Virtual University	**MIDTERM EXAMINATION** **SEMESTER SPRING 2005** **MTH601 – Operational Research**	**Total Marks: 50** **Duration:90mins**

StudentID/LoginID	
Name	
PVC Name/Code	
Date	

Maximum Time Allowed: ($1^{1/2}$ Hour)

Please read the following instructions carefully before attempting any of the questions:

1. Attempt all questions. Marks are written adjacent to each question

2. Paste the bitmap image for the tables, diagrams etc while solving your questions

3. Do not ask any questions about the contents of this examination from anyone

a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

c. Write all steps, missing steps may lead to deduction of marks.

4. The duration of this examination is **90 minutes**.

5. This examination is closed book, closed notes, closed neighbors.

6. Calculator is allowed

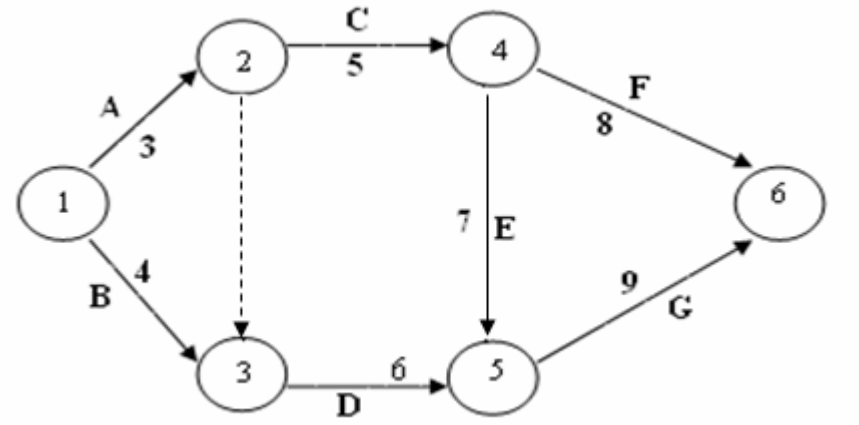
****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.**

For Teacher's use only

	Descriptive				Multiple Choice Questions										Total
Question	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	4+10=14
Marks															

Question No: 1 **Marks: 10**

For the following project diagram find out Earliest Start Time (**EST**), Earliest Finish Time (**EFT**), Latest Start Time (**LST**), Latest Finish Time (**LFT**) for each activity. Also find the **Critical path**.



NOTE: *There is no need of making diagram corresponds to each step, you have to write down only the required terms correspond to each activity and the justification for the critical path.*

Solution:

Activity	EST	EFT	LST	LFT
A(1 – 2)	0	3	0	3
B(1 – 3)	0	4	5	9
D1(2 – 3)	3	3	9	9
C(2 – 4)	3	8	3	8
D(3 – 5)	4	10	9	15
E(4 – 5)	8	15	8	15
F(4 – 6)	8	16	16	24
G(5 – 6)	15	24	15	24

To find the **Critical Path** we'll find the Slack (Float) of each activity.

Activity	Slack
A(1 – 2)	0
B(1 – 3)	5
D1(2 – 3)	6
C(2 – 4)	0
D(3 – 5)	5
E(4 – 5)	0
F(4 – 6)	8
G(5 – 6)	0

Critical Path: A – C – E – G

Question No: 2 **Marks: 10**

Suppose the following estimates of activity times of a project,

Activity	Optimistic	Most likely	Pessimistic
A	1	3	5
B	3	4	5
C	4	5	6
D	3	5	7
E	5	6	13
F	4	7	10
G	6	8	10

If the critical path of the project is B, D, E, F, then calculate the **Expected Completion Time** (Length of the path) and **Variance** for the project

Solution:

Critical Activities	Optimistic Time (t_0)	Most Likely Time (t_m)	Pessimistic Time (t_p)	Expected Duration (t_e)	Variance
B	3	4	5	4	0.11
D	3	5	7	5	0.44
E	5	6	13	7	1.78
F	4	7	10	7	1

Expected Completion time = 4 + 5 + 7 + 7
= 23

Variance (of the project) = 0.11 + 0.44 + 1.78 + 1
= 3.33

Question No: 3 **Marks: 10**

Demand for the product is 600 units per week, and the items are withdrawn at a constant rate. The setup cost for placing an order to replenish inventory is $25. The unit cost of each item is $3, and the inventory holding cost is $0.05 per item per week.

(a). Assuming shortages are not allowed, determine how often to make a production run and what size it should be.

(b). If shortages are allowed but cost $2 per item per month, determine how often to order and what size the order should be.

Solution:

Demand (D) = 600 unit/week
Ordering Cost (C_2) = $25/order
Item Cost (C_1) = $3/item

Holding Cost (C_3) = \$0.05 /item/week

(a). Shortages are not allowed:

Q* =?

t* = ?

$Q^* = \sqrt{2C_2D/C_3}$

Putting all the values in the above formula we have

$Q^* = \sqrt{2*25*600/0.05}$

$Q^* = \sqrt{600000}$

Q*= 774.59

t*=Q*/D

= 774.59/600

= 1.29

(b). Shortages are allowed:

Shortage Cost (C_4) = \$2 /item/month

= 2/4.33

= 0.46 /item/week

Q* =?

t* =?

$Q^* = \sqrt{2C_2D/C_3} * \sqrt{(C_3+C_4)/C_4}$

$= 774.596 * \sqrt{(0.05+0.46)/0.46}$

= 813.32

t* = Q*/D

= 813.32/600

= 1.36

Question No: 4 **Marks: 10**

Marshal Hotel in Islamabad remains open 24 hours a day. Waiters and busboys report for duty at; 3 AM, 11 AM, 3PM, 7 PM or 11 PM and each works an 8-hour shift. The following table shows the minimum number of workers needed during the six periods into which the day is divided. The hotel's scheduling problem is to determine how many waiters and busboys should report for work at the start of each period in order to minimize the total staff required for one day's operation.

Period	Time	Number of waiters and busboys required
1	03 AM - 07 AM	3
2	07 AM - 11 AM	12
3	11 AM - 03 PM	16
4	03 PM - 07 PM	9
5	07 PM - 11 PM	11
6	11 PM - 03 AM	4

Formulate the problem as an LP model.

Solution:

Decision variables

Let

x_1 = number of workers starting work in period 1
x_2 = number of workers starting work in period 2
x_3 = number of workers starting work in period 3
x_4 = number of workers starting work in period 4
x_5 = number of workers starting work in period 5
x_6 = number of workers starting work in period 6

Objective function

In this problem we have to minimize the total staff size

That is objective function is:

Minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

Constraints:

Since there are 8 hour shift so the number of workers in each period consist of those who start work in that period plus those who started in the previous period(as there are 4 hours for each period)

Periods	Minimum Requirements
1 and 2	12
2 and 3	16
3 and 4	9
4 and 5	11
5 and 6	4
6 and 1	3

$x_1 + x_2 \geq 12$
$x_2 + x_3 \geq 16$
$x_3 + x_4 \geq 9$
$x_4 + x_5 \geq 11$
$x_5 + x_6 \geq 4$
$x_6 + x_1 \geq 3$
$x_i \geq 0$

Now the LP problem is

Minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

Subject to,

$x_1 + x_2 \geq 12$
$x_2 + x_3 \geq 16$
$x_3 + x_4 \geq 9$
$x_4 + x_5 \geq 11$
$x_5 + x_6 \geq 4$
$x_6 + x_1 \geq 3$
$x_i \geq 0$

Question No: 5 **Marks:10**

Select the best choice (Only one) from the given four choices against each question.

1. _____ is any stored resource that is used to satisfy a current or future need
   (a). Inventory
   (b). Simulation
   (c). Allocation
   (d). None of the above

2. Which one is best describe Micro Economic Planning?
   (a). Distribution of fertilizer
   (b). Improving the layout of a workshop in a company
   (c). Investment planning of the country
   (d). All of the above

3. The purpose of CPM is
   (a). Obtain least cost schedule
   (b). Strike a balance or an optimum time schedule
   (c). Both a & b
   (d). None of the above

4. _____ = EST + Time required to perform the jobs
   (a). Early Finish Time
   (b). Latest Start Time
   (c). Latest Finish Time
   (d). None of the above

5. Free slack can never exceeds
   (a). Total Slack
   (b). Independent Slack
   (c). Both a & b
   (d). None of the above

6. The formula for calculating expected time t_e is
   (a). $t_e = \frac{(t_m + 4t_0 + t_p)}{6}$
   (b). $t_e = \frac{(t_0 + 4t_m + t_p)}{6}$
   (c). $t_e = \frac{(t_0 + 4t_p + t_m)}{6}$
   (d). None of the above

7. In LP problem, the decision variables can be divided into any fractional levels, so that non-integer values for the decision variables are permitted. This assumption is know as
   (a). Additivity

(b). Proportionality
(c). Divisibility
(d). None of the above

8. “OR is the scientific method of providing executive departments with a quantitative basis for decision regarding the operations under their control”. This definition was given by
(a). Blackett in 1948
(b). Morse and Kimbel in 1946
(c). Churchment Ackoff and Arnoff in 1957
(d). Saaty in 1958

9. Which of the following is a characteristic of critical path scheduling techniques?
(a). The jobs or tasks are dependent on each other.
(b). Interruptions may be permitted if they are documented.
(c). The jobs or tasks must be well defined, and their completion marks the end of the project.
(d). The jobs or tasks do not necessarily have to be in a predetermined sequence

10. If $t_0 = 2$, $t_m = 5$ and $t_p = 14$, then V_t= _____
(a). 2 hrs
(b). 3 hrs
(c). 4 hrs
(d). None of the above