Question No : 1 of 38

The graph of an odd function is symmetrical about ----------------

Answer (Please select your correct option)

- x-axis
- y-axis
- origin ✓
- None of these

even function y axis
odd function origion

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

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Conduct Instructions

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MTH631 Real Analysis II

Question No : 10 of 34

The rate of change of $f(x,y)$ in the direction of a unit vector is called the --------------- of $f(x,y)$.

Answer (Please select your correct option)

- differential
- directional derivative
- mixed derivative
- None of these

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10

Question Summary : (Attempted Question)

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Question No : 11 of 34 Marks:

A function of several variables may have first order partial derivatives at a point X_0 but ——— continuous at X_0.

Answer (Please select your correct option)

- fail to be ✓
- must be
- always not
- None of these

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Question Summary : (Attempted Question)

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Question No : 12 of 34

A function of the form $L(X) = f_{x_1}(X_0)x_1 + f_{x_2}(X_0)x_2 + \dots + f_{x_n}(X_0)x_n$ is called ———

Answer (Please select your correct option)

- quadractic function
- differential of f at X_0 ✓ page 105
- constant function
- None of these

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12

Question Summary : (Attempted Question)

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If $h = f(x,y), x = f(r,\theta)$ and $y = f(r,\theta)$, then using the chain rule, $\frac{\partial h}{\partial \theta}$ = ...

Answer (Please select your correct option)

- $\frac{\partial f}{\partial r}\frac{\partial r}{\partial \theta} + \frac{\partial f}{\partial r}\frac{\partial r}{\partial \theta}$
- $\frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta}$
- $\frac{\partial f}{\partial x} + \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} + \frac{\partial y}{\partial \theta}$
- None of these

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Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 14 of 34

If $h = f(x,y), x = r\cos\theta$ and $y = r\sin\theta$, where then using the chain rule, $\frac{\partial h}{\partial r} =$ ....

Answer (Please select your correct option)

- $\cos\theta \frac{\partial f}{\partial \theta} + \sin\theta \frac{\partial f}{\partial r}$
- $\cos\theta \frac{\partial f}{\partial x} + \sin\theta \frac{\partial f}{\partial y}$
- $\cos\theta \frac{\partial f}{\partial x} - \sin\theta \frac{\partial f}{\partial y}$
- None of these

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14

Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 9 of 36

A point on the graph of a function at which the tangent line is parallel to x-axis is called the ....

Answer (Please select your correct option)

- critical point ✓
- point of inflection
- point of relative extrema
- None of these

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Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 5 of 34

If f and g are continuous functions at the point X_0 in R^n, then $\frac{f}{g}$ is continuous at X_0 provided that ——

Answer (Please select your correct option)

- $g(X_0) = 0$
- $g(X_0) \neq 0$ ✓
- $f(X_0) = g(X_0) = 0$
- None of these

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

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Question No : 3 of 34

__________ is the only region in R.

Answer (Please select your correct option)

- Set of finite points of R
- Interval
- Any finite sequence of R
- None of these

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3

Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 4 of 34

For the function $f(x,y)=\frac{xy}{x^2+y^2}$, the limit of $f(x,y)$ as $(x,y)\rightarrow(0,0)$ along the line $y=x$ is ----

Answer (Please select your correct option)

- $\frac{1}{2}$ ✓ p 87
- $-\frac{1}{2}$
- 0
- 1

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Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 2 of 34

If H is an open covering of a compact subset S then, S can be covered by ----------

Answer (Please select your correct option)

- infinitely many sets from H
- finitely many sets from H ✓ p 79
- atleast one set from H
- None of these

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2

Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 15 of 34

If $p(X)$ is a homogeneous polynomial of degree r in $X - X_0$. If $p(X) \le 0$ for all $X \ne X_0$. Then p is called ______

Answer (Please select your correct option)

- positive semidefinite
- positive definite
- negative semidefinite ✓ p 126
- negative definite

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Question Summary : (Attempted Question)

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Question No : 12 of 34

A function of the form $L(X) = f_{x_1}(X_0)x_1 + f_{x_2}(X_0)x_2 + \ldots + f_{x_n}(X_0)x_n$ is called ———

Answer (Please select your correct option)

- quadractic function
- differential of f at X_0
- constant function
- None of these

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Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 15 of 34

If $p(X)$ is a homogeneous polynomial of degree r in $X - X_0$. If $p(X) \le 0$ for all $X \ne X_0$. Then p is called ______

Answer (Please select your correct option)

- positive semidefinite
- positive definite
- negative semidefinite ✓
- negative definite

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

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Start | VU Examination Syste...

If $h = f(x,y), x = f(r,\theta)$ and $y = f(r,\theta)$, then using the chain rule, $\frac{\partial h}{\partial \theta} =$ ....

Answer (Please select your correct option)

- $\frac{\partial f}{\partial r} \cdot \frac{\partial r}{\partial \theta} + \frac{\partial f}{\partial r} \cdot \frac{\partial r}{\partial \theta}$
- $\frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial \theta}$
- $\frac{\partial f}{\partial x} + \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} + \frac{\partial y}{\partial \theta}$
- None of these

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Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 14 of 34

If $h = f(x,y), x = r\cos\theta$ and $y = r\sin\theta$, where then using the chain rule, $\frac{\partial h}{\partial r} =$ ---------

Answer (Please select your correct option)

- $\cos\theta\frac{\partial f}{\partial\theta} + \sin\theta\frac{\partial f}{\partial r}$
- $\cos\theta\frac{\partial f}{\partial x} + \sin\theta\frac{\partial f}{\partial y}$
- $\cos\theta\frac{\partial f}{\partial x} - \sin\theta\frac{\partial f}{\partial y}$
- None of these

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

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MTH631 Real Analysis II

Question No : 18 of 34 **Marks: 1**

A function f is said to be absolutely integrable on $[a,b)$ if f is locally integrable on $[a,b)$ and ------------

Answer (Please select your correct option)

- $\int_a^b |f(x)|dx < \infty$ ✓ p 137
- $\int_a^b |f(x)|dx = \infty$
- $\int_a^b |f(x)|dx \geq \infty$
- None of these

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Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 19 of 34

An absolutely convergent integral is always ----------------

Answer (Please select your correct option)

- divergent
- convergent ✓
- conditionally convergent
- None of these

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Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 20 of 34

If f is locally integrable on $[a,b)$ and $\int_a^b |f(x)|dx < \infty$, then $\int_a^b f(x)dx$ is ----------

Answer (Please select your correct option)

- divergent
- conditionally convergent
- convergent ✓
- None of these

p 137

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20

Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 34 of 34 Marks: 10 (Budgeted Time Min)

Suppose $U_0 \in T$ is a limit point of T, g is continuous at U_0 and f is continuous at $X_0 = G(U_0)$. Then, show that $f \circ g$ is continuous at U_0.

Answer (Please click here to Add Answer)

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

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MTH631 Real Analysis II

Question No : 33 of 34 **Marks: 5 (Budgeted Time**

Show that $(0,0)$ is a critical point of the function $p(x,y) = x^2 - 2xy + y^2 - x^4 - y^4$ and it has positive semidefinite second differentials at $(0,0$

Answer (Please **click here** to Add Answer)

100%

Start Time: 9:41 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

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MTH631 Real Analysis II

Question No : 32 of 34 Marks:

Define a locally integrable function. Also show that the function $f(x) = \log x$ is locally integrable on $[0, 1)$.

Answer (Please click here to Add Answer)

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Question Summary : (Attempted Question)

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MTH631 Real Analysis II

Question No : 17 of 26

A function f is said to be absolutely integrable on $[a,b)$ if f is locally integrable on $[a,b)$ and ———

page 167

Answer : (Please select your correct option)

- $\int_a^b |f(x)|dx < \infty$
- $\int_a^b |f(x)|dx = \infty$
- $\int_a^b |f(x)|dx \geq \infty$
- None of these

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[illegible]
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MTH631 Real Analysis II

Question No : [illegible]

The function $f(x) = (1-x)^{-p}$ is locally integrable on ———

Answer : (Please select your correct option)

- [0,1]
- [−1,1)
- (0,1)
- [0,1)

✓ p 131

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MTH631 Real Analysis II

Question No : 13 of 26

If value of a becomes zero in the Taylor series $\sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!}(x-a)^{n}$, then the series is called ______

page 44

Answer (Please select your correct option)

- Maclaurin series
- Fourier series
- Power series
- None of these

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MTH631 Real Analysis II

Question No : 11 of 26

If $f(x) = 3x + 5x^3$, then at $x = -1$, df = ______

Answer (Please select your correct option)

- 18dy
- 18dx
- 18

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MTH631 Real Analysis II

Question No : 19 of 26

A function of the form $L(X) = m_1 x_1 + m_2 x_2 + \ldots + m_n x_n$ for some constants m_i's is called ———

page 112

Answer (Please select your correct option)

- linear function
- quadratic function
- constant function
- None of these

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MTH631 Real Analysis II

Question No : 5 of 26

In a vector valued function $G = (g_1, g_2, \ldots, g_n)$. Then $g_1, g_2, \ldots, g_n$ are called the ———— functions of G

page 114

Answer (Please select your correct option)

- bijective
- surjective
- component
- None of these

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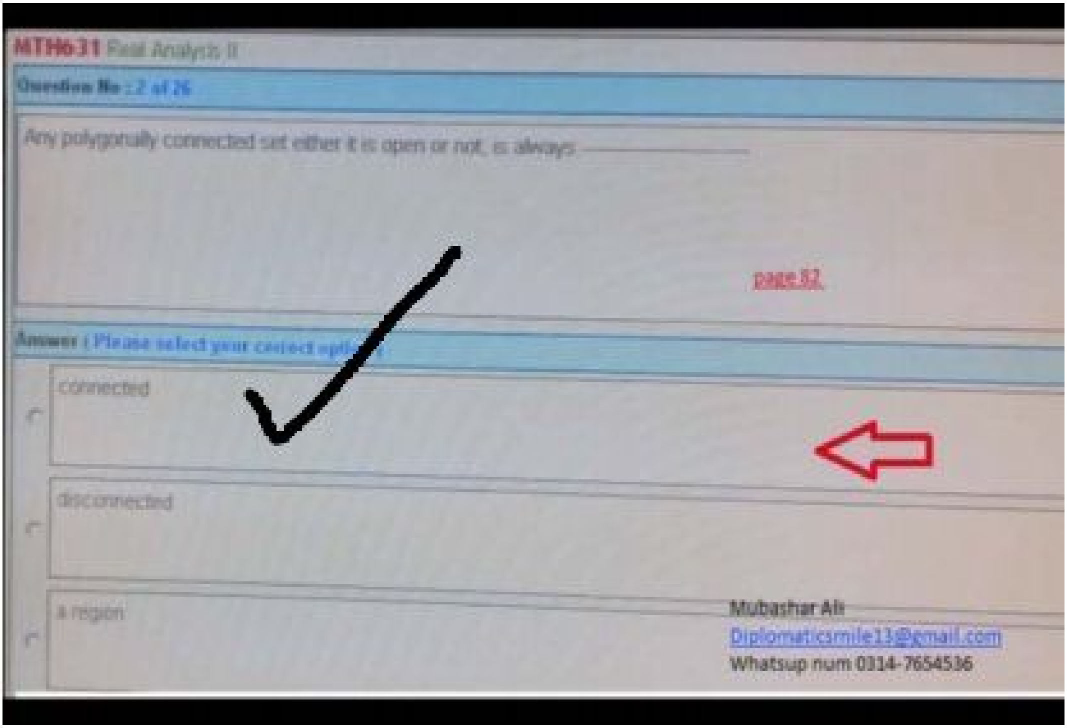
MTH631 Real Analysis II
Question No : 2 of 26
Any polygonally connected set either it is open or not, is always ________
page 82
Answer : Please select your correct option
connected
disconnected
a region
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MTH631 Real Analysis II

Question No : 3 of 26

The set $S = \{(x, y), x^2 + y^2 \le 1 \text{ or } x^2 + y^2 \ge 4\}$ is not a region in R^2, since it is ________

page 90

Answers (Please select your correct option)

- not connected ✓ p 82 ⇐
- connected
- polygonally connected
- None of these

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MTH631 Real Analysis II

Question No : 2 of 26

A set S is polygonally connected if every pair of points in S can be connected by a polygonal path lying ____________

Answer (Please select your correct option)

- outside the set S
- partially in S
- entirely in S ✓ p 81
- None of these

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MTH631 Real Analysis II

Question No : 1 of 26

A set is closed if and only if it always contains all of its ——

Answer (Please select your correct option)

- limit points ✓ p 78
- neighbourhood points
- interior points
- None of these

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MTH631 Real Analysis II

Question No : 26 of 26

Suppose that $f(x,y)=\begin{cases} 0 & \text{if x and y are rational,} \\ 1 & \text{if x is rational and y is irrational,} \\ 2 & \text{if x is irrational and y is rational,} \\ 3 & \text{if x and y are irrational} \end{cases}$

Find

$$\overline{\int_R} f(x,y)\,d(x,y) \quad \text{and} \quad \underline{\int_R} f(x,y)\,d(x,y) \quad \text{if} \quad R=[a,b]\times[c,d].$$

Let $\mathbf{P}=\{R_1,R_2,\ldots,R_k\}$. Let

$$M_j=\sup\{f(x,y)\,|\,\mathbf{X}\in R_j\} \quad \text{and} \quad m_j=\inf\{f(x,y)\,|\,\mathbf{X}\in R_j\}.$$

Let j be arbitrary in $\{1,2,\ldots,k\}$. Since R_j contains a point $(\widehat{x}_j,\widehat{y}_j)$ with $\widehat{x}_j$ and $\widehat{y}_j$ irrational, $M_j=3$. Hence, $\overline{\int_R} f(x,y)\,d(x,y)=3(b-a)(d-c)$. Since R_j contains a point $(\widetilde{x}_j,\widetilde{y}_j)$ with $\widetilde{x}_j$ and $\widetilde{y}_j$ rational, $m_j=0$. Hence, $\underline{\int_R} f(x,y)\,d(x,y)=0$.

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MTH631 Real Analysis II

Question No : 25 of 26

Prove: If a nonempty subset of R^n is both open and closed, then $S=R^n$.

Suppose that $\mathbf{X}_0$ is in S. If $S \neq \mathbb{R}^n$, there is an $\mathbf{X}_1 \notin S$. Let

$$H = \{\tau \mid (1-t)\mathbf{X}_0 + t\mathbf{X}_1 \in S \text{ for } 0 \le t < \tau\}.$$

Since S is open it contains a neighborhood of $\mathbf{X}_0$, so $H \neq \emptyset$. Since $\mathbf{X}_1$ is not in S, $\tau \le 1$ for all τ in H. Let $\rho = \sup H$ and $\overline{\mathbf{X}} = (1-\rho)\mathbf{X}_0 + \rho\mathbf{X}_1$; then $\overline{\mathbf{X}}$ is a limit point of S and so in S, because S is closed. Since S is open, it contains some ϵ-neighborhood of $\overline{\mathbf{X}}$, so $\rho + \epsilon/2$ is in H. This contradicts the definition of ρ. Hence, $S = R^n$.

MTH631 Real Analysis II

Question No : 24 of 26

If R is degenerate, then prove by using the definition of Riemann integral that $\int_R f(X)\,dA = 0$ if f is bounded on R.

Suppose, for example, that $R = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$ with $a_1 = b_1$. Then every partition $\mathbf{P}$ of R is of the form $\mathbf{P} = \{R_1, R_2, \ldots, R_k\}$, where $v(R_j) = 0$, $1 \le j \le k$. Therefore, every Riemann sum of f over R equals zero, so $\int_R f(\mathbf{X})\,d\mathbf{X} = 0$.

Answer (Please click here to Add Answer)

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MTH631 Real Analysis II

Question No : 23 of 26

Determine whether the integral $\int_0^{\infty} \frac{1+\cos^2 x}{\sqrt{1+x^2}}\,dx$ converges or diverges.

$\frac{1+\cos^2 x}{\sqrt{1+x^2}} > \frac{1}{\sqrt{1+x^2}} = \frac{1}{x\sqrt{1+1/x^2}} > \frac{1}{x\sqrt{2}}$ if $x > 1$. Since $\int_1^{\infty} \frac{dx}{x} = \infty$,
$\int_0^{\infty} \frac{1+\cos^2 x}{\sqrt{1+x^2}}\,dx = \infty$

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MTH631 Real Analysis II

Question No : 22 of 26

Calculate $\dfrac{\partial f(X)}{\partial \Phi}$ where $f(x,y,z) = \log(1+x+y+z)$, $\Phi = (0,1,0)$.

$h(t) = f(x+\phi_1 t, y+\phi_2 t, z+\phi_3 t) = \log(1+x+y+z+(\phi_1+\phi_2+\phi_3)t)$; $h'(t) = \dfrac{\phi_1+\phi_2+\phi_3}{1+x+y+z+(\phi_1+\phi_2+\phi_3)t}$; $\dfrac{\partial f(\mathbf{X})}{\partial \mathbf{L}} = h'(0) = \dfrac{\phi_1+\phi_2+\phi_3}{1+x+y+z}$; if $\mathbf{L} = (0,1,0)$ then $\dfrac{\partial f(\mathbf{X})}{\partial \mathbf{L}} = \dfrac{1}{1+x+y+z}$.

Question No : 20 of 26

A ———— surface in $\mathbb{R}^n$ has zero content.

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Answer (Please select your correct option)

- parabolic
- elliptical
- differentiable ✓ p 153
- None of these

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MTH611 Real Analysis II

Question No : 17 of 26

If f is integrable on a rectangle R, then ———

page 150

Answer (Please select your correct option)

- [illegible]
- [illegible] p 150
- [illegible]
- None of these

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MTH631 Real Analysis II

Question No : 18 of 26

Marks: 1

The upper integral $\overline{\int_a^b} f(x)\,d\alpha(x)$ of f over R is the infimum of all the ________

page 1..

Answer: (Please select your correct option)

- lower sums
- upper sums ✓ p 147 ⇦
- average sums
- None of these

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MTH631 Real Analysis II

Question No : 17 of 26 | Marks: 1 (Budgeted Time

f and g are locally integrable on $[a,b)$, $g(x) > 0$ and $f(x) \geq 0$ on some subintervals $[a_1, b)$ of $[a,b)$, and $\lim_{x \to b^-} \frac{f(x)}{g(x)} = M$. If $M = \infty$ and $\int_a^b g(x)dx = \infty$,

page 148

Answer (Please select your correct option)

- $\int_a^b f(x)dx$ and $\int_a^b g(x)dx$ converges together
- $\int_a^b f(x)dx \geq \infty$
- $\int_a^b f(x)dx = \infty$
- $\int_a^b f(x)dx < \infty$

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page 136.

Question No : 14 of 26

If $p(X)$ is a homogeneous polynomial of degree r in $X - X_0$ & $p(X) \geq 0$ for all X, then p is called ______

page 134

Answer (Please select your correct option)

- positive semidefinite ✓
- positive definite
- negative semidefinite
- negative definite

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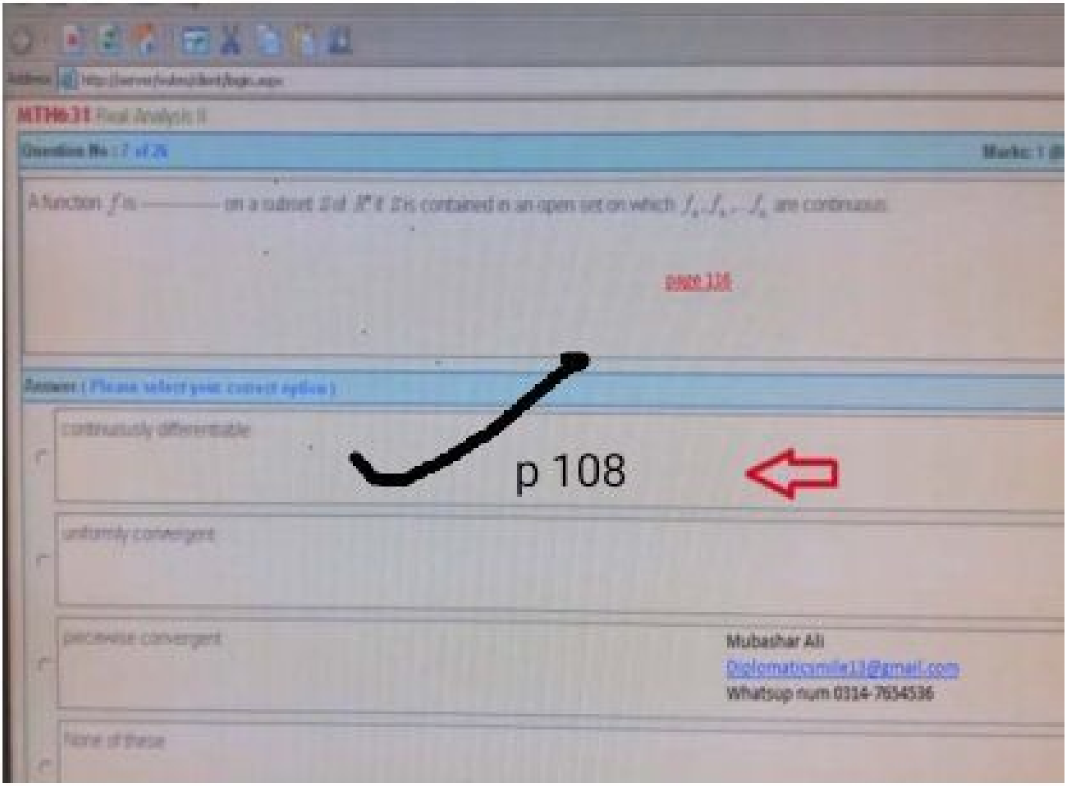

MTH631 Real Analysis II
Question No : 7 of 26
A function f is ———— on a subset S of R^n if S is contained in an open set on which f_x1, f_x2, ..., f_xn are continuous.
page 116
Answer (Please select your correct option)
continuously differentiable
p 108
uniformly convergent
piecewise convergent
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None of these

MTH631 Final Analysis II

Question No : 3 of 26

For the function $f(x,y)=\frac{xy}{x^2+y^2}$, the limit of $f(x,y)$ as $(x,y)\to(0,0)$ along the line $y=x$ is ——

page 87

Answer (Please select your correct option)

- $\frac{1}{2}$
- $-\frac{1}{2}$
- 0
- 1

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MTH631 Real Analysis II

Question No : 1 of 26

The singleton set is always ——

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Answer (Please select your correct option)

- disconnected
- polygonally disconnected
- connected
- None of these

MTH631 Real Analysis II

Question No : 2 of 26

Any polygonally connected set either it is open or not, is always ________

page 82

Answer (Please select your correct option)

- connected
- disconnected
- a region
- None of these

Question No : 3 of 26

Marks: 1 (Budgeted Ti

For the function $f(x,y)=\frac{xy}{x^2+y^2}$, the limit of $f(x,y)$ as $(x,y)\to(0,0)$ along the line $y=x$ is ——

page 87

Answer (Please select your correct option)

- $\frac{1}{2}$
- $-\frac{1}{2}$
- 0
- 1

MTH631 Real Analysis II

Question No : 4 of 26 — **Marks: 1 (Budgeted**

If f and g are continuous functions at the point X_0 in R^n, then $\frac{f}{g}$ is continuous at X_0 provided that ———

page 100

Answer (Please select your correct option)

- $g(X_0) = 0$
- $g(X_0) \neq 0$
- $f(X_0) = g(X_0) = 0$
- None of these

MTH631 Real Analysis II

Question No : 5 of 26 **Marks: 1 (Budgeted Ti**

Suppose $U_0 \in T$ is a limit point of T, g is continuous at U_0 and f is continuous at $X_0 = G(U_0)$. Then ——— is continuous at U_0.

page 102

Answer (Please select your correct option)

p 94

- $f \circ g$
- $\frac{f}{g}$
- $\frac{g}{f}$
- None of these

MTH631 Real Analysis II

Question No : 6 of 26 Marks: 1 (Budgeted T

A set A □ R of real numbers is ———— if there exists a real number M □ R, such that x ≤ M for every x □ A.

Answer (Please select your correct option)

- bounded below
- uniformly continuous
- bounded above
- None of these

MTH631 Real Analysis II

Question No : 7 of 26 **Marks: 1 (Bud**

A function f is ———— on a subset S of R^n if S is contained in an open set on which $f_{x_1}, f_{x_2}, \ldots f_{x_n}$ are continuous

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Answer (Please select your correct option)

- continuously differentiable ✓ ⇦
- uniformly convergent
- piecewise convergent
- None of these

MTH631 Real Analysis II

Question No : 8 of 26 Marks: 1 (Budgeted

A critical point at which a function attains its minimum value among all points where it is defined is called ....

Answer (Please select your correct option)

- global maximum
- global minimum ⇦
- supremum
- None of these

MTH631 Real Analysis II

Question No : 18 of 26

A function of the form $L(X) = m_1 x_1 + m_2 x_2 + \ldots + m_n x_n$ for some constants m_i's is called ———

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Answer (Please select your correct option)

- linear function
- quadratic function
- constant function
- None of these

MTH631 Real Analysis II

Question No : 10 of 26

A function of the form $L(X) = m_1 x_1 + m_2 x_2 + \dots + m_n x_n$ for some constants m_i's is called ———

page 112

Answer (Please select your correct option)

- linear function ⇦
- quadratic function
- constant function
- None of these

2.17.1 Linear Function

A *linear function* is a function of the form

$$L(\mathbf{X}) = m_1x_1 + m_2x_2 + \cdots + m_nx_n,$$

where m_1, m_2, $\ldots$, m_n are constants. From definition of
differentiable at $\mathbf{X}_0$ if and only if there is a linear function L s
can be approximated so well near $\mathbf{X}_0$ by

$$L(\mathbf{X}) - L(\mathbf{X}_0) = L(\mathbf{X} - \mathbf{X}_0)$$

MTH631 Real Analysis II

Question No : 9 of 26

For the function $f(x,y) = 3x^2y^3 + xy$ in two variables, $f_{xy}(0,0) =$ ....

Answer (Please select your correct option)

- 1
- -1
- 0
- None of these

correct

it is an *rth-order partial derivative of f*. The function

$$f(x, y) = 3x^2y^3 + xy$$

has partial derivatives everywhere. Its first-order partial derivatives are

$$f_x(x, y) = 6xy^3 + y, \quad f_y(x, y) = 9x^2y^2 + x.$$

Its second-order partial derivatives are

$$\begin{aligned} f_{xx}(x, y) &= 6y^3, & f_{yy}(x, y) &= 18x^2y, \\ f_{xy}(x, y) &= 18xy^2 + 1, & f_{yx}(x, y) &= 18xy^2 + 1. \end{aligned}$$

There are eight third-order partial derivatives. Some examples are

$$f_{xxy}(x, y) = 18y^2, \quad f_{xyx}(x, y) = 18y^2, \quad f_{yxx}(x, y) = 18y^2.$$

Compute $f_{xx}(0,0)$, $f_{yy}(0,0)$, $f_{xy}(0,0)$, and $f_{yx}(0,0)$ if

$$f(x, y) = \begin{cases} \frac{(x^2y+xy^2)\sin(x-y)}{x^2+y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

If $(x, y) \neq (0, 0)$, the ordinary rules for differentiation, applied sepa
and y, yield

$$\begin{aligned} f_x(x, y) &= \frac{(2xy+y^2)\sin(x-y)+(x^2y+xy^2)\cos(x-y)}{x^2+y^2} \\ &\quad - \frac{2x(x^2y+xy^2)\sin(x-y)}{(x^2+y^2)^2}, \quad (x, y) \neq (0, 0), \end{aligned}$$

and

$$\begin{aligned} f_y(x, y) &= \frac{(x^2+2xy)\sin(x-y)-(x^2y+xy^2)\cos(x-y)}{x^2+y^2} \\ &\quad - \frac{2y(x^2y+xy^2)\sin(x-y)}{(x^2+y^2)^2}, \quad (x, y) \neq (0, 0). \end{aligned}$$

These formulas do not apply if $(x, y) = (0, 0)$, so we find $f_x(0, 0)$ and $f_$
their definitions as difference quotients:

$$\begin{aligned} f_x(0, 0) &= \lim_{x \to 0} \frac{f(x, 0) - f(0, 0)}{x} = \lim_{x \to 0} \frac{0 - 0}{x} = 0, \\ f_y(0, 0) &= \lim_{y \to 0} \frac{f(0, y) - f(0, 0)}{y} = \lim_{y \to 0} \frac{0 - 0}{y} = 0. \end{aligned}$$

Setting $y = 0$ in (2.32) and (2.33) yields

$$f_x(x, 0) = 0, \quad f_y(x, 0) = \sin x, \quad x \neq 0,$$

$$f_y(0,0) = \lim_{y\to 0} \frac{f(0,y)-f(0,0)}{y} = \lim_{y\to 0} \frac{0-0}{y} = 0.$$

Setting $y = 0$ in (2.32) and (2.33) yields

$$f_x(x,0) = 0, \quad f_y(x,0) = \sin x, \quad x \neq 0,$$

2.16. Directional Derivative

so

$$f_{xx}(0,0) = \lim_{x\to 0} \frac{f_x(x,0)-f_x(0,0)}{x} = \lim_{x\to 0} \frac{0-0}{x} = 0.$$

$$f_{yx}(0,0) = \lim_{x\to 0} \frac{f_y(x,0)-f_y(0,0)}{x} = \lim_{x\to 0} \frac{\sin x - 0}{x} = 1.$$

Setting $x = 0$ in (2.32) and (2.33) yields

$$f_x(0,y) = -\sin y, \quad f_y(0,y) = 0, \quad y \neq 0,$$

so

$$f_{xy}(0,0) = \lim_{y\to 0} \frac{f_x(0,y)-f_x(0,0)}{y} = \lim_{y\to 0} \frac{-\sin y - 0}{y} = -1,$$

$$f_{yy}(0,0) = \lim_{y\to 0} \frac{f_y(0,y)-f_y(0,0)}{y} = \lim_{y\to 0} \frac{0-0}{y} = 0.$$

2.16.2 Equality of Mixed Partial Derivatives

Theorem: Suppose that f, f_x, f_y, and f_{xy} exist on a neighborhood N and f_{xy} is continuous at (x_0, y_0).

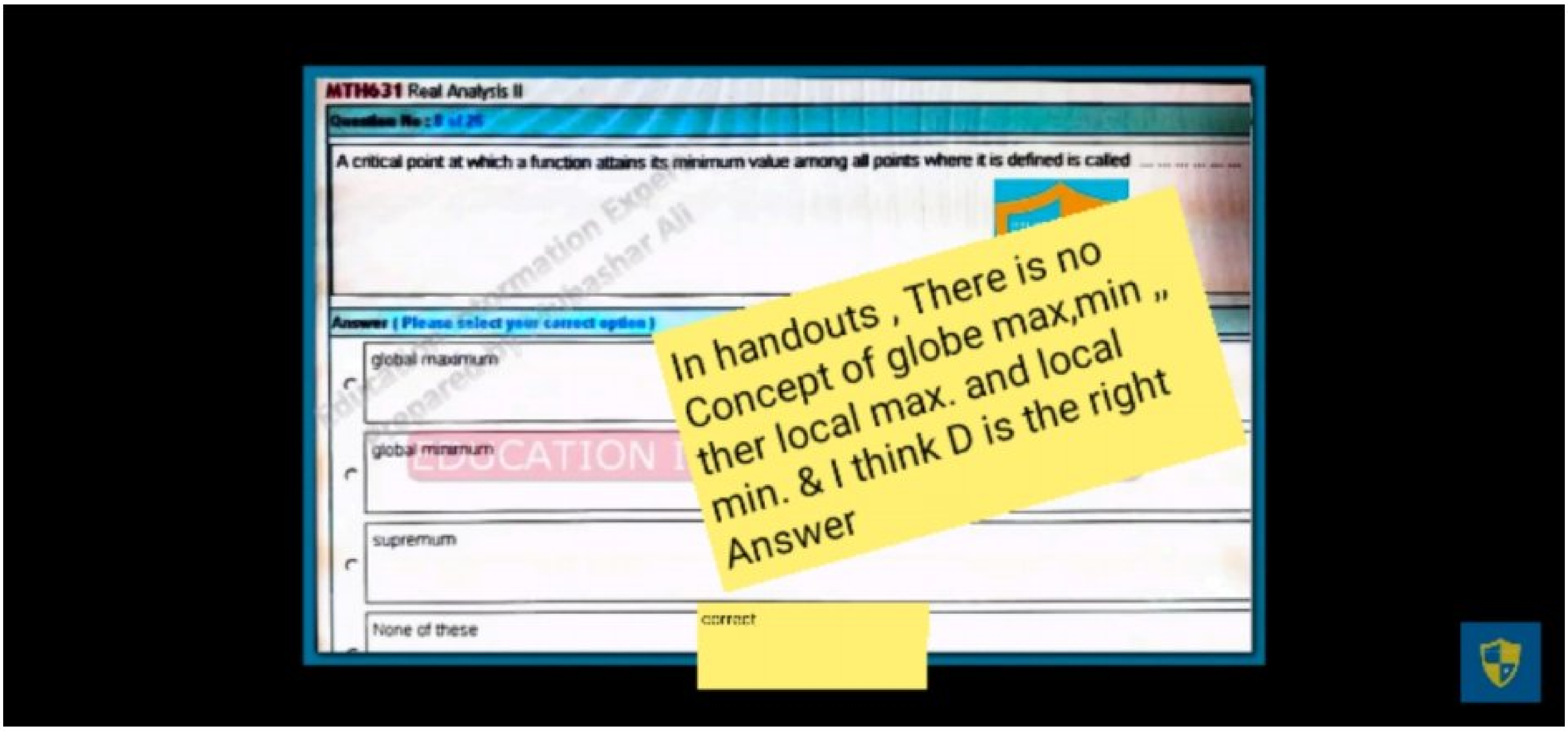
Then $f_{yx}(x_0, y_0)$ exists, and

$$f_{yx}(x_0, y_0) = f_{xy}(x_0, y_0).$$

Proof: Suppose that $\varepsilon > 0$. Choose $\delta > 0$ so that the ope

$$S_\delta = \{(x,y) : |x - x_0| < \delta, |y - y_0| < \delta\}$$

is in N.


MTH631 Real Analysis II
A critical point at which a function attains its minimum value among all points where it is defined is called
Answer (Please select your correct option)
global maximum
global minimum
supremum
None of these
In handouts , There is no Concept of globe max,min ,, ther local max. and local min. & I think D is the right Answer
correct

2.18 Maxima and Minima

We say that $\mathbf{X}_0$ is a *local extreme point* of f if there is a $\delta > 0$ such that

$$f(\mathbf{X}) - f(\mathbf{X}_0)$$

does not change sign in $S_\delta(\mathbf{X}_0) \cap D_f$.

More specifically, $\mathbf{X}_0$ is a *local maximum point* if

$$f(\mathbf{X}) \leq f(\mathbf{X}_0)$$

or a *local minimum point* if

$$f(\mathbf{X}) \geq f(\mathbf{X}_0)$$

for all $\mathbf{X}$ in $S_\delta(\mathbf{X}_0) \cap D_f$.

MTH631 Real Analysis II

Question No : 7 of 26

A $f_{x}, f_{x}, \ldots f_{x}$ exist on a neighborhood of X_0 and are continuous at X_0, then f is ——— at X_0

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Answer (Please select your correct option)

- uniformly converget
- piecewise convergent
- differentiable ✓
- None of these

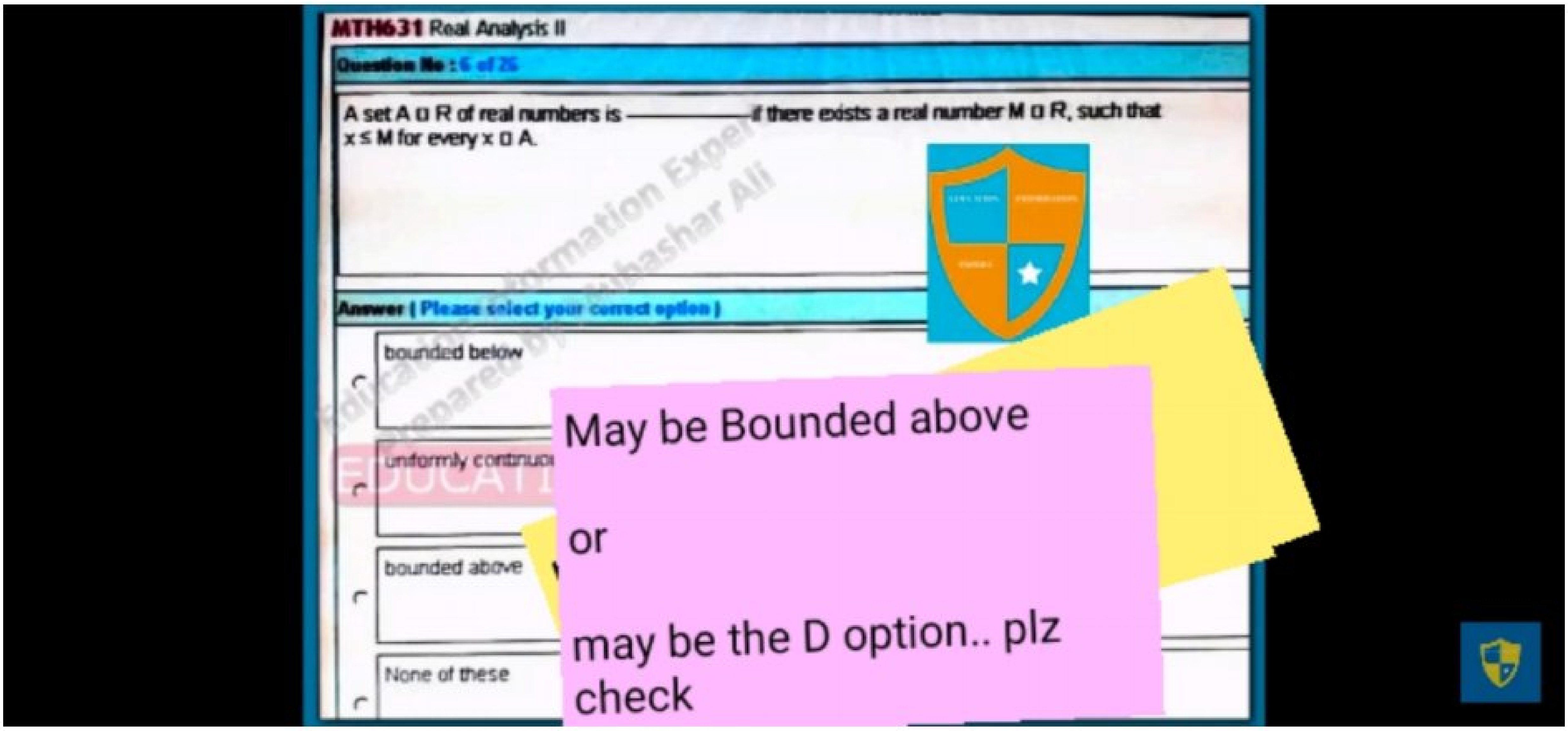
2.17.3 A sufficient Condition for Differentiability

Theorem: If f_{x_1}, f_{x_2}, $\ldots$, f_{x_n} exist on a neighborhood of $\mathbf{X}_0$ and are continuous at $\mathbf{X}_0$, then f is differentiable at $\mathbf{X}_0$.

Proof: Let $\mathbf{X}_0 = (x_{10}, x_{20}, \ldots, x_{n0})$ and suppose that $\varepsilon > 0$. Our assumptions imply that there is a $\delta > 0$ such that $f_{x_1}, f_{x_2}, \ldots, f_{x_n}$ are defined in the n-ball

$$S_\delta(\mathbf{X}_0) = \{\mathbf{X} : |\mathbf{X} - \mathbf{X}_0| < \delta\}$$

and


MTH631 Real Analysis II
Question No : 6 of 26
A set A □ R of real numbers is ———— if there exists a real number M □ R, such that x ≤ M for every x □ A.
Answer (Please select your correct option)
bounded below
uniformly continuo
bounded above
None of these
May be Bounded above
or
may be the D option.. plz check

1.12 Pointwise and Uniform Bounded Functions

A sequence of functions $\{F_n\}$ on the set S is said to be pointwise bounded on S if the sequence of functions is bounded for every $x \in S$, that is, if there exists a finite valued function $\phi(x)$ defined on S such that

$$|F_n(x)| < \phi(x), \quad x \in S, n = 1, 2, 3, ...$$

We say that $\{F_n\}$ is uniformly bounded on S if there exist a number M such that

$$|F_n(x)| < M, \quad x \in S, n = 1, 2, 3, ...$$

conclude that $f(\mathbf{C}) = u$ for some $\mathbf{C}$ in S.

Theorem: A function f is uniformly continuous on a subset S of its domain in $\mathbb{R}^n$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that

$$|f(\mathbf{X}) - f(\mathbf{X}')| < \varepsilon$$

whenever

$$|\mathbf{X} - \mathbf{X}'| < \delta$$

and $\mathbf{X}, \mathbf{X}' \in S$.

Remark: We emphasize that δ must depend only on ε and S, and not on the particular points $\mathbf{X}$ and $\mathbf{X}'$.

Theorem: If f is continuous on a compact set S in $\mathbb{R}^n$, then f is uniformly continuous on S.

$|x - p_1| < \delta$.

Since $\{F_n\}$ is pointwise bounded, there exists $M_i < \infty$ such that

$$|F_n(p_i)| < M_i, n \in \mathbb{N}.$$

If we take

$$M = \max\{M_1, ..., M_r\},$$

then $|F_n(x)| < M + \varepsilon$ for every $x \in S$. This proves the first part of the theorem.

We say that $\{F_n\}$ is uniformly bounded on S if there exist a number M such that

$$|F_n(x)| < M, \quad x \in S, n = 1, 2, 3, \ldots.$$

MTH631 Real Analysis II

Question No : 5 of 26

In a vector valued function $G = (g_1, g_2, \ldots, g_n)$. Then $g_1, g_2, \ldots, g_n$ are called the ———— functions of G .

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Answer (Please select your correct option)

- bijective
- surjective
- component
- None of these

2.19 Differentiable Vector Valued Function

A vector-valued function $\mathbf{G} = (g_1, g_2, \ldots, g_n)$ is *differentiable* at

$$\mathbf{U}_0 = (u_{10}, u_{20}, \ldots, u_{m0})$$

if its component functions g_1, g_2, $\ldots$, g_n are differentiable at $\mathbf{U}_0$.

Lemma: Suppose that $\mathbf{G} = (g_1, g_2, \ldots, g_n)$ is differentiable at

$$\mathbf{U}_0 = (u_{10}, u_{20}, \ldots, u_{m0}),$$

and define

$$M = \left(\sum_{i=1}^{n} \sum_{j=1}^{m} \left(\frac{\partial g_i(\mathbf{U}_0}{\partial u_j} \right)^2 \right)^{1/2}.$$

Then, if $\epsilon > 0$, there is a $\delta > 0$ such that

MTH631 Real Analysis II

Question No : 4 of 26

The quotient of two continuous functions is a ———— function wherever the denominator is non-zero.

Answer (Please select your correct option)

- differentiable
- continuous
- composite
- None of these

MTH631 Real Analysis II

Question No : 3 of 26

The set $S = \{(x, y), x^2 + y^2 \leq 1 \text{ or } x^2 + y^2 \geq 4\}$ is not a region in R^2, since it is ——

page 90

Answer (Please select your correct option)

- not connected ✓
- connected
- polygonally connected
- None of these

Example: Intervals are the only regions in $\mathbb{R}$. The n-ball $B_r(\mathbf{X}_0)$ is a region in $\mathbb{R}^n$, as is its closure $\overline{S}_r(\mathbf{X}_0)$. The set $S = \{(x, y) : x^2 + y^2 \leq 1 \quad \text{or} \quad x^2 + y^2 \geq 4\}$ is not a region in $\mathbb{R}^2$, since it is not connected.

The set S_1 obtained by adding the line segment

$$L_1: \quad \mathbf{X} = t(0, 2) + (1 - t)(0, 1), \quad 0 < t < 1,$$

MTH631 Real Analysis II

Question No : 2 of 26

A set S is polygonally connected if every pair of points in S can be connected by a polygonal path lying ————

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Answer (Please select your correct option)

- outside the set S
- partially in S
- entirely in S
- None of these

2.8 Polygonally Connected Set

A set S is *polygonally connected* if every pair of points in S can be connected by a polygonal path lying entirely in S.

Theorem: An open set S in $\mathbb{R}^n$ is connected if and only if it is polygonally connected.

MTH631 Real Analysis II

Question No : 1 of 26

A set is closed if and only if it always contains all of its ——

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Answer (Please select your correct option)

- limit points
- neighbourhood points
- interior points
- None of these

limit $\overline{\mathbf{X}}$. Since $\overline{\mathbf{X}}$ is a limit point of every S_k and every S_k is closed, $\overline{\mathbf{X}}$ is in e
S_k (**A set is closed if and only if it contains all its limit points**). There
$\overline{\mathbf{X}} \in I$, so $I \neq \emptyset$. Moreover, $\overline{\mathbf{X}}$ is the only point in I, since if $\mathbf{Y} \in I$, then

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MTH631 Real Analysis II

Question No : 19 of 26

The lower integral $\underline{\int_R} f(X)d(X)$ of f over R is the supremum of all the ——

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Answer (Please select your correct option)

- lower sums ⇦
- upper sums
- average sums
- None of these

$$s(\mathbf{P}) = \sum_{j=1}^{k} m_j V(R_j).$$

The *lower integral of f over R*, denoted by

$$\underline{\int_R} f(\mathbf{X})\, d\mathbf{X},$$

is the supremum of all lower sums.

Theorem: Let f be bounded on a rectangle R and le

Then

1. The upper sum $S(\mathbf{P})$ of f over $\mathbf{P}$ is the supremu
   sums of f over $\mathbf{P}$.

MTH631 Real Analysis II

Question No : 20 of 26

If f and g are integrable on S and $f(X) \leq g(X)$ for X in S, then ———

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Answer (Please select your correct option)

- $\int_S f(X)dX \geq \int_S g(X)dX$
- $\int_S f(X)dX \leq \int_S g(X)dX$
- $\int_S f(X)dX > \int_S g(X)dX$
- None of these

Theorem: If f and g are integrable on S and $f(\mathbf{X}) \leq g(\mathbf{X})$ for $\mathbf{X}$ in S, then

$$\int_S f(\mathbf{X})\, d\mathbf{X} \leq \int_S g(\mathbf{X})\, d\mathbf{X}.$$

Theorem: If f is integrable on S, then so is $|f|$, and

$$\left| \int_S f(\mathbf{X})\, d\mathbf{X} \right| \leq \int_S |f(\mathbf{X})|\, d\mathbf{X}.$$

Theorem: If f and g are integrable on S, then so is the product fg.

MTH631 Real Analysis II

Question No : [illegible]

The upper integral $\overline{\int_R} f(X)d(X)$ of f over R is the infimum of all the————

page 135

Answer (Please select your correct option)

- lower sums
- upper sums ✔
- average sums
- None of these

3.9 Upper and Lower Integrals

If f is bounded on a rectangle R in $\mathbb{R}^n$ and $\mathbf{P} = \{R_1, R_2, \ldots, R_k\}$ is a partition of R.

Let

$$M_j = \sup_{\mathbf{X} \in R_j} f(\mathbf{X}), \quad m_j = \inf_{\mathbf{X} \in R_j} f(\mathbf{X}).$$

The *upper sum* of f over $\mathbf{P}$ is

$$S(\mathbf{P}) = \sum_{j=1}^{k} M_j V(R_j).$$

MTH631 Real Analysis II

Question No : 17 of 26

A function f is said to be absolutely integrable on $[a,b)$ if f is locally integrable on $[a,b)$ and ———

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Answer (Please select your correct option)

- $\int_a^b |f(x)|dx < \infty$
- $\int_a^b |f(x)|dx = \infty$
- $\int_a^b |f(x)|dx \geq \infty$
- None of these

3.2 Absolute integrability

We say that f is *absolutely integrable on* $[a, b)$ if f is locally integrable on $[a, b)$ and $\int_a^b |f(x)|\, dx < \infty$. In this case we also say that $\int_a^b f(x)\, dx$ *converges absolutely* or *is absolutely convergent*.

Remark: If f is nonnegative and integrable on $[a, b)$, then f is absolutely integrable on $[a, b)$, since $|f| = f$.

Example: The empty set and singleton sets are connected, because they cannot represented as the union of two disjoint nonempty sets.

2.6 Heine-Borel Theorem

We are going to state and prove the Heine-Borel theorem for $\mathbb{R}^n$.

This theorem concerns *compact* sets. As in $\mathbb{R}$, a compact set in $\mathbb{R}^n$ is a closed and bounded set.

of *supremum* and *infimum* of a function on a set S.

Theorem: If f is continuous on a compact set S in $\mathbb{R}^n$, then f is bounded on S.

MTH631 Real Analysis II

Question No : 1 of 75

The singleton set is always —

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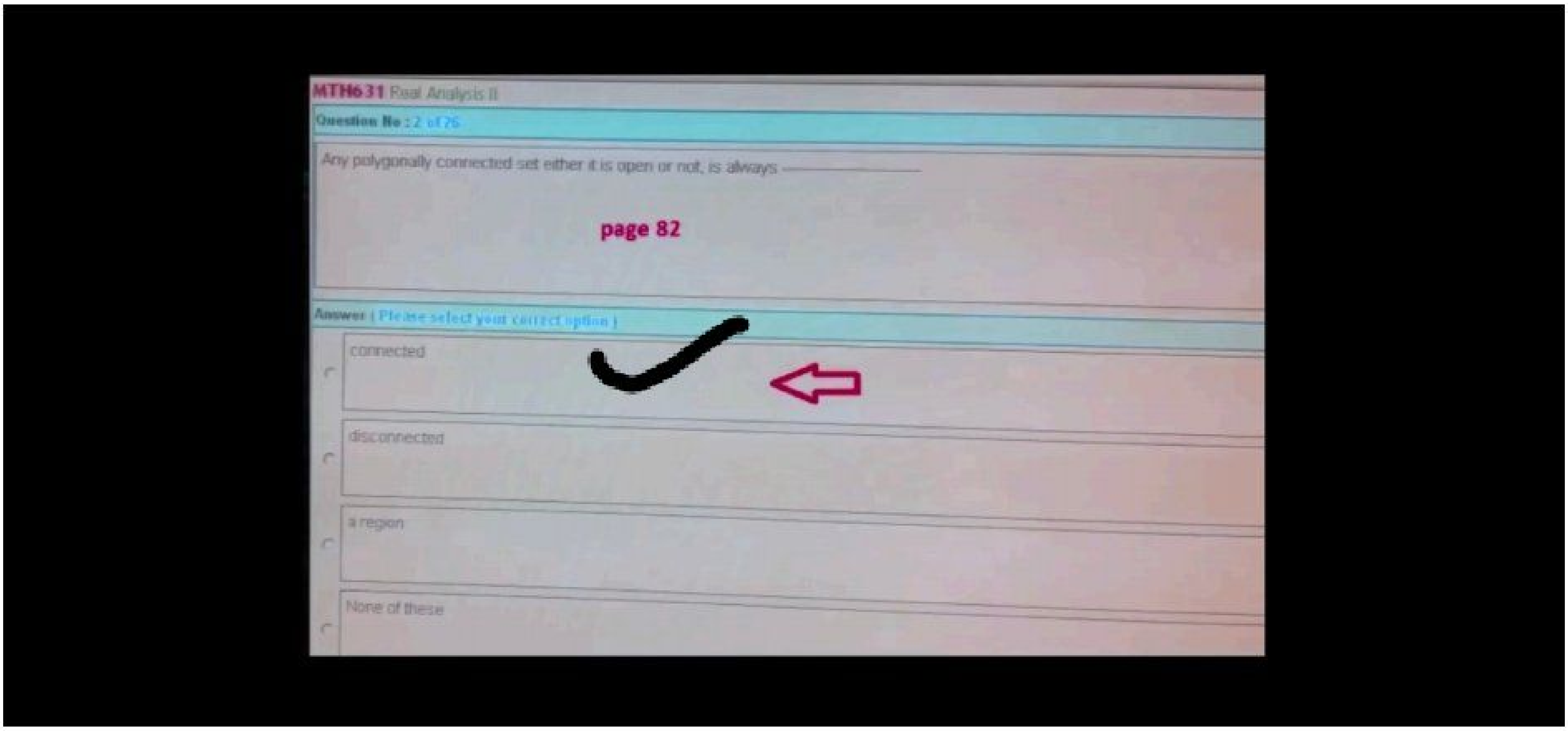
Answer (Please select your correct option)

- disconnected
- polygonally disconnected
- connected ✓ corr
- None of these

f S can be expressed in this way, then S is *disconnected*.

mple: The empty set and singleton sets are connected, because

esented as the union of two disjoint nonempty sets.


MTH631 Real Analysis II
Question No : 2 of 76
Any polygonally connected set either it is open or not, is always ——————
page 82
Answer (Please select your correct option)
connected
disconnected
a region
None of these

e, $A = S$, which completes the proof of necessity.

Any polygonally connected set, open or not, is connected.

A set (not open) may be connected but not polygonally connec

Example: The function

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

is defined everywhere in $\mathbb{R}^2$ except at $(0, 0)$. Does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist?

If we try to answer this question by letting (x, y) approach $(0, 0)$ along the line $y = x$, we see the functional values

$$f(x, x) = \frac{x^2}{2x^2} = \frac{1}{2}$$

and conclude that the limit is $1/2$.

MTH631 Real Analysis II

Question No : 1 of 26 | **Marks: 1 (Budgeted**

If f and g are continuous functions at the point X_0 in R^n, then $\frac{f}{g}$ is continuous at X_0 provided that ————

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Answer (Please select your correct option)

- $g(X_0)=0$
- $g(X_0)\neq 0$ ✓ ⇦
- $f(X_0)=g(X_0)=0$
- None of these

valid for all p and q. The quotient

$$f(x) = \frac{h(x)}{g(x)} \tag{1.51}$$

of two power series

$$\begin{aligned} h(x) &= \sum_{n=0}^{\infty} c_n (x - x_0)^n, \quad |x - x_0| < R_1, \\ g(x) &= \sum_{n=0}^{\infty} b_n (x - x_0)^n, \quad |x - x_0| < R_2, \end{aligned}$$

can be represented as a power series

$$f(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n \tag{1.52}$$

with a positive radius of convergence, provided that

$$b_0 = g(x_0) \neq 0.$$

MTH631 Real Analysis II

Question No : 5 of 25 **Marks:** 1 (Budgeted Ti

Suppose $U_0 \in T$ is a limit point of T, g is continuous at U_0 and f is continuous at $X_0 = G(U_0)$. Then ——— is continuous at U_0.

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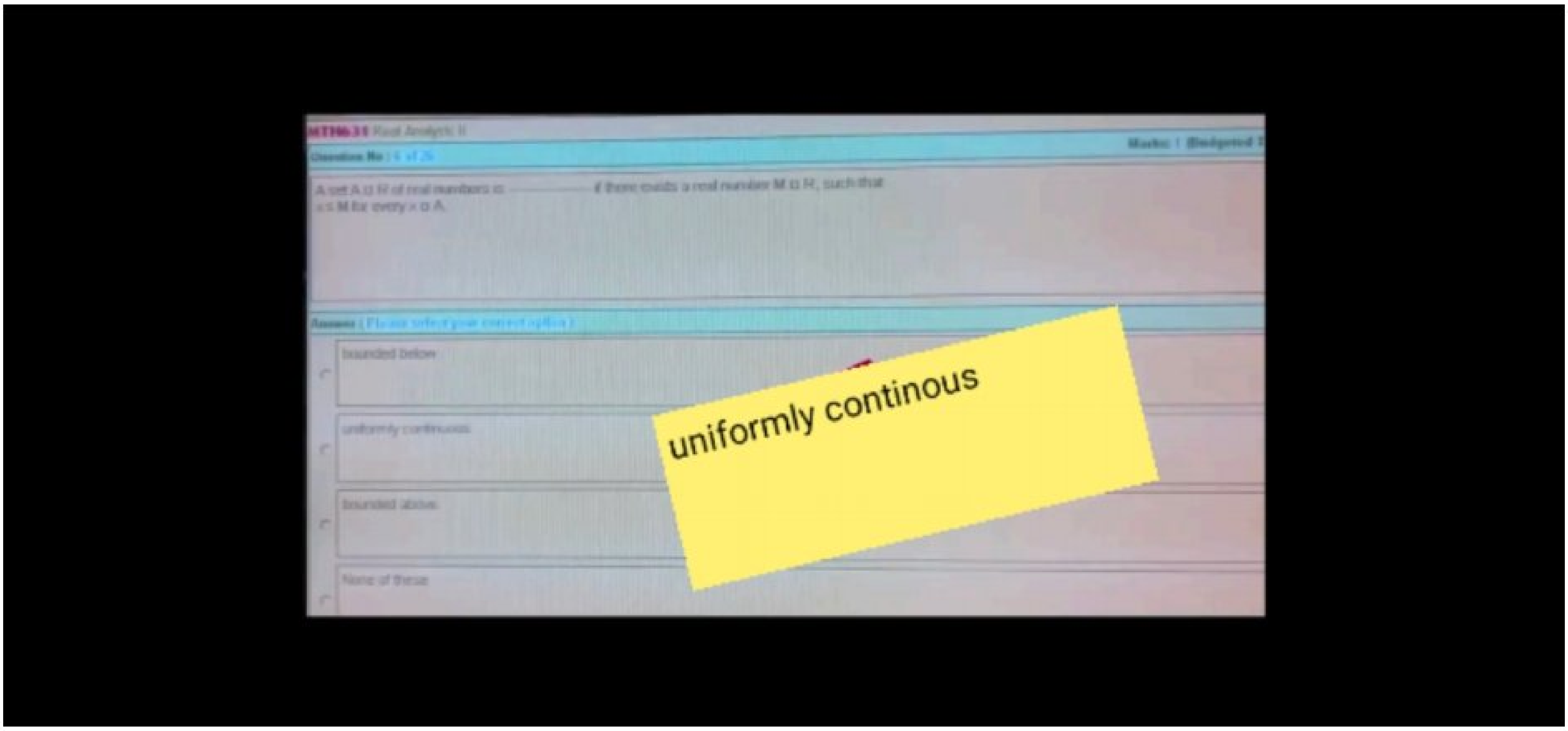
Answer (Please select your correct option)

- $f \circ g$
- $\frac{f}{g}$
- $\frac{g}{f}$
- None of these

Theorem: Suppose that $\mathbf{U}_0$ is in T and is a limit point of T, $\mathbf{G}$ is continuous at $\mathbf{U}_0$, and f is continuous at $\mathbf{X}_0 = \mathbf{G}(\mathbf{U}_0)$. Then $h = f \circ \mathbf{G}$ is continuous at $\mathbf{U}_0$.


bounded below
uniformly continuous
bounded above
None of these
uniformly continous

MTH631 Real Analysis II

Question No : 7 of 25 **Marks: 1**

A function f is ———— on a subset S of R^n if S is contained in an open set on which $f_{x_1}, f_{x_2}, \ldots, f_{x_n}$ are continuous

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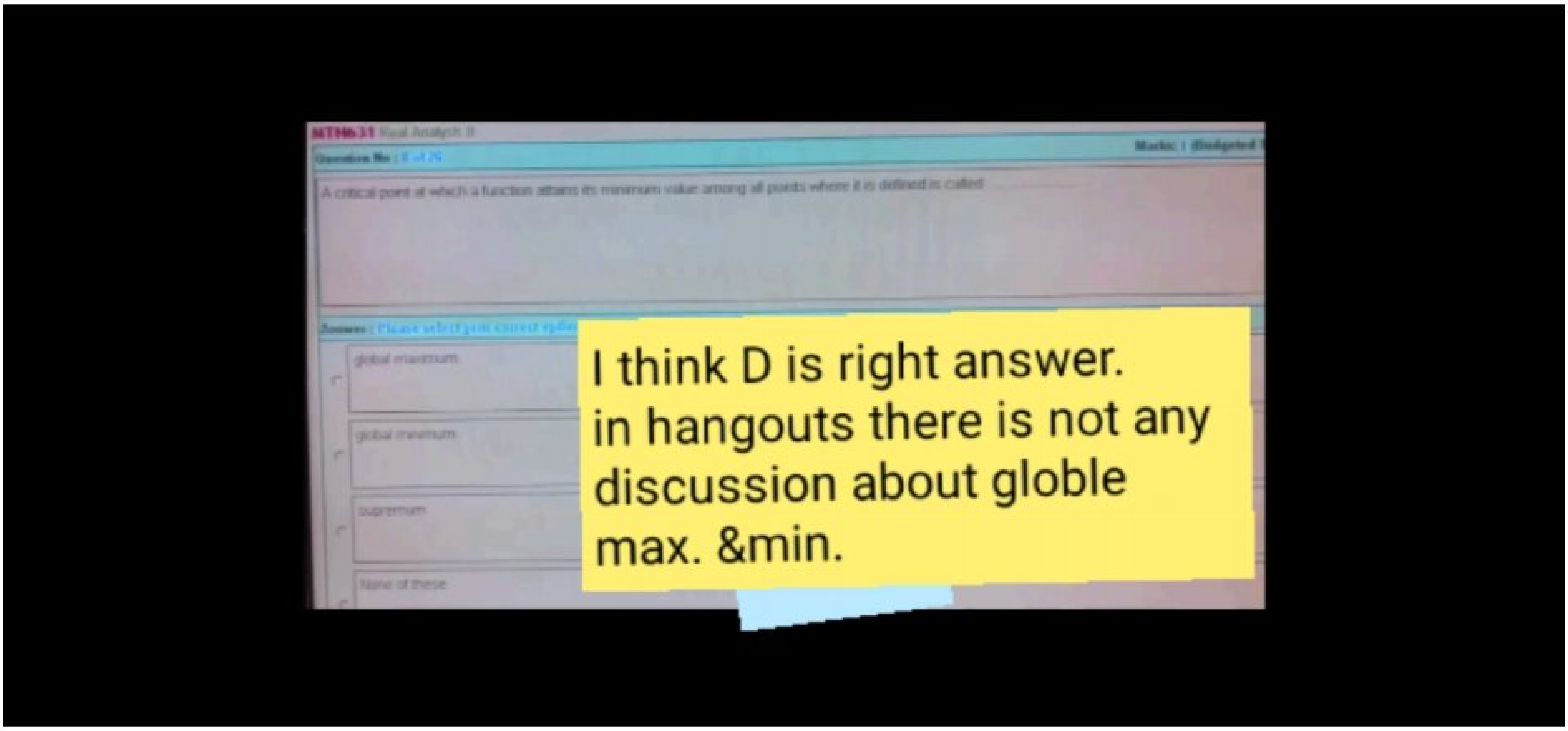
Answer (Please select your correct option)

- continuously differentiable ⇦
- uniformly convergent
- piecewise convergent
- None of these

2.17.4 Continuously Differentiable Function

We say that f is *continuously differentiable* on a subset S of $\mathbb{R}^n$ if S is contained in an open set on which $f_{x_1}, f_{x_2}, \ldots, f_{x_n}$ are continuous.

The above theorem implies that such a function is differentiable at each $\mathbf{X}_0$ in S.


MTH631 Real Analysis II
A critical point at which a function attains its minimum value among all points where it is defined is called
global maximum
global minimum
supremum
None of these
I think D is right answer.
in hangouts there is not any
discussion about globle
max. &min.

Proof: See lecture.

2.16 Directional Derivative

Let $\mathbf{\Phi}$ be a unit vector and $\mathbf{X}$ a point in $\mathbb{R}^n$.

The directional derivative of f at $\mathbf{X}$ in the direction of $\mathbf{\Phi}$ is defined by

$$\frac{\partial f(\mathbf{X})}{\partial \mathbf{\Phi}} = \lim_{t \to 0} \frac{f(\mathbf{X} + t\mathbf{\Phi}) - f(\mathbf{X})}{t}$$

if the limit exists.

Question 1

Prove that $d(S) = d(\overline{S})$ for any set S in $\mathbb{R}^n$.

Proof:

Since $S \subset \overline{S}$, (A) $d(S) \le d(\overline{S})$. If $\mathbf{X}$ and $\mathbf{Y}$ are in $\overline{S}$ and $\epsilon > 0$, there are points $\mathbf{X}_1, \mathbf{Y}_1$ in S such that $|\mathbf{X} - \mathbf{X}_1| < \epsilon$ and $|\mathbf{Y} - \mathbf{Y}_1| < \epsilon$. Then

$$|\mathbf{X} - \mathbf{Y}| \le |\mathbf{X} - \mathbf{X}_1| + |\mathbf{X}_1 - \mathbf{Y}_1| + |\mathbf{Y}_1 - \mathbf{Y}| < d(S) + 2\epsilon.$$

Hence, $d(\overline{S}) < d(S) + \epsilon$. Let $\epsilon \to 0+$ to conclude that $d(\overline{S}) \le d(S)$, which, with (A), implies that $d(S) = d(\overline{S})$.

Question 2

If a nonempty subset S of $\mathbb{R}^n$ is both open and closed, then $S = \mathbb{R}^n$.

Prove:

Suppose that $\mathbf{X}_0$ is in S. If $S \neq \mathbb{R}^n$, there is an $\mathbf{X}_1 \notin S$. Let

$$H = \left\{\tau \,\middle|\, (1-t)\mathbf{X}_0 + t\mathbf{X}_1 \in S \text{ for } 0 \le t < \tau\right\}.$$

Since S is open it contains a neighborhood of $\mathbf{X}_0$, so $H \neq \emptyset$. Since $\mathbf{X}_1$ is not in S, $\tau \le 1$ for all τ in H. Let $\rho = \sup\, H$ and $\overline{\mathbf{X}} = (1-\rho)\mathbf{X}_0 + \rho\mathbf{X}_1$; then $\overline{\mathbf{X}}$ is a limit point of S and so in S, because S is closed. Since S is open, it contains some ϵ−neighborhood of $\overline{\mathbf{X}}$, so $\rho + \epsilon/2$ is in H. This contradicts the definition of ρ. Hence, $S = R^n$.

Question 3

$$S = \{(x, y, x) \mid |x| \le 2, |y| \le 1, |z - 2| \le 2\}$$

Ans:

$d(S)$ is the supremum of $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

where $-2 \le x_1, x_2 \le 2, -1 \le y_1, y_2 \le 1, 0 \le z_1, z_2 \le 4$;

to maximize this function let, for example,
$x_1 = -2, x_2 = 2, y_1 = -1, y_2 = 1, z_1 = 0, z_2 = 4$; thus, $d(S) = \sqrt{4^2 + 2^2 + 4^2} = 6.$

Question 4

$$f(x, y, z) = \log(1 + x + y + z), \quad \Phi = (0, 1, 0)$$

Ans:

$$h(t) = f(x + \phi_1 t, y + \phi_2 t, z + \phi_3 t)$$
$$= \log(1 + x + y + z + (\phi_1 + \phi_2 + \phi_3)t);$$
$$h'(t) = \frac{\phi_1 + \phi_2 + \phi_3}{1 + x + y + z + (\phi_1 + \phi_2 + \phi_3)t};$$

$$\frac{\partial f(\mathbf{X})}{\partial \mathbf{L}} = h'(0) = \frac{\phi_1 + \phi_2 + \phi_3}{1 + x + y + z}; \text{ if } \mathbf{L} = (0, 1, 0)$$

then

$$\frac{\partial f(\mathbf{X})}{\partial \mathbf{L}} = \frac{1}{1 + x + y + z}.$$

Question 5

If $h(u, v) = f(u^2 + v^2)$, then $vh_u - uh_v = 0$.

ANS:

$$h_u = 2uf'$$

$$h_v = 2vf'$$

$$vh_u - uh_v = 2(vu - uv)f'$$

$$vh_u - uh_v = 0.$$

Question 6

If $h(u, v) = f(\sin u + \cos v)$, then $h_u \sin v + h_v \cos u = 0$.

Ans:

$$h_u = f' \cos u$$
$$h_v = -f' \sin v$$

$$h_u \sin v + h_v \cos u = (\sin v \cos u - \cos u \sin v) f'$$
$$h_u \sin v + h_v \cos u = 0.$$

Question 7

If $h(u, v) = f(u/v)$, then $uh_u + vh_v = 0$.

Ans:

$$h_u = \frac{1}{v} f'$$

$$h_v = -\frac{u}{v^2} f'$$

$$uh_u + vh_v = \left(\frac{u}{v} - \frac{u}{v}\right) f'$$

$$uh_u + vh_v = 0.$$

Question 8

If $h(u,v) = f(g(u,v), -g(u,v))$, then $dh = (f_x - f_y)\,dg$.

Ans:

$$h_u = (f_x - f_y)g_u$$

$$h_v = (f_x - f_y)g_v$$

$$dh = h_u\,du + h_v\,dv$$
$$dh = (f_x - f_y)(g_u\,du + g_v\,dv)$$
$$dh = (f_x - f_y)\,dg.$$

Question 9

Let $h(r, \theta, z) = f(x, y, z)$, where $x = r\cos\theta$ and $y = r\sin\theta$.

Find h_r, h_θ, and h_z in terms of f_x, f_y, and f_z.

Ans:

$$h_r = f_x x_r + f_y y_r + f_z z_r$$

$$h_r = f_x \cos\theta + f_y \sin\theta$$

$$h_\theta = f_x x_\theta + f_y y_\theta + f_z z_\theta$$

$$h_\theta = f_x(-r\sin\theta) + f_y(r\cos\theta)$$

$$h_\theta = r(-f_x \sin\theta + f_y \cos\theta)$$

$$h_z = f_x x_z + f_y y_z + f_z = f_z.$$

Question 10

Let $h(r, \theta, \phi) = f(x, y, z)$, where $x = r\sin\phi\cos\theta$, $y = r\sin\phi\sin\theta$, and $z = r\cos\phi$. Find h_r, h_θ, and h_ϕ in terms of f_x, f_y, and f_z.

ANS:

$$h_r = f_x x_r + f_y y_r + f_z z_r$$

$$h_r = f_x \sin\phi\cos\theta + f_y \sin\phi\sin\theta + f_z\cos\phi$$

$$h_\theta = f_x x_\theta + f_y y_\theta + f_z z_\theta$$

$$h_\theta = r\sin\phi(-f_x\sin\theta + f_y\cos\theta)$$

$$h_\phi = f_x x_\phi + f_y y_\phi + f_z z_\phi$$

$$h_\phi = r(f_x\cos\phi\cos\theta + f_y\cos\phi\sin\theta - f_z\sin\phi.$$

Question 11

Let $h(\mathbf{U}) = f(\mathbf{G}(\mathbf{U}))$ and find $d_{\mathbf{U}_0}h$ by writing h explicitly as a function of $\mathbf{U}$.

$$\begin{aligned} f(x,y) &= 3x^2 + 4xy^2 + 3x, \\ g_1(u,v) &= ve^{u+v-1}, \\ g_2(u,v) &= e^{-u+v-1}, \end{aligned} \qquad (u_0, v_0) = (0, 1)$$

Ans:

$$\begin{aligned} h(\mathbf{U}) &= 3v^2e^{2u+2v-2} + 4ve^{u+v-1}e^{-2u+2v-2} + 3ve^{u+v-1} \\ &= 3v^2e^{2u+2v-2} + 4ve^{-u+3v-3} + 3ve^{u+v-1} \end{aligned}$$

$$h_u(\mathbf{U}) = 6v^2e^{2u+2v-2} - 4ve^{-u+3v-3} + 3ve^{u+v-1}$$

$$h_u(\mathbf{U}_0) = 5$$

$$h_v(\mathbf{U}) = (6v + 6v^2)e^{2u+2v-2} + (4 + 12v)e^{-u+3v-3} + (3 + 3v)e^{u+v-1}$$

$$h_v(\mathbf{U}_0) = 34$$

$$d_{\mathbf{U}_0}h = h_u(\mathbf{U}_0)\,du + h_v(\mathbf{U}_0)\,dv = 5\,du + 34\,dv$$

Question 12

Let $h(\mathbf{U}) = f(\mathbf{G}(\mathbf{U}))$ and find $d_{\mathbf{U}_0}h$ by writing h explicitly as a function of $\mathbf{U}$.

$$\begin{aligned} f(x, y, z) &= e^{-(x+y+z)}, \\ g_1(u, v, w) &= \log u - \log v + \log w, \\ g_2(u, v, w) &= -2\log u - 3\log w, \\ g_3(u, v, w) &= \log u + \log v + 2\log w, \end{aligned} \qquad (u_0, v_0, w_0) = (1, 1, 1)$$

Ans:

$$x(\mathbf{U}) + y(\mathbf{U}) + z(\mathbf{U}) = (\log u - \log v + \log w) + (-2\log u - 3\log w) + (\log u + \log v + 2\log w)$$

$$x(\mathbf{U}) + y(\mathbf{U}) + z(\mathbf{U}) = 0$$

hence

$$h(\mathbf{U}) \equiv 1$$

$$d_{\mathbf{U}_0}h = 0.$$

Question 13

Let $h(\mathbf{U}) = f(\mathbf{G}(\mathbf{U}))$ and find $d_{\mathbf{U}_0}h$ by writing h explicitly as a function of $\mathbf{U}$.

$$f(x, y) = (x + y)^2,$$
$$g_1(u, v) = u\cos v, \qquad (u_0, v_0) = (3, \pi/2)$$
$$g_2(u, v) = u\sin v,$$

Ans:

$$h(\mathbf{U}) = u^2(\cos v + \sin v)^2 \quad ;$$
$$= u^2(1 + 2\sin v\cos v) \quad ;$$
$$= u^2(1 + \sin 2v)$$
$$h_u(\mathbf{U}) = 2u(1 + \sin 2v)$$
$$h_u(\mathbf{U}_0) = 6$$
$$h_v(\mathbf{U}) = 2u^2\cos 2v;$$
$$h_v(\mathbf{U}_0) = -18 \quad ;$$
$$d_{\mathbf{U}_0}h = h_u(\mathbf{U}_0)\,du + h_v(\mathbf{U}_0)\,dv$$
$$d_{\mathbf{U}_0}h = 6\,du - 18\,dv.$$

Question 14

Let $h(\mathbf{U}) = f(\mathbf{G}(\mathbf{U}))$ and find $d_{\mathbf{U}_0}h$ by writing h explicitly as a function of $\mathbf{U}$.

$$\begin{aligned} f(x,y,z) &= x^2+y^2+z^2, \\ g_1(u,v,w) &= u\cos v\sin w, \\ g_2(u,v,w) &= u\cos v\cos w, \\ g_3(u,v,w) &= u\sin v; \end{aligned} \qquad (u_0,v_0,w_0) = (4,\pi/3,\pi/6)$$

Ans:

$$h(\mathbf{U}) = u^2\cos^2 v\sin^2 w + u^2\cos^2 v\cos^2 w + u^2\sin^2 v$$

$$h(\mathbf{U}) = u^2$$

$$h_u(\mathbf{U}) = 2u$$

$$h_v(\mathbf{U}) = 0$$

$$h_w(\mathbf{U}) = 0$$

$$h_u(\mathbf{U}_0) = 8;$$

$$h_v(\mathbf{U}_0) = 0;$$

$$h_w(\mathbf{U}_0) = 0$$

$$dh = 8\,du.$$

Question 15

Show that (0, 0) is a critical point of each of the following functions, and that they have positive semidefinite second differentials at (0, 0).

$$p(x, y) = x^2 - 2xy + y^2 + x^4 + y^4;$$

Ans:

$$p(x, y) = x^2 - 2xy + y^2 + x^4 + y^4$$

$$p_x(x, y) = 2x - 2y + 4x^3$$

$$p_y(x, y) = -2x + 2y + 4y^3.$$

Since $p_x(0,0) = p_y(0,0) = 0$, (0, 0) is a critical point of p.

$$p_{xx}(x, y) = 2 + 12x^2$$

$$p_{xy}(x, y) = -2;$$

$$p_{yy}(x, y) = 2 + 12y^2$$

;

$$(d^{(2)}_{(0,0)} p)(x, y) = 2x^2 - 4xy + 2y^2$$

$(d^{(2)}_{(0,0)} p)(x, y) = 2(x - y)^2$ is positive semidefinite.

Question 16

Show that (0, 0) is a critical point of each of the following functions, and that they have positive semidefinite second differentials at (0, 0).

$$q(x, y) = x^2 - 2xy + y^2 - x^4 - y^4.$$

Ans:

$$q(x, y) = x^2 - 2xy + y^2 - x^4 - y^4$$

$$q_x(x, y) = 2x - 2y - 4x^3$$

$$q_y(x, y) = -2x + 2y - 4y^3$$

Since $q_x(0,0) = q_y(0,0) = 0$, (0, 0) is a critical point of q.

$$q_{xx}(x, y) = 2 - 12x^2$$

$$q_{xy}(x, y) = -2$$

$$q_{yy}(x, y) = 2 - 12y^2$$

$$(d^{(2)}_{(0,0)}q)(x, y) = 2x^2 - 4xy + 2y^2$$

$(d^{(2)}_{(0,0)}q)(x, y) = 2(x - y)^2$ is positive semidefinite.

Question 17

If R is degenerate, then implies that $\int_R f(\mathbf{X})\,d\mathbf{X} = 0$ if f is bounded on R.

Proof:

Suppose, for example, that $R = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$ with $a_1 = b_1$.

Then every partition $\mathbf{P}$ of R is of the form $\mathbf{P} = \{R_1, R_2, \ldots, R_k\}$, where $v(R_j) = 0$,

$1 \le j \le k$. Therefore, every Riemann sum of f over R equals zero, so $\int_R f(\mathbf{X})d\mathbf{X} = 0$.

Question 18

Suppose that

$$f(x,y) = \begin{cases} 0 & \text{if } x \text{ and } y \text{ are rational,} \\ 1 & \text{if } x \text{ is rational and } y \text{ is irrational,} \\ 2 & \text{if } x \text{ is irrational and } y \text{ is rational,} \\ 3 & \text{if } x \text{ and } y \text{ are irrational.} \end{cases}$$

Find $\overline{\int_R} f(x,y)\,d(x,y)$ and $\underline{\int_R} f(x,y)\,d(x,y)$ if $R = [a,b] \times [c,d]$.

Ans:

Let $\mathbf{P} = \{R_1, R_2, \ldots, R_k\}$. Let

$$M_j = \sup\{f(x,y) \mid \mathbf{X} \in R_j\} \quad \text{and} \quad m_j = \inf\{f(x,y) \mid \mathbf{X} \in R_j\}.$$

Let j be arbitrary in $\{1,2,\ldots,k\}$. Since R_j contains a point $(\widehat{x}_j, \widehat{y}_j)$ with $\widehat{x}_j$ and $\widehat{y}_j$ irrational, $M_j = 3$. Hence, $\overline{\int_R} f(x,y)\,d(x,y) = 3(b-a)(d-c)$. Since R_j contains a point $(\widetilde{x}_j, \widetilde{y}_j)$ with $\widetilde{x}_j$ and $\widetilde{y}_j$ rational, $m_j = 0$. Hence, $\underline{\int_R} f(x,y)\,d(x,y) = 0$.

Question 19

If S_1 and S_2 have zero content, then $S_1 \cup S_2$ has zero content.

Prove

If $\epsilon > 0$, there are rectangles $T_1, T_2, \ldots, T_r$ and $T'_1, T'_2, \ldots, T'_s$ such that $S_1 \subset \cup_{i=1}^{r} T_i$, $S_2 \subset \cup_{j=1}^{s} T'_j$, $\sum_{i=1}^{r} V(T_r) < \epsilon/2$, and $\sum_{j=1}^{s} V(T'_j) < \epsilon/2$. Then $\{T_1, \ldots, T_m, T'_1, \ldots, T'_r\}$ covers $S_1 \cup S_2$ with total content $< \epsilon$.

Question 20

If S_1 has zero content and $S_2 \subset S_1$, then S_2 has zero content.

prove

If $\epsilon > 0$, there are rectangles $T_1, T_2, \ldots, T_r$ and $T_1', T_2', \ldots, T_s'$ such that
If $S_1 \subset \cup_{i=1}^{r} T_r$ and $S_2 \subset S_1$, then $S_2 \subset \cup_{i=1}^{r} T_r$.