MTH405 lecture 01

Numerical analysis (Virtual University of Pakistan)

Lec-1 Book-405

Group Theory

Out Lines

(i) Introduction → The Term group was used by Galois in 1830 in order to describe set of one-to-one functions on finite set under composition.
Group Theory is a study of symmetry.

(ii) set

(iii) Binary operation

IV Associative Law

V Group

VI Examples

Applications of Group Theory

(i) Group Theory has its applications in Physics chemistry, Material science etc.

(ii) Crystals and molecules of Hydrogen can be modeled with The Help of Group Theory

(iii) It is also used in cryptography.

Set

a set is a collection of objects. under a certain rule.

$N = \{1, 2, 3, \ldots\}$

$W = \{0, 1, 2, 3, \ldots\}$

$Z = \{0, \pm 1, \pm 2, \ldots\}$

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Binary operation:

Let 'X' be a non-empty set. The binary operation '$*$' is a function which assigns each ordered pair (a,b) of '$X * X$' unique element of X.

$$*: X * X \longrightarrow X \qquad *(+,-,\times,\div)$$

Example

Show that multiplication is a binary operation on the set of Natural numbers. N

Solution

$$N = \{1,2,3,\dots\} \quad ; \quad 3,5 \in N$$

$$3\times5 = 5\times3 = 15 \in N$$

Example

Show that division is not a binary operation on the set of Natural numbers N.

$$N = \{1,2,3,\dots\} \quad ; \quad 6,3 \in N$$

$$6 \div 3 = \frac{6}{3} = 2 \in N$$

$$3 \div 6 = \frac{\cancel{3}1}{\cancel{6}2} = \frac{1}{2} \notin N$$

Example

Show that subtraction is not a binary operation on Natural numbers N.

$$N = \{1,2,3,\dots\} \qquad 2,3 \in N$$

$$3-2 = 1 \in N$$

$$2-3 = -1 \notin N$$

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Example

Show That division is a binary operation on Real number $\mathbb{R}$.

$Q = \{x = \frac{p}{q}, p, q \in Z, q \neq 0\}$

$Q' = \{x \neq \frac{p}{q}, p, q \in Z, q \neq 0\}$

$\mathbb{R} = Q \cup Q'$; $\frac{2}{3}, \frac{5}{7} \in \mathbb{R}$

$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{2\times7}{3\times5} = \frac{14}{15} \in \mathbb{R}$

$\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2} = \frac{5\times3}{7\times2} = \frac{15}{14} \in \mathbb{R}$

Example

Show that the operation 'o' defined by (i) $x \circ y = 0$ for all $x, y \in Z$ is a binary operation on Integers Z

$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$3, -5 \in Z$ $(3)\circ(-5) = (-5)\circ(3) = 0 \in Z$

(ii) $x \circ y = x + y - xy$ for all $x, y \in Z$

$3, 5 \in Z$

$(3)\circ(5) = 3 + 5 - (3)(5)$

$= 8 - 15$

$= -7 \in Z$

$-7, 5$

$(-7)\circ(5) = -7 + 5 - (-7)(5)$

$= -2 + 35$

$= 33 \in Z$