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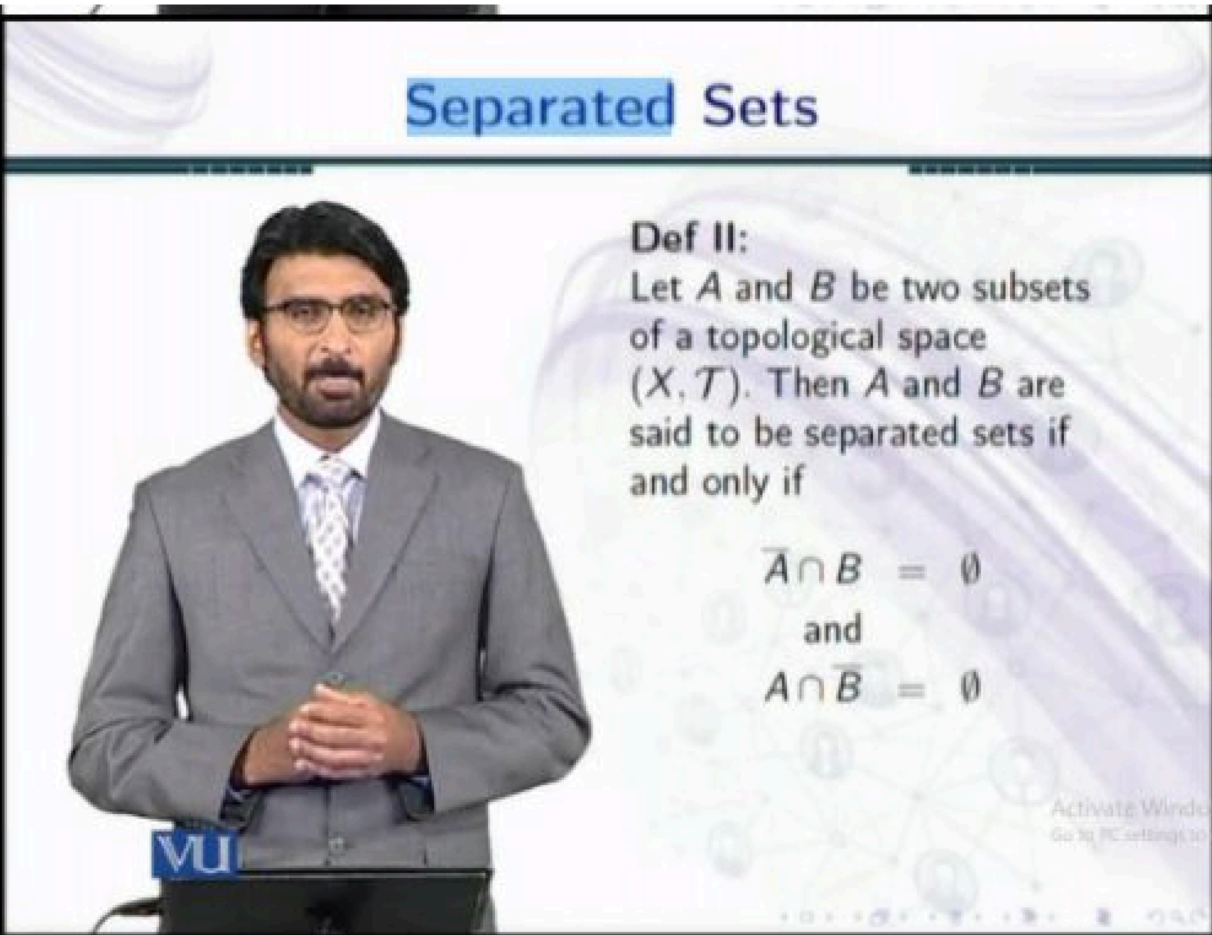
MTH634 Topology

Question No : 34 of 41

Define separated sets.


Separated Sets
Def II:
Let A and B be two subsets of a topological space $(X, \mathcal{T})$. Then A and B are said to be separated sets if and only if
$\overline{A} \cap B = \emptyset$
and
$A \cap \overline{B} = \emptyset$
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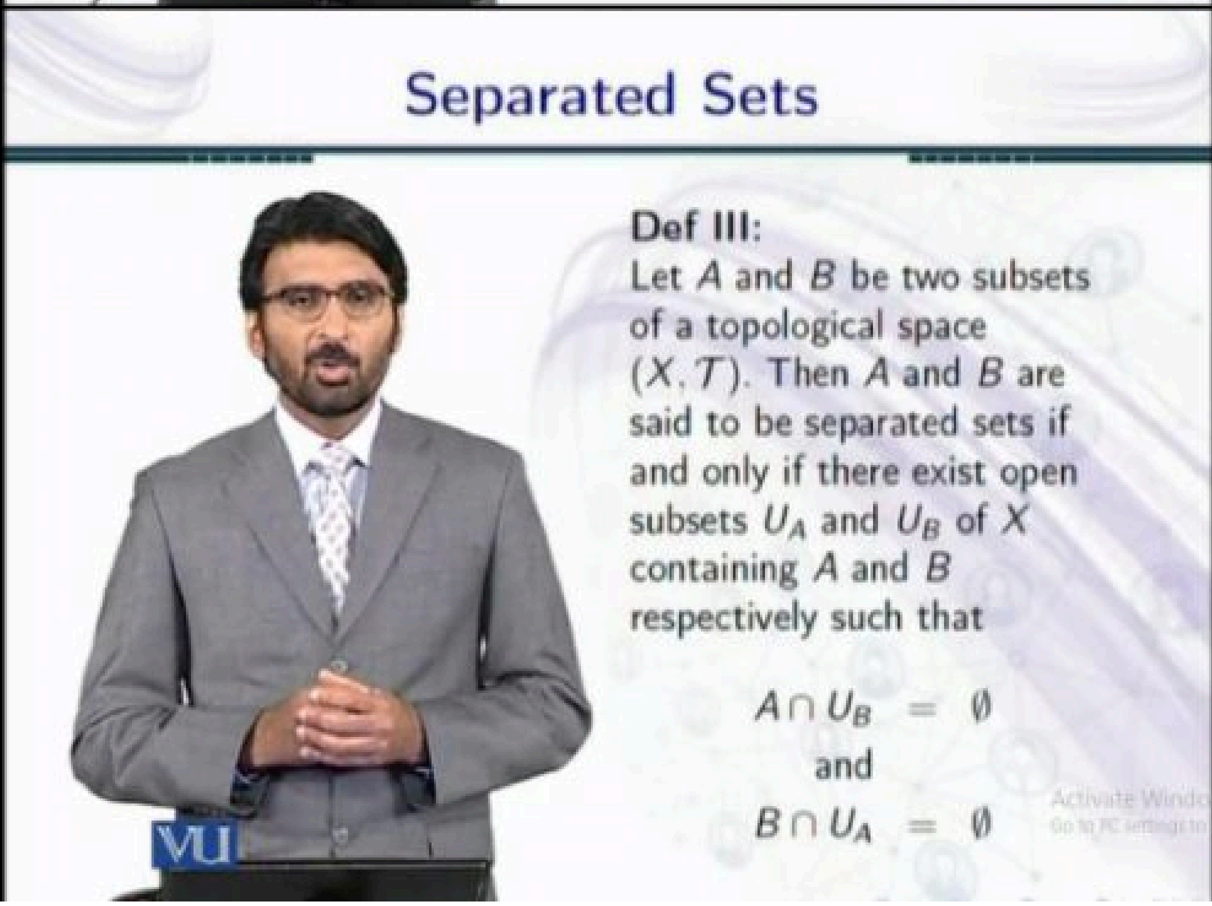
Separated Sets
Def III:
Let A and B be two subsets of a topological space $(X, \mathcal{T})$. Then A and B are said to be separated sets if and only if there exist open subsets U_A and U_B of X containing A and B respectively such that
$A \cap U_B = \emptyset$
and
$B \cap U_A = \emptyset$
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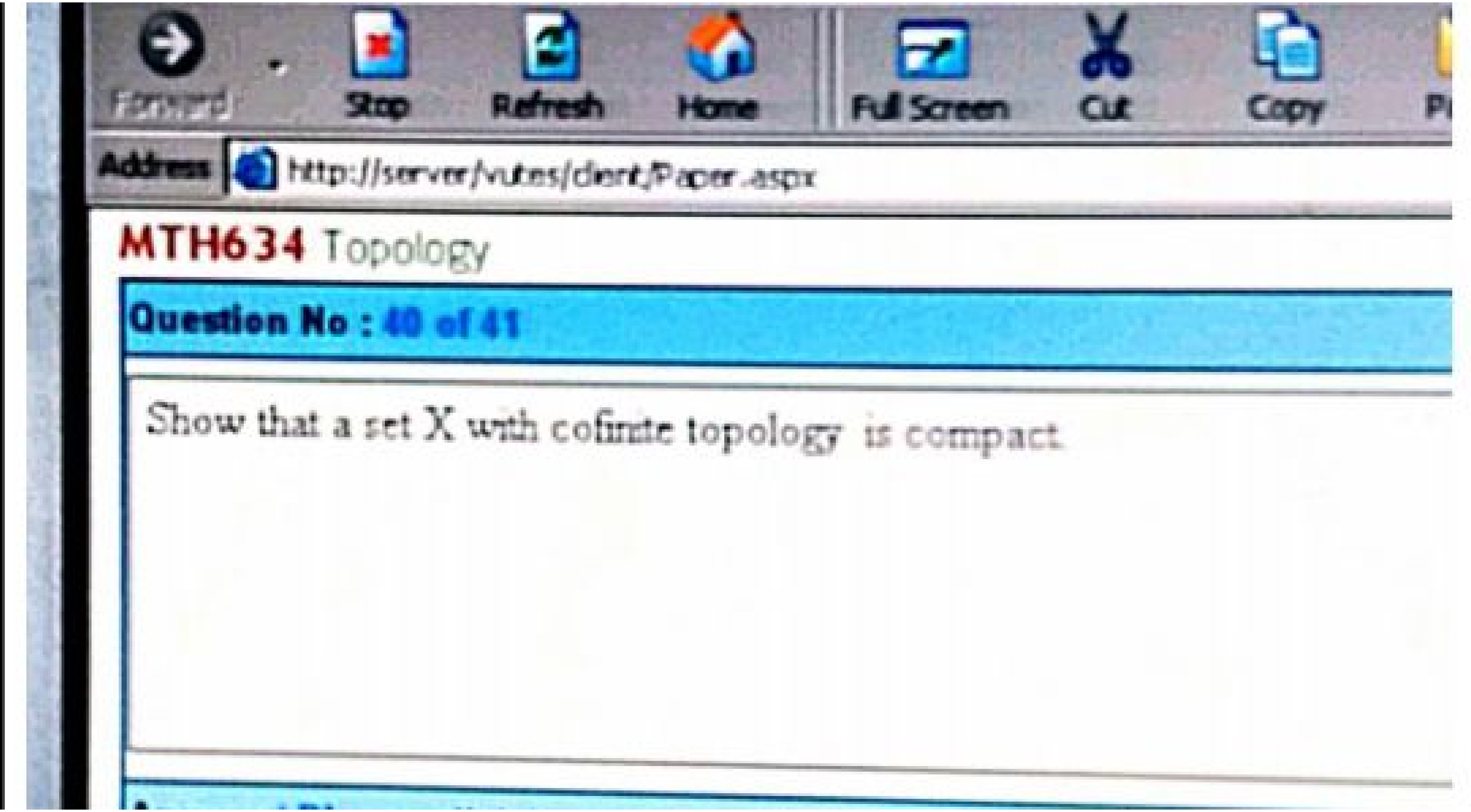
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MTH634 Topology
Question No : 40 of 41
Show that a set X with cofinite topology is compact.

Q Show that a set X with cofinite topology is compact.

Proof: If X is finite, Then

$$\mathcal{T}_{cof} = P(X)$$

$\Rightarrow (X, \mathcal{T}_{cof})$ is compact

But if X is not finite Then consider an open cover D of X.

Now Let $\phi \in D \in D$ $\because D^c$ is finite

Now, for every $x \in D^c$

Choose an element of D. D_x containing x.

$\{D_x \mid x \in D^c\}$ is finite

$\because D^c$ is finite

Consider this subcover of D

~~$X = X \setminus \{D\}$~~

$$C = \{D\} \cup \{D_x \mid x \in D^c\}$$

$\rightarrow C$ is a finite cover of D

$\Rightarrow (X, \mathcal{T}_{cof})$ is compact

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MTH634 Topology

Question No : 37 of 41

Show that a discrete space X is separable if and only if X is countable.

Proposition 1:
A discrete space is separable if and only if it is countable.

Proof: X, $\mathcal{T}_{dis} = P(X)$

We know that the only dense subset in X is X itself.

So the only choice for A is X s.t $\overline{A} = X$

$\Rightarrow$ X is separable iff X is countable.

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MTH634 Topology

Question No : 39 of 41

Show that every second countable space is first countable.

Every second countable space is first countable.

Proof:

Let X be a second countable space.

i.e $\exists$ B (a basis) s.t B is countable

Now let $p \in X$ and consider $B_p \subset B$

$$B_p = \{ B_p \mid p \in B_p \} \subset B$$

Now B_p is countable $\because B$ is countable

Now B_p is local base at p

$B_p \subset B$ (basis) $\forall$ $U \in \mathfrak{T}$ s.t $p \in U$

$\exists$ $B_p \in B$ s.t. $p \in B_p \subset U$

$\rightarrow B_p$ is a local base at p

and this exists $\forall$ $p \in X$

$\rightarrow X$ is first countable

Question No : 36 of 41

Prove that every subspace of a second countable space is second countable.

Answer (Please click here to Add Answer)

Second Countable Spaces

Remark:
Every subspace of a second countable space is second countable.

Proof: Let $A \subset X$ (X is second countable space)

So $\exists$ $\mathcal{B} = \{B_n \mid n \in \mathbb{N}\}$. countable.

Now $\mathcal{B}_A = \{A \cap B_n \mid n \in \mathbb{N}\}$ is a basis for $\mathcal{T}_A$. And $\mathcal{B}_A$ is countable

$\Rightarrow A$ is second countable. $\square$

MTH634 Topology

Question No : 33 of 41

Define T_1 - space.

T_1 Space

Def:
A topological space $(X, \mathcal{T})$ is said to be "T_1 Space" iff for each $x, y \in X$ such that $x \neq y$ there exist open subsets U_x, U_y of X containing x, y respectively such that $y \notin U_x$ and $x \notin U_y$.

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MTH634 Topology

Question No : 32 of 41

Give one example of second countable space.

Examples: Second Countable

Let us see discuss some examples of second countable spaces.

Example 1:

Let X be a countable set with discrete topology. Then $(X, \mathcal{T}_{Dis})$ is second countable. Recall basis for discrete space, i.e.

$$\mathcal{B} = \{\{x\} | x \in X\}$$

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Question No : 27 of 33

Define Normal space.

Normal Spaces

Def:
A topological space $(X, \mathcal{T})$ is said to be "normal" iff for every pair of disjoint closed subsets, $F_1, F_2 \subset X$ there exist open subsets U_{F_1} and U_{F_2} containing F_1 and F_2 respectively such that

$$U_{F_1} \cap U_{F_2} = \emptyset.$$

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Question No : 26 of 33

Define metric topology

Metric Topology

Metric Topology:
Let X be a nonempty set with metric d. The topology $\mathcal{T}$ on X generated by the set of all open balls in X with respect to d is called the "Metric Topology."
We also say that a topology $\mathcal{T}$ on X induced by the metric d.

Metric Space:
Consider X with a metric d, this d induces a topology the "metric topology." (X, d) is called a metric space.

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Question No : 31 of 33

Define connected set.

Answer (Please click here to Add Answer)

Connected Spaces

Connected Set:
A subset A of a topological space $(X, \mathcal{T})$ is said to be connected iff there exists no pair of nonempty open subsets U and V of X such that

$$A \cap U \quad \text{and} \quad A \cap V$$

are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V)$$

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MTH634 Topology

Question No : 30 of 33

Show that a set X with topology T containing finite number subsets of X is compact

xamples: Compact Spaces

Example 2:
A set X with topology $\mathcal{T}$ containing finite number subsets of X is compact.

Reason:
Since every open cover $\mathcal{C}$ of X is a subclass of $\mathcal{T}$ and $\mathcal{T}$ itself finite so $\mathcal{C}$ is finite too and any subcover $\mathcal{S}$ of $\mathcal{C}$ is also finite.

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MTH634 Topology

Question No : 28 of 33

Define connected component in a topological space X.

Connected Component

Def:
Consider a topological space $(X, \mathcal{T})$. Let $a \in X$ and C be a connected subset of X containing a.
Then

$$C_a = \bigcup_{a \in C} C$$

is called the connected component of X containing a.

Connected Component

(4) A space $(X, \mathcal{T})$ is connected iff it has only one connected component.

Remarks:

1. The connected component C_a is the largest connected subset of X containing a.
2. If $b \in C_a$ then $C_b = C_a$.
3. X can be written as disjoint union of its connected components. i.e.

$$X = \coprod_{C_i} C_i$$

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MTH634 Topology

Question No : 29 of 33

Consider the sets $X = \{1,2,3\}$ and $Y = \{a,b,c\}$ with topologies $\tau_X = \{\phi, \{1\}, \{1,2\}, X\}$ and $\tau_Y = \{\phi, \{a,b\}, \{b\}, Y\}$ repectively. Define a map $f : X \rightarrow Y$ as:

$f(1) = f(2) = a, f(3) = c,$

Show that f is a closed map.

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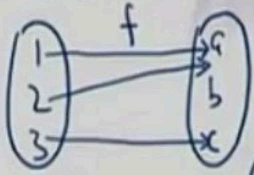
Closed Mapping

Ex: $X = \{1,2,3\}$, $\mathfrak{T}_X = \{\phi, \{1\}, \{1,2\}, X\}$

$Y = \{a,b,c\}$, $\mathfrak{T}_Y = \{\phi, \{a,b\}, \{b\}, Y\}$



$\mathcal{C}_X = \{X, \{2,3\}, \{3\}, \phi\}$

$\mathcal{C}_Y = \{Y, \{c\}, \{a,c\}, \phi\}$

$f(\phi) = \phi$ ✓

$f(\{3\}) = \{c\}$ ✓ | $f(\{2,3\}) = \{a,c\}$ ✓ | $f(X) = \{a,c\}$ ✓

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Question No : 33 of 33 **Marks: 5 (Budg**

Consider $X = \{a, b, c, d, e\}$ with topology $\tau = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, \{a, d, e\}, \{a, b, d, e\}, X\}$. Show that the set $A = \{c, e\}$ is disconnected subset of X.

Answer (Please click here to Add Answer)

Examples: Connected Spaces

Example 2: Consider $X = \{a, b, c, d, e\}$ with
$\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, \{a, d, e\}, \{a, b, d, e\}, X\}$.
Then $A = \{c, e\}$ is disconnected subset of X.

Reason:

Since there exists a pair of nonempty open subsets $U = \{a, b, d, e\}$ and $V = \{a, b, c\}$ of X such that

$$A \cap U = \{e\} \quad \text{and} \quad A \cap V = \{c\}$$

$\{e\} \cap \{c\} = \emptyset$

are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V) = \{e\} \cup \{c\} = \{c, e\}$$