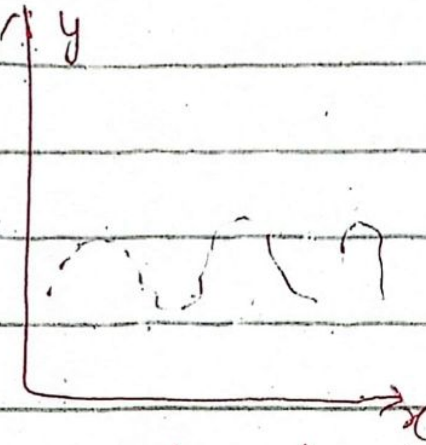
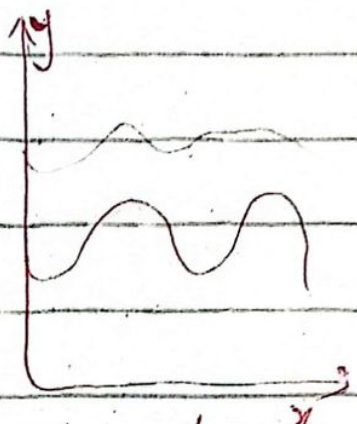
Discrete Math (MTH202)

Discrete   Continuous





* it consist of sequence of individual steps.

* logic:-
It is the study of principle that distinguish b/w a valid and an invalid arguments.

* Statements:-
declarative sentence that is either true or false but not both. also called proposition

example

$2+4=6 \Rightarrow T$

$2+4=7 \Rightarrow F$

Grass is green $\Rightarrow$ T

$\Rightarrow$ Close the door $\rightarrow$ Not proposition

P $x+2=4$ not Propostion

q May i come in not a statement

q ; it is hot today statement

logical connectives:-

And, OR, Not

use to joind to statements

And	$\wedge$	Hat
OR	$\vee$	vel
Not	$\sim$	Tilda
Conditional	$\rightarrow$	Arrow
Bi conditional	$\leftrightarrow$	Double Arrow

Example

P = Islamabad is the Capital of Pakistan.

and

q = 17 is divible by 3

$\boxed{\begin{matrix}\text{AND} \\ P \wedge q\end{matrix}}$

$\boxed{\begin{matrix}\text{OR} \\ P \vee q\end{matrix}}$ OR

$\boxed{\begin{matrix}\text{Not} \\ \sim P\end{matrix}}$ And, But = AND

not, nor, neither = not

but and Nor = AND

- It is not hot $\sim P$
- It is hot and sunny $P \wedge q$
- It is not hot or sunny $\sim P \vee q$
- It is not hot but sunny $\sim P \wedge q$
- It is neither hot nor su $\sim P \wedge \sim q$

★ Negation ($\sim$):- NOT

$\sim P$

P	$\sim P$
F	T
T	F

Conjunction:- ($\wedge$) AND

$p \wedge q$ $\quad 2^2 = 4$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$1+1=2 \quad 2^2=4 \quad p+q=$

★ Disjunction ($\vee$) / OR $\quad 2^2 = 4$

$p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

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Truth tables logical Equivalence and De Morgan's law.

Truth table $\quad 2^2 = 4$

$\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

$p \wedge (q \vee \sim r) \quad 2^3 = 8$

p	q	r	$\sim r$	$q \vee \sim r$	$(p \wedge q \vee \sim r)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

* Exclusive OR: $P \oplus q$

P XOR q

P	q	$P \oplus q$
T	T	F
T	F	T
T	F	T
F	F	F

Do true ya do false
False answer kay ga.

* Double Negation :-

$\sim(\sim P) = P$ — MCQ

P	$\sim P$	$\sim(\sim P)$
T	F	T
F	T	F

Same value.

* Demorgram's law

$\sim(P \land q) \equiv \sim P \lor \sim q$

$\sim(P \lor q) \equiv \sim P \land \sim q$

Example:-

The fan is slow and it is very hot

* Tan fan is not slow or it is not very hot

* $1 < x \leq 4$

$x > -1$ and $x \leq 4$

$x \leq -1$ or $x \geq 4$

* Tautology → "t" jithay true hogy

P	$\sim P$	$P \vee \sim P$
T	F	T
F	T	T

Hence $P \vee \sim P \equiv t$ this is tautology

Contradiction "C"

$P \wedge \sim P \equiv C$

P	$\sim P$	$P \wedge \sim P$
T	F	F
F	T	F

law of logic:-

1. cammulative law = $P \wedge q \equiv q \wedge P$

   $P \vee q \equiv q \vee P$

2. Associative law :- $(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$

3. Distributive law:-

$$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$$

4. Identity law:-

$P \wedge t \equiv P$

$P \vee c \equiv P$

T	T	F	F	-
F	T	F	F	F

5. Negation law

$P \vee \sim P \equiv t$

$P \wedge \sim P \equiv c$

T	F	= T	T
F	T	= T	F

6. idempotent law :-

$P \wedge P \equiv P$

$P \vee P \equiv P$

T	T	= T
F	F	= F

#3

law of logic theis Applications and Implications.

Simplify using law of Logic

① $P \vee [\sim(\sim P \wedge q)]$

$P \vee [\sim(\sim P) \vee \sim q)]$ Degrm law

$\equiv$ ~~$P \vee (P \wedge q)$~~ ~~Degrom law~~ Double negation law

$\equiv P \vee (P \vee \sim q)$ Double negation

$\equiv (P \vee P) \vee (\sim q)$ Aocsitive law

$= P \vee (\sim q)$ idemoy law

② $\sim(\sim P \wedge q) \wedge (P \vee q) \equiv P$ T F / T F

★ $\sim(\sim P) \vee \sim q \wedge (P \vee q)$ D.L

★ $P \vee \sim q \wedge (P \vee q)$ DoubNe law

★ $P \vee (\sim q \wedge q)$ distnd law

★ $P \vee C$ Neg law

★ $\boxed{P}$ Identity law

☆ Implication / conditional Statement

$p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

if p then q or p implies q

Example :- (1) (2)

				result
if	1=1	then	3=3	T
	1=1		2=3	F
	1=0		3=3	T
	1=0		2=3	T

$p \to q = \sim q \to \sim p$

p	q	$\sim q$	$\sim p$	$p \to q$	$\sim q \to \sim p$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	T	T

★ ~~Introduction to~~ ~~<~~ hacking ★

Inverse of conditional Statement $p \rightarrow q$ is $\sim p \rightarrow \sim q$

★ If P is a square then P is a rectangle

★ if P is not a square then P is not a rectangular.

$\sim P \rightarrow \sim q$

Bi-conditional law of logic involving Biconditional and their Application

Double Implication / Biconditional

$P \leftrightarrow q$

if and only if

P	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example:-

(1) $1+1=3$ if and only if earth is flat = True.
F F

(2) Sky is blue iff $1=0$ = false
T F

(3) Mike is whit iff birds do lay eggs = T
T T $\Leftrightarrow$

(4) $x>5$ iff $x^2>25$ False
T $\Leftrightarrow$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

$p \Leftrightarrow q$ also expressed as.
" if p then q and conversely"
" p is necessary and sufficient for q"

② for you to the contest it is necessary and sufficient that you have the only winning ticket.

① you ~~be~~ win the contest if and only if you hold the winning ticket.

②. if you read the news paper every day you will be informed and conversely

② you will be informed if and only if you

p	q	r	$p \leftrightarrow q$	$r \leftrightarrow q$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow q)$
T	F	T	F	T	
T	F	F	F	F	
T	F	T	F	F	
T	F	F	F	T	
F	F	T	F	F	
F	T	F	F	F	
F	F	T	T	F	
F	F	F	F	T	

$2^3 = 8$

p	q	r	$p \leftrightarrow q$	$r \leftrightarrow q$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow q)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
T	F	F	F	T	F
F	T	T	F	T	F
F	T	F	F	F	T
F	F	T	T	F	F
F	F	F	T	T	T

of logic for Bi-conditional:-

* Commutative law:

$$p \leftrightarrow q = q \leftrightarrow p$$

* Implication law

$$p \rightarrow q \equiv \sim p \vee q$$
$$\equiv \sim(p \wedge \sim q)$$

* Expastation law:-

$$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$$

* Equivalence

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

5 Reduction ad absurdum:

$$P \to q \equiv (P \wedge \sim q) \to c$$

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Valid and Invalid Argument

* Argument:-

list of statement called premises or assumption or hypotheses

result of statement called conclusion.

		valid	Invalid
P_1	Premis	T	T
P_2	Premis	T	T
P_3	Premis	T	T
∴	conclusion	T	F

Example:-

- I like Math.
- Math is intresting
- Math is intersting i like it / Math.

Valid Argument:-

* if the conclusion is true when all the premise are True.

An argment is valid if conjunction of its premises imply conclusion

$P_1 \land P_2 \land P_3 \ldots \land P_n) \Rightarrow C$ is a tautology.

Invalid Argument:

if the conculusion is false when all the premises are true

an argment is invalid if conjunction of its premises does not imply conclusion.

$P \rightarrow q$

P

$\therefore q$

p	q	$p \to q$	p	q
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	F

Premis coclusion

$p \to q$	p
T	T
T	F

critical row

all premies are

true

p	q	$p \to q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

$p \to q$

q

$\therefore p$

invalid

Premises Colusion

So coclusion na ek false hai toh Answer bi false.

1) if i got Eid bonus I will by sterro | $e \to s$

2) if I sell my mooter cycle I will buy a stero | $m \to s$

3) if i get an eid bonus or i sell my mota cycel then I will buy a stero

$e \vee m \to s$

$2^3 = 8$

e	S	m	$e \to s$	$m \to s$	$e \vee m$	$\to s$
T	T	T	T	T	T	T
F	T	F	T	T	T	T
T	F	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	F	F	T	F
F	F	F	T	T	F	T