Question 1

Write the expression for total energy of a particle executing simple harmonic motion.

Ans:

If T is the kinetic energy, V the potential energy and $E = T + V$ the total energy of a simple harmonic oscillator, then we have

$$K.E = T = \frac{1}{2}mv^2$$

and

$$V = \frac{1}{2}kx^2$$

Then the total energy of S.H.M will be

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

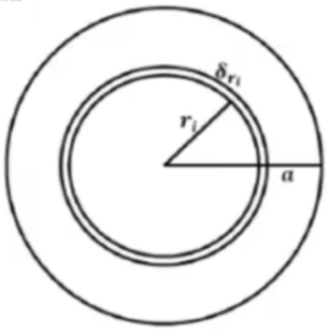
Question 2

Find the moments of inertia of a ring of radius a about an axis through center.

Solution:

Let M be the mass and a the radius of the ring. Then the mass per unit length will be $M/_{2\pi a}$.
We regard this ring to be composed of small elements of mass (δm) each of length δs,
we can write it as

$$\frac{\delta m}{\delta s} = \frac{M}{2\pi a} \Rightarrow \delta m = \frac{M}{2\pi a}\delta s$$



Moment of inertia of the element about an axis through center O and the perpendicular to the plane of the ring equals $\frac{M}{2\pi a}\delta s\, a^2$.
Therefore the M.I of the whole ring will be

$$I_{ring} = \frac{M}{2\pi a}a^2 \sum_{elements} \delta s = \frac{Ma}{2\pi}\int ds = \frac{Ma}{2\pi} \times 2\pi a = Ma^2$$

Hence Ma^2 is the required moment of inertia of the ring.

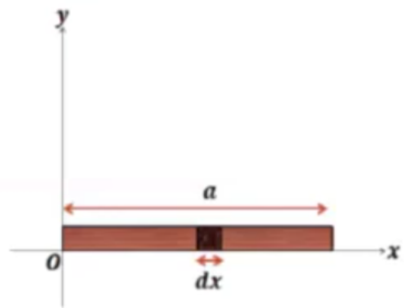
Question 3

Calculate the moment of inertia of a uniform rod of length l about an axis perpendicular to the rod and passing through an end point.

Solution:

Let the $X - axis$ be chosen along the length of the rod, with origin at one end point as shown in the figure. Let M and a be the mass and length of rod respectively. We suppose the rod to be composed of small elements.

Let dm and dx be the mass and the length of the specific element of the rod at a distance x from the end point O.



Then
$$\frac{dm}{dx} = \frac{M}{a}$$
$$\Rightarrow dm = \frac{M}{a} dx$$

Then the moment of inertia of this element about the given axis is

$$I_{element} = dm x^2 = \frac{M}{a} x^2 dx$$

Hence the moment of inertia of the whole rod will be

$$I = \sum_{all\ elements} \frac{M}{a} x^2 dx$$
$$= \frac{M}{a} \int_0^a x^2 dx$$
$$= \frac{M}{a} . \left|\frac{x^3}{3}\right|_0^a = \frac{M}{a}\frac{a^3}{3}$$
$$= \frac{1}{3} M a^2$$

Question 4

To find the inertia of uniform solid cuboid (parallelpiped) at one of its corner, $I_{xx} = \frac{M}{3}(b^2 + c^2)$ if Then find I_{yy} and I_{zz}

Solution:

Similarly, due to symmetry, we can write

$$I_{yy} = \frac{M}{3}(a^2 + c^2)$$

$$I_{zz} = \frac{M}{3}(a^2 + b^2)$$

Question 5

If the moment of inertia of a uniform circular cyclinder of length b and radius a about x-axis through the center and perpendicular to the central axis is :

$$I_x = m\left(\frac{1}{4}a^2 + \frac{1}{12}b^2\right)$$

Solution:

Due to symmetry we have

$$I_x = I_y = m\left(\frac{1}{4}a^2 + \frac{1}{12}b^2\right)$$

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Question 7

Consider the force field $F = -kr^3\vec{r}$ Find the potential energy of the given force field

Solution :

Since the field is conservative there exists a potential V such that

$$F = -\nabla V$$

$$F = -kr^3\vec{r} = -k(x^2 + y^2)^{\frac{3}{2}}(xi + yj)$$

$$= -k(x^2 + y^2)^{\frac{3}{2}}xi - k(x^2 + y^2)^{\frac{3}{2}}yj = -\nabla V$$

$$= -\frac{\partial V}{\partial x}i - \frac{\partial V}{\partial y}j - \frac{\partial V}{\partial z}k$$

By comparing, we get

$$\frac{\partial V}{\partial x} = k(x^2 + y^2)^{3/2}x, \quad \frac{\partial V}{\partial y} = k(x^2 + y^2)^{3/2}y \quad \text{and} \quad \frac{\partial V}{\partial z} = 0$$

From which, by omitting the constants, we get

$$V = \frac{1}{5}\,k(x^2 + y^2)^{5/2} = \frac{1}{5}\,kr^5$$

Question 8

if $r = a\cos\omega t\, i + b\sin\omega t\, j$. and $p = -am\omega\sin\omega t\, i + bm\omega\cos\omega t\, j$

then find angular momentum

Solution :

We known that

$$r \times p$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\cos\omega t & b\sin\omega t & 0 \\ -am\omega\sin\omega t & bm\omega\cos\omega t & 0 \end{vmatrix}$$

$$= [abm\omega\cos^2\omega t + abm\omega\sin^2\omega t]\hat{k}$$

$$= abm\omega\hat{k}$$

$$\tau = \frac{d\Omega}{dt} = \frac{d}{dt}(r \times p) = 0$$

Question 9

What is a degree of freedom of the system of rigid bodies.

Solution :

The number of coordinates required to specify the position of a system of one or more particles is called the number of degrees of freedom of the system.

For example a particle moving freely in space requires 3 coordinates, e.g. (x, y, z), to specify its position. Thus the number of degrees of freedom is 3.

Question 10

Linear Momentum

We define the linear momentum ρ of the system as the vector sum of the linear momentum of the individual particles, namely,

$$p = \sum_i p_i = \sum_i m_i v_i$$

From equation (1)

$$r_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{m} \quad (1)$$

$$\dot{r}_{cm} = v_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{m}$$

it follows that

$$p = m v_{cm}$$

that is, the linear momentum of a system of particles is the product of the velocity of the center of mass and the total mass of the system.

Question 11

formulas for translational and rotational K.E. of a rigid body

Solution:

translational formula

$$T_{tr} = \frac{1}{2}Mv^2$$

rotational formula

$$T_{rot} = \frac{1}{2}\omega . L \qquad \text{eq (1)}$$

We known $L = I\omega$

We k putting the value of L in eq (1) we get

$$T_{rot} = \frac{1}{2}\omega . I\omega = \frac{1}{2}I\omega^2$$

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Question 12

the moment of inertia of a rectangular plate with sides a and b about an X axis $I_x = \frac{1}{3}Mb^2$ and Y axis $I_y = \frac{1}{3}Ma^2$ Find the moment of inertia along z-axis

Solution:

$$I_z = I_x + I_y$$

$$I_z = \frac{1}{3}Mb^2 + \frac{1}{3}Ma^2$$

$$I_z = \frac{1}{3}M(a^2 + b^2)$$

Question 13

Calculate the moment of inertia I_{cm} for a uniform rod of length l and mass M rotating about an axis through the center, perpendicular to the rod.

Soution:

In order to calculate the moment of inertia through the center of mass c.m., we use parallel axes theorem.

In a transparent notation

$$I_l = I_{cm} + Md^2 \qquad (1)$$

where d is the distance between the origin and the center of mass and $d = {}^{l}/_{2}$.

Also I_l is the moment of inertia of rod about one of its end (which we calculated earlier).

Here

$$I_l = \frac{1}{3}Ml^2 \qquad (2)$$

From (1), we have

$$I_{cm} = I_l - Md^2$$

Substituting value from (2) in (1)

$$I_{cm} = \frac{1}{3}Ml^2 - M\left(\frac{l}{2}\right)^2 = \frac{1}{3}Ml^2 - \frac{1}{4}Ml^2$$

$$I_{cm} = \frac{1}{12}Ml^2$$

which is required moment of inertia about its center of mass.