Date:

Mth-603

Q.1

State the sufficient condition of convergence of the iterative solution to the exact solution?

Ans:-

Mid-term Question

M-M

Q.2

Write a formula for finding the value of K_1 in fourth-order R-K method?

Ans:-

The fourth-order R-K method is described in,

$$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2(K_2 + K_3) + K_4)$$

The formula for finding the value of K_1 is

$$K_1 = hf(t_n, y_n)$$

Same as we can write the formula of K_2, K_3, K_4, which are

$$K_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{K_1}{2}\right) \text{ (imp)}$$

$$K_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{K_2}{2}\right)$$

$$K_4 = hf(t_n + h, y_n + K_3)$$

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Q.3

Write Adam-Moultan's Predictor formula for finding the solution of differential equation?

Ans:-

Adam-Moultan's Predictor formula for finding the solution of differential equation is.

M-m

$$\cancel{y_{n+1} = y_n + \frac{h}{24}[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y}$$

$$y_{n+1} = y_n + \frac{h}{24}\left[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right] + \frac{251}{720}h\nabla^4 y'_n$$

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Q.1

Evaluate the integral $\int_3^5 (\log x+1)\,dx$ using Trapezoidal rule Take $h=1$?

Sol:-

	y_0	y_1	y_2
X	3	4	5
$Y=\log(x+1)$	0.602	0.699	0.778

M-m

Given, $h=1$

by using trapezoidal rule

$$I = \frac{h}{2}(y_0+2y_1+y_2)$$

$$= \frac{1}{2}(0.602+2(0.699)+0.778)$$

$$= \frac{1}{2}(0.602+1.398+0.778)$$

$$= \frac{1}{2}(2.778)$$

$$= 1.389 \text{ Ans.}$$

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Q.4

Obtain numerically the solution of $y' = x^2 + 2x + y^2$, $y(0) = 1$ using Euler's method to find y at $x = 1$, $h = 1$

Sol:-

M.M

We Know,

$$y_{m+1} = y_m + hf(x_m, y_m)$$

Let $m = 0$

$$y_1 = y_0 + hf(x_0, y_0)$$

Given $h = 1$, $y(0) = 1 \Rightarrow y = 1$, $x = 0$

thus, the value of y will be

$$y = y_1 = 1 + 1\left[x^2 + 2x + y^2\right]$$

$$= 1 + 1\left[(0)^2 + 2(0) + (1)^2\right]$$

$$= 1 + 1(1)$$

$$= 2$$ Ans.

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Q-5

If, in solving a given differential eq. $y_0 = 1$, $y_1' = 1.1$, $y_2' = 1.2$, $y_3' = 1.3$, $h = 1$ then find y_4 by Milne's Predictor formula?

Sol:-

The Milne's Predictor formula for y_4 is

$$y_4 = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right) \quad \text{M.M}$$

$$= 1 + \frac{4(1)}{3}\left(2(1.1) - 1.2 + 2(1.3)\right)$$

$$= 1 + \frac{4}{3}(2.2 - 1.2 + 2.6)$$

$$= 1 + \frac{4}{3}(4.8 - 1.2)$$

$$= 1 + \frac{4}{3}(3.6)$$

$$= 1 + \frac{14.4}{3}$$

$$= 1 + 4.6$$

$$= 5.6 \quad \text{Ans}$$

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Q.6

Find F(h), using Richardson's extrapolation limit, while finding $y'(0.01)$ to the function $y = 1/x$ with $h = 0.005$?

Sol:-

M-M

~~We ta~~ Given $h = 0.005$

For obtaining F(h), we use formula

$$F(h) = \frac{y(x+h) - y(x-h)}{2h} = \frac{\frac{1}{x+h} - \frac{1}{x-h}}{2h}$$

$$= \frac{\frac{1}{0.01+0.005} - \frac{1}{0.01-0.005}}{2(0.005)}$$

$$= \frac{\frac{1}{0.015} - \frac{1}{0.005}}{0.01}$$

$$= \frac{66.67 - 200}{0.01}$$

$$= \frac{-133.33}{0.01} = -13333 \text{ Ans}$$

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Q.7

Evaluate the integral $\int_0^3 (x^2+x)\,dx$ using Simpson's 3/8 rule?

Sol:-

x	0	1	2	3
$y=(x^2+x)$	0	2	6	12

M.M

$h = x_1 - x_0 = 1-0 = 1$

by using simpson's $\frac{3}{8}$ rule

$$\int_0^3 (x^2+x)\,dx = \frac{3}{8}h\left[y_0+3(y_1+y_2)+y_3\right]$$

$$= \frac{3}{8}(1)\left[0+3(2+6)+12\right]$$

$$= \frac{3}{8}(24+12)$$

$$= \frac{3}{8}\times 36$$

$$= \frac{108}{8} = 13.5 \text{ Ans}$$

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Q.8

Construct a backward difference table from the following values of x and y

x	-1	0	1	2	3
$y=f(x)$	10	2	10	62	80

Sol:- M.M

X	$Y=f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
-1	10				
0	2	-8			
1	10	8	16		
2	62	52	44	28	
3	80	18	-34	-78	-106

نوٹ:

Backward difference ∇

Forward difference Δ

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Q.9

Use R-K method of order four to find the value of K_1, K_2, K_3, K_4 for the initial value problem $y' = \frac{1}{2}(2x^3+y)$, $y(1)=2$, $h=0.1$

Ans:-

M.M

by R-K method

$$K_1 = hf(x_1, y_1) = h\left[\frac{1}{2}(2x^3+y)\right]$$

Given $y(1)=2 \Rightarrow y=2,\ x=1,\ h=0.1$

$$K_1 = h\left\{\frac{1}{2}(2x^3+y)\right\} = 0.1\left[\frac{1}{2}(2(1)^3+2)\right]$$

$$= 0.1(2) = 0.2$$

Now,

$$K_2 = hf\left(x_1+\frac{h}{2}, y_1+\frac{K_1}{2}\right)$$

$$= 0.1\left(1+\frac{0.1}{2}, 2+\frac{0.2}{2}\right) = 0.1(1.05, 2.1)$$

$$= 0.1(1.05+2.1) = 0.315$$

Now,

$$K_3 = hf\left(x_1+\frac{h}{2}, y_1+\frac{K_2}{2}\right)$$

$$= 0.1\left(1+\frac{0.1}{2}, 2+\frac{0.315}{2}\right) = 0.1(1.05, 2.1575)$$

$$= 0.1(1.05+2.1575) = 0.32075$$

Now,

$$K_4 = hf(x_1+h, y_1+K_3)$$

$$= 0.1(1+0.1, 2+0.32075)$$

$$= 0.1(1.1, 2.32075)$$

$$= 0.1(1.1+2.32075) = 0.342075$$