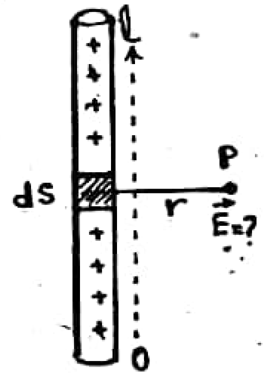
ELECTRIC FIELD DUE TO LINEAR CHARGE DISTRIBUTION.

If positive source charge is distributed uniformly along a line, Then



$$\frac{dq}{dS} = \lambda = \text{Linear charge density.}$$

$$dq = \lambda dS.$$

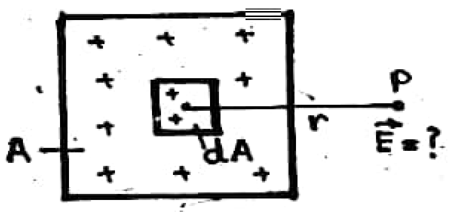
$$\Longrightarrow q = \int_0^l \lambda dS$$

Then $\vec{E} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2}\hat{r}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^l \frac{\lambda dS}{r^2}\hat{r} \longrightarrow ②$$

ELECTRIC FIELD DUE TO SURFACE CHARGE DISTRIBUTION.

If positive source charge is distributed uniformly over a surface, Then



$$\frac{dq}{dA} = \sigma = \text{Surface charge density.}$$

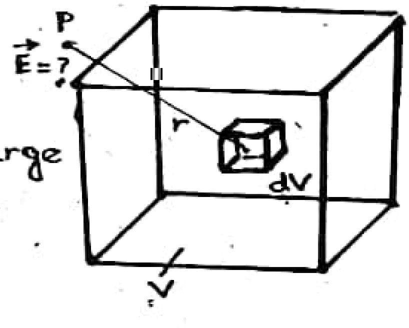
$$dq = \sigma dA \Longrightarrow q = \int_S \sigma dA$$

Then, $\vec{E} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2} \times \hat{r}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma dA}{r^2}\hat{r} \longrightarrow ③$$

ELECTRIC FIELD DUE TO VOLUME CHARGE DISTRIBUTION.

If positive charges are distributed uniformly in a volume V, Then



$$\frac{dq}{dV} = \rho = \text{Volume charge density.}$$

$$dq = \rho dV$$

$$\Longrightarrow q = \int_V \rho dV$$

Then $\vec{E} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2}\hat{r}$

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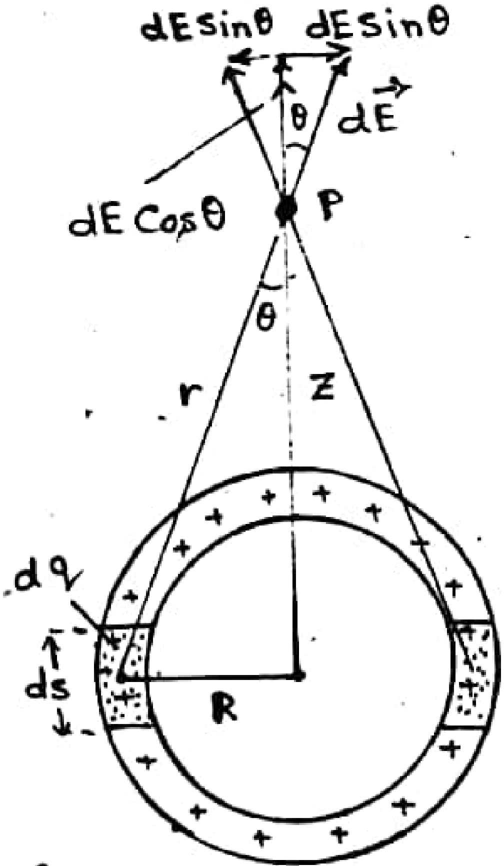
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho dV}{r^2}\hat{r} \longrightarrow ④$$

ELECTRIC FIELD DUE TO RING OF CHARGE.

Q.- Determine the electric field due to ring of charge.



ANS.-
Consider a thin ring of radius R with uniform +ve charge density λ around its circumference.
Let us consider a small element of the ring of Length 'ds'. The small element has a charge 'dq'.

$$\because \lambda = \frac{dq}{ds}$$

$$\Rightarrow dq = \lambda ds \longrightarrow (1)$$

This charge produces an electric field dE at a point P, which is at a distance r from the element 'ds'

$$\because dE = \frac{1}{4\pi\epsilon_0} \times \frac{dq}{r^2} \longrightarrow (2)$$

From the figure, $r^2 = Z^2 + R^2$

$$dE = \frac{1}{4\pi\epsilon_0} \times \frac{dq}{Z^2+R^2} \longrightarrow (3)$$

Resolving dE in to its rect. Comps.

$$dE \begin{cases} dE_Z = dE\cos\theta. \text{ (Parallel to ring axis) (Z-axis)} \\ dE_Y = dE\sin\theta. \text{ (Perp to ring axis)} \end{cases}$$

The only component along ring axis give resultant E, the component ⊥ to ring axis vanishes having equal magnitudes.

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and opposite directions. $-dE_z \longleftarrow \bullet \longrightarrow dE_z$

$$r^2 = Z^2 + R^2$$

$$\Rightarrow \quad r = (Z^2 + R^2)^{1/2}$$

$$\cos\theta = \frac{Z}{r}$$

$$\Rightarrow \quad \cos\theta = \frac{Z}{(Z^2 + R^2)^{1/2}} \longrightarrow (4)$$

From equs. (3) and (4).

$$dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, ds}{(Z^2+R^2)} \times \frac{Z}{(Z^2+R^2)^{1/2}}$$

$$dE\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{Z\lambda}{(Z^2+R^2)^{3/2}}\, ds \longrightarrow (5)$$

Integration gives the resultant E.

$$E = \int_{S=0}^{S=2\pi R} dE\cos\theta = \frac{Z\lambda}{4\pi\epsilon_0 (Z^2+R^2)^{3/2}} \int_{S=0}^{S=2\pi R} ds$$

$ds = 2\pi R$ = Circumference of the circle

$$E = \frac{Z\lambda}{4\pi\epsilon_0 (Z^2+R^2)^{3/2}} \Big| S \Big|_0^{2\pi R}$$

$$E = \frac{Z\lambda}{4\pi\epsilon_0 (Z^2+R^2)^{3/2}} (2\pi R) \longrightarrow (6)$$

$$E = \frac{Z(\lambda 2\pi R)}{4\pi\epsilon_0 (Z^2+R^2)^{3/2}}$$

As $\lambda = q/S = q/2\pi R$

$$\Rightarrow \quad q = \lambda(2\pi R)$$

$$E = \frac{Zq}{4\pi\epsilon_0 (Z^2+R^2)^{3/2}}$$

If point P lies very far off from the ring then, $Z >> R$, so R can be neglected.

$$E = \frac{Zq}{4\pi\epsilon_0 (Z^2)^{3/2}} = \frac{Zq}{4\pi\epsilon_0 Z^{2\times 3/2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{Z^2} \Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \times \frac{q}{Z^2}\hat{Z}}$$