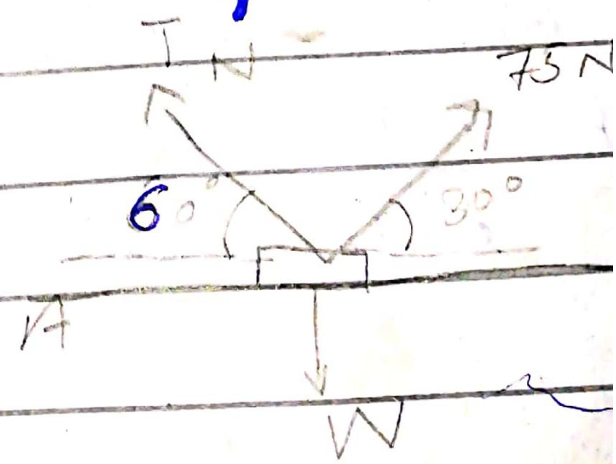
$= F(\cos 50^\circ + 20^\circ)$

$= 28.3(\cos 50^\circ + \cos 20^\circ)$

$G = 44.78N$

Example #02

Two light strings are attached to a block of mass 20kg. The block is in equilibrium on an horizontal surface AB with its strings taut. The strings make angle of 60 and 30 degrees with the horizontal on either sides of the block and the tensions in the strings are TN and 75N respectively. Given that the surface is smooth, Find the value of T and the magnitude of the contact force acting on the block.



Example # 04:-

$m = 20kg$

$W = mg \Rightarrow 20(10) = 200N$

$X = 0 \Rightarrow T\cos 60° = 75\cos 30°$

$T = \frac{75\cos 30°}{\cos 60°} = 129.9N$

$Y = 0$

$R + T\sin 60° + 75\sin 30° = 200$

$R + 129.9\sin 60° + 75\sin 30° = 200$

$R = 200 - 129.9\sin 60 - 75\sin 30°$

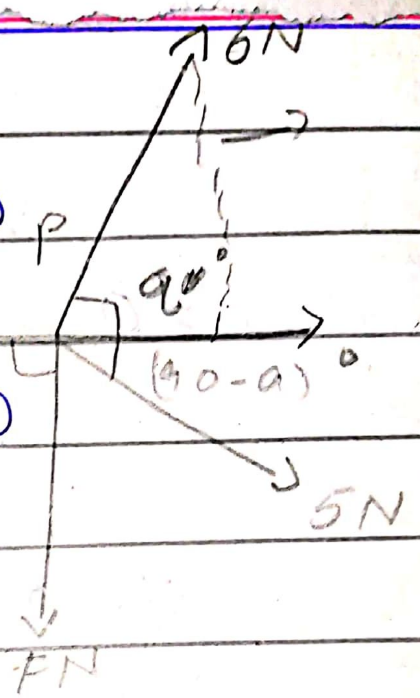
$R = 50N$

Lecture # 14
Applications of Equilibrium of A particle.

Example # 03:-

Equilibrium of a particle.

A particle p is in equilibrium on on a smooth horizontal table under the action of horizontal forces of Magnitude 6N, 5N, FN and FN acting in the direction. Find the value of a & f



$\vec{F} = 6\cos a + 5\sin(90-a)$

$= 6\cos a + 5\sin a \rightarrow (1)$

$\downarrow \; F + 5\sin(90-a) = 6\sin a$

$F + 5\sin a = 6\sin a$

$F = 6\sin a - 5\cos a$

using (1) & (2)

$6\sin a - 5\cos a = 6\cos a + 5\sin a$

$6\sin a - 5\sin a = 6\cos a + 5\cos a$

$\sin a = 11\cos a$

$\frac{\sin a}{\cos a} = 11$

$\tan a = 11$

$a = \tan^{-1}(11)$

$= 84.80$

using (1)

$F = 6\cos 84.8° + 5\sin 84.8°$

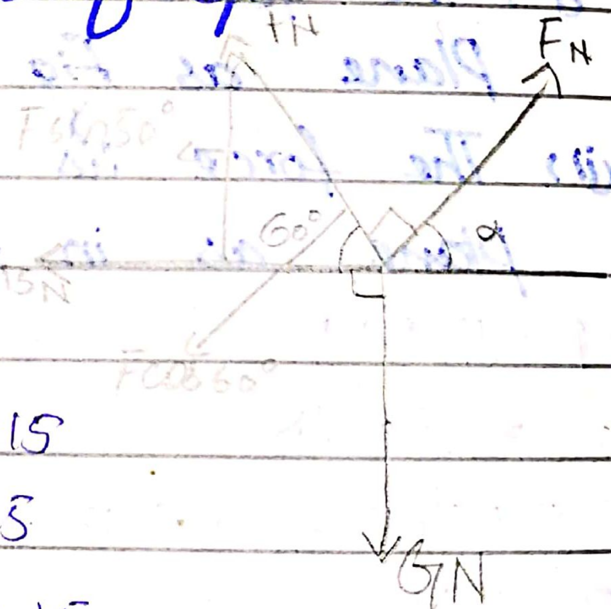
$= 5.52\,N$

Example #04:-

Four horizontal forces act at a point O and are in equilibrium. the Magnitude of the forces are FN, GN 15N, and FN. and the forces act in the direction.

(i) Show that $F = 41.0$ Correct to 3 Significant figures.

(ii) Find the value of G.



Solution:-

$\alpha + 60^\circ + 90^\circ = 180^\circ$

$\alpha = 30^\circ$

(i) $F\cos 30^\circ = F\cos 60^\circ + 15$

$F\cos 30^\circ - F\cos 60^\circ = 15$

$F(\cos 30^\circ - \cos 60^\circ) = 15$

$$F = \frac{15}{\cos 30^\circ - \cos 60^\circ}$$

$F = 40.983$

$F = 41.0$

(ii) $G = F\sin 30^\circ + F\sin 60^\circ$

$= 41 \cdot \sin 30^\circ + 41 \sin 60^\circ$

$= 55.983 \approx 56$

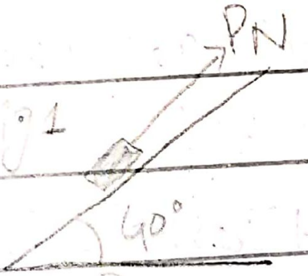
Example #05:-

A small block of weight 12N is at rest on a smooth plane inclined at 40 degree to the horizontal. The block is held in equilibrium by a force of Magnitude PN. Find the value of P when:-

i) The force is parallel to the plane as Fig 1.

(ii) The force is horizontal to the plane as in Fig 2.

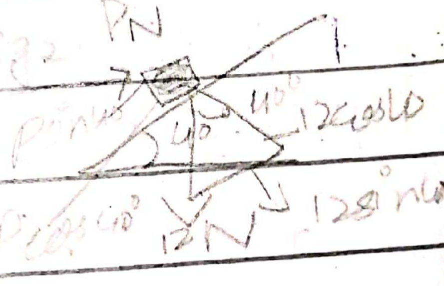
Fig 1



(i) $P = 12\sin 40^\circ$

$= 7.71N$

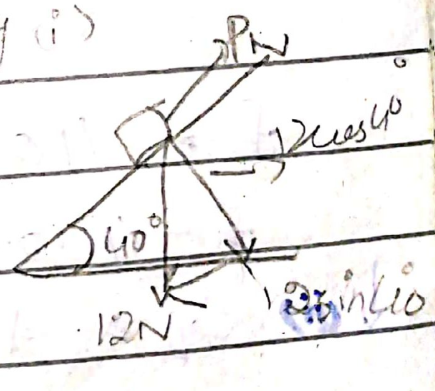
Fig 2



(ii) $P\cos 40^\circ = 12\sin 40^\circ$

$P = \dfrac{12\sin 40^\circ}{\cos 40^\circ}$

fig (ii)

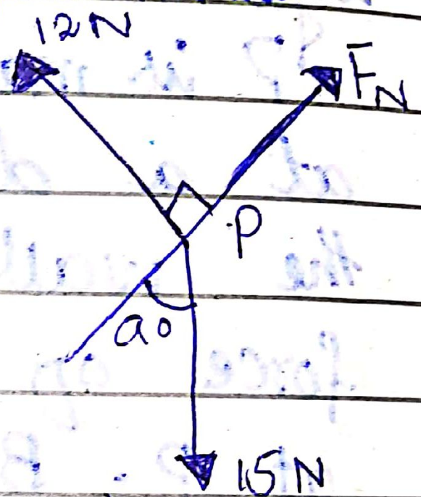


$P = 10.1N$

Lecture #13

Example Equilibrium:-

Three Coplaner forces of Magnitude FN, 12N and 15N are in equilibrium acting at a point P in the direction shown in diagram. Find a & F.

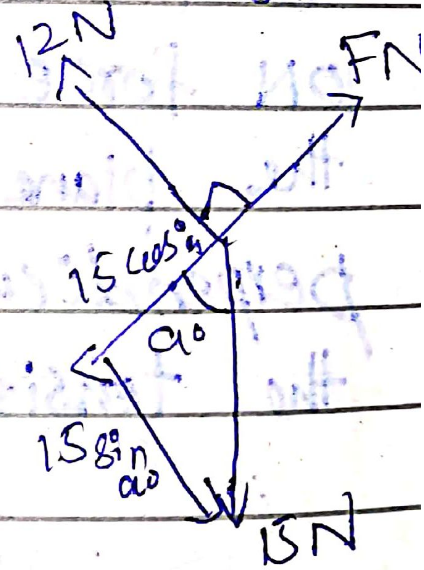


$15 \sin a = 12$

$\sin a = \frac{12}{15}$

$a = \sin^{-1}\left(\frac{12}{15}\right)$

$a = 53.1^\circ$



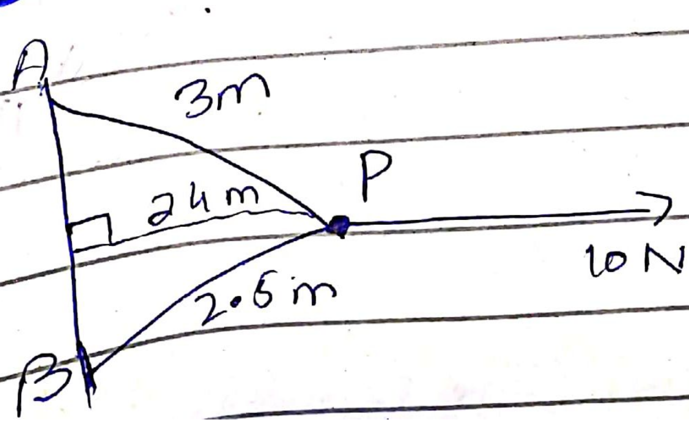
$F = 15 \cos a^\circ$

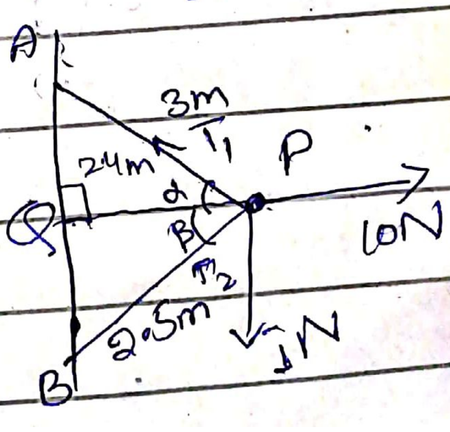
$= 15 \cos(53.1^\circ)$

$F = 9N$

Example:-

A and B are fixed points of a vertical wall with A vertically above B. A particle P of Mass 0.7kg is attached to A by a light inextensible string of length 3m. P is also attached to B by a light inextensible String of length 2.5m. P is maintained in equilibrium at a distance of 2.4m from the wall by a horizontal force of Magnitude 10N acting at P. Both strings are taut and 10N force acts in the plane in the plane ABB which is perpendicular to the wall. Find the tension in the strings.





$m = 0.7\ 45$

$m = 0.7kg$

$W = mg \Rightarrow 0.7 \times 10 = 7N$

$AQ^2 = AP^2 + QP^2$

$AQ = \sqrt{3^2 - (2.4)^2} = 1.8m$

$\cos\alpha = \frac{2.4}{3} = \frac{24}{30} = \frac{4}{5}$

$\sin\alpha = \frac{1.8}{3} = \frac{18}{30} = \frac{3}{5}$

$QB^2 = PB^2 - QP^2 \quad \because \triangle QPB$

$QB = \sqrt{(2.5)^2 - (2.4)^2} = 0.7$

$\cos\beta = \frac{2.4}{2.5} = \frac{24}{25} \Rightarrow \sin\beta = \frac{0.7}{2.5} = \frac{7}{25}$

$\rightarrow$

$T_1 \cos\alpha + T_2 \cos\beta = 10N$

$T_1 \times \frac{4}{5} + T_2 \frac{24}{25} = 10N$

$T_1 = \left(10 - \frac{24}{25} T_2\right) \times 5/4 \longrightarrow (1)$

$\downarrow$

$T_1 \sin\alpha = T_2 \sin\beta + 7$

$T_1 \times \frac{3}{5} = T_2 \left(\frac{7}{25}\right) + 7$

$T_1 = \frac{5}{3} \left(\frac{7T_2}{25} + 7\right) \longrightarrow (2)$

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using 1 and 2

$$\frac{5}{4}\left(10-\frac{24}{55}T_2\right)=\frac{5}{3}\left(\frac{7T_2}{25}+7\right)$$

$$3\left(10-\frac{24}{25}T_2\right)=4\left(\frac{7T_2}{25}+7\right)$$

$$0.8T_2=0.4$$

$$T_2=0.5$$

$$T_1=\frac{5}{3}\left(\frac{7(0.5)}{25}+7\right)$$

$$T_1=11.9N$$

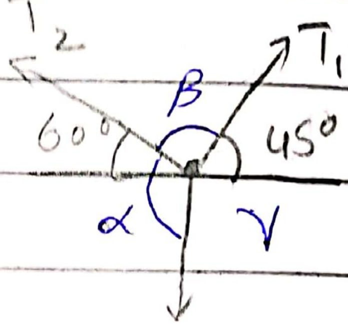
Lecture #16

Lamys Theorem

Example # 01.

A particle of mass 15kg is suspended by two strings. If the angle made by string are 45° and 60° with horizontal. Find the Tensions in the string.

Sol:-



$$60° + \beta + 45° = 180°$$

$$\beta = 180° - 60° - 45°$$

$$= 75°$$

$$\alpha = 60° + 90°$$

$$= 150°$$

$$\gamma = 90° + 45°$$

$$\gamma = 135°$$

$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\gamma} = \frac{W}{\sin\beta}$$

$$W = mg \Rightarrow (15)(10) = 150N$$

$$\frac{T_1}{\sin\alpha} = \frac{W}{\sin\beta} \Rightarrow \frac{T_1}{\sin 150°} = \frac{150}{\sin 75°}$$

$$T_1 = \frac{150 \sin 150^\circ}{\sin 75^\circ} = \frac{150 \times 0.5}{0.9659}$$

$$T_1 = 77.7N$$

$$\frac{T_2}{\sin \gamma} = \frac{W}{\sin B}$$

$$\frac{T_2}{\sin 135^\circ} = \frac{150}{\sin 75}$$

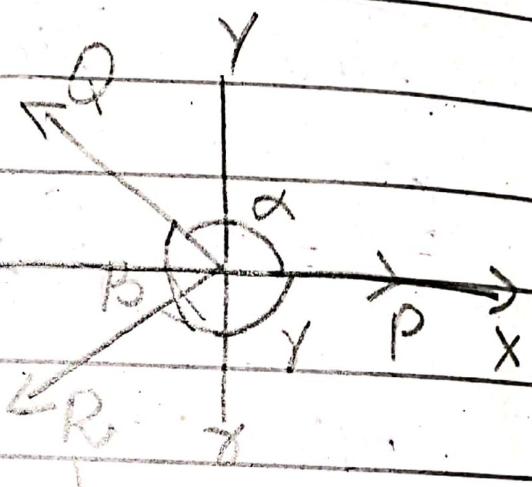
$$T_2 = \frac{150 \times \sin 135^\circ}{\sin 75^\circ}$$

$$= \frac{150 \times 0.707}{0.9659}$$

$$T_2 = 109.8N$$

Lamys Theorem Proof :-

"If a particles in equilibrium under the action of three Concurrent forces, then the magnitude of each force is proportional to the sin angle between other two."



Proof :-

$X = P\cos 0^\circ + Q\cos\alpha + R\cos(\alpha+\beta)$

$X = P(1) + Q\cos\alpha + R\cos\gamma$

$X = P + Q\cos\alpha + R\cos\gamma$

$Y = P\sin 0^\circ + Q\sin\alpha + R\sin(\alpha+\beta)$

$= P(0) + Q\sin\alpha + R\sin(\alpha+\beta)$

$\alpha+\beta+\gamma = 2\pi$

$\alpha+\beta = 2\pi-\gamma$

$\cos(\alpha+\beta) = \cos(2\pi-\gamma)$

$= \cos\gamma$

$\because \alpha+\beta+\gamma = 2\pi$

$\alpha+\beta = 2\pi-\gamma$

$\sin(\alpha+\beta) = \sin(2\pi-\gamma)$

$= -\sin\gamma$

$Y = Q\sin\alpha - R\sin\gamma \Rightarrow$

$X = 0 \quad , \quad Y = 0$

$X = 0$

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$$P + \cos\alpha + R\cos\gamma = 0 \longrightarrow (1)$$

$$y = 0$$

$$Q\sin\alpha - R\sin\gamma = 0$$

$$Q\sin\alpha = R\sin\gamma$$

$$\frac{R}{\sin\alpha} = \frac{Q}{\sin\gamma} \longrightarrow (2)$$

$$(2) \Longrightarrow Q = \frac{R\sin\gamma}{\sin\alpha} \longrightarrow (3)$$

using eq (3) in (1)

$$P + R\frac{\sin\gamma}{\sin\alpha}\cos\alpha + R\cos\gamma = 0$$

$$P + R\left(\frac{\sin\gamma}{\sin\alpha}\cdot\cos\alpha + \cancel{R}\cos\gamma\right) = 0$$

$$P + R\left(\frac{\sin\gamma\cos\alpha + \cos\gamma\sin\alpha}{\sin\alpha}\right) = 0$$

$$P + R\left(\frac{\sin(\alpha+\gamma)}{\sin\alpha}\right) = 0$$

$$P + R\frac{(-\sin\beta)}{\sin\alpha} = 0$$

$$P = R\frac{\sin\beta}{\sin\alpha}$$

$$\frac{P}{\sin\beta} = \frac{R}{\sin\alpha} \rightarrow (4)$$

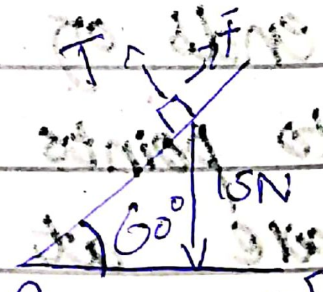
combining eq (2) & (4)

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}.$$

Lecture # 17:-
Applications of Lamys Theorem

Example:-

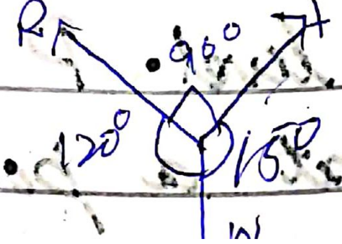
A block of mass 1.5kg rests in Equilibrium in a rough inclined plane at 60° to the horizontal. Find the Magnitude of the Frictional Contact force and normal Contact Force.



$m = 1.5kg$

$w = mg$

$= 1.5 \times 10 = 15N$



$$\frac{F}{\sin 120^\circ} = \frac{R}{\sin 150^\circ} = \frac{15N}{\sin 90^\circ}$$

$$\frac{F}{\sin 120^\circ} = \frac{15}{\sin 90^\circ}$$

$$F = \frac{15}{1} \times \sin 120$$

$$F = 15 \times 0.866$$

$$F = 13N$$

$$\frac{R}{\sin 150} = \frac{15}{\sin 90}$$

$$R = \frac{15 \times \sin 150^\circ}{\sin 90^\circ}$$

$$= 15 \times 0.5/1$$

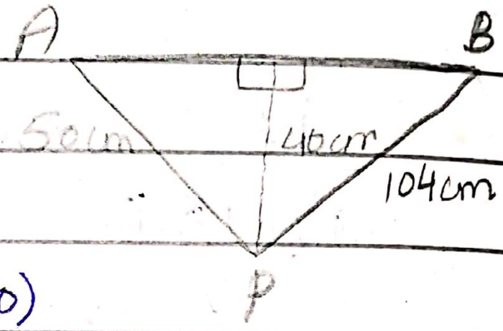
$$R = 7.5N$$

Example:-

A particle P of Mass 2.1 kg is attached to one end of each of two light inextensible strings. The other ends of the string are attached to points A and B which are at the same horizontal level. P hangs in equilibrium ~~above~~ below a point 40m the level of A and B, and the strings PA and PB have lengths 50cm and 104cm respectievely. Show that the tension in the string PA is 20N, and Find the tension in the string PB.

T_1 = Tension in PA

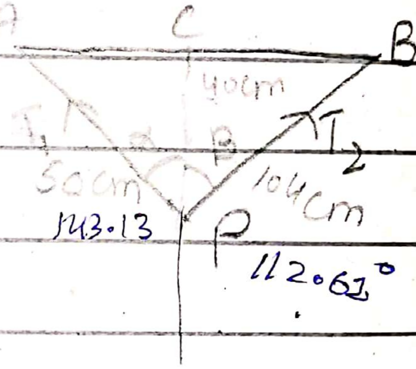
T_2 = Tension in PB



$m = 2.1\ kg$

$W = mg = (2.1)(10)$

$= 21N$



ΔACD; $\cos\alpha = \frac{PC}{PA} = \frac{40}{50}$

$\alpha = \cos^{-1}\left(\frac{40}{50}\right)$

$\alpha = 36.87°$

ΔBCD;- $\cos\beta = \frac{PC}{PB} = \frac{40}{104} \Rightarrow \beta = 67.38$

$\cancel{\alpha + \beta}$ $\alpha + \beta = 36.87° + 67.38° = 104.2$

By lamiys theorem:-

$$\frac{T_1}{\sin 112.62°} = \frac{T_2}{\sin 143.13°} = \frac{W}{\sin 104.25}$$

$$\frac{T_1}{\sin 112.62^\circ} \quad \frac{21}{\sin 104.25^\circ}$$

$$\Rightarrow T_1 = \frac{21 \times \sin 112.62^\circ}{\sin 104.25^\circ}$$

$$T_1 = 20N$$

$$\frac{T_2}{\sin 43.13} = \frac{21}{\sin 104.25^\circ}$$

$$T_2 = \frac{21 \times \sin 43.13}{\sin 104.25^\circ}$$

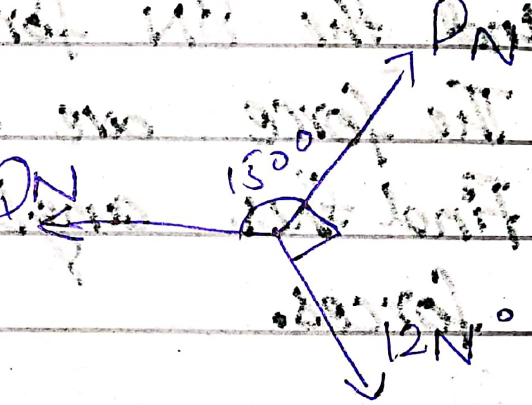
$$T_2 = 13N$$

Lecture # 18

Applications of Lamy's Theorem in daily life.

Example:

Three Coplanar forces PN, QN and 12N are acting on a particle and are in equilibrium. Find the magnitude of the forces P & Q.

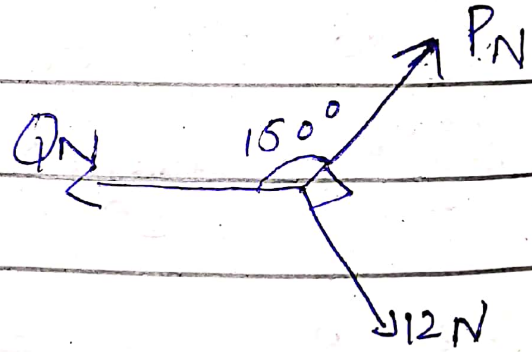


$\alpha + 90° + 150° = 360°$

$\alpha + 240° = 360$

$\alpha = 360° - 240°$

$\alpha = 120°$



$$\frac{P}{\sin 120°} = \frac{Q}{\sin 90°} = \frac{12}{\sin 150°}$$

$$\frac{P}{\sin 120°} = \frac{12}{\sin 150°}$$

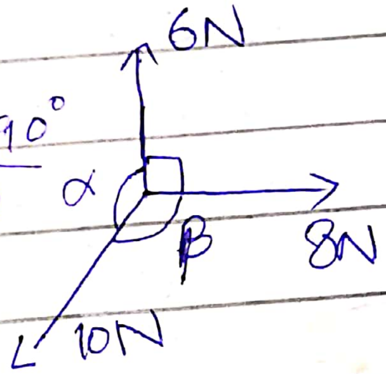
$$P = \frac{\sin 120° \times 12}{\sin 150°} \Rightarrow P = 20.8N$$

$$\frac{Q}{\sin 90^\circ} = \frac{12}{\sin 150^\circ} \Rightarrow Q = \frac{12 \sin 90^\circ}{\sin 150^\circ}$$

$$Q = 24N$$

Example #:-

Three Coplaner Forces are acting at a point O. The magnitudes of the forces are 6N, 8N and 10N respectively. The Angle between the 6N force and the 8N force is 90 degree. The force are in equilibrium. Find ther angles between the forces.



$$\frac{\sin\alpha}{8} = \frac{\sin\beta}{6} = \frac{\sin 90^\circ}{10}$$

$$\frac{\sin\alpha}{8} = \frac{\sin 90^\circ}{10}$$

$$\sin\alpha = \frac{8}{10} = 0.8$$

$$\alpha = 180 - 55.73^\circ = 126^\circ.87^\circ$$

$$\beta + \alpha + 90^\circ = 360^\circ$$

$$\beta + 126.87^\circ + 90^\circ = 360^\circ$$

$$\beta = 360^\circ - 90^\circ - 126.87^\circ$$

$$= 270 - 126.87$$

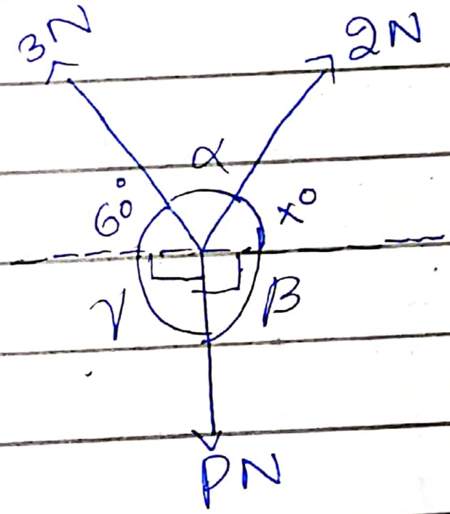
$$\beta = 143.13^\circ$$

Lecture # 19.-

Example of Lamy's Theorem.

Example:-

The Three Concurrent coplanar forces show in the diagram have Magnitude 3N, 2N and PN. Given that the three forces are in equilibrium, find the value of angle x and P.



$\gamma = 60^\circ + 90^\circ = 150$

$$\frac{P}{\sin\alpha} = \frac{3}{\sin\beta} = \frac{2}{\sin 150^\circ}$$

So,
$$\frac{3}{\sin\beta} = \frac{2}{\sin 150}$$

$\Rightarrow 3\sin 150^\circ = 2\sin\beta$

$\because \sin 150^\circ = \frac{1}{2}$

$$\sin\beta = \frac{3}{2\times 2} = \frac{3}{4}$$

$$\beta = \sin^{-1}\left(\frac{3}{4}\right) = 48.6.$$

$\beta = 90^\circ + x'$

$= 154.3^\circ$

$\because \beta$ is obtuse — $\beta = 180 - 48.6$

$= 131.4^\circ$

$\beta - 90^\circ = 131.4^\circ - 90^\circ$

$= 41.4^\circ$

$\alpha + \beta + \gamma = 360^\circ$

$\alpha + 131.4^\circ - 150^\circ = 360^\circ$

$\alpha = 78.6^\circ$

$$\frac{P}{\sin\alpha} = \frac{2}{\sin 150}$$

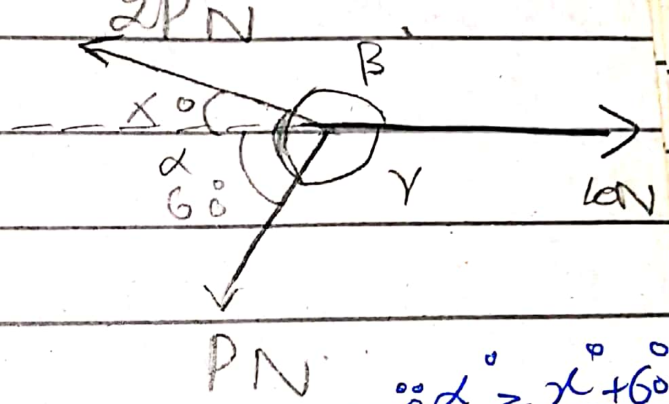
$$P/\sin 78.6^\circ = \frac{2}{\sin 150^\circ}$$

$P = 4 \times \sin 78.6^\circ$

$P = 3.92N$

Example of Lamy's Theorem.

Example :-
The three coplaner forces shown in the diagram are in equilibrium. Find the values of angle x and force P.



$\therefore \alpha^\circ = x^\circ + 60^\circ$

$\gamma + 60^\circ = 180^\circ$

$\gamma = 120^\circ$

$$\frac{P}{\sin\beta} = \frac{2P}{\sin\gamma}$$

$$\frac{P}{\sin\beta} = \frac{2P}{\sin 120} = \frac{10}{\sin\alpha}$$

$$\cancel{\frac{10}{\sin\alpha}} \quad \frac{\cancel{P}}{\sin\beta} = \frac{2\cancel{P}}{\sin 120^\circ}$$

$$\sin\beta = \frac{\sin 120}{2} = \frac{\sqrt{3}}{4}$$

$$\beta = \sin^{-1}\frac{\sqrt{3}}{4}$$

$\beta = 25.7^\circ$

$$\beta = 180^\circ - 25.7^\circ$$
$$= 154.3^\circ$$
$$x = 180 - \beta = 180 - 154.3^\circ$$
$$x^\circ = 25.7^\circ$$
$$\alpha = x^\circ + 60$$
$$\alpha = 60^\circ + 25.7^\circ = 85.7^\circ$$
$$\frac{2P}{\sin 120^\circ} = \frac{10}{\sin \alpha}$$
$$2P = \frac{10 \times \sin 120}{\sin(85.7^\circ)}$$
$$P = 10 \times \frac{\sin 120^\circ}{2 \times \sin 85.7^\circ}$$
$$P = 4.35N$$

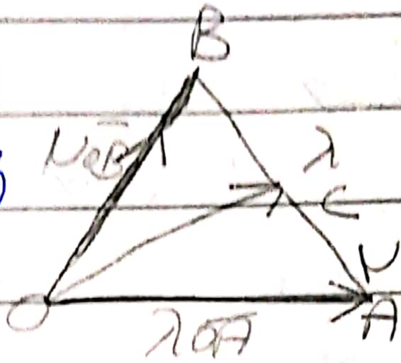
Lecture # 20:-

The (Lambda, mu) Theorem and its proof:-

Proof of Lambda mu theorem:-

(λ, μ) Theorem :-

if two concurrent forces are represented by $\lambda \vec{OA}$ and $\mu \vec{OB}$, their resultant is given by $(\lambda + \mu)\vec{OC}$, where C devides AB such that $AC : CB = \mu : \lambda$



proof :-

$$\vec{OA} = \vec{OC} + \vec{CA} \longrightarrow (1)$$

$$\vec{OB} = \vec{OC} + \vec{CB} \longrightarrow (2)$$

$$(1) \times \lambda \Rightarrow \lambda \vec{OA} = \lambda \vec{OC} + \lambda \vec{CA} \longrightarrow (3)$$

$$(2) \times \mu \Rightarrow \mu \vec{OB} = \mu \vec{OC} + \mu \vec{CB} \longrightarrow (4)$$

$(3) + (4) \Rightarrow$

$$\lambda \vec{OA} + \mu \vec{OB} = \lambda \vec{OC} + \mu \vec{OC} + \lambda \vec{CA} + \mu \vec{CB} \longrightarrow (5)$$

$$AC : CB = \mu : \lambda$$

$$\lambda \vec{AC} = \mu \vec{CB} \Rightarrow \lambda \vec{CA} + \mu \vec{CB} = 0$$

$$\lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu)\vec{OC} - \lambda \vec{AC} + \mu \vec{CB}$$

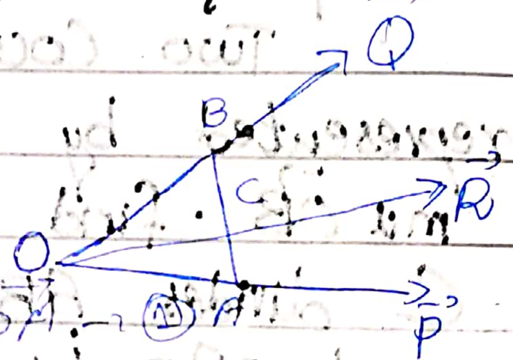
$$R = \lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu)\vec{OC}$$

which is the proof.

Example #02:- λ, μ theorem

forces P & Q act at a point and their resultant is R. if any transversal cuts three lines of action in the points A, B, C respectively. prove that $P/OA + Q/OB = R/OC$

Proof



$$\vec{P} = P\hat{OA} = P\frac{\vec{OA}}{|OA|} \rightarrow (1)$$

$$\vec{Q} = Q\hat{OB} = Q\frac{\vec{OB}}{|OB|} = \frac{Q}{|OB|}\vec{OB} \rightarrow (2)$$

$$\vec{R} = R\hat{OC} = R\frac{\vec{OC}}{|OC|} = \frac{R}{|OC|}\vec{OC} \rightarrow (3)$$

$$\vec{R} = \vec{P} + \vec{Q}$$

$$= \frac{P}{|OA|}\vec{OA} + \frac{Q}{|OB|}\vec{OB}$$

$$R =$$

$$= \left(\frac{P}{OA} + \frac{Q}{OB}\right)\vec{R}$$

$$= 16\hat{i} -$$

$$\frac{R}{|OC|}\vec{OC} = \left(\frac{P}{OA} + \frac{Q}{OB}\right)\vec{R}$$

$$= \left(\frac{P}{OA} + \frac{Q}{OB}\right)\vec{OC}$$

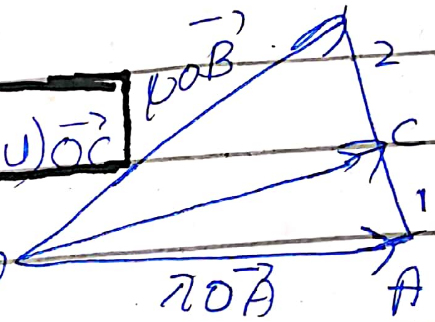
$$\boxed{\frac{\vec{R}}{OC} = \frac{P}{OA} + \frac{\vec{Q}}{OB}}$$ Hence prove.

Example 2

Two Cocurrent forces are represented by lambda $\vec{OA}$ and mu $\vec{OB}$. find the vector $\vec{OC}$ where C divides AB so that AC : CB = 1:2 and OA = i+j- 2K. & OB = i+j+2k.

$\mu = 1$, $\lambda = 2$

$$\boxed{\vec{R} = \lambda\vec{OA} + \mu\vec{OB} = (\lambda+\mu)\vec{OC}}$$



$$\lambda\vec{OA} + \mu\vec{OB} = (\lambda+\mu)\vec{OC}$$

$$2(i+j-2k) + (1)(i+j+2k) = (2+1)\vec{OC}$$

$$2i+2j-4k+i+j+2k = 3\vec{OC}$$

$$3\hat{i} + 3\hat{j} - 2\hat{k} = 3\vec{OC}$$

$$\vec{OC} = \frac{3i+3\hat{j}-2\vec{k}}{3}$$

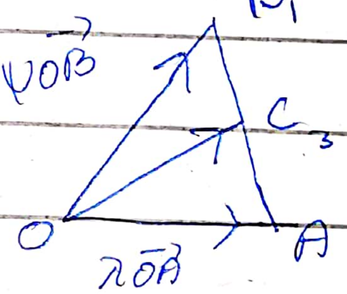
Lecture # 21.

Applications of (λ : μ) Theorem:

Example:- (λ : μ) theorem (3)

Two Concurrent forces are represented by lambda OA and mu OB. Find vector OA where C divides AB so that AC : CB = 3 : 1 and OB = 2i + j − 5k. and OC = 4i + 2j + k. Also find the resultant of these fores.

$\bar{R} = \lambda OA + \mu \vec{OB} = (\lambda + \mu) OC$ — (1)



$\mu = 3$, $\lambda = 1$

by (1)

$\Rightarrow 1 \vec{OA} + 3(2\hat{i} + \hat{j} - 5k) = (1+3)(4\hat{i} + 2\hat{j} + \hat{k})$.

$\Rightarrow \vec{OA} = 6\hat{i} + 3\hat{j} - 15\hat{k} = 4(4\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow OA = 16 + 8\hat{j} + 4k - 6\hat{i} - 3\hat{j} + 15\hat{k}$

$\vec{OA} = 10i + 5\hat{j} + 19\hat{k}$

$\vec{R} = (3+1)\vec{OC}$

$= 4(4\hat{i} + 2\hat{j} + \hat{k})$

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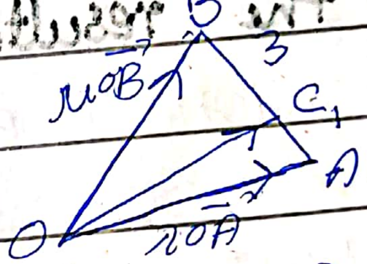
Example # (04)

Two Coucurrent forces are represented by lambda OA and mu OB. Find vector OB where C divides AB so that $AC : CB = 1:3$ and $\vec{OA} = 2\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{OC} = 3\hat{i}+3\hat{j}+\hat{k}$. Also find the resultant of these forces.



By $(\lambda-\mu)$ Theorem:-

$$\vec{R} = \lambda\vec{OA} + \mu\vec{OB} = (\lambda+\mu)\vec{OC} \quad \text{—— (1)}$$

$\mu = 1, \quad \lambda = 3$

$$(1) \Rightarrow \; 3(2\hat{i}+3\hat{j}+5\hat{k}) + \vec{OB} = (3+1)(3\hat{i}+3\hat{j}+\hat{k})$$

$$6\hat{i}+9\hat{j}+15\hat{k} + \vec{OB} = 4(3\hat{i}+3\hat{j}+\hat{k})$$

$$\vec{OB} = 12\hat{i}+12\hat{j}+4\hat{k}-6\hat{i}-9\hat{j}-15\hat{k}$$

$$\vec{OB} = 6\hat{i}+3\hat{j}-11\hat{k}$$

Now $\vec{R}$

$$R = (\lambda+\mu)\vec{OC}$$

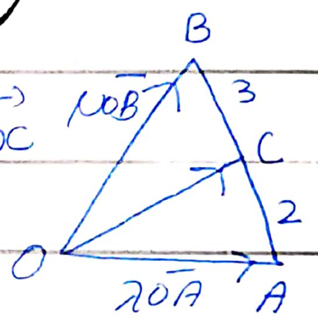
$$= (3+1)(3\hat{i}+3\hat{j}+\hat{k})$$

$\vec{R} = 12\hat{i} + 12\hat{j} + 4\hat{k}$

Example # 05:-

Two Concurrent forces are represented by lambda OA & mu OB. Find the Magnitude of the resultant force where C divides AB so that AC:CB = 2:3 and OA = i+3j+5k & OC = 4i-j+k. Also find the resultant of these forces.

By (λ, μ) Theorem:-



$\vec{R} = \lambda\vec{OA} + \mu\vec{OB} = (\lambda+\mu)\vec{OC}$

$\lambda = 3 \quad , \quad \mu = 2$

$\vec{R} = (\lambda+\mu)\vec{OC}$

$= (3+2)(4i - j + k)$

$= (5)(4i - j + k)$

$\vec{R} = 20i - 5j + 5k$

$R = \sqrt{(20)^2 + (5)^2 + (5)^2}$

$= \sqrt{400 + 25 + 25} = \sqrt{450} = \sqrt{9 \times 25 \times 2}$

$R = 3 \times 5 \times \sqrt{2}$

$R = 15\sqrt{2}$