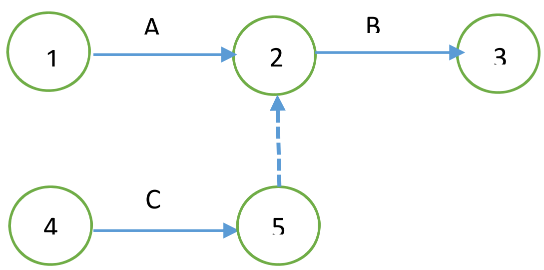
Solution of Practice Questions Lecture 4

Question 1:

Write the relationship between the activities.



Solution

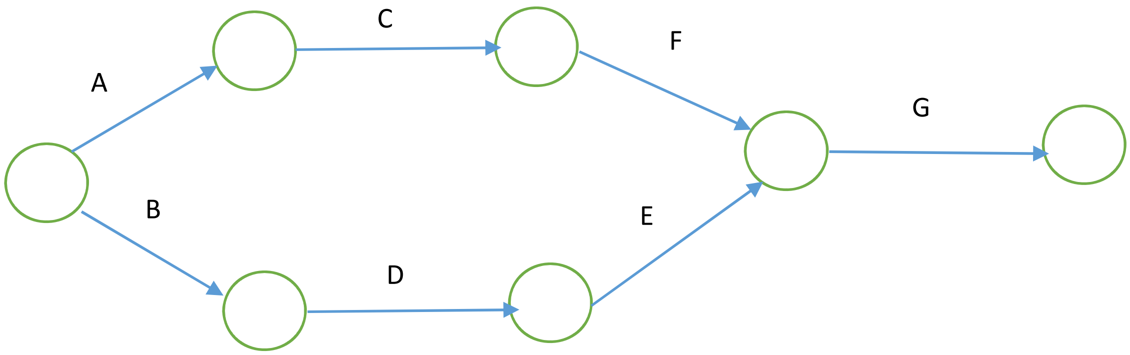
Activity A and activity C are predecessors of activity B.

Question 2:

Construct a network diagram for the following activity data.

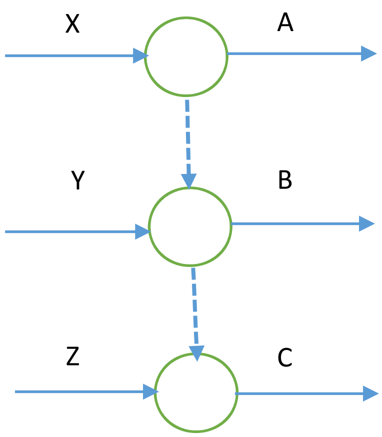
Activity	Predecessor
A	-
B	-
C	A
D	B
E	D
F	C
G	E,F

Solution



Question 3:

Write the relationship between the activities for the following network diagram.



Solution

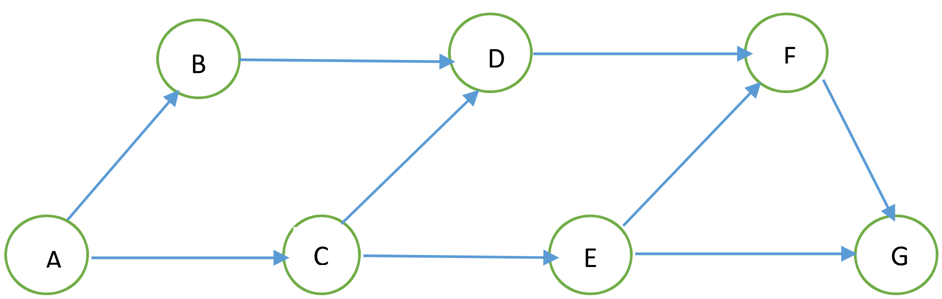
X is predecessor of A, B and C.

Y is predecessor of B and C.

Z is predecessor of C only.

Question 4:

Construct a table that shows the predecessor activities for the following network.



Solution

Event	Predecessor
A	-
B	A
C	A
D	B,C
E	C
F	D,E
G	E,F

Question 5:

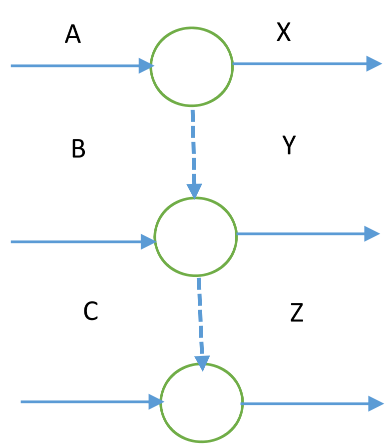
Construct a network diagram for the following relationship.

1) A is predecessor of X, Y and Z.

2) B is predecessor of Y and Z.

3) C is predecessor of Z only

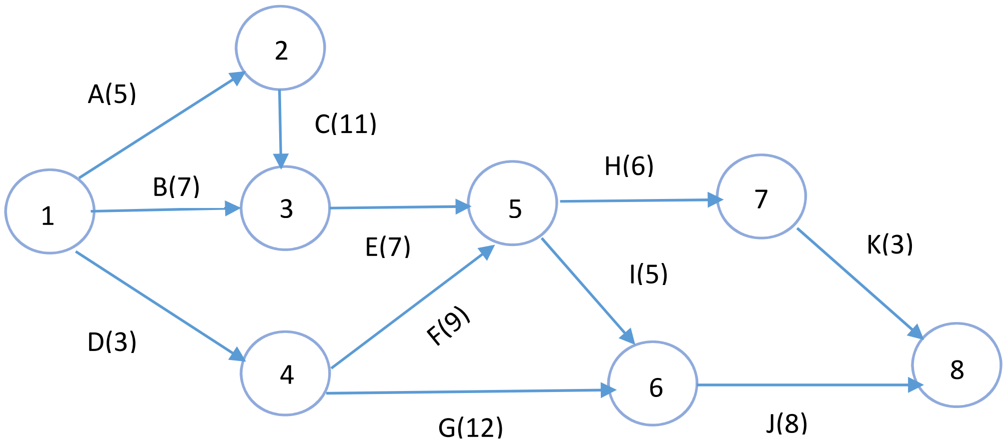
Solution



Solution of Practice Questions Lecture#7-9

Question 1:

Find the critical path and the expected duration of the following project.



Solution

There are 7 possible paths to get from the starting point at node 1 and travel through the network to end at node 8.

Possible Paths

1-2-3-5-7-8=A-C-E-H-K=5+11+7+6+3=32

1-2-3-5-6-8=A-C-E-I-J=5+11+7+5+8=36

1-3-5-7-8=B-E-H-K=7+7+6+3=23

1-3-5-6-8=B-E-I-J=7+7+5+8=27

1-4-5-6-8=D-F-I-J=3+9+5+8=25

1-4-5-7-8=D-F-H-K=3+9+6+3=21

1-4-6-8=D-G-J=3+12+8=23

Hence the critical path is the longest distance with the shortest possible time i.e.

=1-2-3-5-6-8

=A-C-E-I-J

=5+11+7+5+8

=36

Question 2:

A project has the following time schedule.

Activity	Time(hours)
1-2	2
1-3	2
1-4	1
2-5	4
3-6	8
3-7	5
4-6	3
5-8	1
6-9	5
7-8	4
8-9	3

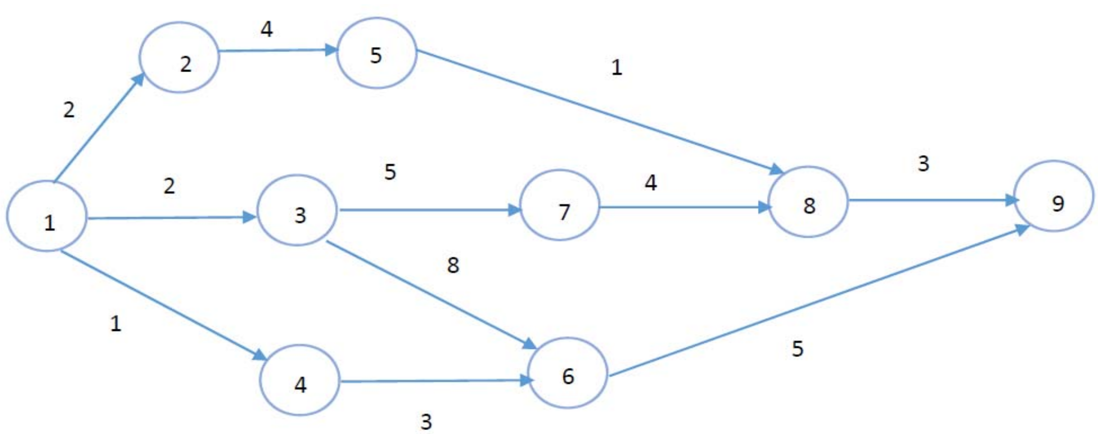
a) Draw the network

b) Calculate the following time estimates.

- Earliest Start Time (EST) of each activity.
- Earliest Finish Time (EFT) of each activity.

Solution

a) Network



b)

Event	EST
1	0
2	2
3	2
4	1
5	6
6	10
7	7
8	11
9	15

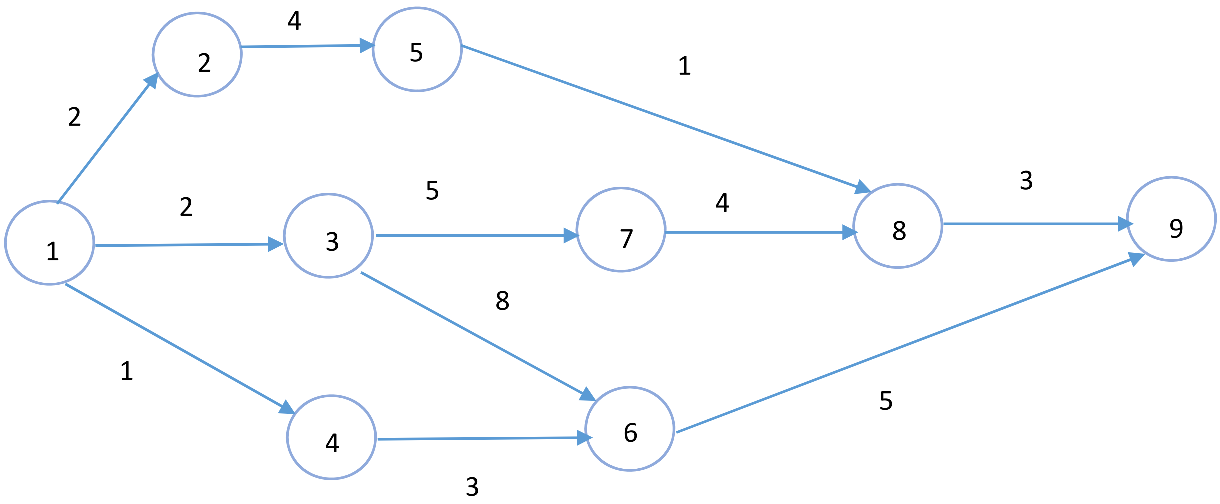
EST of any activity is the EST of its tail event

Activity	Time in Hours	EST	EFT
1-2	2	0	2
1-3	2	0	2
1-4	1	0	1
2-5	4	2	6
3-6	8	2	10
3-7	5	2	7
4-6	3	1	4
5-8	1	6	7
6-9	5	10	15
7-8	4	7	11
8-9	3	11	14

EFT=EST + Duration of the activity

Question 3:

Calculate the LST and LFT for each activity for the network given below.



Find the critical path and the expected duration of the project.

Solution

Event	LFT
1	0
2	7
3	2
4	7
5	11
6	10
7	8
8	12
9	15

LFT of any activity is the LFT of its head event

Activity	Time in Hours	LST	LFT
1-2	2	5	7
1-3	2	0	2
1-4	1	6	7
2-5	4	7	11
3-6	8	2	10
3-7	5	3	8
4-6	3	7	10
5-8	1	11	12
6-9	5	10	15
7-8	4	8	12
8-9	3	12	15

LST=LFT- Duration of the activity

Possible Paths:

- 1-2-5-8-9=2+4+1+3=10
- 1-3-7-8-9-=2+5+4+3=14
- 1-3-6-9=2+8+5=15
- 1-4-6-9=1+3+5=9

Hence 1-3-6-9 is the critical path with duration 15.

Question 4:

A small marketing project consists of the jobs as shown in the table below.

Activity	Predecessor	Duration
X	-	4
Y	-	5
Z	X	6

a) Draw the Network diagram.

b) Calculate the following time estimates.

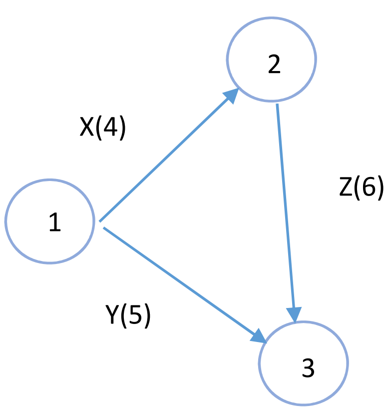
- Earliest Start Time of each activity
- Earliest Finish Time of each activity
- Latest Start Time of each activity
- Latest Finish Time of each activity

c) Critical Path and its duration.

d) Find Total Float, Independent Float and free Float for each activity.

Solution

a) Network:



b) Time Estimates

Activity	Duration	EST	EFT	LST	LFT
1-2	4	0	4	0	4
1-3	5	0	5	5	10
2-3	6	4	10	4	10

c) Possible Paths:

Time taken to complete the activities in two paths are:

- 1-2-3=4+6=10
- 1-3=5

Hence 1-2-3 is the critical path and the duration is 10 days.

d)

Activity	Duration	Total Float	Free Float	Independent Float
1-2	4	0	0	0
1-3	5	5	5	5
2-3	6	0	0	0

Question 5:

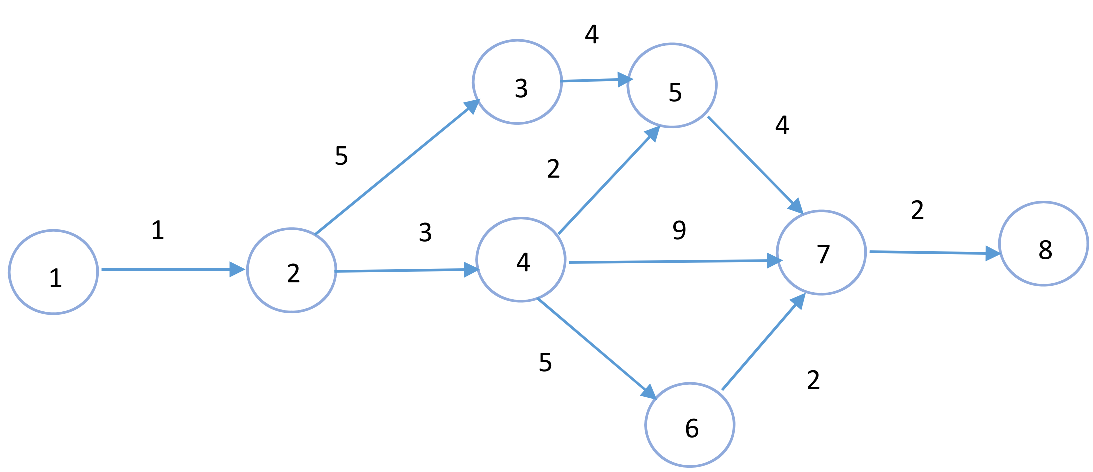
A project with 10 activities is given below

Activity	1-2	2-3	2-4	3-5	4-5	4-6	4-7	5-7	6-7	7-8
Duration	1	5	3	4	2	5	9	4	2	2

a) Construct the Network

b) Find the Critical Path.

Solution a) Network



b) ***Possible Paths:***

- 1-2-3-5-7-8=1+5+4+4+2=16
- 1-2-4-5-7-8=1+3+2+4+2=12
- 1-2-4-7-8=1+3+9+2=15
- 1-2-4-6-7-8-=1+3+5+2+2=13

Hence the critical path is 1-2-3-5-7-8=16

Question 6

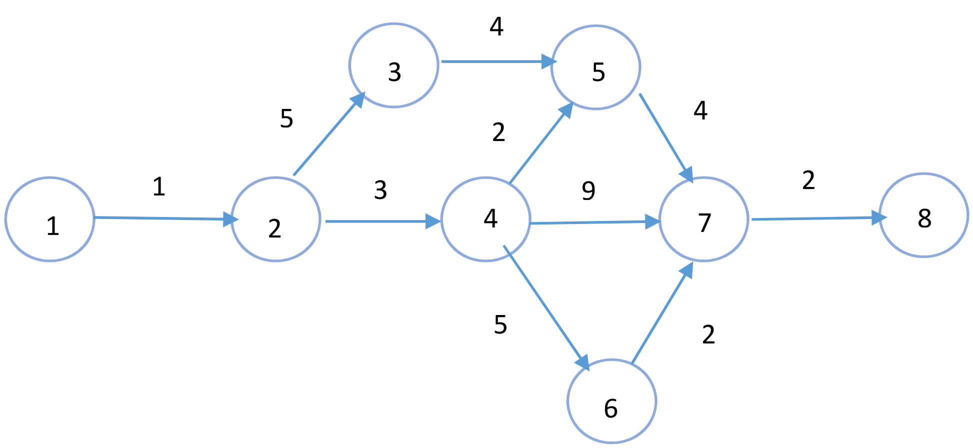
How many types of floats are there? Also mention their formulas.

Solution

1. Independent float
   $I = j_E - i_L - D$
2. Free float
   $F = j_E - i_E - D$
3. Total float
   $I = j_L - i_E - D$.

Question 7:

A project is represented by the network shown below



Find total float and free float.

Solution

Activity	Total Float	Free Float
1-2	0	0
2-3	0	0
2-4	1	0
3-5	0	0
4-5	4	4
4-6	3	0
4-7	1	1
5-7	0	0
6-7	3	3
7-8	0	0

Question 8:

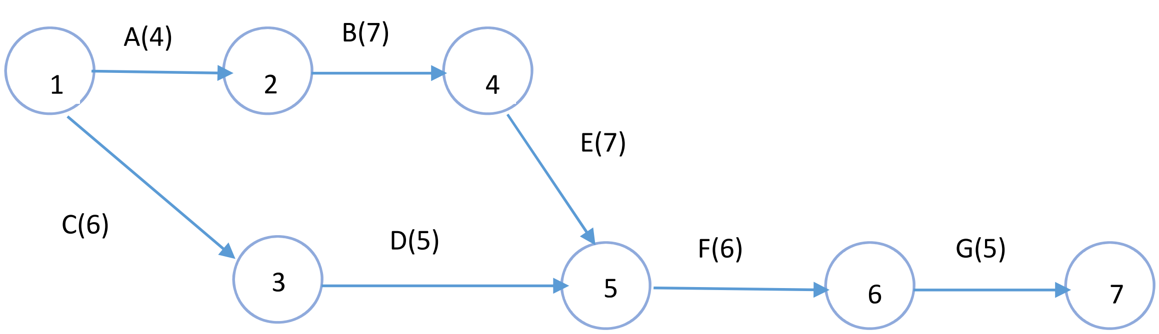
A project consist of the seven activities for which the relevant data are given below:

Activity	Preceding activity	Duration
A	-	4
B	A	7
C	-	6
D	C	5
E	B	7
F	D,E	6
G	F	5

(a) Draw an arrow diagram representing the project and find the project completion time.
(b) Calculate the total float for each activity.

Solution

a)



b)

Activity	Total Float
1-2	0
1-3	7
2-4	0
3-5	7
4-5	0
5-6	0
6-7	0

Critical path:

Path of zero total float through the project is called the critical path. Total float is zero on the activities A, B, E, F and G. Therefore

=A-B-E-F-G

=1-2-4-5-6-7

=4+7+7+6+5

=29

Solution of Practice Questions Lecture 10

Question 1:

If we have 10, 12 and 26 days as optimistic, most likely and pessimistic time for an activity, then calculate the following

a) Expected time of the activity

b) Variance and standard deviation.

Solution

a)

$$T_e = \frac{10 + (12 * 4) + 26}{6}$$

$$= \frac{84}{6} = 14$$

b)

$$V_t = \left[\frac{(t_p - t_o)}{6} \right]^2$$

$$= \left[\frac{(26 - 10)}{6} \right]^2$$

$$= \left[\frac{16}{6} \right]^2 = \left[\frac{8}{3} \right]^2 = [2.67]^2 = 7.11$$

$$\text{Standard } Deviation = \sqrt{Variance}$$

$$= \sqrt{7.11}$$

$$= 2.67$$

Question 2:

A teacher notes that when her student starts reading the optimistic (t_0), most likely (t_m) and pessimistic (t_p) time estimates are 10, 20 and 60.Calculate the following

a) What will be the expected duration of reading?

b) What would be its variance?

Solution

a)

$$T_e = \frac{t_o + 4t_m + t_p}{6}$$
$$= \frac{10 + (4*20) + 60}{6}$$
$$= \frac{150}{6}$$
$$= 25$$

b)

$$V_t = \left[\frac{(t_p - t_o)}{6}\right]^2$$
$$= \left[\frac{(60-10)}{6}\right]^2$$
$$= \left[\frac{50}{6}\right]^2 = [8.334]^2 = 69.45$$

Question 3:

The time estimates for the pert model of a medical institute for each activity are given as follows:

Activity	Time Estimates		
	t_O	t_m	t_p
A	2	4	12
B	10	15	20
D	5	5	5

Compute the expected time for each activity.

Solution

As $T_e = \frac{t_o + 4t_m + t_p}{6}$

For Activity A:

$$T_e = \frac{2 + (4*4) + 12}{6}$$
$$= \frac{30}{6} = 5$$

For Activity B:

$$T_e = \frac{10 + (15 * 4) + 20}{6}$$

$$= \frac{90}{6} = 15$$

For Activity D:

$$T_e = \frac{5 + (4 * 5) + 5}{6}$$

$$= \frac{30}{6} = 5$$

Question 4:

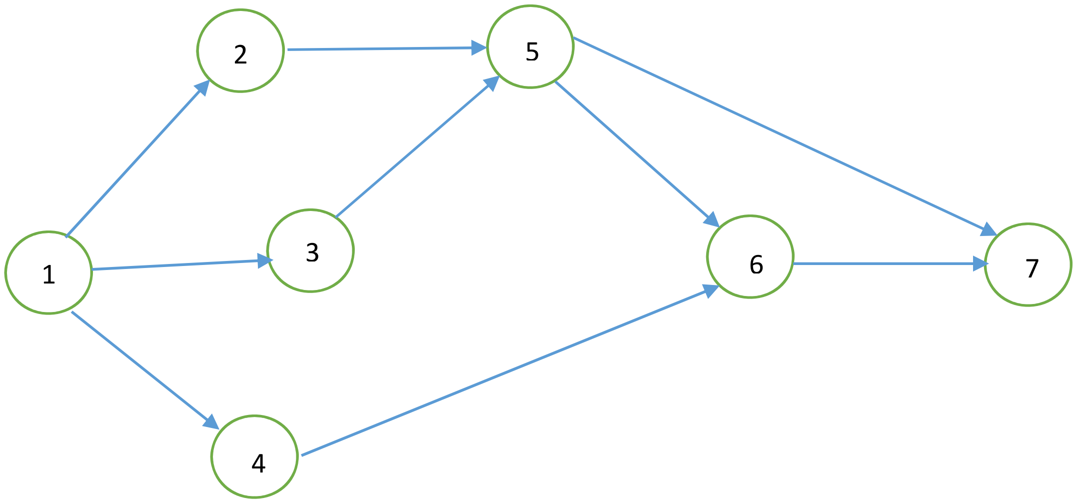
A certain project is composed of nine activities whose time estimates are given below:

Activity	Expected Duration (week)		
	Optimistic	**Most likely**	**Pessimistic**
1-2	1	1	7
1-3	3	5	7
1-4	2	2	8
2-5	1	1	1
3-5	3	6	9
4-6	2	5	8
5-6	4	6	14
5-7	3	7	11
6-7	6	8	10

a) Draw the project network and trace all the possible paths from it.
b) What is the expected project length?
c) Calculate the variance and standard deviation of the project length.

Solution

a)



Possible Paths**:**

1 – 2 – 5 – 7,

1 – 2 – 5 – 6 – 7,

1 – 3 – 5 – 7,

1 – 3 – 5 – 6 – 7,

1 – 4 – 6 – 7.

b)

Activity	**Time Estimates**			**Expected Time**	**Variance**
	t_o	t_m	t_p		
1-2	1	1	7	2	1
1-3	3	5	7	5	0.45
1-4	2	2	8	3	1
2-5	1	1	1	1	0
3-5	3	6	9	6	1
4-6	2	5	8	5	1
5-6	4	6	14	7	2.79
5-7	3	7	11	7	1.77
6-7	6	8	10	8	0.45

The critical path is 1-3-5-6-7.

Expected Project length =5+6+7+8= 26.

c) For the project Length
Variance=Sum of the variances of the critical activities

$=0.45+1+2.79+0.45=4.69$

$\text{Standard } Deviation = \sqrt{Variance}$

$=\sqrt{4.69}$

$=2.17$

Question 5:

Activity	**Expected Duration (week)**		
	Optimistic	**Most likely**	**Pessimistic**
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

If the critical path for the above project is 1-3-5-6, then find the variance and standard deviation for the critical path.

Solution

As the critical path is 1 – 3 – 5 – 6

Activity	Expected Duration	Variance
1-2	2	1
1-3	4	1
1-4	3	1
2-5	1	0
3-5	6	4
4-6	5	1
5-6	7	4

Project Length=4+6+7=17

Variance=sum of the variance of the critical activities

=1+4+4=9

Standard Deviation=square root (variance) =3

Question 6:

A project schedule has the following characteristics:

Activity	Expected Duration (week)		
	Optimistic	**Most likely**	**Pessimistic**
1-2	3	3	3
2-3	3	6	9
2-4	2	4	6
3-5	4	6	8
4-6	4	6	8
5-6	0	0	0
5-7	3	4	5
6-7	2	5	8

a) Determine the critical path and the expected project time.
b) What is the probability that the project will be completed in 20 weeks?
c) What is the probability that the project will be completed in 23 weeks?

Solution

a) Critical path: 1-2-3-5-6-7
Expected project Time=20
Standard deviation of the critical path=2.6666

b)

$$z = \frac{T - T_e}{S.D} = \frac{20 - 20}{2.666} = 0$$

$$P(x < 20) = P\ (z < 0)$$

$$= 0.5$$

$$= 50\%$$

c)

$$z = \frac{T - T_e}{S.D} = \frac{23 - 20}{2.666} = 1.125$$

$$P(x < 23) = P(z < 1.125)$$

$$= 0.5 + P(0 < z < 1.125)$$

$$= 0.5 + 0.3686$$

$$= 0.8686$$

Solution No 1:

$$\begin{aligned} &\textit{Maximize} \quad Z = 6x_1 + 11x_2 \\ &\textit{Subject to} \quad 2x_1 + x_2 \leq 104 \\ &\qquad x_1 + 2x_2 \leq 76 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Solution No 2:

$$\begin{aligned} &\textit{Minimize} \quad Z = 6x_1 + 16x_2 \\ &\textit{Subject to} \quad x_1 + x_2 = 200 \\ &\qquad x_1 \leq 80 \\ &\qquad x_2 \geq 60 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Solution No 3:

$$\begin{aligned} &\textit{Minimize} \quad Z = 250x_1 + 400x_2 \\ &\textit{Subject to} \quad 5x_1 + 7x_2 \geq 56 \\ &\qquad 6x_1 + 9x_2 \geq 32 \\ &\qquad 7x_1 + 5x_2 \geq 64 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Solution No 4:

$$\begin{aligned} &\textit{Maximize} \quad Z = 2500x_1 + 14500x_2 + 12000x_3 \\ &\textit{Subject to} \quad 3x_1 + 11x_2 + 16x_3 \geq 1250 \\ &\qquad 2x_1 + 9x_2 + 7x_3 \leq 850 \\ &\qquad x_1, x, x_3 \geq 0 \end{aligned}$$

Solution No 5:

$$\begin{aligned} &\textit{Minimize} \quad Z = 60x_1 + 84x_2 \\ &\textit{Subject to} \quad 25x_1 + 35x_2 \geq 30 \\ &\qquad 50x_1 + 20x_2 \geq 58 \\ &\qquad 10x_1 + 15x_2 \geq 32 \\ &\qquad 40x_2 \geq 70 \\ &\qquad 24x_1 + 32x_2 \geq 48 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Solution 1:

Curve shows the *inverse relation* b/w the Order Quantity and Ordering cost.
Since the ordering cost per unit item is inversely proportional to the quantity ordered, so it would be maximum whenever the ordered quantity will be minimum.

Solution No 2:

$D = 600 units$

$C_4 = 0$

$C_3 = 0.60$

$C_2 = 80$

$Economic\ Order\ Quantity:$

$$EOQ = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2*80*600}{0.60}}$$

$$= \sqrt{16000} = 400 units$$

Solution No 3:

$D = 200 units$

$C_4 = 0$

$C_3 = 1$

$C_2 = 25$

$Q = ?$

$t = ?$

$Economic\ Order\ Quantity:$

$$EOQ = Q^* = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2*25*200}{1}}$$

$$= \sqrt{10000} = 100$$

$$t = \frac{Q^*}{D} = \frac{100}{200} = 0.5$$

Thus he should produce 100 *units of his product at interval of* 15 *days* .

Solution No 4:

$$D = 15000$$
$$C_1 = Rs4 / Item$$
$$C_2 = Rs100 / order$$
$$C_3 = 0.60 / Item / month$$
$$Economic\ Order\ Quantity:$$
$$EOQ = Q^* = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2*100*15000}{7.2}}$$
$$= \sqrt{416666.66} = 645.49 units$$
$$Time\ between\ order:$$
$$t = \frac{Q^*}{D} = \frac{645.49}{15000} = 0.5164\ per\ month$$
$$Number\ of\ orders\ per\ year:$$
$$N = \frac{D}{Q} = \frac{15000}{645.49} = 23.23$$

Solution No 5:

$$D = 360$$
$$C_2 = 100$$
$$C_3 = 1$$
$$Economic\ Order\ Quantity:$$
$$EOQ = Q^* = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2*100*360}{1}}$$
$$= \sqrt{72000} = 268.32\ units$$
$$Number\ of\ orders\ per\ year:$$
$$N = \frac{D}{Q} = \frac{360}{268.32} = 1.34$$

Solution No 1:

Item No	Annual Usage	Descending Order	Rank
3	75000	95000	9
4	37000	75000	3
2	14000	37000	4
1	36000	36000	1
7	32000	32000	7
8	8000	16000	6
6	16000	14000	2
5	11000	11000	5
9	95000	8000	8
10	4000	4000	10

Cumulative Annual usage	%Cumulative Annual usage	Cumulative % of items	ABC Category
95000	28.96	10	A
170000	51.82	20	A
207000	63.1	30	B
243000	74.08	40	B
275000	83.83	50	B
291000	88.71	60	C
305000	92.98	70	C
316000	96.34	80	C
324000	98.78	90	C
328000	100	100	C

Solution No 2:

Model No	Annual consumption	Unit Price(Rs)	Usage Value	Descending Order
101	30000	0.1	3000	42000
102	280000	0.15	42000	22000
103	3000	0.1	300	9000
104	110000	0.05	5500	5500
105	4000	0.05	200	4000
106	220000	0.1	22000	3000
107	15000	0.05	750	800
108	80000	0.05	4000	750
109	60000	0.15	9000	300
110	8000	0.1	800	200

Cumulative Annual usage	%Cumulative Annual usage	Cumulative % of items	ABC Category
42000	48	10	A
64000	73	20	A
73000	83	30	B
78500	90	40	B
82500	94	50	B
85500	98	60	C
86300	98.6	70	C
87050	99.4	80	C
87350	99.6	90	C
87550	100	100	C

Solution No 3:

Item No	Units	Unit Cost	Usage Value	Descending Order
1	7000	5	35000	72000
2	24000	3	72000	57000
3	15000	10	15000	35000
4	600	22	13200	20000
5	38000	1.5	57000	15000
6	40000	0.5	20000	13200
7	60000	0.2	12000	12000
8	3000	3.5	10500	10500

Cumulative Usage Value	% Cumulative Usage Value
72000	30.67
129000	54.96
164000	69.87
184000	78.39
199000	84.78
212200	90.41
224000	95.44
234700	100

Solution No 4:

Product	Unit	Unit Cost	Usage value	Descending Order
1	40	350	14000	14000
2	30	30	900	10000
3	50	200	10000	1400
4	20	70	1400	1000
5	8	100	800	900
6	20	20	400	800
7	10	100	1000	800
8	40	20	800	600
9	10	60	600	400
10	5	20	100	100

Cumulative Usage Value	%Cumulative Annual usage	ABC Category
14000	46.7	A
24000	80	A
25400	84.7	B
26400	88	B
27300	91	C
28100	93.7	C
28900	96.3	C
29500	98.3	C
29900	99.7	C
30000	100	C

Solution No 1:-

In the time subinterval AC, the area of triangle AOC above the x- axis represent the availability of items

In the time subinterval BC, the area of triangle CEB below the x- axis represent the shortage of items

Solution No 2:-

$$D = 100$$
$$C_3 = 0.02$$
$$C_2 = 100$$
$$C_4 = 5$$
$$EOQ = ?$$
$$EOQ = \sqrt{\frac{2C_2D}{C_3}} * \sqrt{\frac{C_3 + C_4}{C_4}}$$
$$EOQ = \sqrt{\frac{2*100*100}{0.02}} * \sqrt{\frac{0.02+5}{5}}$$
$$EOQ = \sqrt{\frac{20000*5.02}{0.02*5}}$$
$$EOQ = \sqrt{\frac{20000*5.02}{0.02*5}}$$
$$EOQ = \sqrt{1004000}$$
$$EOQ = 1001.998$$

Solution No 3:-

1)
$$EOQ = \sqrt{\frac{2C_2D}{C_3}} * \sqrt{\frac{C_3 + C_4}{C_4}}$$
$$EOQ = \sqrt{\frac{2*400*15000}{1.2}} * \sqrt{\frac{1.2+10}{10}}$$
$$EOQ = \sqrt{\frac{800*15000*11.2}{1.2*10}}$$
$$EOQ = \sqrt{11200000}$$
$$EOQ = 3346.640$$
$$Q^* = 3346.640$$

2) $t = \frac{Q^*}{D}$

$t = \frac{3346.640}{15000} = 0.223109 Years$

$t = 2.67 Months$

3) $N = \frac{12}{t}$

$N = \frac{12}{2.67} = 4.5$

4) $S^* = \sqrt{\frac{2C_2D}{C_4}} * \sqrt{\frac{C_3}{C_3 + C_4}}$

$S^* = \sqrt{\frac{2*400*15000}{10}} * \sqrt{\frac{1.2}{1.2+10}}$

$S^* = \sqrt{80*15000} * \sqrt{\frac{1.2}{11.2}}$

$S^* = \sqrt{128571.4286}$

$S^* = 358.568$

5) $I = Q^* - S^*$

$I = 3346.640 - 358.568$

$I = 2988.072$

6) Total Cost per period:

$$= C_1 Q^* + C_2 + \frac{C_3 (Q^* - S^*)^2}{2D} + \frac{C_4 S^{*2}}{2D}$$

$$= (2*3346.64) + 400 + \frac{1.2(2988.071)^2}{30,000} + \frac{10(358.568)^2}{30,000}$$

$$= 6693.28 + 400 + 357.1429 + 42.857 = 7493.0099$$

Solution No 4:-

$$C(\mathrm{S},\mathrm{Q}) = \mathrm{C}_1 D + \frac{\mathrm{C}_2 D}{Q} + \frac{\mathrm{C}_3(\mathrm{Q}-\mathrm{S})^2}{2Q} + \frac{\mathrm{C}_4 S^2}{2Q}$$

$$\frac{\partial C}{\partial Q} = 0 - \frac{\mathrm{C}_2 D}{Q^2} + \frac{\mathrm{C}_3}{2}\left(\frac{2Q(\mathrm{Q}-\mathrm{S}) - (\mathrm{Q}^2 - 2\,\mathrm{QS} + S^2)}{Q^2}\right) - \frac{\mathrm{C}_4 S^2}{2Q^2}$$

$$= -\frac{\mathrm{C}_2 D}{Q^2} + \frac{\mathrm{C}_3}{2}\left(\frac{2Q^2 - 2Q\mathrm{S} - \mathrm{Q}^2 + 2\,\mathrm{QS} - S^2)}{Q^2}\right) - \frac{\mathrm{C}_4 S^2}{2Q^2}$$

$$= -\frac{\mathrm{C}_2 D}{Q^2} + \frac{\mathrm{C}_3}{2}\left(\frac{\mathrm{Q}^2 - S^2)}{Q^2}\right) - \frac{\mathrm{C}_4 S^2}{2Q^2}$$

$$\frac{\partial C}{\partial S} = 0 + 0 - C_3\left(\frac{(\mathrm{Q}-\mathrm{S})}{Q}\right) + \frac{\mathrm{C}_4 S}{Q}$$

$$= -C_3\left(\frac{\mathrm{Q}-\mathrm{S}}{Q}\right) + \frac{\mathrm{C}_4 S}{Q}$$

Solution No 5:

$$\frac{\partial C}{\partial Q} = -\frac{C_2 D}{Q^2} + \frac{C_3}{2}\left(\frac{Q^2 - S^2)}{Q^2}\right) - \frac{C_4 S^2}{2Q^2}$$

For optimal value putting $\frac{\partial C}{\partial Q}$ and $\frac{\partial C}{\partial S}$ *equals* to zero.

$$-2C_2 D + C_3(Q^2 - S^2) - C_4 S^2 = 0$$

$$C_3 Q^2 = 2C_2 D + C_3 S^2 + C_4 S^2$$

$$C_3 Q^2 = 2C_2 D + S^2(C_3 + C_4)$$

$$Q^2 = \frac{2C_2 D}{C_3} + S^2\left(\frac{C_3 + C_4}{C_3}\right) \qquad \text{(A)}$$

$$\frac{\partial C}{\partial S} = 0 + 0 - C_3\left(\frac{(Q - S)}{Q}\right) + \frac{C_4 S}{Q}$$

$$\frac{\partial C}{\partial S} = -C_3\left(\frac{Q - S}{Q}\right) + \frac{C_4 S}{Q}$$

$$C_3 Q = C_3 S + C_4 S$$

$$S = \frac{C_3 Q}{C_3 + C_4} \qquad \text{(B)}$$

Putting the value of S *in* (A)

$$Q^2 = \frac{2C_2 D}{C_3} + \left(\frac{C_3 Q}{C_3 + C_4}\right)^2\left(\frac{C_3 + C_4}{C_3}\right)$$

$$= \frac{2C_2 D}{C_3} + \left(\frac{C_3 Q^2}{C_3 + C_4}\right)$$

$$Q^2\left(1 - \frac{C_3}{C_3 + C_4}\right) = \frac{2C_2 D}{C_3}$$

$$Q = \sqrt{\frac{2C_2 D}{C_3}}\sqrt{\frac{C_3 + C_4}{C_4}}$$

Putting the value of Q in (B).

$$S = \frac{C_3}{C_3 + C_4}\sqrt{\frac{2C_2 D}{C_3}}\sqrt{\frac{C_3 + C_4}{C_4}}$$

$$S = \sqrt{\frac{2C_2 D}{C_4}}\sqrt{\frac{C_3}{C_3 + C_4}}$$

Solution No.1:-

1) $$t = \frac{Q^*}{D}$$

$$t = \frac{8490}{3000} = 2.83$$

2) $$N = \frac{12}{t}$$

$$t = \frac{12}{2.83} = 4.240$$

3) *The time of manufacturing*

$$t_1 = \frac{Q^*}{R}$$

$$t_1 = \frac{8490}{6000} = 1.415$$

Solution No.2:-

$$Optimum\ Manufacturing\ Quantity = Q^* = \sqrt{\frac{2C_2D}{C_3}} * \sqrt{\frac{R}{R-D}}$$

$$Q^* = \sqrt{\frac{2*400*2500}{1}} * \sqrt{\frac{5000}{5000-2500}}$$

$$Q^* = \sqrt{\frac{2*400*2500*5000}{2500}}$$

$$Q^* = \sqrt{800*5000}$$

$$Q^* = 2000$$

Solution No.3:-

$$\text{(a)}\ Optimum\ Manufacturing\ Quantity = Q* = \sqrt{\frac{2C_2D}{C_3}*\frac{R}{R-D}}*\sqrt{\frac{C_3+C_4}{C_4}}$$

$$Q* = \sqrt{\frac{2*500*6000}{8}*\frac{36000}{36000-6000}}*\sqrt{\frac{8+20}{20}}$$

$$Q* = \sqrt{\frac{1000*6000}{8}*\frac{36000}{30000}}*\sqrt{\frac{28}{20}}$$

$$Q* = \sqrt{1,260,000}$$

$$Optimum\ Manufacturing\ Quantity = Q* = Approx.1123 units$$

$$\text{(b)}\quad S = \sqrt{\frac{2C_2D}{C_4}*\frac{R-D}{R}}*\sqrt{\frac{C_3}{C_3+C_4}}$$

$$S = \sqrt{\frac{2*500*6000}{20}*\frac{36000-6000}{36000}}*\sqrt{\frac{8}{8+20}}$$

$$S = \sqrt{\frac{2*500*6000}{20}*\frac{30000}{36000}}*\sqrt{\frac{8}{28}}$$

$$S = \sqrt{\frac{50*5000}{1}*\frac{2}{7}}$$

$$S = \sqrt{\frac{500,000}{7}}$$

$$S = approx.267$$

(c) *Time between orders*

$$\text{t} = \frac{Q*}{D}$$

$$t = \frac{1123}{6000} = 0.19 Years$$

Manufacturing Time :

$$t* = \frac{Q*}{R}$$

$$t* = \frac{1123}{36000} = 0.03$$

Solution No.4:-

$$Holding\ Cost = Rs.2\ per\ item\ per\ unit\ time$$

$$Holding\ Cost\ for\ period\ AD = C_3 * [Area\ of\ \Delta ACD]$$

$$= C_3 * [\frac{1}{2} * CE * AD]$$

$$= 2 * [\frac{1}{2} * 2 * 2]$$

$$Holding\ Cost\ for\ period\ AD = Rs.4$$

$$Shortage\ Cost = Rs.4\ per\ item\ per\ unit\ time$$

$$Shortage\ Cost\ for\ period\ DB = C_4 * [Area\ of\ \Delta DFB]$$

$$= 4 * [\frac{1}{2} * GF * DB]$$

$$= 4 * [\frac{1}{2} * 2 * 2]$$

$$= Rs.8$$

Solution No.01:

The given constraint is

$$-4x + 2y \leq 11$$

$$As\ the\ point\ (x, 4)\ satisfy\ the\ given\ constraint$$

$$-4(x) + 2(4) \quad \leq 11$$

$$-4x + 8 \quad \leq 11$$

$$-4x \leq 11 - 8$$

$$-4x \leq 3$$

$$x \geq -\frac{3}{4}$$

Hence

$$\mathbf{X_{min} \quad = \quad -3/4}$$

Solution No.02:

The given constraint is

$$4x - 2y \leq 11$$

$$As\ the\ point\ (2, -y)\ satisfy\ the\ given\ constraint$$

$$4(2) - 2(-y) \quad \leq 11$$

$$8 + 2y \quad \leq 11$$

$$2y \leq 11 - 8$$

$$2y \leq 3$$

$$y \leq \frac{3}{2}$$

Hence

$$\mathbf{y_{max} \quad = \quad 3/2}$$

Solution No.03:

Part-a

The associated equation of the given constraint is:

$4x + 2y = 12$

As the feasible region of the given constraints comes only in the 1st quadrant because x and y are positive only in 1st quadrant. ($x, y \geq 0$)

On Y-Axis, **$x = 0$;**

$\Longrightarrow$ $4(0) + 2y = 12$

$2y = 12$

$y = 6$

1st corner point is A(0,6)

At intersection of line with X-Axis, **$y = 0$;**

$\Longrightarrow$ $4x + 2(0) = 12$

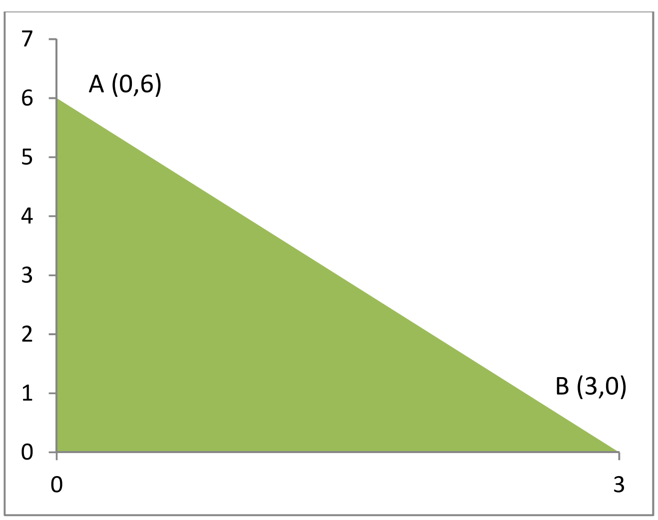
$4x = 12$

$x = 3$

2nd corner point is B(3,0)

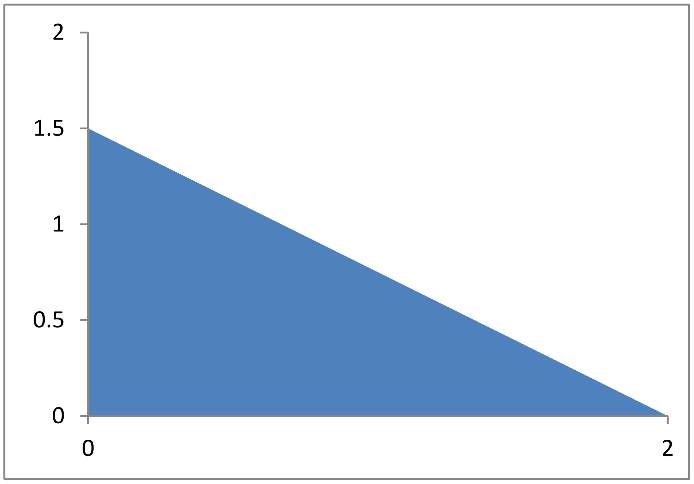
3rd corner point will be origin C(0,0)

The feasible region with corner points is shown below



Do **Part-b&c** by yourself (Similar to part-a)

Solution No.04:



From the graph, it is clear that

X-intercept = a = 2

Y-intercept = b = 1.5

So the associated equation of constraint can be written as:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2} + \frac{y}{1.5} = 1$$

$$\frac{x}{2} + \frac{10y}{15} = 1$$

$$\frac{x}{2} + \frac{2y}{3} = 1$$

$$3x + 4y = 6$$

So the set of constraints that will represent the above shaded region will be:

$3x + 4y \leq 6$

$x, y \geq 0$

Solution No.05:

Part-a

The optimal value of "x" will be on X-Axis where **y = 0;**

$\Longrightarrow$ $4x + 9(0) \leq 36$

$4x \leq 36$

$x \leq 9$

$\mathbf{x_{max} = 9}$

The optimal value of "y" will be on Y-Axis where **x = 0;**

$\Longrightarrow$ $4(0) + 9y \leq 36$

$9y \leq 36$

$y \leq 4$

$\mathbf{y_{max} = 4}$

As **"x"** and **"y"** cannot be negative as given by 2nd constraint equation i.e x, y ≥ 0

So optimal values of "x" and "y" are:

$\mathbf{x_{max} = 9}$ $\mathbf{y_{max} = 4}$

$\mathbf{x_{min} = 0}$ $\mathbf{y_{min} = 0}$

Part-b

The optimal value of "x" will be on X-Axis where **y = 0;**

$\Longrightarrow$ $4x + 9(0) \geq 36$

$4x \geq 36$

$x \geq 9$

$\mathbf{x_{min} = 9}$

The optimal value of "y" will be on Y-Axis where **x = 0;**

$\Longrightarrow$ $4(0) + 9y \geq 36$

$9y \geq 36$

$y \geq 4$

$\mathbf{y_{min} = 4}$

So optimal values of "x" and "y" are:

$\mathbf{x_{max} = \infty}$ $\mathbf{y_{max} = \infty}$

$\mathbf{x_{min} = 9}$ $\mathbf{y_{min} = 4}$

Solution No.06:

First, it is needed to find the corner points of the feasible region for the given constraint equations:

The associated equations of the given constraints are:

x + 4y = 20 …………………(1)

2x + 3y = 30 ………………..(2)

<u>For equation (1)</u>

Putting x=0; we get

(0) + 4y = 20 $\Longrightarrow$ y = 5

A (0, 5)

Putting y=0; we get

x + 4(0) = 20 $\Longrightarrow$ x = 20

B(20, 0)

So the two points of equation (1) are **A (0, 5) and B(20, 0)**

<u>For equation (2)</u>

Putting x=0; we get

2(0) + 3y = 30 $\Longrightarrow$ y = 10

C (0, 10)

Putting y=0; we get

2x + 3(0) = 30 $\Longrightarrow$ x = 15

D(15, 0)

So the two points of equation (2) are **A (0, 10) and B(15, 0)**

The corner points of the feasible region are A(0, 5), D(15, 0), O(0, 0) and E(a, b).

Point "E(a, b)" is the intersection of given two lines.

Multiplying equation (1) by "2" and subtracting from equation (2)

$$\begin{array}{l} 2x+3y= \ 30 \\ \underline{\pm 2x \pm 8y = \pm 40} \end{array}$$

-5y = -10 $\Longrightarrow$ y = 2

Putting value of "y" in equation (1), we get

x + 4(2) = 20

x = 20-8

x = 12

So point E is E(12, 2)

<u>The value of function Z at point "A(0, 5)"</u>

$Z = 3(0) + 2(5) \implies$ **Z = 10**

<u>The value of function Z at point "D(15, 0)"</u>

$Z = 3(15) + 2(0) \implies$ $\mathbf{Z_{max} = 45}$

<u>The value of function Z at point "E(12, 2)"</u>

$Z = 3(12) + 2(2) \implies$ **Z = 40**

So the objective function has maximum value at point D(15, 0).

<u>Question No.07:</u>

First, it is needed to find the corner points of the feasible region for the given constraint equations:

The associated equations of the given constraints are:

x + y = 6 …………………(1)

3x + 8y = 24 ………………..(2)

<u>For equation (1)</u>

Putting x=0; we get

$(0) + y = 6 \implies y = 6$

A (0, 6)

Putting y=0; we get

$x + (0) = 6 \implies x = 6$

B(6, 0)

So the two points of equation (1) are **A (0, 6) and B(6, 0)**

<u>For equation (2)</u>

Putting x=0; we get

$3(0) + 8y = 24 \implies y = 3$

C (0, 3)

Putting y=0; we get

$3x + 8(0) = 24 \implies x = 8$

D(8, 0)

So the two points of equation (2) are **A (0, 3) and B(8, 0)**

The corner points of the feasible region are B(6, 0), C(0, 3), O(0, 0) and E(a, b).

Point "E(a, b)" is the intersection of given two lines.

Multiplying equation (1) by "3" and subtracting from equation (2)

$$\begin{array}{r} 3x+8y=\ \ 24 \\ \pm 3x \pm 3y = \pm 18 \\ \hline \end{array}$$

5y = 6 ⟹ y = 6/5

Putting value of "y" in equation (1), we get

x + 6/5 = 6

x = 6 – 6/5

x = 24/5

So point E is E(24/5, 6/5)

<u>The value of function Z at point "B(6, 0)"</u>

Z = 4(6) + 3(0) ⟹ **Z = 24**

<u>The value of function Z at point "C(0, 3)"</u>

Z = 4(0) + 3(3) ⟹ $\mathbf{Z_{min} = 9}$

<u>The value of function Z at point "E(24/5, 6/5)"</u>

$$Z = 4*\frac{24}{5} + 3*\frac{6}{5}$$
$$Z = \frac{96}{5} + \frac{18}{5}$$
$$Z = \frac{96+18}{5}$$
$$Z = \frac{114}{5}$$

So the objective function has minimum value at point C(0, 3).

<u>Question No.08:</u>

Try yourself