Mth-634

Q-1

Show that the set X with cofinite topology is compact?

Ans:-

If the set X is finite then

$$T_{cof} = P(X)$$

M.m

therefore,

$$(X, T_{cof}) \text{ is compact.}$$

But if X is not finite, then, Let an open cover $\mathcal{D}$ of X. Now ~~chose~~ Let $D \in \mathcal{D}$ since $\mathcal{D}$ is an open cover, this implies that D is open and it is also implies that D^c is finite. $(D \neq \phi)$

Now for every $x \in D^c$ choose an element of $\mathcal{D}$, say D_x containing x.

$$\{D_x \mid x \in D^c\}$$

since D^c is finite, consider this subcover of $\mathcal{D}$. The cover C,

$$C = \{D\} \cup \{D_x \mid x \in D^c\}$$

here $\{D\}$ and $\{D_x \mid x \in D^c\}$ is finite, so, C is finite subcover of $\mathcal{D}$.

This implies that (X, T_{cof}) is compact.

Mth-634

Q.2

Define separate sets?

Ans:-

If A and B be two subsets of a topological space (X,T). Then A and B are said to be separated sets if and only if

$A \cap B = \phi$ and $\bar{A} \cap B = \phi$ and $A \cap \bar{B} = \phi$

Q.3

M.m

Show that every second countable space is first countable?

Ans:-

Let X be a second countable space i.e there exist a basis $\mathcal{B}$ and $\mathcal{B}$ is countable. Now Let $p \in X$ and Let $\mathcal{B}_p \subset \mathcal{B}$ where $\mathcal{B}_p = \{B_p \mid p \in B_p\} \subset \mathcal{B}$

Now $\mathcal{B}_p$ is countable, $\because \mathcal{B}$ is countable

Now $\mathcal{B}_p$ is local base at p.

therefore,

X is first countable.

Date:

Mth-634

Q.4

Show that a discrete space X is separable if and only if X is countable?

Ans:-

since $T_{dis} = P(X)$

We Know that the only dense subset in X is X itself.

So, the only choice for A is X such that $\bar{A} = X$

therefore X is separable iff X is countable.

M.m

Q.5

Let $X = \{a, b, c\}$, Write the basis for the discrete topology on X?

Ans:-

If $X = \{a, b, c\}$, then basis for discrete topology on X will be

$$B = \{\{a\}, \{b, c\}\}$$

Date:

Mth-634

Q.6

Prove that every subspace of a second countable space is second countable?

Ans:-

Let $A \subset X$ (X is 2nd countable space)

Since X is 2nd countable so there exist $\mathcal{B} = \{B_n \mid n \in \mathbb{N}\}$ that is countable

Now basis for subspaces is

$\mathcal{B}_A = \{A \cap B_n \mid n \in \mathbb{N}\}$ is a basis for τ_A and $\mathcal{B}_A$ is countable. This implise that A is 2nd countable.

Q.7

M.m

Give one example ~~for~~ of second countable space?

Ans:-

Let $X = \{a, b, c, d\}$ and $T = \{\phi, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ be a Topology defined on X. A basis for T is

$$B = \{\{a\}, \{b\}, \{c, d\}\}$$

So,

(X, T) is second countable.

Mth-634

Q.8

Define T_1- space?

Ans:-

A topological space (X,T) is said to be "T_1 space" iff for each $x,y \in X$ such that $x \neq y$ there exist open subsets U_x, U_y of X containing x, y respectively such that $y \notin U_x$ and $x \notin U_y$.

Q.9

Define Normal space? M.m

A topological space (X,T) is said to be "Normal space" iff for every pair of disjoint closed subsets $F_1, F_2 \subset X$, there exist open subsets U_{F_1} and U_{F_2} containing F_1 and F_2 such that

$$U_{F_1} \cap U_{F_2} = \phi$$

Q.10

Define Metric Topology?

Ans:-

Let X be a nonempty set with metric d. The T on X generated by the set of all open balls in X with respect to d. is called Metric Topology.

Q-11

Show that a set X with topology T containing finite number subsets of X is compact?

Ans:-

Since every open cover C of X is subclass of T and T itself finite So, C is finite too and any subcover S of C is also finite.

Q.12

Define connected set? M.m

Ans:-

A subset A of a topological space (X,T) is said to be connected iff there exists no pair of nonempty open subsets U and V of X such that

$$A \cap U \text{ and } A \cap V$$

are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V)$$

Mth-634

Q.13

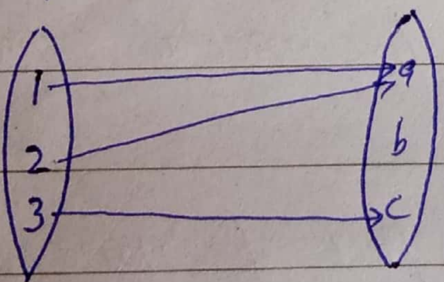
Conside the set $X=\{1,2,3\}$ and $Y=\{a,$
with Topologies $T_x=\{\phi,\{1\},\{1,2\},X\}$ and
$T_y=\{\phi,\{a,b\},\{b\},Y\}$ respectively. Define
a map $f:X\longrightarrow Y$ as $f(1)=f(2)=a, f(3)=$
show that f is a closed map.?

Ans:-

f is called closed map iff the image of each closed subset of X is closed in Y. such that

$$U\subset X \Rightarrow f(U)\subset Y$$ M.m

and map $f:X\longrightarrow Y$ is



therefore,

$$f(1)=f(2)=a$$

and

$$f(3)=c$$

Mth-634

Q-14

Define connected component in a topological space X?

Ans:-

Consider a topological space (X, T). Let $a \in X$ and C be a connected subset of X containing a. Then

M.M

$$C_a = \bigcup_{a \in C} C$$

is called the connected component of X.

Q.15

Consider $X = \{a, b, c, d, e\}$ with topology $T = \{\phi, \{a\}, \{a,b\}, \{a,b,c\}, \{a,d,e\}, \{a,b,d,e\}, X\}$. Show that the set $A = \{c, e\}$ is disconnected subset of X?

Ans:-

Since there exists a pair of nonempty open subsets $U = \{a,b,d,e\}$ and $V = \{a,b,c\}$ of X such that $A \cap U = \{e\}$ and $A \cap V = \{c\}$ and $\{e\} \cap \{c\} = \phi$ are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V) = \{e\} \cup \{c\} = \{e, c\}$$

Date:

Mth-634

Q-16

Define Regular space?

Ans:-

A topological space (X,T) is said to be regular if for any closed set A and a point not in A, there exist two open sets U and V such that $x \in U$, $A \subseteq V$ and $U \cap V = \phi$

Q.17

M.m

Conside $X = \{a,b,c,d,e\}$ with topology $T = \{\phi, \{a,b\}, \{d,e\}, \{a,b,c\}, \{a,b,d,e\}, X\}$. Then show X is a disconnected space?

Ans:-

since there exist nonempty open disjoint subsets

$U = \{a,b,c\}$ and $V = \{d,e\}$

of X such that

$$X = U \cup V$$

Date:

Mth-634

Q.18

Show that every finite subset of T_1-space is closed?

Ans:-

Let (X, T) be a T_1 space and
$A \subset X$

Let $A = \{a_1, a_2, \cdots a_n\}$ then

$$A = \bigcup_{i=1}^{n} \{a_i\} \quad , \text{where } a_i \in A$$

M.m

Since X is T_1 space, So every singleton subset of X is closed in X.

$\{a_i\}$ is closed for all $i = 1, 2, 3 \cdots n$

It implies that A is closed in X.

Q.19

Give any two examples of Lindelof space?

Ans:-

i) $X = \mathbb{R}$ with usual topology is Lindelof

ii) A set X with indiscrete topology then it is Lindelof.

Mth-634

Q.20

Prove $f:(X,T)\to(Y,T')$ is a closed map?

Proof:-

Let A be a closed subset of X, since X is compact and we know that a closed subspace of a compact space is compact. So A is compact.

Now f is continuous and we know image of a compact space under a continuous map is compact so f(A) is compact subspace of Y.

M.M

Now Y is Hausdorff, we know every compact subspace of Housdorff is closed so f(A) is closed.

Thus,

f is a closed map.

Mth-634

Q.21

Consider $X=\{a,b,c\}$ with $T=\{\phi,\{a\},\{b,c\},X\}$ then show that (X,T) is regular space?

Ans:-

$C_T=\{\phi,\{b,c\},\{a\},X\}$

then $U_a=\{a\}$ and $U_F=\{b,c\} \longrightarrow (1)$

$U_c=\{b,c\}$ and $U_F=\{a\} \longrightarrow (2)$

by (1)

$U_a \cup U_F=\phi$

M.M

by (2)

$U_c \cup U_F=\phi$

Thus,

(X,T) is regular space.

Q.22

Consider $(\mathbb{R},T_u)$, then prove $A=(0,1)$ and $B=(4,9]$ are separated sets?

Ans:-

by definition,

$A \cap B=(0,1)\cap(4,9]=\phi$

$A' \cap B=(0,1)'\cap(4,9]=\phi$

and

$A \cap B'=(0,1)\cap(4,9]'=\phi$

Thus A and B are separated sets.

Date:

Mth-634

Q.23

Prove every subspace of a T_1 space is also a T_1 space?

M.m

Proof:-

Let (X,T) be a T_1 space $Y \subset X$ and (Y,T_y) be a subspace of (X,T).

We know that in a space every singleton subset is closed then that space is T_1

Let $p \in Y \subset X$ $\quad \because X$ is T_1

so, $\{p\} \subset X$ closed, then

$\{p\}^c$ is open in X

therefore,

$Y \cap \{p\}^c$ is open in Y

but $Y \cap \{p\}^c = Y / \{p\}$

This implies that $\{p\}$ is closed in Y

and it implies that (Y,T_y) is T_1.

Q.24

Give one example of regular space?

Ans:-

i) Discrete space is a regular space

ii) Indiscrete space is a regular space.