MTH 645 FINAL SLIDES (23-45)

Example 3 : $x \Delta y = xy$, $f(x) = e^{-\left(\frac{1-x}{x}\right)}$

$x \Delta_f y = f^{-1}\left(\left(f(x) \Delta f(y)\right) \vee a\right)$ $\qquad x \Delta_f y = ?$

Let $y = e^{-\left(\frac{1-x}{x}\right)}$

$\ln y = -\left(\frac{1-x}{x}\right)$

$x \ln y = x - 1$

$1 = x - x \ln y$

$1 = x(1 - \ln y)$

$f^{-1}(y) = x = \frac{1}{1-\ln y}$

$f^{-1}(x) = \frac{1}{1-\ln x}$

Lecture 23

$x \Delta_f y = f^{-1}\left[\left(f(x) \Delta f(y)\right) \vee a\right]$

$= f^{-1}\left[\left(f(x) . f(y)\right) \vee a\right]$

$= f^{-1}\left[f(x) f(y) \vee 0\right]$

$f(0) = e^{-\left(\frac{1-0}{0}\right)} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$f(0) = 0$

$\Rightarrow a = 0$

Lecture 23

Example 4: $f(x) = e^{x-1}$, $x \Delta y = xy$, $x \Delta_f y = ?$

Sol: Let $y = e^{x-1}$

$$\ln y = x - 1$$

$$f^{-1}(y) = x = 1 + \ln y$$

$$\Rightarrow f^{-1}(x) = 1 + \ln x$$

$$f(0) = e^{0-1} = e^{-1} = \frac{1}{e}$$

$$\rightarrow a = \frac{1}{e}$$

Lecture 24

$$x \Delta_f y = f^{-1}\left(\left(f(x) \Delta f(y)\right) \vee a\right)$$

$$= f^{-1}\left(\left(f(x) f(y)\right) \vee a\right)$$

Put $a = \frac{1}{e}$

$$= f^{-1}\left(\left(f(x) f(y)\right) \vee \frac{1}{e}\right)$$

$$= f^{-1}\left(\left(e^{x-1} \cdot e^{y-1}\right) \vee \frac{1}{e}\right)$$

$$= f^{-1}\left(e^{x+y-2} \vee \frac{1}{e}\right)$$

Lecture 24

$$x \Delta_f y = f^{-1}\left(e^{x+y-2} \vee e^{-1}\right)$$

$$= 1 + \ln\left(e^{x+y-2} \vee e^{-1}\right)$$

$$= 1 + \left(\ln\left(e^{x+y-2}\right) \vee \ln e^{-1}\right)$$

$$= 1 + \left(x+y-2 \vee -1\right)$$

$$= 1 + x + y - 2 \vee 1 - 1$$

$$= x + y - 1 \vee 0)$$

Lecture 25

Nilpotent t-norm: Δ is nilpotent if for $b \neq 1$, $b^{[n]} = 0$, for some +ve integer n, where n depends on b.

Strict t-norm: Δ is strict if for $b \neq 0$, $b^{[n]} > 0$, for some +ve integer n.

e.g. $b^{[3]} = b \Delta b \Delta b$

Exercise: Prove that Δ_1 is nilpotent t-norm.

Sol: $x \Delta_1 y = \max(x+y-1, 0)$

Let $x \neq 1$

$\Rightarrow x \Delta_1 x = \max(x+x-1, 0) = \max(2x-1, 0)$

if $x \le \frac{1}{2} \Rightarrow x \Delta_1 x = 0$

if $1 > x > 1/2 \Rightarrow x > 2x-1$

$x \Delta_1 x \Delta_1 x = \max(\max(2x-1, 0) + x - 1, 0)$

$= \max(2x-1+x-1, 0) =$

Lecture 25

Exercise: Prove that Δ_1 is nilpotent t-norm.

Sol: $x \Delta_1 y = \max(x+y-1, 0)$

Let $x \neq 1$

$\Rightarrow x \Delta_1 x = \max(x+x-1, 0) = \max(2x-1, 0)$

if $x \le \frac{1}{2} \Rightarrow x \Delta_1 x = 0$

if $1 > x > 1/2 \Rightarrow x > 2x-1$

$x \Delta_1 x \Delta_1 x = \max(\max(2x-1, 0) + x - 1, 0)$

$= \max(2x-1+x-1, 0) =$

Lecture 25

Lecture 25

Ex: Prove that Δ_0 is nilpotent t-norm.

Sol:

$$x \Delta_0 y = \begin{cases} \min(x,y) ; & \text{if } \max(x,y)=1 \\ 0 ; & \text{otherwise} \end{cases}$$

Let $\max(x,y)=1$

$\Rightarrow x \neq 1 \Rightarrow \max(x,x) \neq 1$

$\Rightarrow x \Delta_0 x = 0$

$\Rightarrow x^{(2)} = 0 \Rightarrow \Delta_0$ is nilp... t-norm.

Lecture 26

Strictly monotone t-norm:

t-norm Δ is called strictly monotone t-norm if $x \Delta y < x \Delta z$ whenever $x > 0$ and $y < z$.

Canellation Law in t-norm:

1. If $x \Delta y = x \Delta z \Rightarrow x = 0$ or $y = z$.
2. If $x \Delta y = x \Delta z$ and $x = 0 \Rightarrow y = z$.

Theorem = $\triangle$ is strictly monotone iff it satisfies the cancellation law.

Proof = Let $\triangle$ is strictly monotone and we show that cancellation law holds.

Take $x \triangle y = x \triangle z$ —— (1)

Suppose on contrary $x \neq 0$ and $y \neq z$

$\Rightarrow x > 0$, let $y < z$

$\Rightarrow x \triangle y < x \triangle z$ —— (2)

(1), (2) $\Rightarrow x \triangle y \lneq x \triangle z$.

$\Rightarrow$ If $\triangle$ is strictly monotone then cancellation law holds.

Conversely if it satisfies cancellation law then it is strictly monotone.

We have to show that

If $x \triangle y < x \triangle z$ whenever $x > 0$ and $y < z$.

Let $\triangle$ is not strictly monotone

$x \triangle y \geq x \triangle z$, whenever $x > 0$ and $y < z$

Take $x \leq x$ and $y < z$

$$\Rightarrow x \triangle y \leq x \triangle z \quad \text{—— (2)}$$

(1) . (2) $\Rightarrow$ $x \triangle y = x \triangle z$

$\Rightarrow$ Contradiction to cancellation

Lecture 26

(1) Reflexive Relation : A fuzzy binary relation R is Reflexive if $R(x, x) = 1$

(2) Symmetric Relation : A fuzzy binary relation R is symmetric if $R(x, y) = 1 \Rightarrow R(y, x) = 1$

(3) Transitive Relation : A fuzzy binary relation R is transitive if

$$R(x, y) = 1,\ R(y, z) = 1 \Rightarrow R(x, z) = 1$$

$$x, y, z \in F$$

Lecture 27

$\Rightarrow$ R is equivalence relation if R is reflexive, symmetric

Antisymmetric Relation = R is Antisymmetric binary relation if $R(x,y) = R(y,x) = 1 \Rightarrow x = y$

(1) Reflexive relation = R is reflexive if $R(x,x) = 1$, $R(u,u) = 1$

(2) Symmetric relation = R is symmetric if $R(u,v) = R(v,u)$

(3) Transitive relation = R is transitive if $R(u,w) \geq R(u,v) \wedge R(v,w)$

$R(u,w) \geq R(u,v) \wedge R(v,w)$

$\Rightarrow$ Equivalence

Anti-symmetric = if $R(u,v) > 0$ and $R(v,u) > 0$ $\Rightarrow u = v$

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α-cut in relation =

$$R_\alpha = R^{-1}([\alpha, 1]) = \{(u,v) : R(u,v) \geq \alpha\}$$

$\alpha = 0.5$ $R = \{((x_1, y_1), 0.3), ((x_1, y_2), 0.7), ((x_2, y_1), 0.9)\}$

$$R_\alpha = R_{0.5} = \{((x_1, y_2), 0.7), ((x_2, y_1), 0.9)\}$$

Composition of Relations = Let R and S be fuzzy relations in $U \times V$ and $V \times W$ respectively and let Δ be a t-norm. The composition $R \circ S$ of R and S with respect to Δ is the fuzzy relation on $U \times W$ with membership function

$$(R \circ S)(u,w) = \bigvee_{v \in V} \{ R(u,v) \,\Delta\, S(v,w) \}$$

Lecture 28

$$x \,\Delta\, y = x \wedge y$$

$$x \,\Delta\, y = xy$$

Example - Find the max-min and max-product composition of R and S.

Solution.

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{bmatrix} y_1 & y_2 & y_3 \\ 0.1 & 0.0 & 0.6 \\ 0.2 & 0.3 & 1 \\ 0 & 0.4 & 0.5 \end{bmatrix} \quad \text{and} \quad S = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \end{array} \begin{bmatrix} s_1 & s_2 & s_1 \\ 0.2 & 0.5 & 0 \\ 0.9 & 1 & 0.3 \\ 1 & 0.5 & [illegible] \end{bmatrix}$$

max-min Composition =

$$R \circ S = \max \begin{cases} [\min(0.1, 0.2), \min(0.0, 0.9), \min(0.6, 1)] \; [\min(0.1, 0.5), \wedge(0, 1), \wedge \ldots \\ [\min(0.2, 0.2), \min(0.3, 0.9), \min(1, 1)] \; - \_\_ - \\ \end{cases}$$

Lecture 28

$$R \circ S = \max \begin{bmatrix} (0.1, 0, 0.6) & (0.1, 0, 0.5) & (0, 0, 0.4) \\ (0.2, 0.3, 1) & (0.2, 0.3, 0.5) & (0, 0.3, 0.4) \\ (0, 0.4, 0.5) & (0, 0.4, 0.5) & (0, 0.3, 0.4) \end{bmatrix}$$

$$R = \begin{bmatrix} 0.1 & 0 & 0.6 \\ 0.2 & 0.3 & 1 \\ 0 & 0.4 & 0.5 \end{bmatrix}$$

$$R \circ S = \begin{bmatrix} \max(0.1, 0, 0.6) & \max(0.1, 0, 0.5) & \max(0, 0, 0.4) \\ \max(0.2, 0.3, 1) & \max(0.2, 0.3, 0.5) & \max(0, 0.3, 0.4) \\ \max(0, 0.4, 0.5) & \max(0, 0.4, 0.5) & \max(0, 0.3, 0.4) \end{bmatrix}$$

$$S = \begin{bmatrix} 0.2 & 0.5 & 0 \\ 0.9 & 1 & 0.3 \\ 1 & 0.5 & 0.4 \end{bmatrix}$$

$$R \circ S = \begin{bmatrix} 0.6 & 0.5 & 0.4 \\ 1 & 0.5 & 0.4 \\ 0.5 & 0.5 & 0.4 \end{bmatrix}$$

Lecture 28

Fuzzy Implication: A fuzzy implication is a map

$$\Rightarrow : [0,1]^2 \longrightarrow [0,1]$$

satisfying

$\Rightarrow$	1	0
1	1	0
0	1	1

$(1 \Rightarrow 1) = 1$

$(0 \Rightarrow 1) = 1$

$(0 \Rightarrow 0) = 1$

$(1 \Rightarrow 0) = 0$

Lecture 29

R-Implication = An R-implication is a map

$$\Rightarrow : [0,1]^2 \longrightarrow [0,1] \text{ such that}$$

$$(x \Rightarrow y) = \max\{z \in [0,1] : x \triangle z \leq y\}$$

$$= \vee\{z \in [0,1] : x \triangle z \leq y\}$$

$$(1 \Rightarrow 1) = \max\{z \in [0,1] : 1 \triangle z \leq 1\}$$

Lecture 29

$$= \max\{z \in [0,1] : z \leq 1\}$$

$$(1 \Rightarrow 0) = \max\{z \in [0,1] : 1 \triangle z \leq 0\}$$

$$= \max\{0\} = 0 \Rightarrow (1 \Rightarrow 0) = 0$$

$$(0 \Rightarrow 0) = \max\{z \in [0,1] : 0 \triangle z \leq 0\}$$

$$= \max\{z \in [0,1]\} = 1 \Rightarrow (0 \Rightarrow 0) = 1$$

$$(0 \Rightarrow 1) = \max\{z \in [0,1] : \underline{0 \triangle z} \leq 1\}$$

Lecture 29

$$= 1 \Rightarrow (0 \Rightarrow 1) = 1$$

∇-implication: A ∇-implication is a map

$\Rightarrow [0,1]^2 \to [0,1]$ of the form

$$(x \Rightarrow y) = \eta(x) \nabla y \quad ; \quad \eta(x) = 1 - x$$

∇ is t-conorm

Lecture 30

0:01:47 0:11:48

Example 1: $x \nabla y = \max(x,y) = \vee(x,y) = x \vee y$

$$(x \Rightarrow y) = \eta(x) \nabla y$$
$$= (1-x) \nabla y = \max(1-x, y)$$

Example 2: $x \nabla y = x + y - xy$

$$(x \Rightarrow y) = \eta(x) \nabla y = (1-x) \nabla y$$
$$= 1 - x + y - (1-x)y = 1 - x + y - y + xy$$

Lecture 30

Example 3: $x \nabla y = 1 \wedge (x+y)$

$$(x \Rightarrow y) = \eta(x) \nabla y = (1-x) \nabla y$$
$$= 1 \wedge (1-x+y)$$
$$= \min(1, 1-x+y)$$

$$(0 \Rightarrow 0) = 1 = (1 \Rightarrow 1) = (0 \Rightarrow 1)$$
$$(1 \Rightarrow 0) = \eta(x) \nabla y = (1-1) \nabla 0 = 0 \nabla 0 = 0$$

Lecture 30

Lecture#31

Distance Between Fuzzy Sets

Definition

Let $X = \{x_1, x_2, ..., x_n\}$ be a discourse set, the fuzzy sets (FSs) A on X is represented in terms of a functions $m : X \to [0, 1]$ such as

$$A = \{\langle x_i, m_A(x_i)\rangle \,| x_i \in X\},$$

where $m_A(x_i)$ is called a membership function and always bounded in the closed interval $[0, 1]$

Distance Between Fuzzy Sets

Let A and B be two FSs on $X = \{x_1, x_2, ..., x_n\}$, then the most widely used distances for FSs are:

(1) Euclidean distance $d_e(A, B)$

$$d_e(A, B) = \sqrt{\sum_{i=1}^{n}(m_A(x_i) - m_B(x_i))^2}$$

(2) Normalized Euclidean distance $d_e^n(A, B)$

$$d_e^n(A, B) = \frac{1}{\ldots}\sqrt{\sum^{n}(m_A(x_i) - m_B(x_i))^2}$$

(3) Hamming distance

$$d_h(A, B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

(4) Normalized Hamming distance $d^n(A, B)$

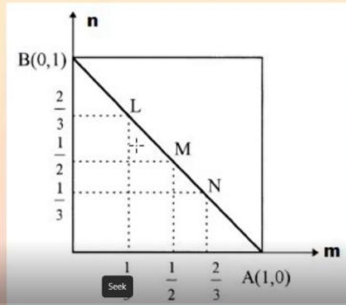
$$d_h(A, B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

(4) Normalized Hamming distance $d^n(A, B)$

$$d^n(A, B) = \frac{1}{n}\sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

Lecture#32

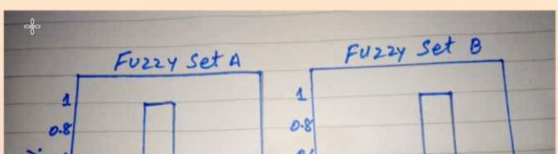
Suppose that $A = (1, 0)$; $B = (0, 1)$; $L = (1, \frac{1}{3})$; $M = (1, \frac{1}{2})$; $N = (1, \frac{2}{3})$ are FSs on $X = \{1\}$, then the geometrical representation of these sets is shown in the diagram below:



Example 2.

Example 2. Find and plot the Hamming distance $d_h(A, B)$ between tow fuzzy sets A and B on the discourse set $X = \{1, 2, 3, 4\}$, such that $A = \{(1, 0.5), (2, 1), (3, 0.3)\}$ and $B = \{(2, 0.4), (3, 0.4), (4, 1)\}$.

Figure: Fuzzy sets A and B



Example 2.

First we find the difference between generating values, that is, $|1-2|=1, |1-3|=2, |1-4|=3, |2-2|=0, |2-3|=1, |2-4|=2, |3-2|=1, |3-3|=0$ and $|3-4|=1$, we get the generating values $(0, 1, 2, 3)$.

Now the Hamming distance with generating value 0 is evaluated as: $d_h(2,2)=|1-0.4|=0.6$ and $d_h(3,3)=|0.3-0.4|=0.1$.

The Hamming distance with generating value 1 is evaluated as: $d_h(1,2)=|0.5-0.4|=0.1$, $d_h(2,3)=|1-0.4|=0.6$ and $d_h(3,4)=|0.3-1|=0.7$,

The Hamming distance with generating value 2 is evaluated as: $d_h(1,3)=|0.5-0.4|=0.1$, $d_h(2,0.5)=0.1$.

Lecture#33

Distance between two fuzzy sets by using max and min operations

$$d(A,B)=\max[\min(m_A(x_i), m_B(x_i))]$$

Consider the $X=\{1,2,3,4\}$, such that $A=\{(1,0.5),(2,1),(3,0.3)\}$ and $B=\{(2,0.4),(3,0.4),(4,1)\}$.

Table: Distance between fuzzy sets A and B.

d	x_A	x_B	$m_A(x_i)$	$m_B(x_i)$	$\min(m_A(x_i), m_B(x_i))$	$\max[\min(m_A(x_i), m_B(x_i))]$
0	2	2	1.0	0.4	0.4	0.4
	3	3	0.3	0.4	0.3	-
1	1	2	0.5	0.4	0.4	
	2	3	1.0	0.4	0.4	0.4
	3	2	0.3	0.4	0.3	-
	3	4	0.3	1.0	0.3	

$$d(A,B) = \max[\min(m_A(x_i), m_B(x_i))]$$

Consider the $X = \{1, 2, 3, 4\}$, such that $A = \{(1, 0.5), (2, 1), (3, 0.3)\}$ and $B = \{(2, 0.4), (3, 0.4), (4, 1)\}$.

Table: Distance between fuzzy sets A and B.

d	x_A	x_B	$m_A(x_i)$	$m_B(x_i)$	$\min(m_A(x_i), m_B(x_i))$	$\max[\min(m_A(x_i), m_B(x_i))]$
0	2	2	1.0	0.4	0.4	0.4
	3	3	0.3	0.4	0.3	-
1	1	2	0.5	0.4	0.4	
	2	3	1.0	0.4	0.4	0.4
	3	2	0.3	0.4	0.3	-
	3	4	0.3	1.0	0.3	-
2	1	3	0.5	0.4	0.4	1.0
	2	4	1.0	1.0	1.0	-

Therefore, $d(A,B) = 0.4 + 0.4 + 1.0 = 1.8$.

Lecture#34

Properties of Distance

Theorem

Let $X = \{x_1, x_2, ..., x_n\}$ be a universe set, then $d(A,B)$, the distance measure between two FSs A and B satisfy the following conditions:

(1) : $0 \le d(A,B) \le 1$;

(2) : $d(A,B) = 0$ *iff* $A = B$;

(3) : $d(A,B) = (B,A)$;

(4) : $d(A,C) \ge d(A,B)$ *and* $d(A,C) \ge d(B,C)$, *for any* $A, B, C \in FSs(X)$.

Distance between two fuzzy sets by using max and min operations

$$d(A,B) = \max[\min(m_A(x_i), m_B(x_i))]$$

Consider the $X = \{1,2,3,4\}$, such that
$A = \{(1,0.5),(2,1),(3,0.3)\}$ and $B = \{(2,0.4),(3,0.4),(4,1)\}$.

Table: Distance between fuzzy sets A and B.

d	x_A	x_B	$m_A(x_i)$	$m_B(x_i)$	$\min(m_A(x_i), m_B(x_i))$	$\max[\min(m_A(x_i), m_B(x_i))]$
0	2	2	1.0	0.4	0.4	0.4
	3	3	0.3	0.4	0.3	-
1	1	2	0.5	0.4	0.4	
	2	3	1.0	0.4	0.4	0.4
	3	2	0.3	0.4	0.3	-
	3	4	0.3	1.0	0.3	-
2	1	3	0.5	0.4	0.4	1.0
	2	4	1.0	1.0	1.0	-

Therefore, $d(A,B) = 0.4 + 0.4 + 1.0 = 1.8$.

Lecture#35

Example 1.

(3) Euclidean distance between M and N is:

$$d_e(M,N) = \sqrt{(\frac{1}{2} - \frac{2}{3}))^2} = \sqrt{(-\frac{1}{6})^2} = \frac{1}{6}$$

(4) Normalized Euclidean distance between M and N is:

$$d_e(M,N) = \frac{1}{1} \cdot \sqrt{(\frac{1}{2} - \frac{2}{3})^2} = \frac{1}{1} \cdot \sqrt{(-\frac{1}{6})^2} = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{6}$$

Lecture#36

Extensions of Fuzzy Sets

- Intuitionistic fuzzy sets (IFSs),
- Picture Fuzzy Sets (PFSs).
- Hesitant Fuzzy Sets (HFSs),

Intuitionistic fuzzy sets

An IFS A on X is given below:

$$A = \{< x_i, (\mu_A(x), \nu_A(x)) > | x_i \in X\},$$

where $\mu_A(x) : X \to [0,1]$; and $\nu_A(x) : X \to [0,1]$ satisfied the following condition, $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$.
The numbers $\mu_A(x), \nu_A(x) \in [0,1]$ denote the degree of membership and non-membership of x to A, respectively.

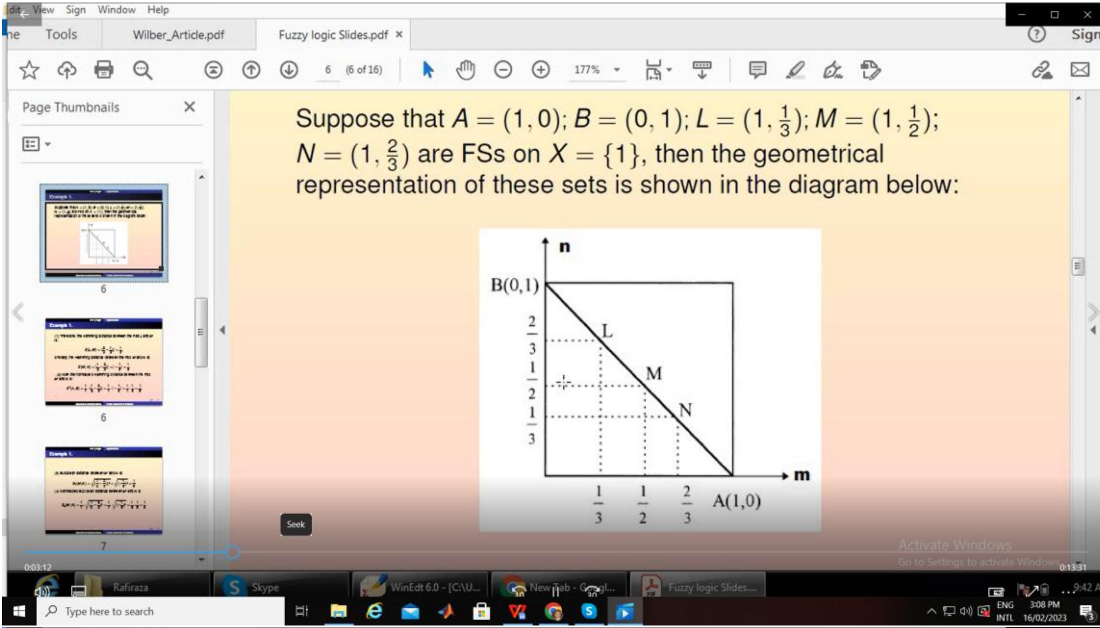
Some Operations on Intuitionistic fuzzy sets

Let $A(\mu_A(x), \nu_A(x))$ and $B(\mu_B(x), \nu_B(x))$ be two IFSs then,
1) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x) \forall x \in X$,
2) $A = B$ iff $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x) \forall x \in X$,
3)
$A \cup B = \{< x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) > | x \in X\}$,
4)
$A \cap B = \{< x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$,

Continued

5) $A \oplus B = \{<$
$x, \mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x), \nu_A(x) \times \nu_B(x)) > | x \in X\}$,
6) $A \otimes B = \{<$
$x, \mu_A(x) \times \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \times \nu_B(x) > | x \in X\}$,
7)
$A - B = \{< x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) > | x \in X\}$,
8) $A^c = \{< x, (\nu_A(x), \mu_A(x)) > | x \in X\}$.

Lecture#37



Tools
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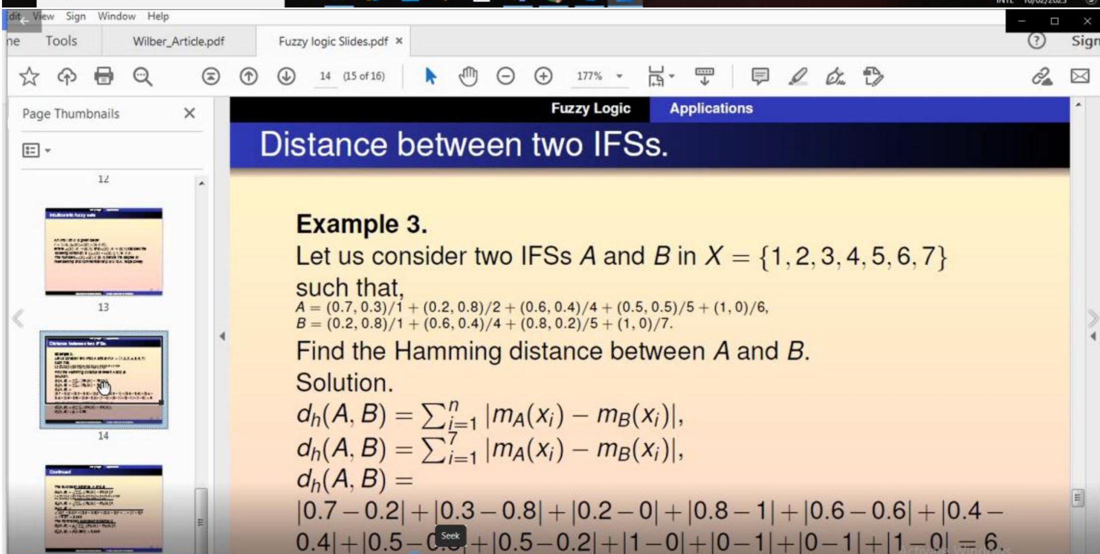
Suppose that $A = (1, 0)$; $B = (0, 1)$; $L = (1, \frac{1}{3})$; $M = (1, \frac{1}{2})$; $N = (1, \frac{2}{3})$ are FSs on $X = \{1\}$, then the geometrical representation of these sets is shown in the diagram below:
n
B(0,1)
L
M
N
m
A(1,0)
2/3
1/2
1/3
Seek



Tools
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Fuzzy Logic
Applications
Distance between two IFSs.
Example 3.
Let us consider two IFSs A and B in $X = \{1, 2, 3, 4, 5, 6, 7\}$ such that,
$A = (0.7, 0.3)/1 + (0.2, 0.8)/2 + (0.6, 0.4)/4 + (0.5, 0.5)/5 + (1, 0)/6$,
$B = (0.2, 0.8)/1 + (0.6, 0.4)/4 + (0.8, 0.2)/5 + (1, 0)/7$.
Find the Hamming distance between A and B.
Solution.
$d_h(A, B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$,
$d_h(A, B) = \sum_{i=1}^{7} |m_A(x_i) - m_B(x_i)|$,
$d_h(A, B) =$
$|0.7 - 0.2| + |0.3 - 0.8| + |0.2 - 0| + |0.8 - 1| + |0.6 - 0.6| + |0.4 -$
$0.4| + |0.5 - 0$ Seek $| + |0.5 - 0.2| + |1 - 0| + |0 - 1| + |0 - 1| + |1 - 0| = 6.$

Let us consider two IFSs A and B in $X = \{1, 2, 3, 4, 5, 6, 7\}$ such that,

$A = (0.7, 0.3)/1 + (0.2, 0.8)/2 + (0.6, 0.4)/4 + (0.5, 0.5)/5 + (1, 0)/6,$
$B = (0.2, 0.8)/1 + (0.6, 0.4)/4 + (0.8, 0.2)/5 + (1, 0)/7.$

Find the Hamming distance between A and B.

Solution.

$$d_h(A, B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|,$$
$$d_h(A, B) = \sum_{i=1}^{7} |m_A(x_i) - m_B(x_i)|,$$
$$d_h(A, B) = |0.7-0.2| + |0.3-0.8| + |0.2-0| + |0.8-1| + |0.6-0.6| + |0.4-0.4| + |0.5-0.8| + |0.5-0.2| + |1-0| + |0-1| + |0-1| + |1-0| = 6.$$

The normalized Hamming distance is

$$d_h^n(A, B) = \frac{1}{n}\sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|,$$
$$d_h^n(A, B) = \frac{6}{7} = 0.86.$$

Continued

The Euclidean distance A and B

$$d_e(A, B) = \sqrt{\sum_{i=1}^{n}(m_A(x_i) - m_B(x_i))^2}$$

$A = (0.7, 0.3)/1 + (0.2, 0.8)/2 + (0.6, 0.4)/4 + (0.5, 0.5)/5 + (1, 0)/6,$
$B = (0.2, 0.8)/1 + (0.6, 0.4)/4 + (0.8, 0.2)/5 + (1, 0)/7.$

$$d_e(A, B) = \sqrt{\sum_{i=1}^{7}(m_A(x_i) - m_B(x_i))^2},$$
$$d_e(A, B) = \sqrt{(0.7-0.2)^2 + (0.3-0.8)^2 + (0.2-0)^2 + \dots + (1-0)^2} = \sqrt{5.21} = 2.283.$$

The normalized Euclidean distance is

$$d_e^n(A, B) = \frac{1}{7}\sqrt{\sum_{i=1}^{n}(m_A(x_i) - m_B(x_i))^2},$$
$$d_e^n(A, B) = \frac{1}{7}(2.283) = 0.326$$

Lecture#38

Picture fuzzy sets

Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set, a picture fuzzy set P on X is defined as:

$$P = \{\langle x, \alpha_P(x), \gamma_P(x), \beta_P(x)\rangle \,|x \in X\},$$

where $\alpha_P(x)$, $\beta_P(x)$, $\gamma_P(x) \in [0, 1]$ are called the acceptance membership, neutral and rejection membership degrees of $x \in X$ to the set P, respectively and $\alpha_P(x)$, $\gamma_P(x)$ and $\beta_P(x)$ fulfil the condition: $0 \leq \alpha_P(x) + \gamma_P(x) + \beta_P(x) \leq 1$, for all $x \in X$. Also $\eta_P(x) = 1 - \alpha_P(x) - \gamma_P(x) - \beta_P(x)$, then $\eta_P(x)$ is said to be a degree of refusal membership of $x \in X$ in P. For our convenience, the picture fuzzy sets over a fixed set X is written as PFSs(X).

Some operations on PFSs

Let $\mathcal{A}$ and $\mathcal{B}$ be two PFSs on X, then the union, intersection and complement are described as follows:

(1) $\mathcal{A} \subseteq \mathcal{B}$ iff $\alpha_{\mathcal{A}}(x) \leq \alpha_{\mathcal{B}}(x)$, $\gamma_{\mathcal{A}}(x) \leq \gamma_{\mathcal{B}}(x)$ and $\beta_{\mathcal{A}}(x) \geq \beta_{\mathcal{B}}(x)$ such that for all $x \in X$;

(2) $\mathcal{A} = \mathcal{B}$ iff $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A}$;

(3) $\mathcal{A} \sqcup \mathcal{B} = \{x, \max(\alpha_{\mathcal{A}}(x), \alpha_{\mathcal{B}}(x)), \min(\gamma_{\mathcal{A}}(x), \gamma_{\mathcal{B}}(x)), \min(\beta_{\mathcal{A}}(x), \beta_{\mathcal{A}}(x))|x \in X\}$;

(4) $\mathcal{A} \sqcap \mathcal{B} = \{x, \min(\alpha_{\mathcal{A}}(x), \alpha_{\mathcal{B}}(x)), \min(\gamma_{\mathcal{A}}(x), \gamma_{\mathcal{B}}(x)), \max(\beta_{\mathcal{A}}(x), \beta_{\mathcal{A}}(x))|x \in X\}$;

(5) $\mathcal{A}^{\mathcal{C}} = \{\langle x, \beta_{\mathcal{A}}(x), \gamma_{\mathcal{A}}(x), \alpha_{\mathcal{A}}(x)\rangle \,|x \in X\}$.

Lecture#39

Let $P_1 = \{(x_1,(0.6,0.2,0.1)),(x_2,(0.4,0.1,0.4))\}$

$P_2 = \{(x_1,(0.7,0.3,0)),(x_2,(0.5,0.2,0.3))\}$

i) $P_1 \subseteq P_2 = ?$

$P_1 \subseteq P_2$, $\alpha_{P_1}(x) \le \alpha_{P_2}(x)$, $\gamma_{P_1}(x) \le \gamma_{P_2}(x)$, $\beta_{P_1}(x) \ge \beta_{P_2}(x)$

$\Rightarrow P_1 \subseteq P_2$.

$P_2 \supseteq P_1 = ?$, from the sets $P_2 \not\subseteq P_1$

Let $P_1 = \{(x_1,(0.6,0.2,0.1)),(x_2,(0.4,0.1,0.4))\}$

$P_2 = \{(x_1,(0.7,0.3,0)),(x_2,(0.5,0.2,0.3))\}$

i) $P_1 \subseteq P_2 = ?$

$P_1 \subseteq P_2$, $\alpha_{P_1}(x) \le \alpha_{P_2}(x)$, $\gamma_{P_1}(x) \le \gamma_{P_2}(x)$, $\beta_{P_1}(x) \ge \beta_{P_2}(x)$

$\Rightarrow P_1 \subseteq P_2$.

$P_2 \supseteq P_1 = ?$, from the sets $P_2 \not\subseteq P_1$

ii) $P_1 = P_2$ iff $P_1 \subseteq P_2$ and $P_2 \subseteq P_1 \Rightarrow P_1 \neq P_2$

Lecture#40

Let P_1 and P_2 be two PFSs s.t.

$$P_1 = \{(x_i, (\alpha_{P_1}(x_i), \gamma_{P_1}(x_i), \beta_{P_1}(x_i)))\}$$

$$P_2 = \{(x_i, (\alpha_{P_2}(x_i), \gamma_{P_2}(x_i), \beta_{P_2}(x_i)))\}$$

$$\Rightarrow \quad P_1 = (\alpha_{P_1}(x_i), \gamma_{P_1}(x_i), \beta_{P_1}(x_i))$$

$$P_2 = (\alpha_{P_2}(x_i), \gamma_{P_2}(x_i), \beta_{P_2}(x_i))$$

Example:- Let P_1 and P_2 be two PFSs defined on a $X = \{x_1, x_2, x_3, x_4\}$ s.t.

$$P_1 = \{(x_1, (0.56, 0.34, 0.1)), (x_2, (0.9, 0.07, 0.03)), (x_3, (0.4, 0.33, 0.19)), (x_4, (0.09, 0.79, 0.03))\}$$

$$P_2 = \{(x_1, (0.7, 0.1, 0.09)), (x_2, (0.1, 0.66, 0.2)), (x_3, (0.06, 0.81, 0.12)), (x_4, (0.72, 0.14, 0.09))\}$$

$$D_h^n(P_1, P_2) = ?$$

$$n = 4$$

$$D_h^n(P_1, P_2) = \frac{1}{4}\{|0.56-0.7| + |0.34-0.1| + |0.1-0.09| +$$
$$|0.9-0.1| + |0.07-0.66| + |0.03-0.2| + |0.4-0.06| +$$
$$|0.33-0.81| + |0.19-0.12| + |0.09-0.72| + |0.79-0.14|$$
$$+ |0.03-0.09|\}$$

$$= \frac{1}{4}[0.14 + 0.24 + 0.01 + 0.8 + 0.59 + 0.17 +$$
$$0.34 + 0.48 + 0.07 + 0.63 + 0.65 + 0.06]$$

$$= \frac{1}{4}[4.18] = 1.045$$ Ans

Lecture#41

Fuzzy sets

IFSs and PFSs

$$X = \{x_1, x_2, \ldots, x_n\}$$

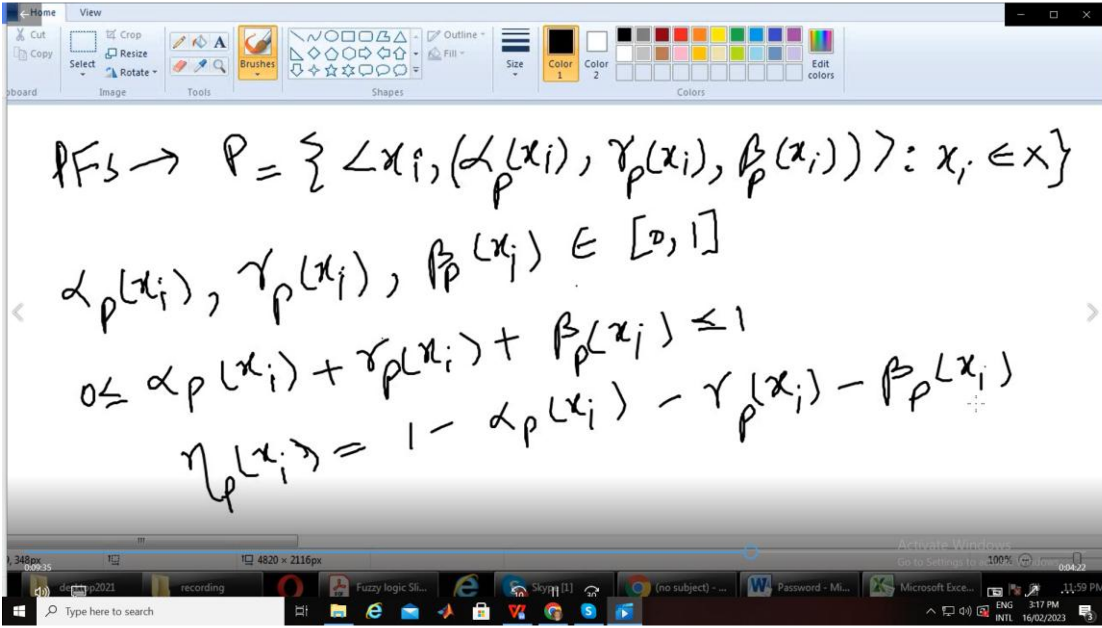
$$A = \{(x_i, \mu_A(x_i)) : x_i \in X\}$$

$$I = \{(\mu_I(x_i), \nu_I(x_i)) : x_i \in X\}$$

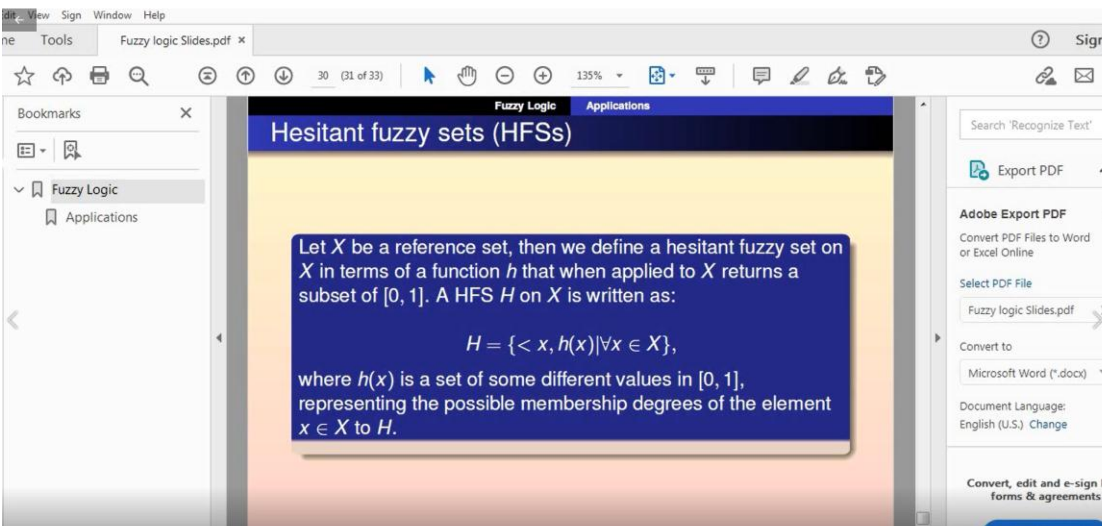
$\mu_I(x_i) \rightarrow$ Membership degree

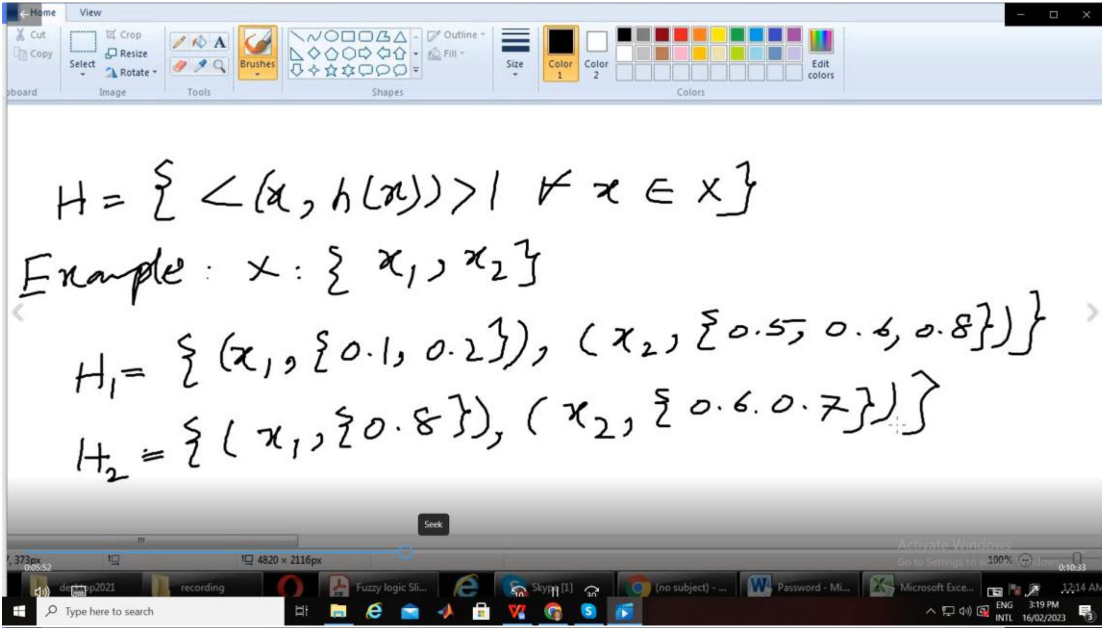
$\nu_I(x_i) \rightarrow$ Non membership degree

$$0 \leq \mu_I(x) + \nu_I(x) \leq 1$$

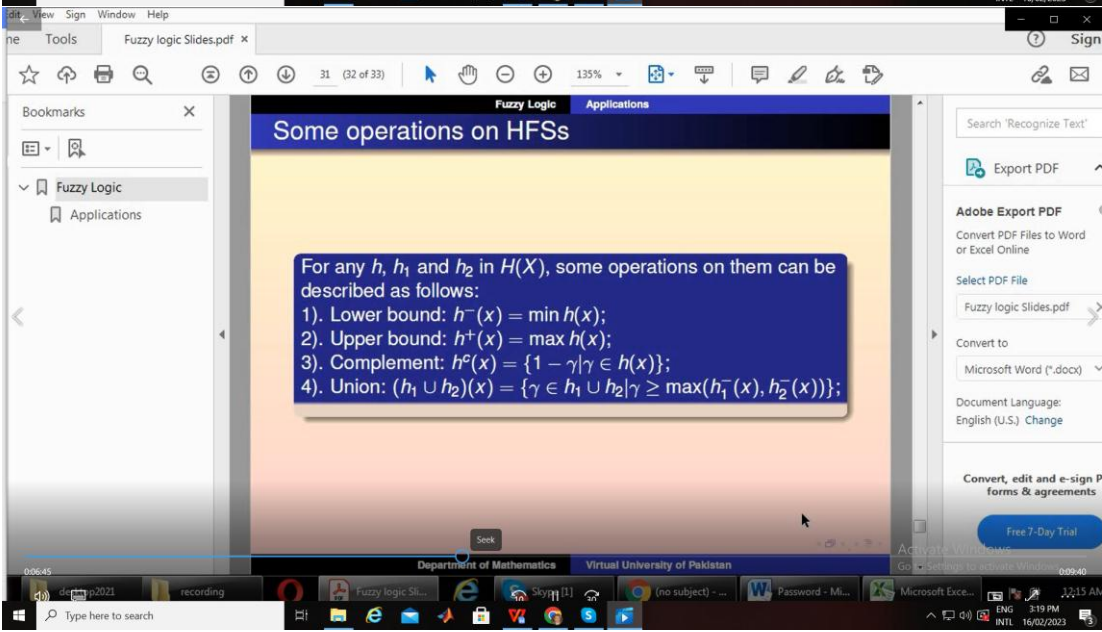

IFS → P = { ⟨xi, (αp(xi), γp(xi), βp(xi))⟩ : xi ∈ X }
αp(xi), γp(xi), βp(xi) ∈ [0, 1]
0 ≤ αp(xi) + γp(xi) + βp(xi) ≤ 1
ηp(xi) = 1 − αp(xi) − γp(xi) − βp(xi)

Lecture#42


Fuzzy Logic
Applications
Hesitant fuzzy sets (HFSs)
Let X be a reference set, then we define a hesitant fuzzy set on X in terms of a function h that when applied to X returns a subset of [0, 1]. A HFS H on X is written as:
H = {< x, h(x)|∀x ∈ X},
where h(x) is a set of some different values in [0, 1], representing the possible membership degrees of the element x ∈ X to H.



$H = \{ \langle (x, h(x)) \rangle \mid \forall x \in X \}$
Example: $X : \{ x_1, x_2 \}$
$H_1 = \{ (x_1, \{0.1, 0.2\}), (x_2, \{0.5, 0.6, 0.8\}) \}$
$H_2 = \{ (x_1, \{0.8\}), (x_2, \{0.6, 0.7\}) \}$



Fuzzy Logic
Applications
Some operations on HFSs
For any h, h_1 and h_2 in $H(X)$, some operations on them can be described as follows:
1). Lower bound: $h^-(x) = \min h(x)$;
2). Upper bound: $h^+(x) = \max h(x)$;
3). Complement: $h^c(x) = \{1 - \gamma | \gamma \in h(x)\}$;
4). Union: $(h_1 \cup h_2)(x) = \{\gamma \in h_1 \cup h_2 | \gamma \geq \max(h_1^-(x), h_2^-(x))\}$;
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Virtual University of Pakistan

Lecture#43

Example:- let $X=\{x_1,x_2,x_3\}$ be a universal set, h_1 and h_2 be two HFSs on X s.t

$$h_1=\{(x_1,\{0.3,0.4\}),(x_2,\{0.6,0.8\}),(x_3,\{0.3,0.4,0.5,0.7\})\}$$

$$h_2=\{x_1,\{0.5,0.6\}),(x_2,\{0.4,0.5\}),(x_3,\{0.2,0.3,0.4,0.6\})\}$$

Then find out the following operations:-

i) $h_1^-(x_3)=?$ $\quad h_1^-(x_3)=\min\{0.3,0.4,0.5,0.7]$

$=0.3$

ii) $h_2^+(x_2)=? \Rightarrow h_2^+(x_2)=\max\{0.4,0.5\}=0.5$

iii) $h_2^c(x_2)=? \Rightarrow \{1-\gamma \mid \gamma\in h_2\}$

$\Rightarrow \{1-0.4,1-0.5\}=\{0.6,0.5\}$

iv) $h_1^\lambda(x_2)=? \Rightarrow \{0.6^\lambda, 0.8^\lambda\}$, if $\lambda=2$

$=\{0.6^2, 0.8^2\}=\{0.36, 0.64\}$

$$\lambda h_1(x_1) = \{1-(1-0.3)^{\lambda}, 1-(1-0.4)^{\lambda}\} = \{1-0.7^2, 1-0.6^2\}$$

$$= \{1-0.49, 1-0.36\} = \{0.51, 0.64\}, \text{ if } \lambda = 2$$

vi) $(h_1 \oplus h_2)(x_2) = ?$

$$(h_1 \oplus h_2) = \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2 \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$$

$$\{0.6, 0.8\} \oplus \{0.4, 0.5\} = \{0.6+0.4-0.6\times0.4, 0.6+0.5-0.6\times0.5, 0.8+0.4-0.8\times0.4, 0.8+0.5-0.8\times0.5\} = \{0.76, 0.8, 0.88, 0.9\}$$

Lecture#44

HFE → hesitant fuzzy element

$$h_1 = \{x_1, \{0.2, 0.3\}\} \Longrightarrow h_1 = (0.2, 0.3)$$

Theorem: Suppose that h_1 and h_2 are two HFEs, then

$$L_{h_1 \oplus h_2} = L_{h_1} \times L_{h_2}$$ and

$$L_{h_1 \otimes h_2} = L_{h_1} \times L_{h_2}$$

Example: Let $h_1 = (0.1, 0.2, 0.7)$ and $h_2 = (0.2, 0.4)$ be two HFEs then

$$h_1 \oplus h_2 = \{0.1+0.2-0.1\times 0.2,\ 0.1+0.4-0.1\times 0.4,\ 0.2+0.2-0.2\times 0.2,\ 0.2+0.4-0.2\times 0.4,\ 0.7+0.2-0.7\times 0.2,\ 0.7+0.4-0.7\times 0.4\} = \{0.28, 0.36, 0.46, 0.52, 0.76, 0.82\}$$

$$h_1 \otimes h_2 = \{0.1\times 0.2,\ 0.1\times 0.4,\ 0.2\times 0.2,\ 0.2\times 0.4,\ 0.7\times 0.2,\ 0.7\times 0.4\}$$
$$= \{0.02, 0.04, 0.04, 0.08, 0.14, 0.28\}$$

$$\therefore L_{h_1 \oplus h_2} = 6 = 3\times 2 = L_{h_1} \times L_{h_2}$$

$$L_{h_1 \otimes h_2} = 6 = 3\times 2 = L_{h_1} \times L_{h_2}$$

Distance b/w HFSs

$$D_e(h_1, h_2) = \sqrt{\sum_{i=1}^{n} (h_{1i} - h_{2i})^2}$$

$$D_h(h_1, h_2) = \sum_{i=1}^{n} |h_{1i} - h_{2i}|$$

Example:- Let $h_1 = (0.7, 0.8, 0.9)$ and $h_2 = (0.5, 0.6, 0.8)$ be two HFEs. Then evaluate i) $D_h(h_1, h_2)$ ii) $D_e(h_1, h_2)$

Sol:- i) $D_h(h_1, h_2) = |0.7-0.5| + |0.8-0.6| + |0.9-0.8|$
$= |0.2| + |0.2| + |0.1| = 0.5$

ii) $D_e(h_1, h_2) = (0.7-0.5)^2 + (0.8-0.6)^2 + (0.9-0.8)^2$
$= (0.2)^2 + (0.2)^2 + (0.1)^2 = 0.04 + 0.04 + 0.01 = 0.09$

Example 2: Let $h_1 = (0.3, 0.4)$ and $h_2 = (0.6, 0.7, 0.8)$ be two HFEs. Then evaluate the normalized Hamming distance and Euclidean distance b/w h_1 and h_2.

Sol. $h_1 = (0.3, 0.3, 0.4)$ and $h_2 = (0.6, 0.7, 0.8)$

$$D_h^n(h_1, h_2) = \frac{1}{3}\left[|0.3-0.6| + |0.3-0.7| + |0.4-0.8|\right]$$

$$= \frac{1}{3}(0.3 + 0.4 + 0.4) = \frac{1}{3}(1.1) = 0.366$$

$$D_e^n(h_1, h_2) = \sqrt{\frac{1}{3}\left((0.3-0.6)^2 + (0.3-0.7)^2 + (0.4-0.8)^2\right)}$$

$$= \frac{1}{3}(0.09 + 0.16 + 0.16) = 0.13667$$

Lecture#45

Similarity Measures (SM)

SM is a process through which we can evaluate most resemblance b/w two objects.

There are many kinds of SMs, like

i) SM based on distance

ii) Cosine SMs iii) Exponential SM

iv) Jaccard and Dice SMs

v) William and Steel SMs, etc.

SMs based on distance

Let S be a SM between two FS is calculated as:-

Suppose that A and B be two FSs then the SM $S(A,B)$ based on distance is written as:

$d_n(A,B)$

$$S(A,B) = 1 - d_n(A,B)$$

$$\text{or} \quad = 1 - d_e(A,B)$$

$$\text{or} \quad = 1 - d_e^n(A,B)$$

$0 \le S(A,B) \le 1$

Theorem:- The SM, $S(A,B)$ b/w two FSs satisfied the following axioms:-

i) $0 \le S(A,B) \le 1$

ii) $S(A,B) = 1$ iff $A = B$

iii) $S(A,B) = S(B,A)$ i.e commutative.

best of luck to all

17-2-23