

**DATA for MID TERM ---- MTH634 ---- Module# 001 to 085****Module : 001 [IMPORTANT IN MY VIEW]**

Topology is combination of two words "Topes (Greeks) means Place" and "Logy (Latin) means Study/Analysis".

Topology is a kind of Math in which we study shapes.

In Geometry we care about exact measurements such as lengths, angles and areas.

In Topology we do not care about measurements.

Geometrically rectangle, triangle, circle and square are different shapes.

By topologically rectangle, triangle, circle and square are same shapes.

In topology we care about arrangement of shapes, relative positions of shapes and Functions/Deformations between the shapes.

By definition topology means "Geometry of positions".

In topology two shapes are considered same if one shape can be deformed to other shape.

For example a circle can be deformed into a square so topologically circle and square are same shapes.

A line and a circle are different shapes.

In fact Topology is a matter of nature of study.

Topological study is more qualitative in nature rather than quantitative.

**Module : 001-007 [IMPORTANT IN MY VIEW]**

Let  $X$  be a set and Power set of  $X$  is denoted by  $P(X)$ .

Power set is the collection of all sub sets of a given set.

Usually we use the word **class** for a set of sets.

$\emptyset$  and set itself are also members of power set.

If  $X$  is a finite set with "n" elements then number of sets in powers set =  $2^n$

Let  $X$  be a non-empty set and  $\tau$  be a subclass of  $P(X)$  then  $\tau$  is called a topology on  $X$  if  $\tau$  satisfied the following axioms. (i)  $\emptyset$  and  $X \in \tau$  (ii) Union of any numbers of sets in  $\tau$  belongs to  $\tau$  (iii). The intersection of finite numbers of sets in  $\tau$  belongs to  $\tau$

The elements of  $\tau$  are called open sets or more specifically  $\tau$  – **Open sets**.

Topology on a set can also be defined by starting only 2<sup>nd</sup> and 3<sup>rd</sup> axioms.

$$\cup\{U \in \tau \mid U \in \varphi\} = \varphi \text{ and } \cap\{U \in \tau \mid U \in \varphi\} = X$$

More than one topology can be defined on a set.

Not every collection of subsets of a set is eligible to be a topology on the set unless that collection does not satisfy all the three stated conditions of given definition.

**Module : 008-010 [IMPORTANT IN MY VIEW]**

For any set there are always two special topologies. (i) Indiscrete Topology (ii) Discrete Topology

**Indiscrete Topology:** A topology on a set  $X$  consists of empty set and the set itself is called Indiscrete Topology. It is also called **Trivial Topology**.

Indiscrete topology is the smallest topology that can be defined on some set  $X$ .

**Discrete Topology:** A topology on a set  $X$  consists of all subsets of  $X$  is called Discrete Topology i.e.  $\tau = P(X)$

Discrete topology is the largest topology that can be defined on some set  $X$ .

**Cofinite Topology:** A topology on a set  $X$  is the collection of all subsets  $U$  of  $X$  such that  $U^c$  is either finite or

complete $X$ i.e. $\tau_{cof} = \{U \subset X \mid U^c \text{ is finite or } X\}$
Cofinite topology is also known as “ <b>Finite Complement Topology</b> ”.
<b>Topological Space:</b> Let $\tau$ be a topology on $X$ then the pair $(X, \tau)$ is a topological space or some times called $\tau$ – <b>Topological space</b> .
<b>Discrete Space:</b> A set $X$ with discrete topology is called “Discrete Topological Space”.
<b>Indiscrete Space:</b> A set $X$ with indiscrete topology is called “Indiscrete Topological Space”.
<b>Module : 011-020</b> [IMPORTANT IN MY VIEW]
Arbitrary intersection of open sets may or may not be open.
Openness is a relative term, $\varphi$ and $X$ are always open.
Closed Set: Let $(X, T)$ be a topological space. A subset “ $A$ ” of “ $X$ ” is closed iff the complement of “ $A$ ” in “ $X$ ” is Open i.e $X - A \in \tau$
Closeness is a relative term, $\varphi$ and $X$ are always closed.
$\varphi$ and $X$ can be open and closed at the same time.
Let $(X, T)$ be a topological space. Consider the collection $A = \{A \subset X \mid A^c \in \tau\}$ i.e. “ $A$ ” is the set of all those subsets of $X$ whose complement is open. Simply: “Collection of all closed subsets”.
Let $X = \{a, b, c, d\}$ and $\tau = \{\varphi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$ then $A = \{X, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \varphi\}$ is the collection of all closed subsets of $X$ .
Intersection of topologies on a set $X$ is always a topology on set $X$ .
Union of topologies on a set $X$ may or may not be a topology on $X$ .
The real number set $\mathbb{R}$ is one of the most studies or dominant object in Mathematics.
<b>Usual Topology on <math>\mathbb{R}</math>:</b> The set of all open sets of $\mathbb{R}$ is $\tau_u$ called Usual Topology on $\mathbb{R}$ .
<b>Real Plan <math>\mathbb{R}^2</math>:</b> The real plan is the Cartesian product of two real lines i.e. $\mathbb{R} \times \mathbb{R}$ . The real plane is written as $\mathbb{R}^2$
Open Disk: An open disk $D(x, y)$ in $\mathbb{R}^2$ centered at $(x, y) \in \mathbb{R}^2$ is the set of all those points $(x', y')$ in $\mathbb{R}^2$ whose distance from $(x, y)$ is less than “ $r$ ” where “ $r$ ” is positive real number.
Let $T_1$ and $T_2$ be two topologies on a non empty set $X$ then $T_1$ is finer than $T_2$ iff $T_1 \subseteq T_2$
Indiscrete topology is the weakest topology on a set $X$ .
Discrete topology is the finest topology on a set $X$ .
Two topologies on a set $X$ are comparable iff one is finer (weaker) than the other.
Not all topologies on a set $X$ are comparable.
Limit Point: Let $(X, T)$ be a topological space and “ $A$ ” be a subset of $X$ . A point $x \in X$ is a limit point of $A$ iff every open set $U_x$ containing $x$ contains a point of $A$ different from $x$ .
Limit Point is also called accumulation point or derived point. [ALSO PREPARE PAGE#50-52]
<b>Module : 021-030</b> [IMPORTANT IN MY VIEW]
Derived Set: Let $(X, T)$ be a topological space and $A$ be a subset of $X$ then the derived set of $A$ is denoted by $\dot{A}$ and it is the set of all limit points of $A$ i.e. $\dot{A} = \{x \in X \mid \forall U_x \in T, A \cap (U_x \setminus \{x\}) \neq \varphi\}$
Derived set of $A$ may or may not be subset of $A$ .
Derived set of empty set is empty.

In a discrete space $X$ the derived set $\dot{A}$ of any subset $A$ of $X$ is empty.
Let $(X,T)$ be a topological space and Let $A, B \subset X$ . If $A \subset B$ then $\dot{A} \subset \dot{B}$
Let $(X,T)$ be a topological space and Let $A, B \subset X$ then $A \subset B$ then $(A \cup B)' = \dot{A} \cup \dot{B}$
Prepare Page # 61-62
A subset "A" of a topological space $X$ is closed iff $A' \subset A$
Let $(X,T)$ be a topological space and "A" be a subset of $X$ . The closure of "A" is denoted by $\bar{A}$ and it is the intersection of all closed subsets of $X$ containing $A$ i.e. $\bar{A} = \bigcap_{C \in c_A} C$ where $c_A$ is the class of all closed subsets of $X$ containing $A$ .
Remember: $A \subset \bar{A}$ , $\bar{\emptyset} = \emptyset$ , $\bar{X} = X$
Let $A$ be a subset of a topological space $X$ then $\bar{A} = A \cup \dot{A}$
Let $A$ be a subset of a topological space $X$ then (i) $\bar{A}$ is closed subset of $X$ (ii) "A" is closed iff $\bar{A} = A$ (iii) $\overline{\bar{A}} = \bar{A}$
Prepare Page#78-79
Let $X$ be a topological space and $A, B \subset X$ then $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$
Let $A$ be a subset of a topological space $X$ then $A$ is said dense in $X$ iff $\bar{A} = X$
Let $X = \mathbb{R}$ with usual topology and consider $Q$ then $\bar{Q} = \mathbb{R}$
$Q$ is dense in $\mathbb{R}$ .
$\mathbb{N}$ is not dense in $\mathbb{R}$ .
$X$ is dense in itself.
A proper closed subset of $X$ can never dense in $X$ .
<b>Module : 031-040 [IMPORTANT IN MY VIEW]</b>
Boundary points are also called frontier Points.
$bd(A)$ is closed.
Boundary of a set $A$ is empty iff $A$ is both open and close.
Let $P$ be a point in a topological space $(X,T)$ . A subset $N$ of $X$ is a neighborhood of $p$ iff there exists an open set $U_p$ such that $p \in U_p \subset N$
Let $P$ be a point in a topological space $(X,T)$ then $N$ is neighborhood of "p" iff "p" is an interior point of $N$ .
A neighborhood $N$ of a point "p" is called an open neighborhood iff $N$ is a neighborhood of all its points i.e. $N$ is an open set.
The class of all neighborhoods of $p \in X$ is called neighborhood system of "p". It is denoted by " $N_p$ ".
Let $N_p$ is the neighborhood system of "p" then $N_p$ is non-empty.
Let $N_p$ is the neighborhood system of "p" then $N_p$ is closed under finite set intersection.
Let $N_p$ is the neighborhood system of "p" then every superset of a member of $N_p$ belongs to $N_p$ .
<b>Module : 041-050 [IMPORTANT IN MY VIEW]</b>
Subspace Topology: Let $A$ be a nonempty subset of a topological space $(X,T)$ Then the collection $T_A = \{V = U \cap A \mid U \in T\}$ of subsets of defines a topology on $A$ . $T_A$ is called subspace topology or relative topology on $A$ relatives to $(X,T)$ .

Let  $(X, T)$  be a topological space. A sub collection  $B \subset T$  is called basis for  $T$  iff for all  $U \in T$  and for all  $x \in U$  there exists  $B \in B$  such that  $x \in B \subset U$ .

If  $B$  is basis for a topology  $T$  on  $X$  then every element of  $T$  can be written as union of the elements of  $B$ .

Prepare p# 113

The lower limit topology on  $\mathbb{R}$  is denoted by  $\mathbb{R}_L$  and  $\mathbb{R}_L = \{[a, b) | a, b \in \mathbb{R}\}$

The Upper limit topology on  $\mathbb{R}$  is denoted by  $\mathbb{R}_{up}$  and  $\mathbb{R}_{up} = \{(a, b] | a, b \in \mathbb{R}\}$

**Module : 051-085 [IMPORTANT IN MY VIEW]**

Any class  $A$  of subsets of a non empty set  $X$  is a sub basis for a unique topology  $T$  on  $X$ .

Let  $T_1$  and  $T_2$  be two topologies on non empty set  $X$  then  $T_1 \cup T_2$  may or may not be a topology on  $X$ . If  $T_1 \cup T_2$  is topology on  $X$  then it will be the smallest topology containing both  $T_1$  and  $T_2$ .

Let  $T_1$  and  $T_2$  be two topologies on non empty set  $X$  then  $T_1 \cap T_2$  is topology on  $X$  then it will be the largest topology containing both  $T_1$  and  $T_2$ .

Continuous Function: Let  $(X, T_x)$  and  $(Y, T_y)$  be two topological spaces. A function  $f: x \rightarrow y$  is continuous iff for all  $u \in T_y$  there exist  $v \in T_x$  such that  $f^{-1}(u) \subset v$ .

Prepare P#147

Constant Function: A function whose image is singleton i.e. whose output is the same for every point is called constant function.

A map  $f : (X, T_x) \rightarrow (Y, T_y)$  is called an open map iff the image of each open subset of  $X$  is open in  $Y$ .

**Important Short/Long Questions**

Open mapping theorem Prove.....5 marks.....P#183

Dense Subset.....5 marks.....P#81

Investigate Continuity..... 5 marks

Limit Point.....2 marks.....P#50

Inverse of image continuity ..... 2 marks

Define neighborhood of a point.....2 marks.....P#100

Explain indiscrete space is not Metrizable ..... 2 marks

Show there is no homeomorphism in open and close interval of  $\mathbb{R}$ ..... 2 marks

Derived set..... 3 marks.....P#54

Find continuity of point ..... 3 marks.....P#172

Define relative Open set .....2 marks

Find exterior, interior and boundary open of given set.....5 marks.....P#87,93
