

MCQS

The standard normal distribution is:

- a) skewed to the right.
- b) also called the t distribution.
- c) an approximation of the binomial distribution.
- d) defined by a mean equal to 0 and a standard deviation equal to 1.**

X is a random normal variable, with mean μ and variance σ^2 . The “standardized form” of X is

$Z = \frac{X - \mu}{\sigma}$. What are the mean and variance, respectively, of Z?

- a) 2, 0
- b) 1, 0
- c) 0, 1**
- d) 2, 1

Let $X \sim N(4, 2^2)$. Which of the following is a standard normal variable?

a) $Z = \frac{X - 4}{4}$

b) $Z = \frac{X - 2}{4}$

c) $Z = \frac{X - 4}{2}$

d) $Z = \frac{X - 2}{2}$

The normal distribution is a _____ distribution.

- a) Symmetrical**
- b) Positively Skewed
- c) Negatively Skewed
- d) Uniform

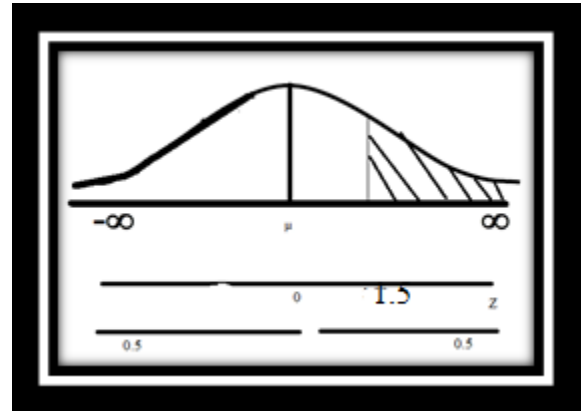
Let the random variable Z have the standard normal distribution. Find $P(0 \leq Z \leq 2)$.

$$P(0 \leq Z \leq 2) = P(0 \leq Z \leq 2)$$

$$= 0.4772.$$

Let the random variable Z have the standard normal distribution. Find $P(Z \geq 1.5)$.

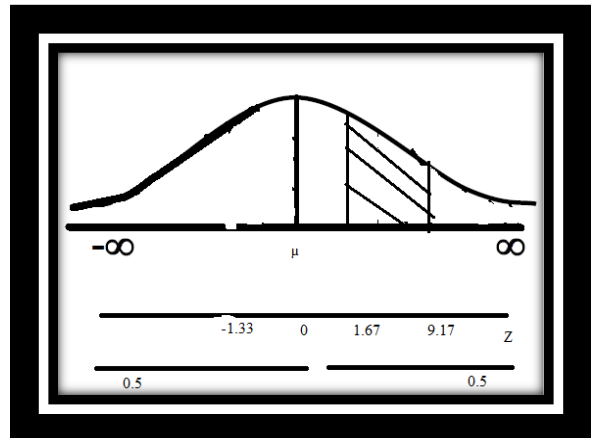
$$P(Z \geq 1.5) = P(0 < Z < \infty) - P(0 < Z < 1.5)$$



$$= 0.5 - 0.4332 = 0.0668.$$

A random variable X is normally distributed with $\mu = 45$ and $\sigma^2 = 36$. Find the probability that it will lie between 55 and 100.

$$\begin{aligned} P(55 < X < 100) &= P\left(\frac{55 - 45}{6} < \frac{X - \mu}{\sigma} < \frac{100 - 45}{6}\right) \\ &= P(1.67 < Z < 9.17) \end{aligned}$$

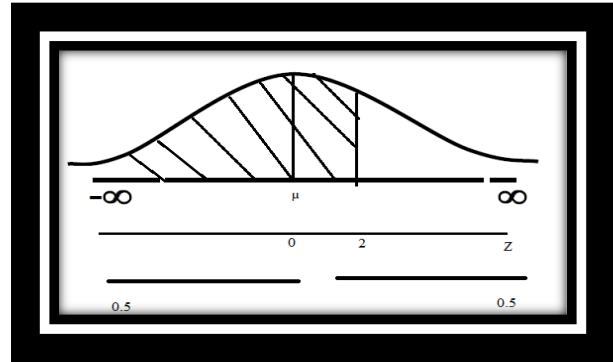


$$= P(0 < Z < 9.17) - P(0 < Z < 1.67)$$

$$= 0.5 - 0.4525 = 0.0475.$$

A random variable X is normally distributed with $\mu = 45$ and $\sigma^2 = 36$. Find the probability that it will be smaller than 57.

$$\begin{aligned} P(X < 57) &= P\left(\frac{X - \mu}{\sigma} < \frac{57 - 45}{6}\right) \\ &= P(Z < 2) \end{aligned}$$



$$= P(-\infty < Z < 0) + P(0 < Z < 2)$$

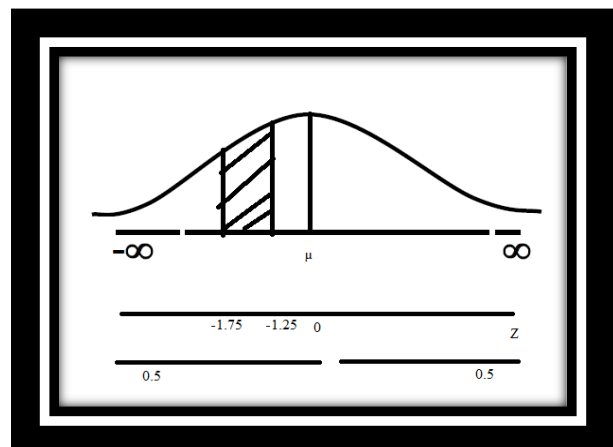
$$= 0.5 + 0.4772 = 0.9772.$$

Suppose that weights of 2000 male students are normally distributed with mean 155 pounds and standard deviation 20 pounds. Find the probability that:

- (i) The students weight lie between 120 and 130 pounds.
- (ii) The students weight less than or equal to 100 pounds.

Solution:

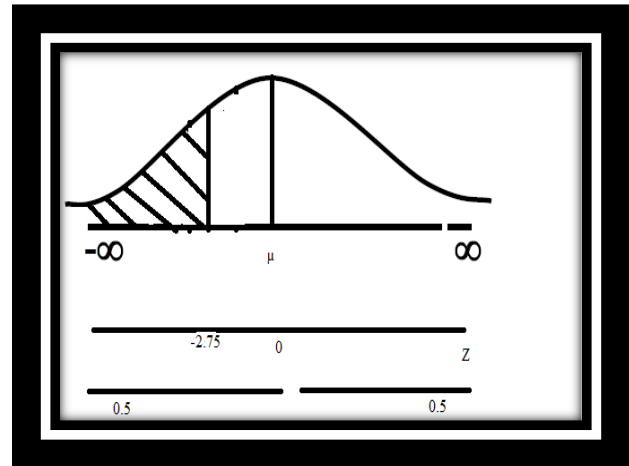
$$\begin{aligned} P(120 < X < 130) &= P\left(\frac{120 - 155}{20} < \frac{X - \mu}{\sigma} < \frac{130 - 155}{20}\right) \\ &= P(-1.75 < Z < -1.25) \end{aligned}$$



$$\begin{aligned}
&= P(Z < -1.75) - P(Z < -1.25) \\
&= 0.4599 - 0.3944 = 0.0655
\end{aligned}$$

(ii)

$$\begin{aligned}
P(X < 100) &= P\left(\frac{X - \mu}{\sigma} < \frac{100 - 155}{20}\right) \\
&= P(Z < -2.75)
\end{aligned}$$



$$\begin{aligned}
&= P(-\infty < Z < 0) - P(-2.75 < Z < 0) \\
&= 0.5 - 0.4970 = 0.003.
\end{aligned}$$

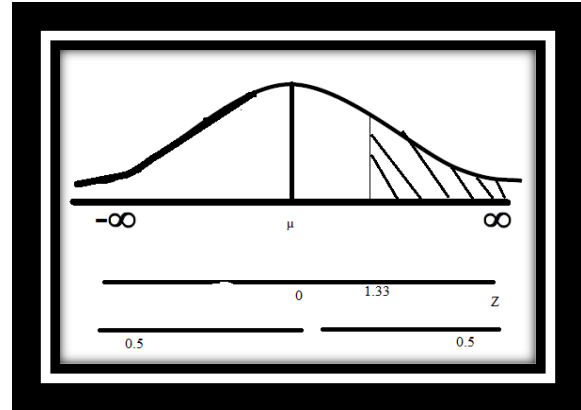
A soft drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

- What fraction of the cups will contain more than 220 milliliters?
- What is the probability that a cup contains between 180 and 200 milliliters?

(Probability of z equal to 1.33 is 0.4082)

Solution:

$$\begin{aligned}
P(X > 220) &= P\left(\frac{X - \mu}{\sigma} > \frac{220 - 200}{15}\right) \\
&= P(Z > 1.33)
\end{aligned}$$



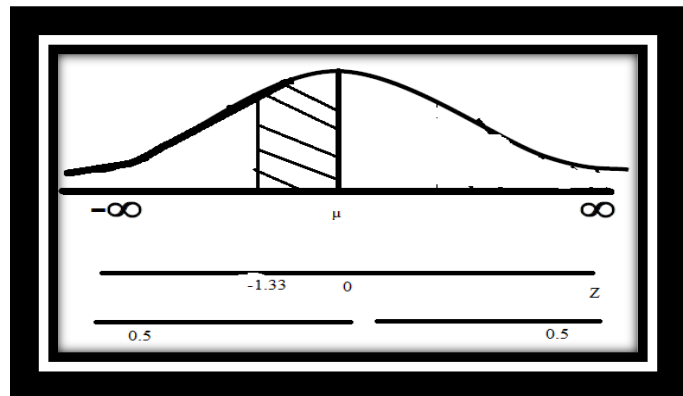
$$= P(0 < Z < \infty) - P(0 < Z < 1.33)$$

$$= 0.5 - 0.4082 = 0.0918.$$

b)

$$P(180 < X < 200) = P\left(\frac{180 - 200}{15} < \frac{X - \mu}{\sigma} < \frac{200 - 200}{15}\right)$$

$$= P(-1.33 < Z < 0)$$



$$=.4082$$