# MTH645 Current papers Questions (Fall 2021) Final Term

Solved by

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## Question # 1

If $f(x) = e^{-\left(\frac{1-x}{x}\right)}$, Find "a" or $f^{-1}(x) = ?$ (2, 3 or 5 Marks)

## Solution:

let $y = e^{-\left(\frac{1-x}{x}\right)} \Rightarrow \ln y = \ln e^{-\left(\frac{1-x}{x}\right)}$

$\Rightarrow \ln y = -\left(\frac{1-x}{x}\right) \ln e \qquad \because \ln e = 1$

$\Rightarrow \ln y = -\left(\frac{1-x}{x}\right)$

$\Rightarrow x \ln y = x - 1$

$\Rightarrow 1 = x - x\ln y = x(1 - \ln y)$

$\Rightarrow x = \frac{1}{1-\ln y} = f^{-1}(y)$

$\Rightarrow \boxed{f^{-1}(x) = \frac{1}{1-\ln x}}$ Ans.

$a = f(0) = e^{-\left(\frac{1-0}{0}\right)} = e^{-\frac{1}{0}} = e^{-\infty} = \frac{1}{e^{\infty}}$

$a = f(0) = \frac{1}{\infty} = 0$

$\boxed{a = 0}$ Ans

Note: The question can be asked by further changing the value of f(x).

## Question # 2

If $\Delta$ is strictly monotone then it has only trivial idempotent element. (5 Mark)

### Solution:

Let $\Delta$ has non trivial idempotent ~~element~~ element $x$ and $y$.

Let $x, y \in (0,1)$

$$x \Delta y < x \Delta z \quad ; \quad x > 0 \ , \ y < z$$

Take $x \leq x$ and $x < 1$

$$\Rightarrow x \Delta x < x \Delta 1 = x$$

$$\Rightarrow x \Delta x < x$$

which is contradiction.

So $\Delta$ has only trivial idempotent element.

## Question # 3

Define Fuzzy Set. (2 Marks)

### Solution:

A fuzzy set consist of two components a regular set and a membership function associated with it.

$$A = \{(x, \mu_A(x)) \mid x \in U\}$$

## Question # 4

Define ∇- Implication (Nebla Implication).

### Solution:

(2 Marks)

A ∇-Implication is a map $\Rightarrow [0,1]^2 \to [0,1]$ of the form $(x \Rightarrow y) = \eta(x) \nabla y$ ; $\eta(x) = 1 - x$

∇ is t-conorm. $\eta$ ↓ Eta

## Question # 5

Define Symmetric Relation.

### Solution:

(2 Marks)

A fuzzy binary relation "R" is symmetric if $R(x,y) = 1 \Rightarrow R(y,x) = 1$

## Question # 6

Define Antisymmetric Relation. (2 Marks)

### Solution:

A fuzzy binary relation R is antisymmetric if $R(x,y) = \& R(y,x) = 1 \Rightarrow x = y$

## Question # 7

Define Reflexive Relation. (2 Marks)

### Solution:

A fuzzy binary relation R is reflexive if $R(x,x) = 1$

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## Question # 8

Find Hamming distance between two fuzzy sets A and B. If $X = \{1, 2, 3, 4\}$

$A = \{(1, 0.5), (2, 1.0), (3, 0.3)\}$ and

$B = \{(2, 0.4), (3, 0.4), (4, 1.0)\}$.

(5 Marks)

**Solution:**

$d_h(1,2) = |0.5 - 0.4| = 0.1$

$d_h(1,3) = |0.5 - 0.4| = 0.1$

$d_h(1,4) = |0.5 - 1.0| = 0.5$

$d_h(2,2) = |1.0 - 0.4| = 0.6$

$d_h(2,3) = |1.0 - 0.4| = 0.6$

$d_h(2,4) = |1.0 - 1.0| = 0$

$d_h(3,2) = |0.3 - 0.4| = 0.1$

$d_h(3,3) = |0.3 - 0.4| = 0.1$

$d_h(3,4) = |0.3 - 1.0| = 0.7$

$D_h = 0.1 + 0.1 + 0.5 + 0.6 + 0.6 + 0 + 0.1 + 0.1 + 0.7$

$D_h = 2.8$ Ans

**Note:** The question can be asked by further changing the sets X, A and B.

## Question # 9

Define Nilpotent t-norm. (2 Marks)

**Solution:**

$\Delta$ is nilpotent t-norm if for $b \neq 1$, $b^{[n]} = 0$ for some positive integers "n", where n depends on b.

## Question # 10

Define strictly monotone t-norm. (2 Marks)

**Solution:**

A t-norm $\Delta$ is called strictly monotone t-norm if $x \Delta y < x \Delta z$, whenever $x > 0$ and $y < z$.

## Question # 11

Define Transitive Relation. (2 Marks)

**Solution:**

A fuzzy binary relation R is transitive if $R(x, y) = 1$, $R(y, z) = 1 \Rightarrow R(x, z) = 1$

where $x, y, z \in$ fuzzy numbers.

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## Question # 12

Write down the types of fuzzy sets.

(5 Marks)

### Solution:

(i) Intutionistic Fuzzy Sets

(ii) Picture Fuzzy Sets

(iii) Hasitent Fuzzy Sets

(iv) Normal Fuzzy Sets

(v) Empty Fuzzy Sets.

## Question # 13

Define $\alpha$-cut in fuzzy binary relation and give example.

(2 or 3 Marks)

### Solution:

$\alpha$-cut in fuzzy binary relation is

$$R_\alpha = R^{-1}([\alpha, 1]) = [(u,v) : R(u,v) \geq \alpha]$$

Example: $\alpha = 0.5$, $R = [((x_1,y_1), 0.3), ((x_2,y_2), 0.7), ((x_3,y_3), 0.9)]$

$$R_\alpha = R_{0.5} = [((x_2,y_2), 0.7), ((x_3,y_3), 0.9)]$$

## Question # 14

Define zero divisor.

(2 Marks)

### Solution:

A fuzzy number $a \in (0,1)$ is called zero divisor of t-norm $\Delta$, if there exist $b \in (0,1)$ such that $a \Delta b = 0$

## Question # 15

If $f(x) = e^{x-5}$, $f^{-1}(x) = 5 + \ln x$ and $a = \frac{1}{e^5}$, $x \Delta y = xy$ then find the value of generating t-norm $x \Delta_f y$.

## Solution: (5 Marks)

$f(x) = e^{x-5}$, $f^{-1}(x) = 5 + \ln x$, $a = \frac{1}{e^5}$, $x \Delta y = xy$

We know that:

$$x \Delta_f y = f^{-1}\left((f(x) \Delta f(y)) \vee a\right)$$

$$= f^{-1}\left((f(x) \cdot f(y)) \vee a\right)$$

$$= f^{-1}\left((e^{x-5} \cdot e^{y-5}) \vee \frac{1}{e^5}\right)$$

$$= f^{-1}\left((e^{x+y-10}) \vee \frac{1}{e^5}\right)$$

$$= f^{-1}\left(e^{x+y-10} \vee e^{-5}\right)$$

$$= \cancel{f^{-1}}\, 5 + \ln(e^{x+y-10} \vee e^{-5})$$

$$= 5 + (\ln e^{x+y-10} \vee \ln e^{-5})$$

$$= 5 + ((x+y-10) \vee (-5))$$

$$= (5 + x + y - 10) \vee (5 - 5)$$

$$x \Delta_f y = (x + y - 5) \vee (0)$$

$$x \Delta_f y = x + y - 5 \vee 0 \quad \underline{\text{Ans}}$$

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# Question #16

Find the max-min composition of R and S where

$$R = \begin{Bmatrix} 0.1 & 0 & 0.6 \\ 0 & 0.3 & 1 \\ 0.2 & 0.4 & 0.4 \end{Bmatrix}, \quad S = \begin{Bmatrix} 0.2 & 0.5 & 0 \\ 0.9 & 1 & 0.3 \\ 1 & 0.5 & 0 \end{Bmatrix}$$

(5 Marks)

## Solution:

$$\left.\begin{aligned} \min(0.1, 0.2) &= 0.1 \\ \min(0, 0.9) &= 0 \\ \min(0.6, 1) &= 0.6 \end{aligned}\right\} \max = 0.6$$

$$\left.\begin{aligned} \min(0.1, 0.5) &= 0.1 \\ \min(0, 1) &= 0 \\ \min(0.6, 0.5) &= 0.5 \end{aligned}\right\} \max = 0.5$$

$$\left.\begin{aligned} \min(0.1, 0) &= 0 \\ \min(0, 0.3) &= 0 \\ \min(0.6, 0) &= 0 \end{aligned}\right\} \max = 0$$

$$\left.\begin{aligned} \min(0, 0.2) &= 0 \\ \min(0.3, 0.9) &= 0.3 \\ \min(1, 1) &= 1 \end{aligned}\right\} \max = 1$$

$$RoS = \begin{bmatrix} 0.6 & 0.5 & 0 \\ 1 & 0.5 & 0.3 \\ 0.4 & 0.4 & 0.3 \end{bmatrix}$$ Ans

$$\left.\begin{aligned} \min(0, 0.5) &= 0 \\ \min(0.3, 1) &= 0.3 \\ \min(1, 0.5) &= 0.5 \end{aligned}\right\} \max = 0.5$$

$$\left.\begin{aligned} \min(0, 0) &= 0 \\ \min(0.3, 0.3) &= 0.3 \\ \min(1, 0) &= 0 \end{aligned}\right\} \max = 0.3$$

$$\left.\begin{aligned} \min(0.2, 0.2) &= 0.2 \\ \min(0.4, 0.9) &= 0.4 \\ \min(0.4, 1) &= 0.4 \end{aligned}\right\} \max = 0.4$$

$$\left.\begin{aligned} \min(0.2, 0.5) &= 0.2 \\ \min(0.4, 1) &= 0.4 \\ \min(0.4, 0.5) &= 0.4 \end{aligned}\right\} \max = 0.4$$

$$\left.\begin{aligned} \min(0.2, 0) &= 0 \\ \min(0.4, 0.3) &= 0.3 \\ \min(0.4, 0) &= 0 \end{aligned}\right\} \max = 0.3$$

**Note:** The question can be asked by further changing the values of R and S.

## Question # 17

Let $X = \{x_1\}$ be a universal set $h_1 = \{\langle x_1, \{0.3, 0.4\}\rangle\}$ and $h_2 = \{\langle x_1, \{0.5, 0.7\}\rangle\}$ are two hesitent fuzzy sets on X evaluate: $h_1 \oplus h_2$ (5 Marks)

### Solution:

$$h_1 \oplus h_2 = \{0.3, 0.4\} \oplus \{0.5, 0.7\}$$

$$= \{0.3+0.5-(0.3)(0.5),\ 0.3+0.7-(0.3)(0.7),\ 0.4+0.5-(0.4)(0.5),\ 0.4+0.7-(0.4)(0.7)\}$$

$$= \{0.8-0.15,\ 1.0-0.21,\ 0.9-0.2,\ 1.1-0.28\}$$

$$= \{0.65,\ 0.79,\ 0.7,\ 0.82\}$$

## Question # 18

Define Picture Fuzzy Set. (2 Marks)

### Solution:

Let X be a fixed set, a picture fuzzy set P on X is defined as $P = \{\langle x, \alpha_p(x), \gamma_p(x), \beta_p(x)\rangle \mid x \in X\}$ where $\alpha_p(x)$, $\beta_p(x)$ and $\gamma_p(x) \in [0,1]$ are called the acceptence, nautral and rejection memembership degrees of $x \in X$ to the set P respectively and fullfil the condition: $0 \leq \alpha_p(x) + \gamma_p(x) + \beta_p(x) \leq 1$, $\forall\ x \in X$.

# Question # 19

Let $X = \{x_1, x_2, x_3\}$ be a fixed set and $A = \{(x_1, 0.3), (x_2, 0.5), (x_3, 0.9)\}$ and $B = \{(x_1, 0.5), (x_2, 0.8), (x_3, 1)\}$ are two fuzzy sets on X evaluate the normalized Hamming distance between A and B.

## Solution: (5 Marks)

The Hamming distance with all generating values is evaluated as:

$d_h(x_1, x_1) = |0.3 - 0.5| = 0.2$

$d_h(x_1, x_2) = |0.3 - 0.8| = 0.5$

$d_h(x_1, x_3) = |0.3 - 1.0| = 0.7$

$d_h(x_2, x_1) = |0.5 - 0.5| = 0$

$d_h(x_2, x_2) = |0.5 - 0.8| = 0.3$

$d_h(x_2, x_3) = |0.5 - 1.0| = 0.5$

$d_h(x_3, x_1) = |0.9 - 0.5| = 0.4$

$d_h(x_3, x_2) = |0.9 - 0.8| = 0.1$

$d_h(x_3, x_3) = |0.9 - 1.0| = 0.1$

The Normalized Hamming Distance is

$$D_h^n(A,B) = \frac{1}{n}\sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

$$= \frac{1}{3}(0.2+0.5+0.7+0+0.3+0.5+0.4+0.1+0.1)$$

$$= \frac{1}{3}(2.8) = 0.93 \quad \text{Ans}$$

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## Question # 20

Let $A = (0.3, 0.4, 0.2)$ and $B = (0.3, 0.5, 0.1)$ be two PFSs, then find (i) $A \subseteq B$ (ii) $A \cup B$ (iii) $A \cap B$

## Solution: (5 Marks)

(i) $A \subseteq B = ?$

$A \subseteq B$ if $\alpha_A(x) \leq \alpha_B(x)$, $\gamma_A(x) \leq \gamma_B(x)$ and $\beta_A(x) \geq \beta_B(x)$, $\forall x \in X$

$0.3 \leq 0.3$, $0.4 \leq 0.5$ and $0.2 \geq 0.1$

Hence $A \subseteq B$.

(ii) $A \cup B = ?$

$$A \cup B = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \min(\gamma_A(x), \gamma_B(x)), \min(\beta_A(x), \beta_B(x))\rangle \mid x \in X\}$$

$$A \cup B = \{\langle x, \max(0.3, 0.3), \min(0.4, 0.5), \min(0.2, 0.1)\rangle\}$$

$$= \{\langle x, 0.3, 0.4, 0.1\rangle\} = (0.3, 0.4, 0.1)$$

(iii) $A \cap B = ?$

$$A \cap B = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\gamma_A(x), \gamma_B(x)), \max(\beta_A(x), \beta_B(x))\rangle \mid x \in X\}$$

$$A \cap B = \{\langle x, \min(0.3, 0.3), \min(0.4, 0.5), \max(0.2, 0.1)\rangle\}$$

$$= \{\langle x, 0.3, 0.4, 0.2\rangle\} = (0.3, 0.4, 0.2)$$

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# Question # 21

In R-implication find the values of 1 implies 1 by using formula of R-implication

## Solution: (2 Marks)

We know that $(x \Rightarrow y) = \max\{z \in [0,1] : x \Delta z \leq y\}$

$$(1 \Rightarrow 1) = \max\{z \in [0,1] : 1 \Delta z \leq 1\}$$
$$= \max\{z \in [0,1] : z \leq 1\}$$
$$= \max\{z \in [0,1]\}$$
$$(1 \Rightarrow 1) = 1 \quad \text{Ans}$$

# Question # 22

Let $x$ be a fuzzy number and $\Delta$ be an idemponent t-norm then find the value of $(x \Delta x) \Delta x$.

## Solution: (3 Marks)

We know that $\Delta$ is an idemponent t-norm if $x \Delta x = x$, $\forall\ x \in [0,1]$

Now we find the value of $(x \Delta x) \Delta x$

$$(x \Delta x) \Delta x = x \Delta x \qquad \because x \Delta x = x$$
$$(x \Delta x) \Delta x = x$$

## Question#23

let $X = \{x_1\}$ be a fixed set and $A = \{\langle x_1, (0.5, 0.3)\rangle\}$ and $B = \{\langle x_1, (0.6, 0.2)\rangle\}$ are two IFSs on X. Evaluate $A \cup B$.

## Solution: (3 Marks)

We know that

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle \mid x \in X\}$$

$$A \cup B = \{\langle x_1, \max(0.5, 0.6), \min(0.3, 0.2)\rangle\}$$

$$A \cup B = \{\langle x_1, (0.6, 0.2)\rangle\}$$

## Question#24

let $X = \{x_1\}$ be a fixed set and $A = \{\langle x_1, (0.5, 0.3)\rangle\}$ and $B = \{\langle x_1, (0.6, 0.2)\rangle\}$ are two IFSs on X. Evaluate $A \otimes B$.

## Solution: (3 Marks)

We know that

$$A \otimes B = \{\langle x, \mu_A(x) \times \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \times \nu_B(x)\rangle \mid x \in X\}$$

$$A \otimes B = \{\langle x_1, 0.5 \times 0.6, 0.3 + 0.2 - 0.3 \times 0.2\rangle\}$$

$$A \otimes B = \{\langle x_1, 0.3, 0.5 - 0.06\rangle\}$$

$$A \otimes B = \{\langle x_1, (0.3, 0.44)\rangle\}$$ Ans

## Question # 25

Define the composition of $RoS$ of fuzzy relation $R$ and $S$ with respect to t-norm $\Delta$.

## Solution: (2 Marks)

Let $R$ and $S$ be fuzzy relation in $U \times V$ and $V \times W$ respectively and let $\Delta$ be a t-norm. Then composition $RoS$ of $R$ and $S$ with respect to $\Delta$ is the fuzzy relation on $U \times W$ with a membership function

$$(R \circ S)(u,w) = \vee [R(u,v) \, \Delta \, S(v,w)]$$

## Question # 26

Evaluate the ~~Ha~~ Euclidean distance between two HFSs $h_1 = \{\langle x_1, \{0.8, 0.9\}\rangle\}$, $h_2 = \{\langle x_1, \{0.6, 0.7\}\rangle\}$

## Solution: (2 Marks)

$$De(h_1, h_2) = \sqrt{\sum_{i=1}^{n} (h_{1i} - h_{2i})^2}$$

$$De(h_1, h_2) = \sqrt{(0.8-0.6)^2 + (0.9-0.7)^2}$$

$$= \sqrt{(0.2)^2 + (0.2)^2}$$

$$= \sqrt{0.04 + 0.04}$$

$$= \sqrt{0.08}$$

$$D_e(h_1, h_2) = 0.2828$$

## Question # 27

Let $X = \{x_1\}$ be a fixed set and $A = \{\langle x_1, (0.8, 0.2)\rangle\}$ and $B = \{\langle x_1, (0.7, 0.3)\}$ are two IFSs on $X$. Then find $A - B$.

**Solution:** (3 Marks)

We know that

$A - B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\mu_B(x), \nu_A(x))\rangle | x \in X\}$

$A - B = \{\langle x_1, \min(0.8, 0.3), \max(0.7, 0.2)\rangle\}$

$A - B = \{\langle x_1, (0.3, 0.7)\rangle\}$ Ans

## Question # 28

Let $\Delta$ be archemedian t-norm and $f$ is generator of t-norm. Then $(x \Rightarrow y) = f^{-1}\left[\frac{f(y)}{f(x)} \wedge 1\right]$.

**Solution:** (5 Marks)

$x \Delta y = f^{-1}(f(x) \cdot f(y) \vee f(0))$

$(x \Rightarrow y) = \max\{z \in [0,1] : x \Delta z \leq y\}$

$= \max\{z \in [0,1] : f^{-1}(f(x) \cdot f(z) \vee f(0)) \leq y\}$

Now $f^{-1}(f(x) \cdot f(z) \vee f(0)) \leq y$

$\Rightarrow f(x) \cdot f(z) \vee f(0) \leq f(y)$

$\Rightarrow f(x) \cdot f(z) \vee a \leq f(y)$

$\Rightarrow f(x) f(z) \leq f(y)$

$\Rightarrow f(z) \leq f(y)/f(x)$

$\Rightarrow z \leq f^{-1}\left(\frac{f(y)}{f(x)} \wedge 1\right)$

So

$(x \Rightarrow y) = f^{-1}\left[\frac{f(y)}{f(x)} \wedge 1\right]$ proved

## Question # 29

Find Hamming distance between two Hesitent fuzzy sets $h_1$ and $h_2$, I?

$h_1 = \{\langle x_1, (0.3, 0.4, 0.4, 0.65)\rangle\}$, $h_2 = \{\langle x_1, (0.5, 0.7, 0.8, 0.95)\rangle\}$

### Solution: (5 Marks)

$$D_h(h_1, h_2) = \sum_{i=1}^{n} |h_{1i} - h_{2i}|$$

$$D_h(h_1, h_2) = |0.3-0.5| + |0.4-0.7| + |0.4-0.8| + |0.65-0.95|$$

$$= 0.2 + 0.3 + 0.4 + 0.3$$

$$D_h(h_1, h_2) = 1.2 \quad \text{Ans}$$

## Question # 30

Find Euclidean distance between two PFSs $A = (0.4, 0.5, 0.1)$ and $B = (0.2, 0.6, 0.2)$

### Solution: (3 Marks)

$$d_e(A,B) = \sqrt{\sum_{i=1}^{n} \left(m_A(x_i) - m_B(x_i)\right)^2}$$

$$d_e(A,B) = \sqrt{(0.4-0.2)^2 + (0.5-0.6)^2 + (0.1-0.2)^2}$$

$$= \sqrt{(0.2)^2 + (-0.1)^2 + (-0.1)^2}$$

$$= \sqrt{0.04 + 0.01 + 0.01} = \sqrt{0.06}$$

$$d_e(A,B) = 0.245 \quad \text{Ans}$$

## Question #31

Find Hamming distancen between two fuzzy sets.

$A = \{(x_1, 0.3), (x_2, 0.5)\}$, $B = \{(x_1, 0.5), (x_2, 0.8)\}$.

**Solution:** (3 Marks)

$$d_h(A,B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

$$d_h(x_1, x_1) = |0.3 - 0.5| = 0.2$$

$$d_h(x_1, x_2) = |0.3 - 0.8| = 0.5$$

$$d_h(x_2, x_1) = |0.5 - 0.5| = 0$$

$$d_h(x_2, x_2) = |0.5 - 0.8| = 0.3$$

$$d_h(A,B) = 0.2 + 0.5 + 0 + 0.3$$

$$d_h(A,B) = 1.0 \text{ Ans}$$

## Question #32

If $f(x) = e^{x-5}$ then find the value of "a".

**Solution:** (2 Marks)

$$f(x) = e^{x-5}$$

$$a = f(0) = e^{0-5}$$

$$a = f(0) = e^{-5}$$

$$\boxed{a = \frac{1}{e^5}} \text{ Ans}$$

## Question # 33

Show that ~~proved that~~ $A \subset B$ of two PFSs, if
$A = (0.2, 0.5, 0.3)$, $B = (0.3, 0.6, 0.1)$

### Solution: (2 Marks)

$A \subseteq B$ if $\alpha_A(x) \le \alpha_B(x)$, $\gamma_A(x) \le \gamma_B(x)$
and $\beta_A(x) \ge \beta_B(x)$, $\forall\ x \in X$

$0.2 \le 0.3$, $0.5 \le 0.6$ and $0.3 \ge 0.1$

Hence $A \subseteq B$ (proved)

## Question # 34

Let $A = \{(x_1, 0.6)\}$, $B = \{(x_1, 0.9)\}$ be two fuzzy sets on $X = \{x_1\}$. Determine normalized Hamming distance between A and B.

### Solution: (2 Marks)

$$d_h^n(A,B) = \frac{1}{n}\sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

$$= \frac{1}{1}(|0.6 - 0.9|)$$

$$d_h^n(A,B) = 0.3 \quad \text{Ans}$$

## Question # 35

Define Idempotent element. (2 Marks)

### Solution:

Let $a \in (0,1)$ is idempotent element if $a \Delta a = a$.
For example (i) $0 \Delta 0 = 0 \Rightarrow 0$ is idempotent element
(ii) $1 \Delta 1 = 1 \Rightarrow 1$ is idempotent element

## Question # 36

Prove that $\Delta_0$ is nilpotent t-norm.

**Solution:** (5 Marks)

$$x \Delta_0 y = \begin{cases} \min(x,y) & \text{if } \max(x,y) = 1 \\ 0 & ; \text{ otherwise.} \end{cases}$$

let $\max(x,y) = 1$

$\Rightarrow x \neq 1$ , $\max(x,x) \neq 1$

$\Rightarrow x \Delta_0 x = 0$

$\Rightarrow x^{[2]} = 0$

$\Rightarrow \Delta_0$ is nilpotent t-norm.

## Question # 37

If $h(x) = \{0.6\}$ and $\lambda = 2$ then find $\lambda h(x)$.

**Solution:** (2 or 3 Marks)

We know that

$\lambda h(x) = \{1 - (1-\gamma)^{\lambda}\}$

$\gamma = 0.6$ , $\lambda = 2$

$\lambda h(x) = \{1 - (1 - 0.6)^2\}$

$= \{1 - (0.4)^2\} = \{1 - 0.16\}$

$\lambda h(x) = \{0.84\}$ Ans

## Question # 38

If $A = \{\langle x, (0.4, 0.2, 0.3)\rangle\}$ is a PFS on $X$, Determine $A^c$:

## Solution: (2 Marks)

We know that, If $A = \{\langle x, (\alpha_A(x), \gamma_A(x), \beta_A(x))\rangle\}$ is a PFS then $A^c = \{\langle x, (\beta_A(x), \gamma_A(x), \alpha_A(x))\rangle\}$

$$A = \{\langle x, (0.4, 0.2, 0.3)\rangle\}$$

$$A^c = \{\langle x, (0.3, 0.2, 0.4)\rangle\}$$ Ans

## Question # 39

Let $X = \{x_1\}$ be a universel set, $\lambda = 3$ and $h = \{\langle x_1, (0.2, 0.8)\rangle\}$ is a HFS on $X$, Evaluate $\lambda h(x_1)$.

## Solution: (2 or 3 Marks)

We know that

$$\lambda h(x) = \{1 - (1 - \gamma)^{\lambda}\}$$

$$\lambda h(x_1) = \{1 - (1 - 0.2)^3, 1 - (1 - 0.8)^3\}$$

$$\lambda h(x_1) = \{1 - (0.8)^3, 1 - (0.2)^3\}$$

$$\lambda h(x_1) = \{1 - 0.512, 1 - 0.008$$

$$\lambda h(x_1) = \{0.488, 0.992\}$$ Ans

## Question # 40

Find the $\nabla$-implication whe $x \nabla y = x + y - xy$

### Solution: (3 Marks)

$$(x \Rightarrow y) = \eta(x) \nabla y$$

$$= (1-x) \nabla y \qquad \because \eta(x) = 1-x$$

$$= (1-x) + y - (1-x)y$$

$$= 1 - x + y - y + xy$$

$$(x \Rightarrow y) = 1 - x + xy \quad \text{Ans}$$

## Question # 41

let $X = \{x_1, x_2\}$ be a universal set,

$A = \{\langle x_1, (0.4, 0.25, 0.35)\rangle, \langle x_2, (0.15, 0.45, 0.4)\rangle\}$ and

$B = \{\langle x_1, (0.3, 0.4, 0.3)\rangle, \langle x_2, (0.25, 0.15, 0.6)\rangle\}$ are

two PFSs on $X$. Find Normalized Hamming distance.

### Solution: (5 Marks)

$$d_h^n(A,B) = \frac{1}{n} \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

$$d_h^n(A,B) = \frac{1}{2}\Big[|0.4 - 0.3| + |0.25 - 0.4| + |0.35 - 0.3| + |0.15 - 0.25| + |0.45 - 0.15| + |0.4 - 0.6|\Big]$$

$$= \frac{1}{2}[0.10 + 0.15 + 0.05 + 0.10 + 0.30 + 0.20]$$

$$d_h^n(A,B) = \frac{1}{2}[0.9] = 0.45 \quad \text{Ans}$$

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## Question #42

Let $X = \{x_1, x_2, x_3, x_4\}$ be a fixed set and $A = \{(x_1, 0.1), (x_2, 0.1), (x_3, 0.3)\}$ and $B = \{(x_2, 0.4), (x_3, 0.4), (x_4, 1)\}$ are two fuzzy sets ~~Subset~~ on $X$, Evaluate the distance between $A$ and $B$ by using max and min.

### Soluation: (5 Marks)

Distance between fuzzy sets A and B

| d | $x_A$ | $x_B$ | $m_A(x_i)$ | $m_B(x_i)$ | $\min(m_A(x_i), m_B(x_i))$ | $\max[\min(m_A(x_i), m_B(x_i))]$ |
|---|---|---|---|---|---|---|
| 0 | $x_2$ | $x_2$ | 0.1 | 0.4 | 0.1 | 0.3 |
| | $x_3$ | $x_3$ | 0.3 | 0.4 | 0.3 | — |
| 1 | $x_1$ | $x_2$ | 0.1 | 0.4 | 0.1 | 0.3 |
| | $x_2$ | $x_3$ | 0.1 | 0.4 | 0.1 | — |
| | $x_3$ | $x_2$ | 0.3 | 0.4 | 0.3 | — |
| | $x_3$ | $x_4$ | 0.3 | 1 | 0.3 | — |
| 2 | $x_1$ | $x_3$ | 0.1 | 0.4 | 0.1 | 0.1 |
| | $x_2$ | $x_4$ | 0.1 | 1 | 0.1 | — |

Therefore $d(A, B) = 0.3 + 0.3 + 0.1 = 1.0$ Ans

## Question #43

If $x \Delta y = \min(x, y) = x \wedge y$ then find $(x \Rightarrow y)$.

### Solution: (5 Marks)

We know that

$$(x \Rightarrow y) = \begin{cases} \max\{z \in [0,1] : x \Delta z \leq y\} \end{cases}$$

If $x \Delta y = \min(x, y)$

$$(x \Rightarrow y) = \max\{z \in [0,1] : \min(x, z) \leq y\}$$

$$(x \Rightarrow y) = \begin{cases} 1 & ; x \le y \\ y & ; x > y \end{cases}$$

## Question# 44

Define Similarity Measures based on distance.

Solution: (2 or 3 Marks)

Let S be a similarity Measures between two fuzzy sets is calculated as:

Suppose that A and B two fuzzy sets then the similarity measures S(A,B) based on distance d(A,B) is written as:

$$S(A,B) = 1 - d_h(A,B)$$

or $S(A,B) = 1 - d_e(A,B)$

or $S(A,B) = 1 - d_h^n(A,B)$

or $S(A,B) = 1 - d_e^n(A,B)$

Similarity Measures satisfied the condition.

$$0 \le S(A,B) \le 1$$

Solved By Mahar Afaq Safdar Muhammadi

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## Question# 45

$H_1(x) = \{0.125, 0.035, 0.245, 0.063\}$ and $H_2(x) = \{0.225, 0.365, 0.415\}$ be two HFSs then Compute (i) $H_1 \cup H_2$ (ii) $H_1 \cap H_2$ **(5 Marks)**

## Solution:

$$H_1 \cup H_2 \geq \max\{\min\{0.125, 0.035, 0.245, 0.063\}, \min\{0.225, 0.365, 0.415\}\}$$
$$\geq \max\{0.035, 0.225\}$$
$$\geq 0.225$$
$$= \{0.225, 0.245, 0.365, 0.415\}$$

$$H_1 \cap H_2 \leq \min\{\max\{0.125, 0.035, 0.245, 0.063\}, \max\{0.225, 0.365, 0.415\}\}$$
$$\leq \min\{0.245, 0.415\}$$
$$\leq 0.245$$
$$= \{0.035, 0.063, 0.125, 0.225, 0.245\}$$

## Question# 46

Define Normal Fuzzy Set. **(2 Marks)**

## Solution:

A fuzzy set is said to be normal fuzzy set if there exist atleast one element whose membership is one.

For example $A = \{(x_1, 0), (x_2, 0.1), \boxed{(x_3, 1)}, (x_4, 0.5)\}$

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## Question # 47

Define Empty Fuzzy Set.

### Solution: (2 Marks)

A fuzzy set is said to be empty fuzzy set if all elements of universal set maps on zero. i.e. the membership value of all elements are zero.

For Example: $A = \{(x_1, 0), (x_2, 0), (x_3, 0)\}$

## Question # 48

Name of any five similarity measure. (5 Marks)

### Solution:

(i) Similarity measures based on distance.
(ii) Cosine similarity measures
(iii) Exponential similarity measures
(iv) Jaccart and Dice Similarity measures.
(v) William and Steel similarity measures.

## Question # 49

List the most used distance formulae b/w FSs. (2 marks)

### Solution:

(i) Hamming distance. $d_h(A,B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$
(ii) Euclidean distance. $de(A,B) = \sqrt{\sum_{i=1}^{n} (m_A(x_i) - m_B(x_i))}$
(iii) Normalized Hamming distance $d_h^n(A,B) = \frac{1}{n} \cdot \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$
(iv) Normalized Euclidean distance $de^n(A,B) = \frac{1}{n}\sqrt{\sum_{i=1}^{n} (m_A(x_i) - m_B(x_i))}$

# Question#50

Writen in matrix form.

$$R=\left\{\frac{(x_1,y_1)}{0.3},\frac{(x_1,y_2)}{0.7},\frac{(x_1,y_3)}{0.4},\frac{(x_2,y_1)}{0.1},\frac{(x_2,y_2)}{0.5},\frac{(x_2,y_3)}{0.9}\right\}$$

## Solution:

(3 Marks)

$$R=\begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.3 & 0.7 & 0.4 \\ 0.1 & 0.5 & 0.9 \end{bmatrix} \end{array}$$