

QNO:- 01

The intersection of some subgroups H_i of a group G for $i \in I$ is again a subgroup of G .

Solution:-

Let's show closure property. Let $a \in \bigcap_{i \in I} H_i$ and

$b \in \bigcap_{i \in I} H_i$, so that $a \in H_i \forall i \in I$ and $b \in H_i \forall i \in I$.

Then $ab \in H_i \forall i \in I$, since H_i is a group. Thus,

$ab \in \bigcap_{i \in I} H_i$. Since H_i is a subgroup for all $i \in I$ and hence

$a^{-1} \in \bigcap_{i \in I} H_i$. Finally, for $a \in \bigcap_{i \in I} H_i$, we have $a \in H_i \forall i \in I$. So, $a^{-1} \in H_i$

$\forall i \in I$ which implies that $a^{-1} \in \bigcap_{i \in I} H_i$. Let G be a group and let

$a_i \in G$ for $i \in I$. There is at least one subgroup of G containing all

the elements a_i for $i \in I$, namely G is itself. The above statement

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assures that if we take the intersection of all subgroups of G containing all a_i for $i \in I$, we will obtain a subgroup H of G . This subgroup H is the smallest subgroup of G containing all the a_i for $i \in I$.

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Let G be a group. Then the commutator subgroup C of G is a normal subgroup of G .

Solution:-

We must show that C is normal in G .

We also know that C consists of all finite product of the commutators. For $x \in C$, we must show that $g^{-1}xg \in C \forall g \in G$, or that if x is a product of the commutators, so is $g^{-1}xg \forall g \in G$. By inserting $e = gg^{-1}$ between each product of commutators occurring in x , we see that it is sufficient to show for each commutator $cdc^{-1}d^{-1}$ that $g^{-1}(cdc^{-1}d^{-1})g$ is in C . But,

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$$\begin{aligned} g^{-1}(cdc^{-1}d^{-1})g &= (g^{-1}cdc^{-1})(e)(d^{-1}g) \\ &= (g^{-1}cdc^{-1})(gd^{-1}dg^{-1})(d^{-1}g) \\ &= [(g^{-1}c)d(c^{-1}g)^{-1}][dg^{-1}d^{-1}g], \end{aligned}$$

which is in C . Thus C is normal in G .

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Q.No:- 03*

If N is a normal subgroup of G , then G/N is abelian if and only if $C \leq N$.

Solution:-

If N is a normal subgroup of G and G/N is abelian, then $(a^{-1}N)(b^{-1}N) = (b^{-1}N)(a^{-1}N)$; that is, $aba^{-1}b^{-1}N = N$,

so $aba^{-1}b^{-1} \in N$ and $C \leq N$. Finally, if $C \leq N$, then

$$(aN)(bN) = abN = ab(b^{-1}a^{-1}ba)N = (abb^{-1}a^{-1})baN = baN = (bN)(aN).$$

Q No:- 04 *

Let G be a finite abelian group of order n , and let m be a positive real integer relatively prime to n . Then the mapping $\phi: x \rightarrow x^m$ is an automorphism of G .

Solution:-

$(m, n) = 1 \Rightarrow$ there exists integers u and v such that

$$mu + nv = 1 \Rightarrow \forall x \in G,$$

$$x^{mu + nv} = x^{mu} \cdot x^{nv} = x^{um}, \text{ since } o(G) = n.$$

Now for all $x \in G$, $x = (x^u)^m$ implies that ϕ is onto.

Further $x^m = e \Rightarrow x^{mu} = e \Rightarrow x = e$, showing that ϕ is 1-1.

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That ϕ is a homo-morphism follows from
the fact that G is abelian. Hence,
 ϕ is an automorphism of G .