MTH401 – Differential Equations - Midterm Paper

Session 4 – Fall 2005

MTH401 - Differential Equations - Q. No. 1 (M - 1)

The differential equation $(3x^2$

$y + 2)\,dx + (x^3 + y)\,dy = 0$

is

►Exact

►Linear

►Homogenous

►Separable

MTH401 - Differential Equations - Q. No. 2 (M - 1) The assumed particular solution for the U.C (Undetermined Coefficient) differential equation

$y' - y = x^2 e^{2x}$

is

► $y_p = c_1 e^{x^2} + c_2 x^2$

► $y_p = (Ax + B)e^{2x}$

► $y_p = (Ax^2 + Bx + c)e^{2x}$

►None of these.

MTH401 - Differential Equations - Q. No. 3 (M - 1) The differential equation

$$x\frac{dy}{dx}+y=y^2\ln x$$

is an example of

► Separable

► Homogenous

► Exact

► None of these.

MTH401 - Differential Equations - Q. No. 4 (M - 1)

For the differential equation y' −2xy=x Integrating factor is

► $-x^2$ ►

e^{x2}

► e^{-x2}

► $x2$

MTH401 - Differential Equations - Q. No. 5 (M - 1)

$$\frac{dy}{dx}=\frac{x+3y-5}{x-y-1}$$

Identify the ordinary differential equation

► Homogenous

► Separable

► Exact

►None of these.

MTH401 - Differential Equations - Q. No. 6 (M - 5)
Solve the differential equation

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$$

MTH401 - Differential Equations - Q. No. 7 (M - 10) Solve

$(y\text{Sec}^2x+\text{Sec}x\tan x)\,dx+(\tan x+2y)dy=0$

MTH401 - Differential Equations - Q. No. 8 (M - 10)

Find the equation of orthogonal trajectories of the curve
$x^2+y^2=cx$

MTH401 - Differential Equations - Q. No. 9 (M - 10)
Solve the differential equation by method of variations of parameters

$$\frac{d^2y}{dx^2}+y=\tan x\sec x$$

	MIDTERM EXAMINATION	
	SPRING 2007	Marks: 40
	MTH401 - DIFFERENTIAL EQUATIONS (Session - 4)	Time: 90min

StudentID/LoginID: ______________________

Student Name: ______________________

Center Name/Code: ______________________

Exam Date: Friday, May 18, 2007

Please read the following instructions carefully before attempting any of the questions:

1. Attempt all questions. Marks are written adjacent to each question.

2. Do not ask any questions about the contents of this examination from anyone.

a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

c. Write all steps, missing steps may lead to deduction of marks.

**WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.

For Teacher's use only											
Question	1	2	3	4	5	6	7	8	9		Total
Marks											

Question No: 1 (Marks: 1) - Please choose one

The differential equation
$(3x^2y + 2)dx + (x^3 + y)dy = 0$ is

- Exact
- Linear
- Homogenous
- Separable

Question No: 2 (Marks: 1) - Please choose one

The assumed particular solution for the U.C(Undetermined Coefficient) differential equation $y' - y = x^2 e^{2x}$ is

- $y_p = c_1 e^{x^2} + c_2 x^2$
- $y_p = (Ax + B)e^{2x}$
- $y_p = (Ax^2 + Bx + c)e^{2x}$
- None of these.

Question No: 3 (Marks: 1) - Please choose one

The differential equation $x\dfrac{dy}{dx} + y = y^2 \ln x$ is an example of

- Separable
- Homogenous
- Exact
- None of these.

Question No: 4 (Marks: 1) - Please choose one

For the differential equation

$y' - 2xy = x$

Integrating factor is

- $-x^2$
- e^{x^2}

- e^{-x^2}
- x^2

Question No: 5 (Marks: 1) - Please choose one

$$\frac{dy}{dx}=\frac{x+3y-5}{x-y-1}$$

Identify the ordinary differential equation

- Homogenous
- Separable
- Exact
- None of these.

Question No: 6 (Marks: 5)

Solve the differential equation

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}} = 0$$

Question No: 7 (Marks: 10)

Solve

$$(ySec^2 x + Secx \tan x)\, dx + (\tan x + 2y)\, dy = 0$$

Question No: 8 (Marks: 10)

Find the equation of orthogonal trajectories of the curve

$$x^2 + y^2 = cx$$

Question No: 9 (Marks: 10)

Solve the differential equation by method of variations of parameters

$$\frac{d^2y}{dx^2} + y = \tan x \sec x$$

	FINALTERM EXAMINATION FALL 2006 MTH401 - DIFFERENTIAL EQUATIONS (Session - 1)	Marks: 53 Time: 120min

StudentID/LoginID: ______________________

Student Name: ______________________

Center Name/Code: ______________________

Exam Date: Monday, February 12, 2007

Please read the following instructions carefully before attempting any of the questions:

1. Attempt all questions. Marks are written adjacent to each question.

2. Do not ask any question about the contents of this examination from anyone.

a. If you think that there is something wrong with any of the question, attempt it to the best of your understanding.

b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

c. Write all steps, missing steps may lead to deduction of marks.

3. Calculator is allowed.

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For Teacher's use only											
Question	1	2	3	4	5	6	7	8	9	10	Total
Marks											
Question	11										
Marks											

Question No: 1 (Marks: 2) - Please choose one

If the variation of the path of the curves can be describrd by the concept of differential equations then which of the following differential equation describe the path for $y-axis$.

- ► $\frac{dy}{dx}=1$
- ► $\frac{dy}{dx}=0$
- ► $\frac{dy}{dx}=-1$
- ► $\frac{dy}{dx}=\infty$

Question No: 2 (Marks: 2) - Please choose one

Suggestive form of the constant input function for the non homogeneous differential equation under the method entitled as "**Method of the undetermined coefficient**" is

- ► $f(x)=e^{x}$
- ► $f(x)=a$
- ► $f(x)=e^{ax}\left(ACosx+BSinx\right)$
- ► Suggestive form is impossible.

Question No: 3 (Marks: 2) - Please choose one

Which of the following function is linearly dependant to the exponential function e^x ?

- ► $-e^x$
- ► e^{-x}
- ► xe^x
- ► $-xe^{-x}$

Question No: 4 (Marks: 2) - Please choose one

Eigen values for the system of the differential equations $X' = AX$ are evaluated for the

- ► Solution vector X
- ► Coefficient matrix A
- ► Differentiated solution vector X'
- ► Transpose of the Coefficient matrix A

Question No: 5 (Marks: 2) - Please choose one

Fundamental set of the solution vectors $X_1, X_2, \cdots\cdots, X_n$ for any system of the differential equations are obtained by

- ► Developing the singleton set $\{X\} = \{c_1X_1, c_2X_2, \ldots\ldots, c_nX_n\}$ of the linear combinitions of the solution vectors.
- ► Taking derivative of the each solution vector and forming the set $\{X_1', X_2', \cdots\cdots, X_n'\}$

▶ Taking Integral of the each solution vector and forming the set

$$\left\{\int X_1dx, \int X_2dx, \cdots\cdots, \int X_ndx\right\}$$

▶ Just verifying their linear independance and establishing the set

$$\left\{X_1, X_2, \cdots\cdots, X_n\right\}$$

.

Question No: 6 (Marks: 5)

For the family with parameter '**a**', of rectangular hyperbola $y = \frac{a}{x}$, find its corresponding orthogonal trajectorie and induce the parameter of the new family.

Question No: 7 (Marks: 5)

(a) Justify whether $D^2 + 9$ annihilates the function $f(x) = \sin 3x$ **(3)**

(b) Evaluate Wronskian of functions $Sin|x|$ and $|x|$ for $x < 0$ provided that $|x| = -x$ for $x < 0$ **(2)**

Question No: 8 (Marks: 8)

(a) Determine the interval of convergence of power series $\sum_{k=0}^{\infty} \frac{2^k}{k} x^k$
(5)

(b) If $\sum_{n=0}^{\infty} nc_n x^n$ *and* $\sum_{n=1}^{\infty} nc_{n-1} x^{n+1}$ are power series solutions for a differential equation then find their sum
by introducing a single summation symbol.
(3)

Question No: 9 (Marks: 10)

(a) Develop a recurrence relation for the differential equation $y'' - xy = 0$ by applying power series method. **(8)**

(b) Discuss shortly the linear independence of the power series solutions

$$y_1 = A\sqrt{x}\left[2+\sum_{k=1}^{\infty}\frac{x^k}{(k+1)!5.8.11..........(3k+2)}\right]$$

and

$$y_2 = Bx\left[3+\sum_{k=1}^{\infty}\frac{x^k}{k!1.4.7..........(3k-2)}\right]$$

$$y''+\frac{1}{3x}(y'-y)=0$$

of the differential equation

(2)

Question No: 10 (Marks: 10)

$$y''-\frac{1}{x}y'+\frac{1}{(x-1)^3}y=0$$

(a) Determine the singular points of differential equation .Also classify each singular point as regular or irregular.

(5)

$$P_n(x)=\frac{1}{2^n\,n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

(b) Using Rodrigues Formula; to generate fourth Legendre's

$$P_3(x)$$

polynomial . **(5)**

Question No: 11 (Marks: 5)

$$\frac{dx}{dt}=3x-4y$$
$$\frac{dy}{dt}=4x-7y$$

$$X=\begin{bmatrix}1\\2\end{bmatrix}e^{-5t}$$

Verify that the vector is a solution of the system **;** .

MTH401 Deferential Equations

Mid Term Examination – Spring 2006

Time Allowed: 90 Minutes

1. The duration of this examination is 90 minutes.
2. Symbols by using math type should be pasted on the paper direct from the math type not from the word document otherwise it would not be visible.
3. Do not ask any questions about the contents of this examination from anyone.
   a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.
   b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.
   c. Write all steps, missing steps may lead to deduction of marks.
4. This examination is closed book and closed notes.
5. Use of Calculator is allowed.
6. Attempt all questions. Marks are written adjacent to each question.

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Question No. 1	**Marks : 1**

The method of undetermined coefficient is limited to homogeneous linear differential equation

- ★ True
- ★ False

Question No. 2	Marks : 1

In the homogeneous differential equation after substitution v=y/x the equation reduces to.

- ★ Separable differential equation.
- ★ Exact differential equation.
- ★ Remain homogeneous equation.
- ★ None of the other

Question No. 3	Marks : 10

Solve the differential equation by the variation of parameters

$$y'' - 9y' + 9y = xe^{3x}$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 x e^{3x}$$

Then just find the particular solution.

Question No. 4	Marks : 5

Determine whether the functions are linearly independent or dependent on $(-\infty, \infty)$

$$f_1(x) = 0,\ f_2(x) = x,\ f_3(x) = e^x$$

Question No. 5	Marks : 10

Solve

$$\frac{dy}{dx} + xy = xy^2$$

Question No. 6	Marks : 10

Solve the differential equation by integrating factor technique

$$y^2 dx + xydy = 0$$

Question No. 7	Marks : 1

If the Wronskian W of three function f(x),g(x),h(x) is zero, what can be said about the dependency of the functions

★ May or may not be dependent
★ Always dependent
★ Never dependent
★ None of the other

Question No. 8	Marks : 1

If $a_n(x) = 0$ in the differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + a_{n-2}(x)\frac{d^{n-2}y}{dx^{n-2}} + ... + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

for some $x \in I$ then

I. Solution of initial value problem may not unique.
II. Solution of initial value problem may not even exist.
III. Solution of initial value problem should exist.
IV. Solution of initial value problem is unique.

★ I is correct only
★ I and II are correct
★ I and III are correct
★ IV is correct only

Question No. 9	Marks : 1

Equation of the form $\frac{dy}{dx} + y = x^2 y^2$ is called

★ First order linear differential equation
★ Bernoulli equation
★ Separable equation
★ None of the other.

	MIDTERM EXAMINATION SEMESTER FALL 2004 MTH401- Differential Equations	Total Marks: 50 Duration: 60min

Instructions

1. Attempt all questions.
2. The Time allowed for this paper is 60 minutes.
3. This examination is closed book, closed notes, closed neighbors; any one found cheating will get zero grades in the course MTH401 Differential Equations.
4. You are not allowed to use any type of Table for Formulae of Differentiation and integration during your exam.
5. Each objective type question carries 2 marks and each Descriptive question carries 10 marks. So write every step of the solution of descriptive question to get maximum marks.
6. Do not ask any questions about the contents of this examination from anyone. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

Question No: 1 **Marks: 2**

A differential equation said to be ordinary differential equation if it contains only ordinary derivatives with respect to single variable.

- **T**
- F

Question No: 2 **Marks: 2**

A solution of the differential equation of the form y= f (x) is called the implicit solution.

- T
- **F**

Question No: 3 **Marks: 2**

Logistic equations are applications of non-linear equations.

- **T**
- F

Question No: 4 **Marks: 2**

The given functions $f_1(x)=5, f_2(x)=\cos^2 x, f_3(x)=\sin^2 x$ are linearly independent.

T

F

Question No: 5 **Marks: 2**

A set of functions whose wronskian is zero then set of functions may or may not be dependent.

T

F

Question No: 6 **Marks: 10**

(a) Define separable form. Just separate the variables of the given differential equation.

$$\sec y\frac{dy}{dx}+\sin(x-y)=\sin(x+y)$$

Solution

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form

d y / d x = h(x)g(y)

$$\sec y\frac{dy}{dx}+\sin(x-y)=\sin(x+y)$$

$$\sec y\frac{dy}{dx}=\sin(x+y)-\sin(x-y)$$

$$\sec y\frac{dy}{dx}=\sin(x+y)-\sin(x-y)$$

$$\sec y\frac{dy}{dx}=\sin x\cos y+\cos x\sin y-\sin x\cos y+\cos x\sin y$$

$$\sec y\frac{dy}{dx}=2\cos x\sin y$$

$$\frac{dy}{2\cos y\sin y}=\cos xdx$$

$$\frac{dy}{\sin 2y}=\cos xdx$$

$$\cos ec2ydy=\cos xdx$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further).**

$$e^x dx+\left(e^x\cot y+2y\cos ecy\right)dy=0$$

Solution

$$e^x dx + \left(e^x \cot y + 2y \cos ecy\right) dy = 0$$

$$M(x,y) = e^x, \quad N(x,y) = e^x \cot y + 2y \cos ecy$$

$$M_y = 0, \quad N_x = e^x \cot y$$

$$M_y \neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$e^x dx + \left(e^x \cot y + 2y \cos ecy\right) dy = 0$$

$$\frac{N_x - M_y}{M} = \frac{e^x \cot y - 0}{e^x} = \cot y$$

$$I.F = e^{\int \cot y dy} = e^{\ln \sin y} = \sin y$$

$$e^x \sin y dx + \left(e^x \cos y + 2y\right) dy = 0$$

$$M(x,y) = e^x \sin y, \quad N(x,y) = e^x \cos y + 2y$$

$$M_y = e^x \cos y, N_x = e^x \cos y$$

$$M_y = N_x$$

Which shows that equation is exact

Question No: 7 **Marks: 10**

(a) Solve Bernoulli equation $y + 2\frac{dy}{dx} = y^3(x-1)$ **(Just make the given equation linear in v, do not integrate)**

Solution

$$xy - \frac{dy}{dx} = y^3(x-1)$$

$$xy^{-2} - \frac{dy}{dx} y^{-3} = (x-1)$$

$$put\ y^{-2} = v$$

$$-2\frac{dy}{dx} y^{-3} = \frac{dv}{dx}$$

$$-\frac{dy}{dx} y^{-3} = \frac{1}{2}\frac{dv}{dx}$$

Then

$$\frac{1}{2}\frac{dv}{dx} + vx = (x-1)$$

$$\frac{dv}{dx} + 2vx = 2(x-1)$$

(b) The radioactive isotope of the lead, Pb-209, decay at a rate proportional to the amount present at any time and has a half-life of 4 hours. If 2 grams of the lead is present initially, how long will it take for 80% of the lead to decay? **(Just make the model of the radioactive decay as well as describe the given conditions do not solve further)**

Solution

Suppose that A_0 is the initial amount of isotope, as given A_0 is 100 and A (t) be the amount present at time t it governed by the differential equation.

$$\frac{dA}{dt}\alpha A$$

$$\frac{dA}{dt} = kA$$

$$\frac{dA}{A} = kdt$$

Integrate both sides

$$\ln A = kt + c$$

$$A = e^{kt+c}$$

$$A = e^{kt}e^{c}$$

$$A = P_0 e^{kt} \qquad say \;\; A_0 = e^{c}$$

Where A_0 is the initial amount of isotope

A_0 =2 = A(0)

A(4)=2/2=1

Then we have to find time when radioactive isotope will take 80% decay. So as A initially given 2 and 80% of 2 is 8/5 so decay would be 1-8/5 = -3/5

P (t) = -3/5 , t=?

Question No: 8 **Marks:10**

(a) Find a second solution of following differential equations where the first solution is given **(also write the formulae)**.

$$x^2 y'' - 4xy' + 6y = 0; \quad y_1 = x^2$$

Solution

$$x^2 y'' - 4xy' + 6y = 0 ; y_1 = x^2$$

differential equation can be written as

$$y'' - \frac{4}{x} xy' + \frac{6}{x^2} = 0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int pdx}}{y_1^{\,2}} dx$$

$$y_2 = x^2 \int \frac{e^{-\int -4/x\,dx}}{\left(x^2\right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\int 4/x\,dx}}{\left(x^3 \ln x\right)^2} dx$$

$$y_2 = x^2 \int \frac{e^{4 \ln x}}{x^4} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\ln x^4}}{x^4} dx$$

$$y_2 = x^2 \int \frac{x^4}{x^4} dx$$

$$y_2 = x^2 \int dx$$

$$y_2 = x^3$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = 4\sin\theta$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

Then just find particular solution.

Solution

We find a particular solution of non-homogeneous differential equation.

Suppose input function

$$y_p = A\cos\theta + B\sin\theta$$

Then

$$y'_p = -A\sin\theta + B\cos\theta$$

$$y''_p = -A\cos\theta - B\sin\theta$$

Substituting in the given differential equation

$$-A\cos\theta - B\sin\theta - 2\left(-A\sin\theta + B\cos\theta\right) - 3\left(A\cos\theta + B\sin\theta\right) = 4\sin\theta$$

$$\left(-A - 2B - 3A\right)\cos\theta + \left(-B + 2A - 3B\right)\sin\theta = 4\sin\theta$$

From the resulting equations

$-A-2B-3A=0;$ $\quad$ $-B+2A-3B=4$

$-4A-2B=0$; $\quad$ $2A-4B=4$

$2A+B=0;$ $\quad$ $A-2B=2 \rightarrow A=2+2B$

$\rightarrow 2(2+2B)+B=0$

$\rightarrow 4+5B=0$

$\rightarrow B=\dfrac{-4}{5}$

$\rightarrow A=2+2\left(\dfrac{-4}{5}\right)=\dfrac{2}{5}$

$y_p=\dfrac{2}{5}\cos\theta-\dfrac{4}{5}\sin\theta$

Question No: 9 **Marks: 10**

(a) Solve differential equation by the undetermined coefficient **(annihilator operator).**

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}=e^{2x}$$

If complimentary solution is given below

$y_c=c_1+c_2e^{4x}$

Then just find general solution.

Solution

In this case of input function is

g(x)= e^{2x}

further

$(D-2)(g(x))=(D-2)(e^{2x})=0$

Therefore the differential operator D^3 annihilates the function g. operating on both sides

$(D-2)(D^2-4D)y=(D-2)(e^{2x})$

$(D-2)(D^2-4D)y=0$

This is the homogeneous equation of order 3. Next we solve this higher order equation.

Thus auxiliary equation is

$(m-2)(m^2-4m)=0$

$m(m-2)(m-4)=0$

$m=0,2,4$

Thus its general solution of the differential equation must be

$y=c_1+c_2e^{4x}+c_3e^{2x}$

(b) Solve the differential equation by the variation of parameters

$y'' - 4y + 3 = \cos x$

If complimentary solution is given below

$y_c = c_1 e^{3x} + c_2 e^{x}$

Then just find particular solution **(do not integrate).**

Solution

$y'' - 4y + 3 = \cos x$

This equation is already in standard form

y''+ P(x)y' +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = $\cos x$

We construct the determinants

Since $y_1 = e^{3x}$, $y_2 = e^{x}$ so

$$W(y_1, y_2) = \begin{vmatrix} e^{3x} & e^{x} \\ 3e^{3x} & e^{x} \end{vmatrix} = e^{3x+x} - 3e^{3x+x} = -2e^{4x}$$

$$W_1 = \begin{vmatrix} 0 & e^{x} \\ \cos x & e^{x} \end{vmatrix} = \cos x e^{x}$$

$$W_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & \cos x \end{vmatrix} = \cos x e^{3x}$$

We determine the derivatives of the function u_1 and u_2

$$u'_1 = \frac{W_1}{W} = \frac{\cos x e^{x}}{-2e^{4x}} \rightarrow u_1 = \int \frac{\cos x}{-2} e^{-3x} dx$$

$$u'_2 = \frac{W_2}{W} = \frac{\cos x e^{3x}}{-2e^{4x}} \rightarrow u_2 = \int \frac{\cos x}{-2} e^{-x} dx$$

$$y_p = u_1 e^{3x} + u_2 e^{x}$$

is required particular solution

	MIDTERM EXAMINATION SEMESTER FALL 2004 MTH401- Differential Equations	Total Marks: 50 Duration: 60min

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Question No: 1 Marks: 2

The differential equation $\frac{dy}{dx} - y = e^x y^2$ is not Bernoulli equation.

- T
- **F**

Question No: 2 Marks: 2

$f(x, y) = \frac{x}{x^2 + y^2}$ Is homogeneous.

- T
- **F**

Question No: 3 Marks: 2

Population dynamics are not practical application of the first order differential equations.

- T
- **F**

Question No: 4	**Marks: 2**

A set

$$\{y_{1,} y_2, \dots, y_n\}$$

Of n linearly dependent solutions, on interval I, of the homogeneous linear nth-order differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

Is said to be a fundamental set of solutions on the interval I.

- T
- **F**

Question No: 5	**Marks: 2**

The differential operator that annihilates $10x^3 - 2x$ is D^4.

- **T**
- F

Question No: 6	**Marks:10**

(a) Define separable form. Just separate the variables of the given differential equation.

$$(3r\theta - 3\theta + r - 1)dr - (2r\theta + 4\theta - r - 2)d\theta = 0$$

Solution

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form

d y / d x = h(x)g(y)

$$(3r\theta - 3\theta + r - 1)dr - (2r\theta + 4\theta - r - 2)d\theta = 0$$

$$(3r\theta - 3\theta + r - 1)dr = (2r\theta + 4\theta - r - 2)d\theta$$

$$[3\theta(r-1) + 1(r-1)]dr = [2\theta(r+2) - 1(r+2)]d\theta$$

$$(r-1)(3\theta+1)dr = (r+2)(2\theta-1)d\theta$$

$$\frac{(r-1)}{(r+2)}dr = \frac{(2\theta-1)}{(3\theta+1)}d\theta$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further)**.

$$\left(\frac{3y^2 - x^2}{y^5}\right)\frac{dy}{dx} + \frac{x}{2y^5} = 0$$

Solution

It can also be written as

$$xdx+2\left(3y^2-x^2\right)dy=0$$
$$M(x,y)=x,\quad N(x,y)=6y^2-2x^2$$
$$M_y=0,\quad N_x=-4x$$
$$M_y\neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$xdx+\left(6y^2-2x^2\right)dy=0$$
$$\frac{N_x-M_y}{M}=\frac{-4x}{x}=-4=g(y)$$
$$I.F=e^{\int -4dy}=e^{-4y}$$
$$xe^{-4y}dx+e^{-4y}\left(3y^2-2x^2\right)dy=0$$
$$M(x,y)=xe^{-4y},\quad N(x,y)=e^{-4y}\left(3y^2-2x^2\right)$$
$$M_y=-4xe^{-4y},N_x=-4xe^{-4y}$$
$$M_y=N_x$$

Which shows that equation is exact

Question No: 7 **Marks:10**

(a) Solve the Bernoulli equation $x^3\frac{dy}{dx}+2xy=y^5$ (**Just make the given equation linear in v, do not integrate**)

Solution

$$x^3\frac{dy}{dx}+2xy=y^5$$
$$\frac{dy}{dx}y^{-5}+\frac{2}{x^2}y^{-4}=\frac{1}{x^3}$$
$$put\ y^{-4}=v$$
$$-4\frac{dy}{dx}y^{-5}=\frac{dv}{dx}$$
$$\frac{dy}{dx}y^{-5}=-\frac{1}{4}\frac{dv}{dx}$$

Then

$$-\frac{1}{4}\frac{dv}{dx}+\frac{2v}{x^2}=\frac{1}{x^3}$$
$$\frac{dv}{dx}-\frac{8v}{x^2}=\frac{-4}{x^3}$$

Thus it is linear in “v”.

(b) Initially there were 200 milligrams of a radioactive substance present. After 8 hours the mass increased by 4%. If the rate of decay is proportional to the amount of the substance present at any time, determine half-life of the radioactive substance? **(Just make the model of the radioactive decay as well as describe the given conditions do not solve further)**

Solution

Suppose that A_0 is the initial amount, as given A_0 is 200 and A(t) be the amount present at time t then its governed by the differential equation

$$\frac{dA}{dt}\alpha A$$

$$\frac{dA}{dt}=kA$$

$$\frac{dA}{A}=kdt$$

Integrate both sides

$$\ln A=kt+c$$

$$A=e^{kt+c}$$

$$A=e^{kt}e^{c}$$

$$A=A_0e^{kt} \qquad say\ A_0=e^{c}$$

Where A_0 is the initial amount

A_0=200 = A(0)

A(8)=200+$\frac{4}{100}(200)$

A(8)=208

A (T) =100

Question No: 8 **Marks:10**

(a) Find a second solution of following differential equations where the first solution is given **(also write the formulae)**. $(1+2x)y''+2xy'-4=0 \quad ,y_1=e^{-2x}$

Solution

$$(1+2x)y''+2xy'-4=0 \quad ,y_1=e^{-2x}$$

differential equation can be written as

$$y''+\frac{2x}{1+2x}y'-\frac{4}{1+2x}=0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int p dx}}{y_1^{\,2}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-\int \frac{2x}{1+2x} dx}}{\left(e^{-2x}\right)^2} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-\int \frac{1+2x-1}{1+2x} dx}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{\int (\frac{1}{1+2x} - 1) dx}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-x} e^{\ln(1+2x)}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{2\ln(1+2x)}}{e^{-3x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{\ln(1+2x)}}{e^{-3x}} dx$$

$$y_2 = e^{-2x} \int (1+2x) e^{3x} dx$$

$$y_2 = e^{-2x} \left[(1+2x)\frac{e^{3x}}{3} - \int (1+2x)' \frac{e^{3x}}{3} dx \right]$$

$$y_2 = e^{-2x} \left[(1+2x)\frac{e^{3x}}{3} - \int \frac{2e^{3x}}{3} dx \right]$$

$$y_2 = (1+2x)\frac{e^{x}}{3} - \frac{2e^{x}}{9}$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = e^{4x}$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

Then just find particular solution.

Solution

We find a particular solution of non-homogeneous differential equation.

Suppose input function

$y_p = Ae^{4x}$

Then

$y'_p = 4Ae^{4x}$

$y''_p = 16Ae^{4x}$

Substituting in the given differential equation

$16Ae^{4x} - 2\left(4Ae^{4x}\right) - 3Ae^{4x} = e^{4x}$

$e^{4x}\left(16A - 8A - 3A\right) = e^{4x}$

From the resulting equations

$5A = 1$

$A = \frac{1}{5}$

$y_p = \frac{1}{5}e^{4x}$

Question No: 9 **Marks: 10**

(a) Solve differential equation by the undetermined coefficient **(annihilator operator).**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = x\cos x$$

If complimentary solution is given below

$y_c = c_1 + c_2e^{4x}$

Then just find general solution.

Solution

In this case of input function is

g(x)= $x\cos x$

further

$$(D^2+1)^2(g(x)) = (D^2+1)^2(x\cos x) = 0$$

Therefore the differential operator $(D^2+1)^2$ annihilates the function g. operating on both sides

$$(D^2+1)^2(D^2-4D)y = (D^2+1)^2(x\cos x)$$

$$(D^2+1)^2(D^2-4D)y = 0$$

This is the homogeneous equation of order 6. Next we solve this higher order equation.

Thus auxiliary equation is

$(m^2+1)^2(m^2-4m)y=0$

$m(m-4)(m^2+1)^2=0$

$m=0,4,i,i,-i,-i$

Thus its general solution of the differential equation must be

$$y=c_1+c_2e^{4x}+(c_3+c_4x)\cos x+(c_5+c_6x)\sin x$$

(b) Solve the differential equation by the variation of parameters

$y''-y=x^2$

Complimentary solution is given below If

$y_c=c_1e^x+c_2e^{-x}$

Then just find particular solution **(do not integrate).**

Solution

$y''-y=x^2$

This equation is already in standard form

y''+ P(x)y' +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = x^2

We construct the determinants

Since $y_1=e^x$, $y_2=e^{-x}$ so

$$W(y_1,y_2)=\begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}=-e^{x-x}-e^{x-x}=-2$$

$$W_1=\begin{vmatrix} 0 & e^{-x} \\ x^2 & -e^{-x} \end{vmatrix}=-x^2e^{-x}$$

$$W_2=\begin{vmatrix} e^x & 0 \\ e^x & x^2 \end{vmatrix}=x^2e^x$$

We determine the derivatives of the function u_1 and u_2

$$u'_1=\frac{W_1}{W}=\frac{-x^2e^{-x}}{2}\rightarrow u_1=\int\frac{x^2}{2}e^{-x}dx$$

$$u'_2=\frac{W_2}{W}=\frac{x^2e^{-x}}{2}\rightarrow u_2=-\int\frac{x^2}{2}e^{-x}dx$$

$$y_p=u_1e^x+u_2e^{-x}$$

is required particular solution

MTH401 MID TERM PAST PAPERS (FILE PART II) SOLVED

BY MASOOM FAIRY

Note:

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- **There is an other file because of Large size of this one.**

MTH401 Deferential Equations

Mid Term Examination – Spring 2006

Time Allowed: 90 Minutes

Question No. 1	**Marks : 1**

The method of undetermined coefficient is limited to homogeneous linear differential equation

- True
- False Page 148

The Method of Undetermined Coefficient

The method of undetermined coefficients developed here is limited to non-homogeneous linear differential equations

Question No. 2 **Marks : 1**

In the homogeneous differential equation after substitution v=y/x the equation reduces to.

- Separable differential equation.
- **Exact differential equation.** **Lecture 5**
- Remain homogeneous equation.
- None of the other

Question No. 7 **Marks : 1**

If the Wronskian W of three function f(x),g(x),h(x) is zero, what can be said about the dependency of the functions

- **May or may not be dependent** page 113
- Always dependent
- Never dependent
- None of the other

A Vanishing Wronskian does not guarantee linear dependence of functions.

Question No. 8 **Marks : 1**

If $a_n(x) = 0$ in the differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + a_{n-2}(x)\frac{d^{n-2}y}{dx^{n-2}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

for some $x \in I$ then

I. Solution of initial value problem may not unique.
II. Solution of initial value problem may not even exist.
III. Solution of initial value problem should exist.
IV. Solution of initial value problem is unique.

- I is correct only
- I and II are correct
- I and III are correct
- IV is correct only

Question No. 9 **Marks : 1**

Equation of the form $\frac{dy}{dx} + y = x^2 y^2$ is called

- First order linear differential equation
- Bernoulli equation
- Separable equation

❺ **None of the other.**
According to all above equations.

FINALTERM EXAMINATION
FALL 2006
MTH401 - DIFFERENTIAL EQUATIONS (Session - 1)

Q: 1: If the variation of the path of the curves can be described by the concept of differential equations then which of the following differential equation describe the path for $y\ axis$.

► $\frac{dy}{dx} = 1$

► $\frac{dy}{dx} = 0$ **Not confirm**

► $\frac{dy}{dx} = -1$

► $\frac{dy}{dx} = \infty$

Q: 2: Suggestive form of the constant input function for the non homogeneous differential equation under the method entitled as "**Method of the undetermined coefficient**" is

1 $f(x) = e^x$

2 $f(x) = a$

3 $f(x) = e^{ax}\left(ACosx + BSinx\right)$

4 **Suggestive form is impossible. PAGE 148**

Q: 3: Which of the following function is linearly dependant to the exponential function e^x ?

- ► $-e^x$
- ► e^{-x} ***not confirm***
- ► xe^x
- ► $-xe^{-x}$

Q: 4: Eigen values for the system of the differential equations $X = AX$ are evaluated for the

- ► Solution vector X
- ► Coefficient matrix A
- ► Differentiated solution vector X
- ► Transpose of the Coefficient matrix A

Q: 5: Fundamental set of the solution vectors $X_1, X_2, \ldots, X_n$ for any system of the differential equations are obtained by

► **Developing the singleton set $\{X\} = \{c_1 X_1, c_2 X_2, \ldots\ldots, c_n X_n\}$ of the linear combinitions of the solution vectors.** **Page 389**

► Taking derivative of the each solution vector and forming the set

$X' \quad X' \quad X'$

- ▶ Taking Integral of the each solution vector and forming the set

$$\left\{\int X_1 dx, \int X_2 dx, \cdots\cdots, \int X_n dx\right\}$$

- ▶ Just verifying their linear independance and establishing the set

$$\left\{X_1, X_2, \cdots\cdots, X_n\right\}$$

MID TERM EXAMINATION
SPRING 2007
MTH401_ SESSION 4

Question No: 1 (Marks: 1) - Please choose one
The differential equation $(3x^2 y + 2)dx + (x^3 + y)dy = 0$ is

PAGE 26

- ▶ **Exact**
- ▶ Linear
- ▶ Homogenous
- ▶ Separable

Question No: 2 (Marks: 1) - Please choose one

The assumed particular solution for the U.C(Undetermined Coefficient) differential equation $y'' - y = x^2 e^{2x}$ is

- ▶ $y_p = c_1 e^{x_2} + c_2 x^2$
- ▶ $y_p = (Ax + B)e^{2x}$
- ▶ $y_p = (Ax^2 + Bx + c)e^{2x}$
- ▶ None of these.

Question No: 3 (Marks: 1) - Please choose one

The differential equation $x\frac{dy}{dx} + y = y^2 \ln x$ is an example of

- ▶ Separable
- ▶ Homogenous
- ▶ Exact
- ▶ **None of these.**

Question No: 4 (Marks: 1) - Please choose one

For the differential equation

$y' - 2xy = x$

Integrating factor is

► x^2

PAGE 34

► e^{x^2}

► e^{x^2}

► x^2

Question No: 5 (Marks: 1) - Please choose one

$$\frac{dy}{dx} = \frac{x + 3y - 5}{x - y - 1}$$

Identify the ordinary differential equation

► Homogenous

► Separable

► **Exact** **PAGE 26**

► None of these.

MIDTERM EXAMINATION
(Solution File)

SEMESTER SPRING 2004
MTH401- Differential Equations

Q: 1: The differential equation $\sec y \frac{dy}{dx} + \sin(x - y) = \sin(x + y)$ is

➔ **Separable** **PAGE 7**

Q: 2: The integrating factor of the differential equation $(x^2 + 1)\frac{dy}{dx} + 2xy = 1$ is

➔ **$x^2 + 1$** PAG 34

Q: 3: The form of the particular solution for the differential equation

$$y'' - y = x^4$$

➔ $y_p = A_4 x^4 + A_3 x^3 + A_2 x^2 + A_1 x + A_0$

Q: 4: Determine which of the given functions are linearly independent.

➔ $f_1(x) = 1 + x, f_2(x) = x, f_2(x) = x^2$

PAGE 110

Q: 5: The differential operator that annihilates $10x^3 \;\; 2x$ is:

➔ D^4

PAGE 167

Question No: 6	**Marks:10**

Solve the following differential equation by using an appropriate substitution.

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

Solution

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y}$$

$$\frac{dy}{dx} = \frac{y^2 + x^2}{xy}$$

$$Homogeneous\ equation,\ so\ put\ y = vx,\ \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 + x^2}{x^2v}$$

$$v + x\frac{dv}{dx} = v + \frac{1}{v}$$

$$x\frac{dv}{dx} = \frac{1}{v} \qquad vdv = \frac{1}{x}dx$$

$$\int vdv = \int \frac{1}{x}dx \qquad \frac{v^2}{2} = \ln x + \ln C$$

$$5 \qquad \frac{y^2}{x^2} = 2\ln xC$$

Question No: 7	**Marks:10**

The population of a town grows at a rate proportional to the population at any time. Its initial population of 500 increases by 15% in 10 years. What will be the population in 30 years?

Solution:

Let $P(t)$ be the population at any time t, then rate of grows will be

---

$$\frac{dP}{dt} P$$

$$\frac{dP}{dt} = kP$$

Here k is constant of proportionality. Since initially population was 500, therefore $P(0) = 500$. Also this population increases by 15% in 10 years. The 15% of 500 is $\frac{15}{100}(500) = 75$, therefore population after 10 years is (initial population + increase in 10 years) = 500+75 = 575 i.e. $P(10) = 575$. So we have the boundary value problem

$$\frac{dP}{dt} = kP \text{ subject to boundary conditions } P(0) = 500,\ P(10) = 575.$$

This first order differential equation. Its solution is given by

$P = Ce^{kt}$ where C is constant of integration.

Applying boundary conditions, we get $C = 500, k = 0.0139$. So the solution is

$$P(t) = 500e^{(0.0139)t}$$

Thus population after 30 years is obtained by putting t =30 in above equation i.e.

$$P(30) = 500e^{(0.0139)30}$$
$$\approx 760.$$

Question No: 8 **Marks:10**

Find a second solution of following differential equations where the first solution is given. You can use any method (reduction of order or formula given in handouts).

$$x^2 y'' + 2xy' - 6y = 0;\ y_1 = x^2$$

MIDTERM EXAMINATION (Solution File)

SEMESTER SPRING 2004
MTH401- Differential Equations

Question No: 1 **Marks: 2**

The differential equation $\frac{dy}{dx} = \frac{x+3y}{3x+y}$ is

Homogeneous

Question No: 2 **Marks: 2**

The integrating factor of the differential equation $\frac{dy}{dx} - y = e^{3x}$ is

e^x

Question No: 3 **Marks: 2**

The form of the particular solution for the differential equation

$$y' - y = \cos 2x$$

$y_p = A\cos 2x + B\sin 2x$ *repeat*

Question No: 4 **Marks:2**

Determine which of the given functions are linearly independent.

$f_1(x) = x,\ f_2(x) = x^2,\ f_2(x) = 4x - 3x^2$ **Repeated**

Question No: 5	Marks:2

The differential operator that annihilates $4e^{x/2}$ is:

- **2 D 1**

Question No: 6	Marks:10

Solve the following differential equations.

$$1 + \ln x + \frac{y}{x}\, dx = (1 - \ln x) dy$$

Solution:

Here

$$M = 1 + \ln x + \frac{y}{x}, \; N = -(1 - \ln x)$$

$$M_y = \frac{\delta M}{\delta y} = \frac{1}{x}, \qquad N_x = \frac{\delta N}{\delta x} = \frac{1}{x}$$

$$M_y = N_x$$

So the given equation is an exact equation. Thus there exists a function $f(x, y)$ such that

$$\frac{\delta f}{\delta x} = M \quad and \quad \frac{\delta f}{\delta y} = N$$

$$\frac{\delta f}{\delta x} = 1 + \ln x + \frac{y}{x} \; --(1) \quad and \quad \frac{\delta f}{\delta y} = \ln x - 1 \; --(2)$$

(1) $f = x + x \ln x - x + y \ln x + H(y) = x \ln x + y \ln x + H(y)$ $\frac{\delta f}{\delta y} = \ln x + H'(y)$

6 $2)$ $\ln x - 1 = \ln x + H'(y)$

1. $-1 = H'(y)$
2. $H(y) = -y$

Hence $f(x, y) = x \ln x + y \ln x - y$

Initially there were 100 milligrams of a radioactive substance present. After 6 hours the mass decreased by 3%. If the rate of decay is proportional to the amount of the substance present at any time, find the amount remaining after 24 hours.

Solution:

Let $A(t)$ be amount present at any time t. Then by given conditions, we have

$$\frac{dA}{dt}$$

$$A \quad \frac{dA}{dt} = kA$$

Initially there were 100 milligrams, therefore $A(0)=100$. Moreover, decreased by 3% will give us $100 - \frac{3}{100}(100) = 97$ milligrams after 6 hours i.e. $A(6) = 97$. So we have boundary value problem

$\frac{dA}{dt} = kA$ subject to boundary conditions $A(0)=100, A(6)= 97$

The solution of this equation is given by

$A(t) = Ce^{kt}$ where C is constant of integration.

Applying boundary conditions, we get

$C = 100, \quad k = -0.005076$

$$A(t)=100e^{-0.005076t}$$

Amount remaining after 24 hours is obtained by putting $t = 24$ in above equation i.e.

2. $A(t)=100e^{-0.005076(24)}$

$1\ 88.529 \ mg.$

Question No: 8 **Marks:10**

Find a second solution of following differential equations where the first solution is given. You can use any method (reduction of order or formula given in handouts).

$$x^2 y'' + y' = 0; \ y_1 = \ln x$$

Solution:

Comparing this equation with $y'' + P(x)y' + Q(x)y = 0$, we get

$$P(x) = \frac{1}{x^2}.$$

But second solution is given by

$$y_2 - y_1 \frac{e^{-\int P(x)dx}}{y^2}dx$$

1

MIDTERM EXAMINATION (Solution File)

SEMESTER SPRING 2004

MTH401- Differential Equations

Question No: 1 **Marks: 2**

The differential equation $(x+y)(x-y)dx + x(x-2y)dy = 0$ is

Exact **PAGE 26**

Question No: 2 **Marks: 2**

The integrating factor of the differential equation $(2y^2+3x)dx + 2xydy = 0$ is

x ***not confirm***

Question No: 3 **Marks: 2**

The form of the particular solution for the differential equation

$y'' - y = \cos x + e^x$ is:

$y_p = Ae^x + B\cos x + C\sin x$ ***Repeated***

Question No: 4 **Marks:2**

Determine which of the given functions are linearly independent.

$f_1(x) = x, f_2(x) = x^2, f_2(x) = 4x - 3x^2$ **REPEATED**

The differential operator that annihilates $4e^{2x}$ is:

$(D-2)(D+5)$

Question No: 6 **Marks:10**

Find the general solution of the given differential equation.

$$\frac{dy}{dx} + 2xy = x^3$$

Solution:

It is of the form $\frac{dy}{dx} + P(x)y = Q(x)$ i.e. Linear First Order Differential Equation with

$P(x) = 2x,\ Q(x) = x^3$

Thus integration factor is given by

$$I.F = u(x) = e^{\int P(x)dx}$$

$$= e^{\int 2xdx} = e^{x^2}$$

But the solution in this case is

$$y = \frac{\int u(x)Q(x)dx + C}{u(x)} \quad \text{--------------------(1.}$$

Now

$$\int u(x)Q(x)dx = \int x^3 e^{x^2}$$

$$= \frac{1}{2}\int (e^{x^2} 2x)x^2 dx$$

$$= \frac{1}{2}\left\{e^{x^2}x^2 - \int e^{x^2} 2xdx\right\} \quad \textit{int egration by parts}$$

$$= \frac{1}{2}\left\{e^{x^2}x^2 - e^{x^2}\right\}$$

So the solution is

$$y = \frac{\frac{1}{2}\{x^2 - 1\} e^{x^2} + C}{e^{x^2}}$$

$$= \frac{1}{2}(x^2 - 1) + Ce^{x^2}$$

Question No: 7 **Marks:10**

A thermometer is taken from an inside room to the outside where the air temperature is $5°F$. After 1 minute the thermometer reads $55°F$, and after 5 minutes the reading is $30°F$. What is the initial temperature of the room?

Solution:

Let $T(t)$ be temperature at any time t and T_0 be the temperature of the surroundings. Then by Newton's Method, we know that

$$\frac{dT}{dt} = k(T - T_0)$$

Where k is constant of proportionality. Here we are given $T_0 = 5$ and $T(1) = 55, T(5) = 30$. Solving above equation we get

$$T = T_0 + Ce^{kt}$$

$$7 \quad T = 5 + Ce^{kt}$$

Using above conditions we get

$$k = -0.173, C\ 59.44.$$

So the initial temperature is given by

$$\blacktriangleright \quad = 5 + Ce^0$$

$$5 + C$$

$$5 + 59.44 = 64.44^o F.$$

MTH401 MID TERM PAST PAPERS (FILE PART II) SOLVED

BY MASOOM FAIRY

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MIDTERM EXAMINATION

MTH401- Differential Equations

Question No: 1 **Marks: 2**

A differential equation said to be ordinary differential equation if it contains only ordinary derivatives with respect to single variable.

T

F

Question No: 2 **Marks: 2**

A solution of the differential equation of the form y= f (x) is called the implicit solution.

T

F

Question No: 3 **Marks: 2**

Logistic equations are applications of non-linear equations. **T**

F

Question No: 4	Marks: 2

The given functions $f_1(x) = 5, f_2(x) = \cos^2 x, f_3(x) = \sin^2 x$ are linearly independent.

T

F

Question No: 5 **Marks: 2**

A set of functions whose wronskian is zero then set of functions may or may not be dependent.

T

F

Question No: 6 **Marks: 10**

(a) Define separable form. Just separate the variables of the given differential equation.

$$\sec y\frac{dy}{dx} + \sin(x-y) = \sin(x+y)$$

Solution

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form d y / d x = h(x)g(y)

$$\sec y\frac{dy}{dx} + \sin(x-y) = \sin(x+y)$$

$$\sec y\frac{dy}{dx} = \sin(x+y) - \sin(x-y)$$

$$\sec y\frac{dy}{dx} = \sin(x+y) - \sin(x-y)$$

$$\sec y\frac{dy}{dx} = \sin x\cos y + \cos x\sin y - \sin x\cos y + \cos x\sin y$$

$$\sec y\frac{dy}{dx} = 2\cos x\sin y$$

$$\frac{dy}{2\cos y\sin y} = \cos x dx$$

$$\frac{dy}{\sin 2y} = \cos x dx$$

$$cosec 2y dy = \cos x dx$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further)**

$$e^x dx + (e^x \cot y + 2y \operatorname{cosec} y) dy = 0$$

Solution

$$e^x dx + \left(e^x \cot y + 2y \operatorname{cosec} y\right) dy = 0$$
$$M(x,y) = e^x, \quad N(x,y) = e^x \cot y + 2y \operatorname{cosec} y$$
$$M_y = 0, \quad N_x = e^x \cot y$$
$$M_y \neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$e^x dx + \left(e^x \cot y + 2y \operatorname{cosec} y\right) dy = 0$$
$$\frac{N_x - M_y}{M} = \frac{e^x \cot y - 0}{e^x} = \cot y$$
$$I.F. = e^{\int \cot y dy} = e^{\ln \sin y} = \sin y$$
$$e^x \sin y dx + \left(e^x \cos y + 2y\right) dy = 0$$
$$M(x,y) = e^x \sin y, N(x,y) = e^x \cos y + 2y$$
$$M_y = e^x \cos y, N_x = e^x \cos y$$
$$M_y = N_x$$

Which shows that equation is exact

Question No: 7 **Marks: 10**

(a) Solve Bernoulli equation $y + 2\frac{dy}{dx} = y^3(x-1)$ (**Just make the given equation linear in v, do not integrate**)

Solution

$$xy - \frac{dy}{dx} = y^3(x-1)$$
$$xy^{-2} - \frac{dy}{dx} y^{-3} = (x-1)$$
$$put\ y^{-2} = v$$
$$-2\frac{dy}{dx} y^{-3} = \frac{dv}{dx}$$
$$-\frac{dy}{dx} y^{-3} = \frac{1}{2}\frac{dv}{dx}$$

Then

$$\frac{1}{2}\frac{dv}{dx} + vx = (x-1)$$
$$\frac{dv}{dx} + 2vx = 2(x-1)$$

(b) The radioactive isotope of the lead, Pb-209, decay at a rate proportional to the amount present at any time and has a half-life of 4 hours. If 2 grams of the lead is present initially, how long will it take for 80% of the lead to decay? (**Just make the model of the radioactive decay as well as describe the given conditions do not further solve**)

Solution

Suppose that A_0 is the initial amount of isotope, as given A_0 is 100 and A (t) be the amount present at time t it governed by the differential equation.

$$\frac{dA}{dt} \alpha A$$

$$\frac{dA}{dt} = kA$$

$$\frac{dA}{A} = kdt$$

Integrate both sides

$$\ln A = kt + c$$

$$A = e^{kt + c}$$

$$A = e^{kt} e^{c}$$

$$A = P_0 e^{kt} \qquad say\ A_0 = e^c$$

Where A_0 is the initial amount of isotope

A_0 =2 = A(0)

A(4)=2/2=1

Then we have to find time when radioactive isotope will take 80% decay. So as A initially given 2 and 80% of 2 is 8/5 so decay would be 1-8/5 = -3/5

P (t) = -3/5 , t=?

Question No: 8 **Marks:10**

(a) Find a second solution of following differential equations where the first solution is given **(also write the formulae)** $x^2 y'' - 4xy' + 6y = 0; \quad y_1 = x^2$

Solution

$$x^2 y'' - 4xy' + 6y = 0; \ y_1 = x^2$$

differential equation can be written as

$$y'' - \frac{4}{x} xy' + \frac{6}{x^2} = 0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int pdx}}{y_1} dx$$

$$y_2 = x^2 \int \frac{e^{-\int 4/x\,dx}}{\left(x^2\right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\int 4/x\,dx}}{\left(x^3 \ln x\right)^2} dx$$

$$y_2 = x^2 \int \frac{e^{4 \ln x}}{x^4} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\ln x^4}}{x^4} dx$$

$$y_2 = x^2 \int \frac{x^4}{x^4} dx$$

$$y_2 = x^2 \int dx$$

$$y_2 = x^3$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = 4\sin\theta$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

Then just find particular solution.

Solution

We find a particular solution of non-homogeneous differential equation

Suppose input function

$$y_p = A\cos\theta + B\sin\theta$$

Then

$$y'_p = -A\sin\theta + B\cos\theta$$

$$y''_p = -A\cos\theta - B\sin\theta$$

Substituting in the given differential equation

$$-A\cos\theta - B\sin\theta - 2(-A\sin\theta + B\cos\theta) - 3(A\cos\theta + B\sin\theta) = 4\sin\theta$$

$$(-A \quad 2B - 3A)\cos\theta + (+B \quad 2A - 3B)\sin\theta = 4\sin\theta$$

From the resulting equations

$-A-2B-3A=0;$ $\quad -B+2A-3B=4$

$-4A-2B=0;$ $\quad 2A-4B=4$

$2A+B=0;$ $\quad A-2B=2 \rightarrow A=2+2B$

$\rightarrow 2(2+2B)+B=0$

$\rightarrow 4+5B=0$

$\rightarrow B=\frac{-4}{5}$

$\rightarrow A=2+2\left(\frac{-4}{5}\right)=\frac{2}{5}$

$y_p=\frac{2}{5}\cos\theta-\frac{4}{5}\sin\theta$

Question No: 9 **Marks: 10**

(a) Solve differential equation by the undetermined coefficient **(annihilator operator).**

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}=e^{2x}$$

If complimentary solution is given below

$y_c=c_1+c_2e^{4x}$

Then just find general solution.

Solution

In this case of input function is

g(x)= e^{2x}

further

$(D-2)(g(x))=(D-2)(e^{2x})=0$

Therefore the differential operator D^3 annihilates the function g. operating on both sides

$(D-2)(D^2-4D)y=(D-2)(e^{2x})$

$(D-2)(D^2-4D)y=0$

This is the homogeneous equation of order 3. Next we solve this higher order equation.

Thus auxiliary equation is

$(m-2)(m^2-4m)=0$

$m(m-2)(m-4)=0$

$m=0,2,4$

Thus its general solution of the differential equation must be

$y=c_1+c_2e^{4x}+c_3e^{2x}$

(b) Solve the differential equation by the variation of parameters

$y''-4y+3=\cos x$

If complimentary solution is given below

$y_c = c_1e^{3x} + c_2e^x$

Then just find particular solution**(do not integrate).**

Solution

$y''-4y+3=\cos x$

This equation is already in standard form

y″+ P(x)y′ +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = $\cos x$

We construct the determinants

Since $y_1 = e^{3x}$, $y_2 = e^x$ so

$$W(y_1, y_2) = \begin{vmatrix} e^{3x} & e^x \\ 3e^{3x} & e^x \end{vmatrix} = e^{3x+x} - 3e^{3x+x} = -2e^{4x}$$

$$W_1 = \begin{vmatrix} 0 & e^x \\ \cos x & e^x \end{vmatrix} = \cos x e^x$$

$$W_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & \cos x \end{vmatrix} = \cos x e^{3x}$$

We determine the derivatives of the function u_1 and u_2

$$u'_1 = \frac{W_1}{W} = \frac{\cos x e^x}{-2e^{4x}} \rightarrow u_1 = \int \frac{\cos x}{-2} e^{-3x} dx$$

$$u'_2 = \frac{W_2}{W} = \frac{\cos x e^{3x}}{-2e^{4x}} \rightarrow u_2 = \int \frac{\cos x}{-2} e^{-x} dx$$

$y_p = u_1e^{3x} + u_2e^x$

is required particular solution

MTH401 Deferential Equations

Mid Term Examination – Spring 2006

Question No. 1	**Marks : 1**

The method of undetermined coefficient is limited to homogeneous linear differential equation

- True
- False

Question No. 2	**Marks : 1**

In the homogeneous differential equation after substitution v=y/x the equation reduc

- Separable differential equation.
- Exact differential equation.
- Remain homogeneous equation.
- None of the other

Question No. 3	**Marks : 10**

Solve the differential equation by the variation of parameters $y''-9y'+9y = xe^{3x}$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 xe^{3x}$$

Then just find the particular solution.

Question No. 4	**Marks : 5**

Determine whether the functions are linearly independent or dependent on $(-\infty, \infty)$

$$f_1(x) = 0, f_2(x) = x, f_3(x) = e^x$$

Question No. 5	Marks : 10

Solve

$$\frac{dy}{dx} + xy = xy^2$$

Question No. 6	Marks : 10

Solve the differential equation by integrating factor technique

$$y^2 dx + xy dy = 0$$

Question No. 7	Marks : 1

If the Wronskian W of three function f(x),g(x),h(x) is zero, what can be said about the dependency of the functions

- May or may not be dependent
- Always dependent
- Never dependent
- None of the other

Question No. 8	Marks : 1

If $a_n(x) = 0$ in the differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + a_{n-2}(x)\frac{d^{n-2} y}{dx^{n-2}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

for some $x \in I$ then

I. Solution of initial value problem may not unique.
II. Solution of initial value problem may not even exist.
III. Solution of initial value problem should exist.
IV. Solution of initial value problem is unique.

- I is correct only
- I and II are correct
- I and III are correct
- IV is correct only

Question No. 9	**Marks : 1**

Equation of the form $\frac{dy}{dx} + y = x^2y^2$ is called

- First order linear differential equation
- Bernoulli equation
- Separable equation
- None of the other.

	MIDTERM EXAMINATION SEMESTER FALL 2004 MTH401- Differential Equations	

Question No: 1 **Marks: 2**

The differential equation $\frac{dy}{dx} - y = e^x y^2$ is not Bernoulli equation.

T

F

Question No: 2 **Marks: 2**

$f(x,y) = \frac{x}{x^2 + y^2}$ Is homogeneous.

T

F

Question No: 3 **Marks: 2**

Population dynamics are not practical application of the first order differential equations.

T

F

Question No: 4	Marks: 2

A set

$\{y_1, y_2, \ldots, y_n\}$

Of n linearly dependent solutions, on interval I, of the homogeneous linear nth-order differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

Is said to be a fundamental set of solutions on the interval I.

T

F

Question No: 5	Marks: 2

The differential operator that annihilates $10x^3 - 2x$ is D^4.

T

F

Question No: 6	Marks:10

(a) Define separable form. Just separate the variables of the given differential equation. $(3r\theta - 3\theta + r - 1)\,dr - (2r\theta + 4\theta - r - 2)\,d\theta = 0$

Solution

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form d y / d x = h(x)g(y)

$$(3r\theta - 3\theta + r - 1)\,dr - (2r\theta + 4\theta - r - 2)\,d\theta = 0$$

$$(3r\theta - 3\theta + r - 1)\,dr = (2r\theta + 4\theta - r - 2)\,d\theta$$

$$[3\theta(r-1) + 1(r-1)]\,dr = [2\theta(r+2) - 1(r+2)]\,d\theta$$

$$(r-1)(3\theta + 1)\,dr = (r+2)(2\theta - 1)\,d\theta$$

$$\frac{(r-1)}{(r+2)}\,dr = \frac{(2\theta - 1)}{(3\theta + 1)}\,d\theta$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further)**.

$$\left(\frac{3y^2 - x^2}{y^5}\right)\frac{dy}{dx} + \frac{x}{2y^5} = 0$$

Solution

It can also be written as

$$xdx + 2(3y^2 - x^2)dy = 0$$

$$M(x,y) = x, N(x,y) = 6y^2 - 2x^2$$

$$M_y = 0, \quad N_x = -4x$$

$$M_y \neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$xdx + (6y^2 - 2x^2)dy = 0$$

$$\frac{N_x - M_y}{M} = \frac{-4x}{x} = -4 = g(y)$$

$$I.F. = e^{\int -4dy} = e^{-4y}$$

$$xe^{-4y}dx + e^{-4y}(3y^2 - 2x^2)dy = 0$$

$$M(x,y) = xe^{-4y}, \quad N(x,y) = e^{-4y}(3y^2 - 2x^2)$$

$$M_y = -4xe^{-4y}, N_x = -4xe^{-4y}$$

$$M_y = N_x$$

Which shows that equation is exact

Question No: 7 **Marks:10**

(a) Solve the Bernoulli equation $x^3\frac{dy}{dx} + 2xy = y^5$ (**Just make the given equation linear in v, do not integrate**)

Solution

$$x^3\frac{dy}{dx} + 2xy = y^5$$

$$\frac{dy}{dx}y^{-5} + \frac{2}{x^2}y^{-4} = \frac{1}{x^3}$$

$$put\, y^{-4} = v$$

$$-4\frac{dy}{dx}y^{-5} = \frac{dv}{dx}$$

$$\frac{dy}{dx}y^{-5} = -\frac{1}{4}\frac{dv}{dx}$$

Then

$$-\frac{1}{4}\frac{dv}{dx} + \frac{2v}{x^2} = \frac{1}{x^3}$$

$$\frac{dv}{dx} - \frac{8v}{x^2} = \frac{-4}{x^3}$$

(b) Initially there were 200 milligrams of a radioactive substance present. After 8 hours the mass increased 4%. If the rate of decay is proportional to the amount of the substance present at any time, determine half-life of the radioactive substance? **(Just make the model of the radioactive decay as well as describe the conditions do not solve given further)**

Solution

Suppose that A is the initial amount, as given A_0 is 200 and A(t) be the amount present at time t then its governed by the differential equation

$$\frac{dA}{dt} \alpha A$$

$$\frac{dA}{dt} = kA$$

$$\frac{dA}{A} = kdt$$

Integrate both sides

$$\ln A = kt + c$$

$$A = e^{kt+c}$$

$$A = e^{kt} e^{c}$$

$$A = A_0 e^{kt} \qquad say \; A_0 = e^c$$

Where A_0 is the initial amount

A_0 =200 = A(0)

$A(8)=200+\frac{4}{100}(200)$

A(8)=208

A (T) =100

Question No: 8 **Marks :10**

(a) Find a second solution of following differential equations where the first solution is given. **(also write the formulae)** $(1+2x)y''+4xy'-4y=0, \quad y_1 = e^{-2x}$

Solution

$(1+2x)y''+4xy'-4y=0, \quad y_1 = e^{-2x}$

differential equation can be written as

$$y''+\frac{2x}{1+2x}y'-\frac{4}{1+2x}=0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int p dx}}{y_1} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-\int \frac{2x}{1+2x} dx}}{\left(e^{-2x}\right)^2} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-\int \frac{1+2x-1}{1+2x} dx}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{\int (\frac{1}{1+2x} - 1) dx}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-x} e^{\ln(1+2x)}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{2\ln(1+2x)}}{e^{-3x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{\ln(1+2x)}}{e^{-3x}} dx$$

$$y_2 = e^{-2x} \int (1+2x) e^{3x} dx$$

$$y_2 = e^{-2x} \left[(1+2x) \frac{e^{3x}}{3} - \int (1+2x)' \frac{e^{3x}}{3} dx \right]$$

$$y_2 = e^{-2x} \left[(1+2x) \frac{e^{3x}}{3} - \int \frac{2e^{3x}}{3} dx \right]$$

$$y_2 = (1+2x) \frac{e^{x}}{3} - \frac{2e^{x}}{9}$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = e^{4x}$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

Then just find particular solution.

Solution

We find a particular solution of non-homogeneous differential equation

Suppose input function

$y_p = Ae^{4x}$

Then

$y'_p = 4Ae^{4x}$

$y''_p = 16Ae^{4x}$

Substituting in the given differential equation

$16Ae^{4x} - 2(4Ae^{4x}) - 3Ae^{4x} = e^{4x}$

$e^{4x}(16A - 8A - 3A) = e^{4x}$

From the resulting equations

$5A = 1$

$A = \frac{1}{5}$

$y_p = \frac{1}{5}e^{4x}$

Question No: 9 **Marks: 10**

(a) Solve differential equation by the undetermined coefficient **(annihilator operator).**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = x\cos x$$

If complimentary solution is given below

$y_c = c_1 + c_2e^{4x}$

Then just find general solution.

Solution

In this case of input function is

g(x)= $x\cos x$

further

$(D^2+1)^2 g(x) = (D^2+1)^2(x\cos x) = 0$

Therefore the differential operator $(D^2+1)^2$ annihilates the function g. operating on both sides

$(D^2+1)^2(D^2-4D)y = (D^2+1)^2(x\cos x)$

$(D^2+1)^2(D^2-4D)y = 0$

This is the homogeneous equation of order 6. Next we solve this higher order equation.

Thus auxiliary equation is

$(m^2+1)^2(m^2-4m)y=0$

$m(m-4)(m^2+1)^2=0$

$m=0,4,i,i\ -i,-i$

Thus its general solution of the differential equation must be

$y=c_1+c_2e^{4x}+(c_3+c_4x)\cos x+(c_5+c_6x)\sin x$

(b) Solve the differential equation by the variation of parameters

$y''-y=x^2$

Complimentary solution is given below If

$y_c=c_1e^x+c_2e^{-x}$

Then just find particular solution **(do not integrate).**

Solution

$y''-y=x^2$

This equation is already in standard form

y''+ P(x)y' +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = x^2

We construct the determinants,

Since $y_1=e^x$, $y_2=e^{-x}$ so

$$W(y_1,y_2)=\begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}=-e^{x-x}-e^{x-x}=-2$$

$$W_1=\begin{vmatrix} 0 & e^{-x} \\ x^2 & -e^{-x} \end{vmatrix}=-x^2e^{-x}$$

$$W_2=\begin{vmatrix} e^x & 0 \\ e^x & x^2 \end{vmatrix}=x^2e^x$$

We determine the derivatives of the function u_1 and u_2

$$u'_1=\frac{W_1}{W}=\frac{-x^2e^{-x}}{2}\rightarrow u_1=\int\frac{x^2}{2}e^{-x}dx$$

$$u'_2=\frac{W_2}{W}=\frac{x^2e^{-x}}{2}\rightarrow u_2=-\int\frac{x^2}{2}e^{-x}dx$$

$y_p=u_1e^x+u_2e^{-x}$

is required particular solution