Question # 5 of 5 (Start time: 08:07:25 PM, 07 December 2022) Total Marks: 1

If tilde coordinate system corresponds to polar coordinates and non tilde Cartesian coordinates then find $\frac{\partial x^1}{\partial \tilde{x}^1}$?

Select the correct option

Reload Math Equations

- ○ $\cos\theta$
- ○ $-\cos\theta$

Click to Save Answer & Move to Next Question

If tilde coordinate system corresponds to polar coordinates and non tilde Cartesian coordinates then find $\frac{\partial x^1}{\partial y^1}$?

Select the correct option

Reload Math Equations

- ( ) $-\cos\theta$
- (•) $\cos\theta$

Click to Save Answer & Move to Next Question

The transformation law for a covariant vector is given by

Select the correct option

- $\bar{A}_i = A_i \frac{\partial x^i}{\partial \bar{x}^j}$
- $\bar{A}_i = A_j \frac{\partial x^j}{\partial \bar{x}^i}$

MC220201510: RUBI NASEER

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Sta

Question # 5 of 5 (Start time: 08:08:54 PM, 07 December 2022)

Covariant components of a vector represent its ________

Select the correct option

- parallel projections
- perpendicular projections

Click to Save

MC220201510: RUBI NASEER

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 08:0

Question # 4 of 5 (Start time: 08:08:18 PM, 07 December 2022)

In Einstein notation, covariant components of a vector are denoted with ________

Select the correct option

- lower indices
- upper indices

Click to Save Answer & M

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 2 of 5 (Start time: 08:07:18 PM, 07 December 2022)

____________ of a vector represent its parallel projections onto coordinate axes.

Select the correct option

- Covariant components
- Contravariant components

Which one of the following is the covariant tensor of order 3?

Select the correct option

Reload Math Equations

- T_{ijk}
- A^{l}_{ij}
- A^{m}_{kl}
- T^{ijk}

Click to Save Answer & Move to Next Question

MC220201510: RUBI NASEER

Time Le

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 08:06 PM, 07 Dece

Question # 1 of 5 (Start time: 08:06:42 PM, 07 December 2022)

To

In Cartesian coordinate system the covariant and contravariant components of a vector are different

Select the correct option

Reload Math E

- false
- true

Click to Save Answer & Move to Next Que

Question # 4 of 5 (**Start time: 08:06:36 PM, 07 December 2022**) **Total Marks: 1**

The transformation law for a covariant vector is given by

Select the correct option

Reload Math Equations

- ◯ $\bar{A}_i = A_j \frac{\partial x^j}{\partial \bar{x}^i}$
- ◯ $\bar{A}_i = A_i \frac{\partial x^j}{\partial \bar{x}^j}$

Click to Save Answer & Move to Next Question

Question # 3 of 5 (Start time: 08:06:04 PM, 07 December 2022)

Total Marks: 1

Using the transformation law $\bar{x}_1 = 2x_1$ and $\bar{x}_2 = x_2$ find $\frac{\partial \bar{x}^1}{\partial x^2}$

Select the correct option

Reload Math Equations

- 1/2
- 1
- 2
- 0

Click to Save Answer & Move to Next Question

Question # 2 of 5 (Start time: 08:05:25 PM, 07 December 2022) **Total Marks: 1**

Which of the following is the example of a contravariant vector.

Select the correct option

Reload Math Equations

- ○ Velocity vector to a curve
- ○ Gradient vector to a surface

Click to Save Answer & Move to Next Question

Question # 1 of 5 (Start time: 08:04:13 PM, 07 December 2022) Total Marks: 1

A contravariant vector is often abbreviated as_____________.

Select the correct option

Reload Math Equations

- vector
- velocity vector

Click to Save Answer & Move to Next Question

Using the transformation law $\bar{x}_1 = 2x_1$ and $\bar{x}_2 = x_2$ find $\frac{\partial x^1}{\partial \bar{x}^1}$

Select the correct option

Reload Math Equations

◉	1/2
○	1

Click to Save Answer & Move to Next Question

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Remove chrome extension Manually

Quiz

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MC210401061: TANZEELA ARSHAD

Time Left

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 07:50 PM, 07 Decemb

Question # 5 of 5 (Start time: 07:55:37 PM, 07 December 2022)

Total

If tilde coordinate system corresponds to polar coordinates and non tilde Cartesian coordinates then find $\frac{\partial y^1}{\partial \bar{y}^1}$?

Select the correct option

Reload Math Equa

- $\cos\theta$
- $-\cos\theta$

Click to Save Answer & Move to Next Ques

Quiz
vulms.vu.edu.pk

MC210401061: TANZEELA ARSHAD

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Time Left 17 sec(s)

Quiz Start Time: 07:50 PM, 07 December 2022

Question # 4 of 5 (Start time: 07:53:52 PM, 07 December 2022)

Total Marks: 1

Which of the following is the example of a contravariant vector

Select the correct option

Reload Math Equations

- Gradient vector to a surface
- Velocity vector to a curve

Click to Save Answer & Move to Next Question

7:54

Quiz

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MC210401061: TANZEELA ARSHAD

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 07:50 PM,

Question # 4 of 5 (Start time: 07:53:52 PM, 07 December 2022)

Which of the following is the example of a contravariant vector

Select the correct option

Gradient vector to a surface

Reload

Velocity vector to a curve

Click to Save Answer & Move to Next

7:54

Quiz
vulms.vu.edu.pk

MC210401061: TANZEELA ARSHAD

Time Left 73 sec(s)

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 07:50 PM, 07 December 2022

Total Marks: 1

Question # 4 of 5 (Start time: 07:53:52 PM, 07 December 2022)

Which of the following is the example of a contravariant vector

Select the correct option

Reload Math Equations

- Gradient vector to a surface
- Velocity vector to a curve

Click to Save Answer & Move to Next Question

Quiz

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MC210401061: TANZEELA ARSHAD

Tim

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 07:50 PM, 07 De

Question # 3 of 5 (Start time: 07:53:06 PM, 07 December 2022)

The transformation law for a contravariant vector is given by

Select the correct option

Reload Math

- $\bar{A}^i = A^j \frac{\partial \bar{x}^i}{\partial x^j}$
- $\bar{A}^i = A^j \frac{\partial x^i}{\partial \bar{x}^j}$

Click to Save Answer & Move to Next Qu

Quiz
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MC210401061 TANZEELA ARSHAD

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 07:50 PM

Question # 2 of 5 (Start time: 07:52:03 PM, 07 December 2022)

Which one of the following is the contravariant tensor of order 2

Select the correct option

- A^{ij}
- T^{ij}
- A^{i}_{j}
- T_{ij}

Reload

Click to Save Answer & Move to Next

MC210401061 TANZEELA ARSHAD

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 2 of 5 (Start time: 07:52:03 PM, 07 December 2022)

Which one of the following is the contravariant tensor of order 2.

Select the correct option

- A^{ij}
- T^{ijk}
- A^i_j
- T_{ij}

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MC210401061: TANZEELA ARSHAD

Time Left 92

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 07:50 PM, 07 December 2022

Question # 1 of 5 (Start time: 07:50:48 PM, 07 December 2022)

Total Marks:

Covariant components of a vector represent its ....

Reload Math Equations

Select the correct option

- perpendicular projections
- parallel projections

Click to Save Answer & Move to Next Question

BC210417713: SABER ALI

Tim

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 07:43 PM, 07 De

Question # 5 of 5 (Start time: 07:46:37 PM, 07 December 2022)

Which of the following is the example of a contravariant vector.

Select the correct option

Reload Mat

◯	Gradient vector to a surface
◯	Velocity vector to a curve

Click to Save Answer & Move to Ne

Search the web and Windows

vulms.vu.edu.pk/Quiz/QuizQuestion.aspx?ver=f6cac9cd-bb24-4da1-92c8-a9b298c205ab

BC210417713: SABER ALI

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Time Left 87 sec(s)

Quiz Start Time: 07:43 PM, 07 December 2022

Question # 4 of 5 (Start time: 07:46:11 PM, 07 December 2022)

Total Marks: 1

Which one of the following is the covaraint tensor of order 3?

Select the correct option

Reload Math Equations

- A_{kl}^{m}
- A_{ij}^{j}
- T_{ijk}
- T^{ijk}

Click to Save Answer & Move to Next Question

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 07:43 PM, 07 December 20

Total Mar

Question # 3 of 5 (Start time: 07:44:56 PM, 07 December 2022)

------------- of a vector represent its parallel projections onto coordinate axes.

Reload Math Equatio

Select the correct option

- Covariant components
- Contravariant components

Click to Save Answer & Move to Next Qu

Search the web and Windows

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Time Left 88 sec(s)

Quiz Start Time: 07:43 PM, 07 December 2022

Question # 2 of 5 (Start time: 07:44:43 PM, 07 December 2022)

Total Marks: 1

Covariant components of a vector represent its _________

Select the correct option

Reload Math Equations

- parallel projections
- perpendicular projections

Click to Save Answer & Move to Next Question

Search the web and Windows

7:44 PM
12/7/2022

…SABER ALI

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Sta

Question # 1 of 5 (Start time: 07:43:45 PM, 07 December 2022)

If tilde coordinate system corresponds to polar coordinates and non tilde Cartesian coordinates then

Select the correct option

- $\cos\theta$
- $-\cos\theta$

Click to Save

Search the web and Windows

A vector whose ____________transform in a contravariant fashion
is a contravariant vector.

Select the correct option

Reload Math Equations

◯	direction
◯	components

Click to Save Answer & Move to Next Question

Question # 3 of 5 (Start time: 07:38:39 PM, 07 December 2022)

On transforming a vector the___________ change and the______

Select the correct option

- ○ magnitude, basis vectors
- ○ components, basis vectors
- ○ basis vectors, unit vectors
- ○ components, unit vectors

Which one of the following is the contravariant tensor of order 2.

Select the correct option

Reload Math Equations

- ○ T_{ij}
- ○ A^l_k
- ○ A^{ii}
- ○ T^{ij}

Click to Save Answer & Move to Next Question

_____________ of a vector represent its parallel projections onto coordinate axes.

Select the correct option

Reload Math Equations

- Covariant components
- Contravariant components

Click to Save Answer & Move to Next Question

Question # 2 of 5 (Start time: 07:37:32 PM, 07 December 2022)

Which one of the following is the contravariant tensor of order 2.

Select the correct option

○	A_k^l
○	T_{ij}
○	T^{ij}
○	A^{ii}

In Cartesian coordinate system the covariant and contravariant components of a vector are different.

Select the correct option

Reload Math Equations

- true
- false

Click to Save Answer & Move to Next Question

Which of the following is the example of a contravariant vector.

Select the correct option

Reload Math Equations

- ○ Velocity vector to a curve
- ○ Gradient vector to a surface

Click to Save Answer & Move to Next Question

Question # 4 of 5 (Start time: 07:33:02 PM, 07 December 2022)

In Einstein notation, covariant components of a vector are denoted with___

Select the correct option

- upper indices
- lower indices

Covectors are also called__________.

Select the correct option

- ○ dual vectors
- ○ unit vectors

Using the transformation law $\bar{x}_1 = 2x_1$ and $\bar{x}_2 = x_2$ find $\frac{\partial \bar{x}^1}{\partial x^1}$

Select the correct option

- 1
- 2
- 0
- 1/2

On transforming a vector the____________ change and the____________change however, the vector itself does not change.

Select the correct option

Reload Math Equ

- ○ basis vectors, unit vectors
- ○ magnitude, basis vectors
- ○ components, unit vectors
- ○ components, basis vectors

Click to Save Answer & Move to Next Que

Question # 5 of 5 (Start time: 07:07:02 PM, 07 December 2022)

The transformation law for a contravariant vector is given by

Select the correct option

- $\bar{A}^i = A^j \frac{\partial \bar{x}^i}{\partial x^j}$
- $\bar{A}^i = A^j \frac{\partial x^j}{\partial \bar{x}^i}$

Question # 4 of 5 (Start time: 07:05:50 PM, 07 December 2022)

The transformation rule for mixed tensor of order 2 is________

Select the correct option

- $\bar{T}^{i}_{j} = \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{n}}{\partial \bar{x}^{j}} T^{m}_{n}$
- $\bar{T}^{i}_{j} = \frac{\partial \bar{x}^{m}}{\partial x^{i}} \frac{\partial x^{j}}{\partial \bar{x}^{n}} T^{m}_{n}$

Question # 3 of 5 (Start time: 07:04:58 PM, 07 December 2022)

The order of ________ is 3.

Select the correct option

- B^{lm}_{mnk}
- A^{ij}_{j}

Quiz

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Using the transformation law $\bar{x}_1 = 2x_1$ and $\bar{x}_2 = x_2$ find $\frac{\partial \bar{x}^1}{\partial x^1}$

Select the correct option

- 2
- 1
- 1/2
- 0

Question # 1 of 5 (Start time: 07:03:09 PM, 07 December 2022)

__________ is a mixed tensor of order 3

Select the correct option

○	T^{l}_{mn}
○	T^{ij}_{j}
○	T^{l}_{kn}
○	T^{k}_{m}

1. A liquid does not form a free surface.
2. Gas expands to fill the entire available space.

Select the correct option

- ○ 1 only
- ○ Neither 1 nor 2
- ○ 1 and 2 both
- ○ 2 only

Click to Save Answer & Mo

If tilde coordinate system corresponds to polar coordinates and non tilde Cartesian coordi

Select the correct option

○	$-r\sin\theta$
○	$r\sin\theta$

Question # 2 of 5 (Start time: 07:25:51 PM, 07 December 2022)

Covariant components of a vector represent its _________

Select the correct option

- ○ perpendicular projections
- ○ parallel projections

____________ of a vector represent its parallel projections onto coordinate axes.

Select the correct option

○	Covariant components
○	Contravariant components

The transformation law for a contravariant vector is given by

Select the correct option

Reload Math Equations

○	$\bar{A}^i = A^i \frac{\partial \bar{x}^j}{\partial x^j}$
●	$\bar{A}^i = A^j \frac{\partial \bar{x}^i}{\partial x^j}$

Click to Save Answer & Move to Next Question

Which one of the following is the covaraint tensor of order 3?

Select the correct option

Reload Math Equations

○	T_{ijk}
○	A_{kl}^{m}
○	A_{ij}^{j}
○	T^{ijk}

Click to Save Answer & Move to Next Question

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2

BC210407871: KIRAN SHAHZADI

Time Left 56 sec(s)

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 06:32 PM, 07 December 2022

Question # 4 of 5 (Start time: 06:34:42 PM, 07 December 2022)

Total Marks: 1

The transformation rule for mixed tensor of order 2 is_____________.

Select the correct option

Reload Math Equations

- $\bar{T}^{i}_{j} = \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{n}}{\partial \bar{x}^{j}} T^{m}_{n}$
- $\bar{T}^{i}_{j} = \frac{\partial \bar{x}^{m}}{\partial x^{i}} \frac{\partial x^{i}}{\partial \bar{x}^{n}} T^{m}_{n}$

The transformation rule for mixed tensor of order 2 is_____________.

Select the correct option

Reload Math Equations

- $\bar{T}^{i}_{j} = \frac{\partial \bar{x}^{i}}{\partial x^{m}} \frac{\partial x^{n}}{\partial \bar{x}^{j}} T^{m}_{n}$
- $\bar{T}^{i}_{j} = \frac{\partial \bar{x}^{m}}{\partial x^{i}} \frac{\partial x^{j}}{\partial \bar{x}^{n}} T^{m}_{n}$

Click to Save Answer & Move to Next Question

Using the transformation law $\bar{x}_1 = 2x_1$ and $\bar{x}_2 = x_2$ find $\frac{\partial x^1}{\partial \bar{x}^1}$

Select the correct option

Reload Math Equations

○	1
○	1/2

Click to Save Answer & Move to Next Question

If tilde coordinate system corresponds to polar coordinates and non tilde Cartesian coordinates then find $\frac{\partial x^1}{\partial x^1}$?

Select the correct option

Reload Math Equations

○	$\cos\theta$
○	$-\cos\theta$

Click to Save Answer & Move to Next Question

Select the correct option

Reload Math Equations

- Velocity vector to a curve
- Gradient vector to a surface

Click to Save Answer & Move to Next Question

A vector whose ____________transform in a contravariant fashion
is a contravariant vector.

Select the correct option

Reload Math Equations

- ○ components
- ○ direction

Click to Save Answer & Move to Next Question

Which one of the following is the covaraint tensor of order 3?

Select the correct option

Reload Math Equations

- ( ) A_{kl}^{m}
- ( ) T_{ijk}
- ( ) T^{ijk}
- ( ) A_{ij}^{j}

Click to Save Answer & Move to Next Question

Select the correct option

Reload Math Equations

◯	A^{ii}
◯	A^{l}_{h}
◯	T_{ij}
◯	T^{ij}

Click to Save Answer & Move to Next Question

The transformation law for a contravariant vector is given by

ect the correct option

Reload Math Equation

$\tilde{A}^{\bar{i}} = A^{j}\frac{\partial x^{\bar{i}}}{\partial x^{j}}$

$\tilde{A}^{\bar{i}} = A^{i}\frac{\partial x^{j}}{\partial x^{\bar{i}}}$

Click to Save Answer & Move to Next Question

In Cartesian coordinate system the covariant and contravariant components of a vector are different.

Select the correct option

Reload Math Equations

- true
- false

Click to Save Answer & Move to Next Question

Reload Math Equations

○	upper indices
○	lower indices

Click to Save Answer & Move to Next Question

____________ of a vector represent its parallel projections onto coordinate axes.

Select the correct option

- Covariant components
- Contravariant components

Select the correct option

Reload Math Equations

○	$\cos\theta$
○	$-\cos\theta$

Click to Save Answer & Move to Next Question

Question # 5 of 5 (Start time: 06:25:18 PM, 07 December 2022)

If $\tilde{x}^1 = 2x^1$ and $\tilde{x}^2 = x^2$ then $x^1 =$ ___________ in terms of tilde coordinates.

Select the correct option

○	$\frac{1}{2}\tilde{x}^1$
○	$2\tilde{x}^1$

Select the correct option

○	A_{kl}^{m}
○	A_{ij}^{j}
○	T_{ijk}
○	T^{ijk}

Click to S

___________ is a mixed tensor of order 3.

Select the correct option

Reload Math Equations

- T^{k}_{m}
- T^{ij}_{j}
- T^{l}_{mn}
- T^{l}_{in}

Click to Save Answer & Move to Next Question

Select the correct option

○	true
○	false

Click to Save A

stion # 2 of 5 (Start time: 06:23:20 PM, 07 December 2022)

tilde coordinate system corresponds to polar coordinates and non tilde Cartesian coordinates then find $\frac{\partial x^1}{\partial \tilde{x}^1}$?

ct the correct option

- $-\cos\theta$
- $\cos\theta$

Click to Save Answ

Covectors are also called___________

Select the correct option

Reload Math Equations

○	dual vectors
○	unit vectors

Click to Save Answer & Move to Next Question

Which statement(s) is(are) true about the following sequence $a_1 = 1$ and $a_n = 1 - e^{-a_{n-1}}$

Select the correct option

Reload Math Equations

- $0 < a_n \leq 1$ for all n.
- Sequence is convergent.
- All of the above.
- $a_{n+1} - a_n = -\left(e^{-a_n} - e^{-a_{n-1}}\right)$

Click to Save Answer & Move to Next Question

BC210407344: MADEEHA RUBAB

Time Left 86 sec(s)

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 08:16 PM, 07 December 2022

Question # 5 of 5 (Start time: 08:20:23 PM, 07 December 2022)

Total Marks: 1

_____________ of a vector represent its parallel projections onto coordinate axes.

Select the correct option

Reload Math Equations

- Covariant components
- Contravariant components

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 1 of 5 (Start time: 08:20:34 PM, 07 December 2022)

A vector whose _____________ transform in a contravariant fashion is a contravariant vector.

Select the correct option

- ○ components
- ○ direction

Type here to search

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 1 of 5 (Start time: 08:20:34 PM, 07 December 2022)

A vector whose ____________ transform in a contravariant fashion is a contravariant vector.

Select the correct option

- components
- direction

Type here to search

Question # 2 of 5 (Start time: 08:21:28 PM, 07 December 2022)

Covariant components of a vector represent its ----------

Select the correct option

- ○ perpendicular projections
- ○ parallel projections

Type here to search

BC210427623: NOUREEN JAVAD

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 3 of 5 (Start time: 08:22:22 PM, 07 December 2022)

A contravariant vector is often abbreviated as_____________.

Select the correct option

- ○ vector
- ○ velocity vector

Type here to search

BC210427623: NOUREEN JAVAD

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 3 of 5 (Start time: 08:22:22 PM, 07 December 2022)

A contravariant vector is often abbreviated as_____________.

Select the correct option

- ○ vector
- ○ velocity vector

Type here to search

BC210427623: NOUREEN JAVAD

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 4 of 5 (Start time: 08:23:33 PM, 07 December 2022)

Using the transformation law $\tilde{x}_1 = 2x_1$ and $\tilde{x}_2 = x_2$ find $\frac{\partial \tilde{x}^1}{\partial x^2}$

Select the correct option

- ○ 2
- ○ 0
- ○ 1
- ○ 1/2

Type here to search

Question # 3 of 5 (Start time: 08:24:15 PM, 07 December 2022) Total Marks: 1

____________ is a mixed tensor of order 3.

Select the correct option

Reload Math Equations

- T^{l}_{lm}
- T^{k}_{m}
- T^{ij}_{j}
- T^{l}_{mn}

Click to Save Answer & Move to Next Question

BC210427623: NOUREEN JAVAD

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 3 of 5 (Start time: 08:22:22 PM, 07 December 2022)

A contravariant vector is often abbreviated as_____________.

Select the correct option

- vector
- velocity vector

Type here to search

Quiz
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BC210407344: MADEEHA RUBAB

Time Left 86 sec(s)

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Quiz Start Time: 08:16 PM, 07 December 2022

Question # 5 of 5 (Start time: 08:20:23 PM, 07 December 2022)

Total Marks: 1

_____________ of a vector represent its parallel projections onto coordinate axes.

Select the correct option

Reload Math Equations

- Covariant components
- Contravariant components

MTH623 - Tensor Analysis and its Applications (Quiz #1)

Question # 1 of 5 (Start time: 08:20:34 PM, 07 December 2022)

A vector whose ____________ transform in a contravariant fashion is a contravariant vector.

Select the correct option

- ○ components
- ○ direction

Type here to search

Question # 2 of 5 (Start time: 08:21:28 PM, 07 December 2022)

Covariant components of a vector represent its ----------

Select the correct option

- ○ perpendicular projections
- ○ parallel projections

Which of the following is the example of a contravariant vector.

Select the correct option

Reload Math Equations

- ○ Gradient vector to a surface
- ○ Velocity vector to a curve

Click to Save Answer & Move to Next Question

MC210401424: FARHAT FATIMA

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 3 of 10 (Start time: 12:43:20 PM, 25 January 2023)

The covariant differentiation of a tensor with constant components is zero.

Select the correct option

- False
- True

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Quiz Start Time: 12:

Question # 4 of 10 (Start time: 12:43:56 PM, 25 January 2023)

In generalized curvilinear coordinates the divergence of vector point function $A^i = A^i(x^i)$ is defined as___________

Select the correct option

- $div(A^i) = A^i_{,i} + \Gamma^i_{ik}$
- $div(A^i) = A_{i,i} + \Gamma^i_{ik} A^k$
- $div(A^i) = A^i_{,i} + \Gamma^k_{ii} A^k$
- $div(A^i) = A^i_{,i} + \Gamma^i_{ik} A^k$

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Quiz Start Time: 12:42

Question # 1 of 10 (Start time: 12:42:19 PM, 25 January 2023)

The ordinary derivative of a tensor is a tensor if and only if coordinate changes are restricted to ___________________.

Select the correct option

Relo

- none of these
- quadratic transformations
- non-linear transformations
- linear transformations

correct

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 10 of 10 (Start time: 12:50:44 PM, 25 January 2023)

The covariant differentiation of a scalar is equal to ___________________.

Select the correct option

○	scalar field itself correct
○	partial derivative of scalar field

210411522: ANEESA BATOOL

4623 - Tensor Analysis and its Applications (Quiz # 3) Quiz Start Time: 01:19 F

stion # 2 of 10 (Start time: 01:19:40 PM, 25 January 2023)

he ordinary derivative of a tensor is a tensor if and only if coordinate changes are restricted to ____________________

ct the correct option

Relo

- linear transformations
- quadratic transformations
- non-linear transformations
- none of these

Question # 10 of 10 (Start time: 01:21:17 PM, 25 January 2023)

The Christoffel symbols of first and second kind are equal in any coordinate system.

Select the correct option

- True
- False

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MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 10 of 10 (Start time: 01:21:17 PM, 25 January 2023)

The Christoffel symbols of first and second kind are equal in any coordinate system.

Select the correct option

◉	True
○	False

Question # 2 of 10 (Start time: 12:42:53 PM, 25 January 2023)

In the notation Γ^{k}_{ij} of Christoffel symbol of second, the symbol k tells____________

Select the correct option

- none of these
- which basis vector points in the direction of this component of the derivative vector

  correct
- which coordinate is being varied to cause a change in the basis vector
- which basis vector's change is being considered

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 3 of 10 (Start time: 12:43:20 PM, 25 January 2023)

The covariant differentiation of a tensor with constant components is zero.

Select the correct option

- False
- True

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Quiz Start Time: 12:42

Question # 4 of 10 (Start time: 12:43:56 PM, 25 January 2023)

In generalized curvilinear coordinates the divergence of vector point function $A^i = A^i(x^i)$ is defined as___________.

Select the correct option

Relo

◯	$div(A^i) = A^i_{,i} + \Gamma^i_{ik}$
◯	$div(A^i) = A_{i,i} + \Gamma^i_{ik} A^k$
◯	$div(A^i) = A^i_{,i} + \Gamma^k_{ii} A^k$
◯	$div(A^i) = A^i_{,i} + \Gamma^i_{ik} A^k$

MTH623 - Tensor Analysis and its Applications (Quiz # 3) Quiz Start Time

Question # 1 of 10 (Start time: 12:42:19 PM, 25 January 2023)

The ordinary derivative of a tensor is a tensor if and only if coordinate changes are restricted to ______________

Select the correct option

- none of these
- quadratic transformations
- non-linear transformations
- linear transformations

Question # 7 of 10 (Start time: 12:47:06 PM, 25 January 2023)

The Christoffel symbols of first kind Γ_{ijk} are symmetric in_____________.

Select the correct option

○	first two indices correct
○	second and third indices
○	first and third indices
○	all indices

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 5 of 10 (Start time: 12:45:22 PM, 25 January 2023)

The Christoffel symbols of second kind are symmetric in__________.

Select the correct option

○	all indices
○	lower indices correct

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 9 of 10 (Start time: 12:48:57 PM, 25 January 2023)

The Christoffel symbols of first kind Γ_{ijk} are defined by ___________

Select the correct option

○	$\Gamma_{jki} = \frac{1}{2}(-g_{jk,i} + g_{ki,j} + g_{ki,j})$
○	$\Gamma_{ijk} = \frac{1}{2}(g_{jk,i} + g_{ki,j} + g_{ki,j})$
○	$\Gamma_{ij} = \frac{1}{2}(g_{jk,i} + g_{ki,j} + g_{ki,j})$
○	$\Gamma_{ijk} = \frac{1}{2}(-g_{jk,i} + g_{ki,j} + g_{ki,j})$ correct

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 10 of 10 (Start time: 12:50:44 PM, 25 January 2023)

The covariant differentiation of a scalar is equal to __________________.

Select the correct option

- scalar field itself
- partial derivative of scalar field

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 1 of 10 (Start time: 01:19:32 PM, 25 January 2023)

The covariant differentiation of metric tensor is ______________________

Select the correct option

- zero in all coordinate systems
- zero in rectangular Cartesian coordinates only

MTH623 - Tensor Analysis and its Applications (Quiz # 3) Qu

Question # 2 of 10 (Start time: 01:19:40 PM, 25 January 2023)

The ordinary derivative of a tensor is a tensor if and only if coordinate changes are restricted to ______

Select the correct option

- (selected) linear transformations
- quadratic transformations
- non-linear transformations
- none of these

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 8 of 10 (Start time: 01:20:57 PM, 25 January 2023)

In curvilinear coordinates the derivative of a vector also includes___________

Select the correct option

- [x] the derivatives of basis vectors
- [ ] the derivatives of unit vectors

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 10 of 10 (Start time: 01:21:17 PM, 25 January 2023)

The Christoffel symbols of first and second kind are equal in any coordinate system.

Select the correct option

- [x] True
- [ ] False

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 2 of 10 (Start time: 12:42:53 PM, 25 January 2023)

In the notation Γ^k_{ij} of Christoffel symbol of second, the symbol k tells____________

Select the correct option

- none of these
- which basis vector points in the direction of this component of the derivative vector
- which coordinate is being varied to cause a change in the basis vector
- which basis vector's change is being considered

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 3 of 10 (Start time: 12:43:20 PM, 25 January 2023)

The covariant differentiation of a tensor with constant components is zero.

Select the correct option

- False
- True

Question # 4 of 10 (Start time: 12:43:56 PM, 25 January 2023)

In generalized curvilinear coordinates the divergence of vector point function $A^i = A^i(x^i)$ is defined as__

Select the correct option

○	$div(A^i) = A^i_{,i} + \Gamma^i_{ik}$
○	$div(A^i) = A_{i,i} + \Gamma^i_{ik} A^k$
○	$div(A^i) = A^i_{,i} + \Gamma^k_{ii} A^k$
○	$div(A^i) = A^i_{,i} + \Gamma^i_{ik} A^k$

MTH623 - Tensor Analysis and its Applications (Quiz # 3) Quiz

Question # 1 of 10 (Start time: 12:42:19 PM, 25 January 2023)

The ordinary derivative of a tensor is a tensor if and only if coordinate changes are restricted to ______

Select the correct option

- none of these
- quadratic transformations
- non-linear transformations
- linear transformations

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 7 of 10 (Start time: 12:47:06 PM, 25 January 2023)

The Christoffel symbols of first kind Γ_{ijk} are symmetric in__________

Select the correct option

○	first two indices
○	second and third indices
○	first and third indices
○	all indices

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 8 of 10 (Start time: 12:47:54 PM, 25 January 2023)

The covariant derivative and partial derivatives coincide when __________ are constant.

Select the correct option

- all components of metric tensor
- Christoffel symbols

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 5 of 10 (Start time: 12:45:22 PM, 25 January 2023

The Christoffel symbols of second kind are symmetric in__

Select the correct option

○	all indices
○	lower indices

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 6 of 10 (Start time: 12:46:25 PM, 25 January 2023)

The Christoffel symbols of first and second kind are equal in any coordinate system.

Select the correct option

- [ ] False
- [ ] True

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 9 of 10 (Start time: 12:48:57 PM, 25 January 2023)

The Christoffel symbols of first kind Γ_{ijk} are defined by ____________

Select the correct option

- $\Gamma_{jki} = \frac{1}{2}(-g_{jk,i} + g_{ki,j} + g_{ki,j})$
- $\Gamma_{ijk} = \frac{1}{2}(g_{jk,i} + g_{ki,j} + g_{ki,j})$
- $\Gamma_{ij} = \frac{1}{2}(g_{jk,i} + g_{ki,j} + g_{ki,j})$
- $\Gamma_{ijk} = \frac{1}{2}(-g_{jk,i} + g_{ki,j} + g_{ki,j})$

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 10 of 10 (Start time: 12:50:44 PM, 25 January 2023)

The covariant differentiation of a scalar is equal to ___________________.

Select the correct option

○	scalar field itself
○	partial derivative of scalar field

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 1 of 10 (Start time: 01:19:32 PM, 25 January 2023)

The covariant differentiation of metric tensor is ________________________

Select the correct option

- zero in all coordinate systems
- zero in rectangular Cartesian coordinates only

MTH623 - Tensor Analysis and its Applications (Quiz # 3) Quiz Start

Question # 2 of 10 (Start time: 01:19:40 PM, 25 January 2023)

The ordinary derivative of a tensor is a tensor if and only if coordinate changes are restricted to ___________

Select the correct option

- [x] linear transformations
- [ ] quadratic transformations
- [ ] non-linear transformations
- [ ] none of these

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 3 of 10 (Start time: 01:19:46 PM, 25 January 2023)

The Christoffel symbols of first kind satisfy the transformation law of a third rank tensor.

Select the correct option

- [x] False
- [ ] True

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 8 of 10 (Start time: 01:20:57 PM, 25 January 2023)

In curvilinear coordinates the derivative of a vector also includes_____________

Select the correct option

- [x] the derivatives of basis vectors
- [ ] the derivatives of unit vectors

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 7 of 10 (Start time: 01:20:51 PM, 25 January 2023)

Christoffel symbols of first and second kind include derivatives of ___________

Select the correct option

- [x] basis vectors
- [ ] unit vectors
- [ ] metric tensor
- [ ] normal vectors

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 6 of 10 (Start time: 01:20:31 PM, 25 January 2023)

The Christoffel symbols of first kind Γ_{ijk} are defined by ____________

Select the correct option

- ( ) $\Gamma_{jki} = \frac{1}{2}(-g_{jk,i} + g_{ki,j} + g_{ki,j})$
- ( ) $\Gamma_{ij} = \frac{1}{2}(g_{jk,i} + g_{ki,j} + g_{ki,j})$
- (•) $\Gamma_{ijk} = \frac{1}{2}(-g_{jk,i} + g_{ki,j} + g_{ki,j})$
- ( ) $\Gamma_{ijk} = \frac{1}{2}(g_{jk,i} + g_{ki,j} + g_{ki,j})$

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 10 of 10 (Start time: 01:21:17 PM, 25 January 2023)

The Christoffel symbols of first and second kind are equal in any coordinate system.

Select the correct option

- True
- False

Rank of a tensor derivative:

The derivative of a zero order tensor becomes a first order tensor.
Generally, the derivative of an m- order tensor forms an $m+1$ order tensor.

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Question # 5 of 10 (Start time: 03:06:34 PM, 25 January 2023)

In any coordinate system x^i, the covariant derivative with respect to x^k of a covariant vector T_i is defined as_____.

Select the correct option

- $T_{i;k} = \frac{\partial T_i}{\partial x^k} - \Gamma^t_{ik} T_t$
- $T_{i;k} = \frac{\partial T_i}{\partial x^k} - \Gamma^i_{ik} T_t$
- $T_{i;k} = \frac{\partial T_i}{\partial x^k} - \Gamma^t_{ik}$
- $T_{i;k} = \frac{\partial T_i}{\partial x^k} + \Gamma^t_{ik} T_t$

The covariant derivative of an arbitrary tensor is a tensor.

Select the correct option

- False
- True

The Christoffel symbols of first kind satisfy the transformation law of a third rank tensor.

lect the correct option

True

False

Question # 3 of 10 (Start time: 12:16:16 PM, 25 January 2023)

In generalized curvilinear coordinates the gradient of scalar function $\phi = \phi(x^i)$ is defined as __

Select the correct option

○	$\nabla\phi = \frac{\partial\phi}{\partial x^i}$
○	$\nabla\phi = \partial\phi\hat{e}_i$
○	$\nabla\phi = \frac{\partial\phi}{\partial x^i}\hat{e}_i$ correct
○	$\nabla\phi = \frac{\partial x^i}{\partial\phi}$

Cl

For a diagonal metric which of the following is true?

Select the correct option

- $\Gamma_{iii} = \frac{1}{2} g_{ij,j}$
- $\Gamma_{iii} = \frac{1}{2} g_{ii,i}$

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 7 of 10 (Start time: 08:32:40 PM, 24 January 2023)

The Christoffel symbols of first kind satisfy the transformation law of a third rank tensor.

Select the correct option

- False

correct

- True

Question # 4 of 10 (Start time: 08:26:32 PM, 24 January 2023)

Christoffel symbols of first and second kind include derivatives of ___________.

Select the correct option

- unit vectors
- basis vectors

correct

- metric tensor
- normal vectors

Question # 2 of 10 (Start time: 08:23:11 PM, 24 January 2023)

The covariant differentiation of a tensor with constant components is zero.

Select the correct option

- True
- False

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 1 of 10 (Start time: 08:21:12 PM, 24 January 2023)

In Cartesian coordinate system all the Christoffel symbols are ______

Select the correct option

- zero
- vanish
- equal
- all options are true

Question # 2 of 10 (Start time: 08:23:11 PM, 24 January 2023)

The covariant differentiation of a tensor with constant components is zero.

Select the correct option

True

False

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 1 of 10 (Start time: 08:21:12 PM, 24 January 2023)

In Cartesian coordinate system all the Christoffel symbols are ______

Select the correct option

○	zero
○	vanish
○	equal
○	all options are true

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 1 of 10 (Start time: 08:21:12 PM, 24 January 2023)

In Cartesian coordinate system all the Christoffel symbols are _____

Select the correct option

- zero
- vanish
- equal
- all options are true

Question # 2 of 10 (Start time: 08:23:11 PM, 24 January 2023)

The covariant differentiation of a tensor with constant components is zero.

Select the correct option

○	True
○	False

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 3 of 10 (Start time: 08:24:55 PM, 24 January 2023)

In the notation Γ^k_{ij} of Christoffel symbol of second, the symbol k tells____________

Select the correct option

- none of these
- which coordinate is being varied to cause a change in the basis vector
- which basis vector's change is being considered
- which basis vector points in the direction of this component of the derivative v

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 3 of 10 (Start time: 08:24:55 PM, 24 January 2023)

In the notation Γ_{ij}^{k} of Christoffel symbol of second, the symbol k tells____________

Select the correct option

○	none of these
○	which coordinate is being varied to cause a change in the basis vector
○	which basis vector's change is being considered
○	which basis vector points in the direction of this component of the derivative v

Question # 4 of 10 (Start time: 08:26:32 PM, 24 January 2023)

Christoffel symbols of first and second kind include derivatives of ___________

Select the correct option

- unit vectors
- basis vectors
- metric tensor
- normal vectors

Question # 5 of 10 (Start time: 08:28:43 PM, 24 January 2023)

The covariant differentiation of metric tensor is ______________________.

Select the correct option

- zero in all coordinate systems
- zero in rectangular Cartesian coordinates only

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 7 of 10 (Start time: 08:32:40 PM, 24 January 2023)

The Christoffel symbols of first kind satisfy the transformation law of a third rank tensor.

Select the correct option

- False
- True

The Christoffel symbols of first kind satisfy the transformation law of a third rank tensor.

Select the correct option

- False
- True

The covariant differentiation of metric tensor is ______________________.

Select the correct option

- ○ zero in all coordinate systems
- ○ zero in rectangular Cartesian coordinates only

MTH623 - Tensor Analysis and its Applications (Quiz # 3)

Question # 9 of 10 (Start time: 04:31:08 PM, 25 January 2023)

In generalized curvilinear coordinates the gradient of scalar function $\phi = \phi(x^i)$ is defined as _

Select the correct option

- $\nabla\phi = \frac{\partial\phi}{\partial x^i}$
- $\nabla\phi = \frac{\partial x^i}{\partial\phi}$
- $\nabla\phi = \partial\phi \hat{e_i}$
- $\nabla\phi = \frac{\partial\phi}{\partial x^i}\hat{e_i}$

$$C: x^i = x^i(t)$$

Consider the transformation law:

$$\boxed{\bar{T}^i = T^r \frac{\partial \bar{x}^i}{\partial x^r}} \quad -1$$

$$\bar{T}^i =$$

Differentiating w.r.t 't' gives:

$$\boxed{\frac{d\bar{T}^i}{dt} = \frac{dT^r}{dt}\,\frac{\partial \bar{x}^i}{\partial x^r} + \left[T^r\,\frac{\partial^2 \bar{x}^i}{\partial x^s \partial x^r}\cdot\frac{dx^s}{dt}\right]} \quad -2$$

$$\frac{d}{dt} = \frac{\partial}{\partial x^s}\cdot\frac{dx^s}{dt}$$

which shows that the ordinary derivative of tensor T along the curve is a when the x' are the linear

functions of the x^r. —3

Theorem:

The derivative of a tensor is a tensor if and only if coordinate changes are restricted to linear transformations. —4

Example: Let C is a curve defined by

$C: x^i = x^i(t)$ then

$\vec{T} = \frac{d\vec{r}}{dt}$ is tangent field along curve C,

and $k = \left\| \frac{d\vec{T}}{dt} \right\|$ is the curvature.

but $\boxed{\text{'k' is not an invariant}}$ in curvilinear coordinates since $d\vec{T}/dt$ is not a general tensor. Clearly, we require a general concept of tensor differentiation.

Covariant Derivative :

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differentiation.

Covariant Derivative :

The covariant derivative of an
-6
arbitrary tensor is a tensor of which the covariant order exceeds that of the original tensor by exactly one.

becomes a first order tensor.

Generally, the derivative of an

$\boxed{m\text{- order tensor forms an } m+1 \text{ order tensor.}}$

-7

However, if the derivative index is a dummy index, then the rank of the derivative will be lower than that of the original tensor. For example of the derivative will be lower than that of the original tensor. For example

$$A_{i j, j} \rightarrow \text{rank is one.}$$

b/c there is only one free index i.

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Now recall that

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

$$\boxed{\begin{aligned} \tilde{e}_r &= \cos\theta\,\vec{e}_1 + \sin\theta\,\vec{e}_2 = \vec{\tilde{e}}_1 \qquad -10 \\ \tilde{e}_\theta &= -r\sin\theta\,\vec{e}_1 + r\cos\theta\,\vec{e}_2 = \vec{\tilde{e}}_2 \end{aligned}}$$

$$\Rightarrow \frac{\partial \vec{\tilde{e}}_1}{\partial \tilde{x}^1} = 0 \quad ,$$

So, in general

$$\boxed{\frac{\partial \vec{V}}{\partial x^i} = \frac{\partial v^j}{\partial x^i}\,\vec{e}_j + v^j\,\frac{\partial \vec{e}_j}{\partial x^i}} \quad -11$$

where

$$\boxed{\frac{\partial \vec{e}_j}{\partial x^i} = \Gamma^k_{ij}\,\vec{e}_k} \quad -12.$$

$$\boxed{\frac{\partial \vec{V}}{\partial x^{i}} = \frac{\partial V^{j}}{\partial x^{i}}\,\tilde{\vec{e}}_{j} + V^{j}\,\frac{\partial \vec{e}_{j}}{\partial x^{i}}} \quad -11$$

where

$$\boxed{\begin{aligned}\frac{\partial \tilde{\vec{e}}_{j}}{\partial x^{i}} &= \Gamma^{k}_{ij}\,\tilde{\vec{e}}_{k} \\ &= \Gamma^{1}_{ij}\,\tilde{\vec{e}}_{1} + \Gamma^{2}_{ij}\,\tilde{\vec{e}}_{2}\end{aligned}} \quad -12.$$

$$\Rightarrow \boxed{\frac{\partial \vec{V}}{\partial x^j} = \frac{\partial V^i}{\partial x^j}\vec{e}_i + V^i \Gamma^k_{ij}\vec{e}_k} \qquad 13$$

These Γ^k_{ij} are the christoffel

Symbols and the derivative on

functions involving derivatives of the components of the fundamental tensor 'g_{ij}'. These functions are called christoffel symbols or an affine connection.

Basically, these quantities represent

Definition:

The quantities or functions defined by

$$\boxed{\Gamma_{ijk} = \frac{1}{2}\left[\frac{\partial}{\partial x^i}(g_{jk}) + \frac{\partial}{\partial x^j}(g_{ki}) - \frac{\partial}{\partial x^k}(g_{ij})\right]} \qquad 15$$

are the Christoffel Symbols of the first kind.

Consider:

$$\boxed{\bar{\Gamma}_{ijk} = \frac{1}{2}\left(-\bar{g}_{ij,k} + \bar{g}_{jk,i} + \bar{g}_{ki,j}\right)} \quad -16$$

now replacing all values of the metric tensor derivative, we have:

$$\bar{\Gamma}_{ijk} = \frac{1}{2}\left[-g_{rs,t} + g_{st,r} + g_{tr,s}\right] \frac{\partial x^r}{\partial \bar{x}^i} \frac{\partial x^s}{\partial \bar{x}^j} \frac{\partial x^t}{\partial \bar{x}^k}$$

(by interchanging the dummy indices r and s in last term.)

$\Rightarrow$ Christoffel Symbol is not a general tensor.

– 16

Question: Compute the christoffel symbols corresponding to the Euclidean metric for spherical coordinates.

$$[g_{\mu\nu}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (x^1)^2 & 0 \\ 0 & 0 & (x^1)^2 \sin^2 x^2 \end{bmatrix}$$

A_{21}

-17

Solution: The formula for christoffel symbols is:

$\Gamma_{111} = \Gamma_{333}$

$$\Gamma_{ijk} = \frac{1}{2}\left[-g_{ij,k} + g_{jk,i} + g_{ki,j}\right]$$

$$= \frac{1}{2}\left(2x^1\right) = x^1 \quad \text{or} \quad \boxed{\Gamma_{212} = r}$$

Find 18

$$\boxed{\Gamma_{122} = \frac{1}{2}\left(-g_{12,2}^{\to 0} + g_{22,1} + g_{21,2}^{\to 0}\right)}$$

$$= \frac{1}{2}\left(x^1\right) = x^1$$

$$\Gamma_{223} = \frac{1}{2}\left(-g_{22,3} + g_{23,2} + g_{32,2}\right) = 0$$

first Kind:

i) $\Gamma_{ijk} = \Gamma_{jik}$ (symmetric in first two indices)

ii) All $\Gamma_{ijk} = 0$ vanish if all g_{ij} are constant.

ib

14

matrix, then for all fixed subscripts

α and $\beta \neq \alpha$ in the range $1, 2, \ldots, n$,

1) $\boxed{\Gamma_{\alpha\alpha\alpha} = \frac{1}{2} g_{\alpha\alpha,\alpha}}$ (not summed on α)

Z.O

2) $-\Gamma_{\alpha\alpha\beta} = \Gamma_{\alpha\beta\alpha} = \Gamma_{\beta\alpha\alpha} = \frac{1}{2} g_{\alpha\alpha\beta}$ (not summed on α)

3) All remaining Christoffel symbols Γ_{ijk} are zero.

implying that

$$\Gamma_{ijk} = 0$$

$$\boxed{\begin{gathered}\Gamma_{ijk} = \frac{1}{2}\left[-g_{ij,k} + g_{jk,i} + g_{ki,j}\right] \\ i \neq j \neq k\end{gathered}}$$

21

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$$\boxed{\Rightarrow g_{ik,j} = \Gamma_{ijk} + \Gamma_{jki}} \qquad 22$$

Now, the conclusion would be valid if the Γ_{ijk} vanished in every coordinate system. But Γ_{ijk} is not a tensor and the conclusion is false. For instance, all $\Gamma_{ijk} = 0$ for the Euclidean metric in rectangular

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43 (43 / 441)

View and annotate

Christoffel Symbols of the Second Kind:

The functions

62

$$\boxed{\Gamma^i_{jk} = g^{ih}\,\Gamma_{jkh}}$$

23

are the Christoffel symbols of the second kind. It is simply the result of raising the third subscript of the Christoffel symbol of the first kind.

24

43 / 441

100%

1) $\Gamma^{i}_{jk} = \Gamma^{i}_{kj}$ (Symmetric in the lower indices.)

25

2) All Γ^{i}_{jk} vanish if all g_{ij} are constant.

This Christoffel symbol gives you the magnitude of one

Tells you which basis vector points in the direction of this

Transformation law:

2^{6}

Starting with:

$$\boxed{\tilde{\Gamma}^{i}_{jk} = \tilde{g}^{ir}\,\tilde{\Gamma}_{jkr} = \left(g^{st}\frac{\partial \tilde{x}^{i}}{\partial x^{s}}\frac{\partial \tilde{x}^{r}}{\partial x^{t}}\right)\tilde{\Gamma}_{jkr}}$$

Substitute for $\tilde{\Gamma}_{jkr}$ from the transformation law of Christoffel symbol of first kind, we have of the second kind for the Euclidean metric in polar coordinates.

$$\boxed{[g_{\mu\nu}] = \begin{bmatrix} 1 & 0 \\ 0 & (x^1)^2 \end{bmatrix}} \quad \begin{pmatrix} x^1 \to r \\ x^2 \to \theta \end{pmatrix}$$

r, θ

Solution:

Using the formula:

$$\Gamma^i_{jk} = \frac{g^{ir}}{2}\left[-g_{jk,r} + g_{kr,j} + g_{rj,k}\right]$$

Show that for all fixed indices in the range $1, 2, \ldots, n$.

a) $\boxed{\Gamma^{\alpha}_{\alpha\beta} = \Gamma^{\alpha}_{\beta\alpha} = \frac{\partial}{\partial x^{\beta}}\left(\frac{1}{2}\ln|g_{\alpha\alpha}|\right)}$ 29

consider

$$\Gamma^{\alpha}_{\alpha\beta} = \frac{g^{\alpha r}}{2}\left[-g_{\alpha\beta,r} + g_{\alpha r,\beta} + g_{\beta r,\alpha}\right]$$

$$= \frac{g^{\alpha\alpha}}{2}\left[g_{\alpha\alpha,\beta} + g_{\beta\alpha,\alpha}\right] = \frac{1}{2} g^{\alpha\alpha} g_{\alpha\alpha,\beta}$$

$$= \frac{1}{2}\frac{g_{\alpha\alpha,\beta}}{g} = \frac{\partial}{\partial x^{\beta}}\left(\frac{1}{2}\ln g_{\alpha\alpha}\right)$$ by $\div$

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b) $\boxed{\Gamma^{\alpha}_{\beta\beta} = \frac{-1}{2g_{\alpha\alpha}}\, g_{\beta\beta,\alpha}}$

$$\Gamma^{\alpha}_{\beta\beta} = \frac{g^{\alpha\lambda}}{2}\left[-g_{\beta\beta,\lambda} + g_{\beta\lambda,\beta} + g_{\beta\lambda,\beta}\right]$$

$$= \frac{g^{\alpha\alpha}}{2}\left[-g_{\beta\beta,\alpha} + \underset{\to 0}{g_{\beta\alpha,\beta}} + \underset{\to 0}{g_{\beta\alpha,\beta}}\right]$$

$$= -g^{\alpha\alpha}\, g_{\beta\beta} \ldots$$

55 / 441

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56 (56 / 441)

Question: Christoffel symbols of the second kind for the Euclidean metric in spherical coordinates.

here, $[g_{\alpha\beta}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (x^1)^2 & 0 \\ 0 & 0 & (x^1)^2 \sin^2 x^2 \end{bmatrix}$ ✓ 3

Then the non zero Christoffel symbols of second kind are:

$$\Gamma^2_{21} = \Gamma^2_{12} = \frac{\partial}{\partial x^1}\left(\frac{1}{2}\ln (x^1)^2\right) = \frac{1}{x^1}$$

56 / 441 100%

$\Gamma_{11} = 1$, $\Gamma_{22} = 2$ and $\Gamma_{33} = $ ...

all other are zero.

Solution: $x^i \to$ rectangular Coordinates

$$\Rightarrow [g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \Gamma^r_{st} = 0 \qquad 32$$

So Eq.★ implies:

$$\frac{\partial^2 x^r}{\partial \tilde{x}^i \partial \tilde{x}^j} = \tilde{\Gamma}^s_{ij} \frac{\partial x^r}{\partial \tilde{x}^s}$$

of T_i itself, the result is a tensor.

Definition:

In any coordinate system x^i, the covariant derivative with respect to x^k of a covariant vector T_i is the tensor

$$T_{i;k} = \frac{\partial T_i}{\partial x^k} - \Gamma^t_{ik} T_t$$

34

* here the second index 'k' emphasize that the

derivative with respect to x^k of a contravariant vector T^i is the tensor

$$\boxed{T^i_{;k} = \frac{\partial T^i}{\partial x^k} + \Gamma^i_{jk} T^j} \quad 34$$

35

★ $$T^{\alpha}{}_{\beta;\gamma} = T^{\alpha}{}_{\beta,\gamma} + \Gamma^{\alpha}_{\gamma\delta} T^{\delta}{}_{\beta} - \Gamma^{\delta}_{\beta\gamma} T^{\alpha}{}_{\delta}$$

$$T_{\alpha\beta;\gamma} = T_{\alpha\beta,\gamma} - \Gamma^{\delta}_{\alpha\gamma} T_{\delta\beta} - \Gamma^{\delta}_{\beta\gamma} T_{\alpha\delta}$$

$$T^{\alpha\beta}{}_{;\gamma} = T^{\alpha\beta}{}_{,\gamma} + \Gamma^{\alpha}_{\delta\gamma} T^{\delta\beta} + \Gamma_{\gamma\delta} T^{\alpha\delta}$$

★ The covariant derivative of a Scalar function is just the partial derivative.

★ The covariant derivative of a Scalar function is just the partial derivative.

$$\boxed{\phi_{;\alpha} = \phi_{,\alpha}}$$

36

Question: Consider polar coordinates and find the covariant derivative $V^{\alpha}{}_{;\alpha}$ of $V^{\alpha} = r^2 \cos\theta \, \hat{e}_r - \sin\theta \, \hat{e}_\theta$.

65

Question: The covariant derivative of Kronecker delta is zero.

66

37

Solution: $\boxed{\delta^i_{j;k} = ?}$

$$\delta^i_{j;k} = \underbrace{\delta^i_{j,k}}_{0} + \Gamma^i_{lk}\,\delta^l_j - \Gamma^l_{jk}\,\delta^i_l$$

$$= \Gamma^i_{jk} - \Gamma^i_{jk}$$

$$= 0$$

Question: Show that $\boxed{g_{\alpha\beta;\gamma} = 0}$ 38

Solution:

$$g_{\alpha\beta;\gamma} = g_{\alpha\beta,\gamma} - \Gamma^{s}_{\alpha\gamma}\, g_{s\beta} - \Gamma^{s}_{\beta\gamma}\, g_{\alpha s}$$

using the formulas:

$$\Gamma^{s}_{\alpha\gamma} = \frac{1}{2} g^{st} \left(g_{t\gamma,\alpha} + g_{t\alpha,\gamma} - g_{\alpha\gamma,t} \right)$$

$$= A^{i}_{;i} + \Gamma^{i}_{ik} A^{k}$$

and similarly:

$$\boxed{\operatorname{div}(A_i) = A_{i;i} = A_{i,i} - \Gamma^{k}_{ii} A_k}$$

Question: Prove that

Question: Prove that

$$\boxed{\operatorname{div} A^i = \frac{1}{\sqrt{g_{\alpha\alpha}}} \frac{\partial\left(\sqrt{g_{\alpha\alpha}}\, A^k\right)}{\partial x^k}}$$

4

Solution:

$$\operatorname{div} A^i = A^i_{;i} = A^i_{,i} + \Gamma^i_{ki} A^k$$

So in terms of covariant derivative we have

$$\boxed{[\nabla \times \vec{A}]_i = \varepsilon_{ijk} \, A_{k;j}}$$ 42

where

$$A_{k;j} = A_{k,j} - \Gamma^{s}_{kj} A_s$$

Laplacian:

Implies

$$\boxed{\nabla.\nabla\phi = \left(\frac{\partial\phi}{\partial x^i}\right)_{;i}}$$ 43

Absolute Differentiation Along a curve: 68

inner product

As $T^i_{;j}$ is a tensor, the inner product of $T^i_{;j}$ with another tensor is also a tensor.

44

dx^i

This tensor is known as the absolute derivative of T^i along the curve C, with components written as

$$\boxed{\frac{\delta T^i}{\delta t} = \frac{dT^i}{dt} + \Gamma^i_{rs} T^s \frac{dx^r}{dt} \; ; \quad T^i = T^i(x^i(t))}$$

It is clear that, in coordinate systems in which the g_{ij} are constant, absolute differentiation reduces to ordinary differentiation.

$T^i_{,r}\dfrac{dx^r}{dt}$ is a tensor of the same type and order as the original tensor T^i.

پراڈکٹ میں Rank ایک جیسا رہتا

This tensor is known as the absolute derivative of T^i along the curve C, with components written as

$$\boxed{\frac{\delta T^i}{\delta t}=\frac{dT^i}{dt}+\Gamma^i_{rs}T^s\frac{dx^r}{dt}\ ; \quad T^i=T^i(x^i(t))}$$

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Question: Prove that absolute differentiation $\frac{\delta T^i}{\delta t}$ is the result of inner product of $T^i_{;j}$ with the tangent vector $\frac{dx^i}{dt}$ of the curve.

Proof:

$$T^i_{;j} = \frac{\partial T^i}{\partial x^j} + \Gamma^i_{rj} T^r$$

Inner product:

$$T^i_{;j}\frac{dx^j}{dt} = \left(\frac{\partial T^i}{\partial x^j} + \Gamma^i_{rj} T^r\right)\frac{dx^j}{dt} = \frac{\partial T^i}{\partial x^j}\frac{dx^j}{dt} + \Gamma^i_{rj} T^r \frac{dx^j}{dt}$$

106 / 441

The Torsion Tensor:

The $\boxed{\text{torsion tensor}}$ is defined as: tensor

$$\boxed{T^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\beta\gamma} - \Gamma^{\alpha}_{\gamma\beta}}$$

Not Symm

Even though Christoffel Symbols are not tensors their difference is a tensor.

In torsion free spaces $\Gamma^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\gamma\beta}$

The length of this vector at time t is
the instantaneous acceleration of the particle.

$$\boxed{a = \sqrt{\delta_{ij}\, a^i a^j}} \quad 49$$

The generalization of $\frac{d}{dt}\left(\frac{dx^i}{dt}\right)$

$\vec{a} =$

$$\frac{\delta}{\delta t}\left(\frac{dx^i}{dt}\right) = \frac{d}{dt}\left(\frac{dx^i}{dt}\right) + \Gamma^i_{rs} \frac{dx^r}{dt} \frac{dx^s}{dt}$$

from Formula

Hence.

Leo

Remember me in prayer

from Formula

Hence,

$$\boxed{\vec{a} = (a^i) = \left(\frac{d^2x^i}{dt^2} + \Gamma^i_{rs} \frac{dx^r}{dt} \frac{dx^s}{dt} \right)}$$

and $a = \sqrt{|g_{ij} a^i a^j|}$

<u>Question</u>: A particle is in motion along the circular arc given parametrically in Spherical derivative of $(x^i(s))$:

$$\boxed{K(s) = \sqrt{\delta_{ij} \frac{d^2x^i}{ds^2} \frac{d^2x^j}{ds^2}}}$$

where

$$\boxed{s(t) = \int_0^t \sqrt{\delta_{ij} \left(\frac{dx^i}{dt}\right)\left(\frac{dx^j}{dt}\right)}\, dt \text{ gives the arc length parameter.}}$$

The generalized formula for curvature is:

$$K(s) = \sqrt{|g_{ij}\, b^i b^j|}$$

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$$b^2 = \frac{d^2x^2}{ds^2} + \Gamma^2_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds} \qquad \frac{d}{ds} = \frac{dx}{dt}\frac{dt}{ds}$$

where $\boxed{\dfrac{ds}{dt} = \sqrt{g_{ij}\dfrac{dx^i}{dt}\dfrac{dx^j}{dt}}}$ 54

$$\Rightarrow \frac{ds}{dt} = \left[g_{11}\frac{dx^1}{dt}\frac{dx^1}{dt} + g_{22}\frac{dx^2}{dt}\frac{dx^2}{dt}\right]^{1/2}$$

$$\frac{ds}{dt} = \left[(x^1)^2(1)(1)\right]^{1/2} = x^1$$

Now: $b^i = \frac{d^2x^2}{}$

in curvilinear coordinates is the following.

Suppose that we seek those curves for which $k = 0$, i.e., the straight lines or geodesics.

$$K = 0 \Rightarrow b^i = 0 \Rightarrow \frac{d^2 x^i}{ds^2} + \Gamma^i_{pq} \frac{dx^p}{ds} \frac{dx^q}{ds} = 0 \qquad 55$$

The Solution of this System of second order differential equations will define the geodesics $x^i = x^i(s)$

Solution: (a) Given $u = (x^1)^2$ and $v = x^2$

and $$\boxed{(g_{ij}) = \begin{bmatrix} (x^1)^2 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow (g^{ij}) = \begin{bmatrix} \frac{1}{(x^1)^2} & 0 \\ 0 & 1 \end{bmatrix}}$$

Inverse 5½

The geodesic equation is:

$$\frac{d^2 x^i}{ds^2} + \Gamma^i_{pq} \frac{dx^p}{ds} \frac{dx^q}{ds} = 0 \quad ; i = 1, 2$$

So first we will find the Christoffel

Now using the formula for absolute derivative:

$$\boxed{\frac{\delta T^i}{\delta t} = \frac{d}{dt}(T^i) + \Gamma^i_{jk} T^j \frac{dx^k}{dt}}$$

56

$$\Rightarrow \frac{\delta T^1}{\delta t} = \frac{dT^1}{dt} + \Gamma^1_{jk} T^j \frac{dx^k}{dt}$$

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based on three simple postulates:

Postulate 1: The principle of relativity. 58

The laws of physics are the same in all inertial reference frames.

Postulate 2: The speed of light is invariant.

All observers in inertial frames will measure the same speed of light, regardless of their state of motion. 59

Postulate 3: Uniform motion is invariant.

A particle at rest or with constant velocity in one inertial frame will be at rest or have constant velocity in all inertial frames.

Applications of Tensors in Special Relativity: 74

In 1905, Albert Einstein proposed the special theory of relativity based on three simple postulates:

Postulate 1: The principle of relativity. 58

The laws of physics are the same in all inertial reference frames.

Postulate 2: The speed of light is invariant.

All observers in inertial frames will measure the same speed of light, regardless of their state of motion. 59

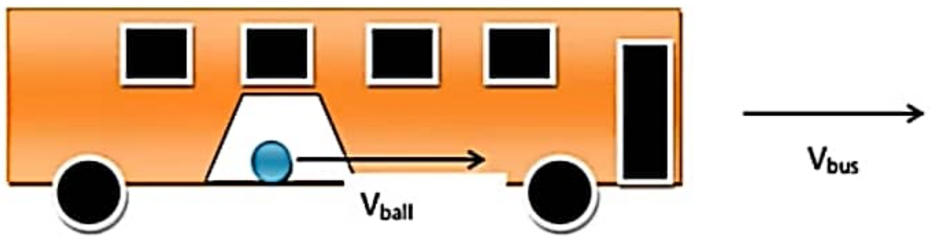
Postulate 3: Uniform motion is invariant.

Inertial Frame of Reference

- Non accelerating
- Newton's 1st law and other laws of physics are valid

For example:

Inside a bus moving at constant velocity with a ball inside



combined. Thus each event is assigned four coordinates (t, x, y, z) where t is the time of the event and (x, y, z) is the location of the event in ordinary rectangular coordinates. Such coordinates

are called SpaceTime Coordinates. 61

Definition: An event space is an $\mathbb{R}^4$ space

* They are identical if
$x_1^i = x_2^i$ for all i

* They are simultaneous time
if $x_1^0 = x_2^0$ -62

* They are copositional
if $x_1^i = x_2^i$ for $i = 1, 2, 3$

distance b/w E_1 and E_2 is

For an arbitrary event E(x), the relativistic

distance from origin to $E(x^i)$ is the real

number $s \geqslant 0$ such that $E(r, i \ldots)$

$$\boxed{s^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2} \quad 63$$

More generally, the length of interval or relativistic distance between $E_1(x_1^i)$ and $E_2(x_2^i)$ is:

$$\boxed{(ds)^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2} \quad 64$$

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$$\Rightarrow (dx^1)^2 + (dx^2)^2 + (dx^3)^2 > (dx^0)^2$$

65

(2) light like if $(ds)^2 = 0$

$$\Rightarrow (dx^0)^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

(3) time like if $(ds)^2 > 0$

$$\Rightarrow (dx^0)^2 > (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

Spacelike Interval: occure same time but

The interval between $E_1(x_1^i)$ and $E_2(x_2^i)$ is

75

(1) Space like if $(dS)^2 < 0$

$$\Rightarrow (dx^1)^2 + (dx^2)^2 + (dx^3)^2 > (dx^0)^2$$

65

(2) light like if $(dS)^2 = 0$

$$\Rightarrow (dx^0)^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

(3) time like if $(ds)^2 > 0$

$$\Rightarrow (dx^0)^2 > (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

In its setting the three ordinary dimensions of space are combined with a single dimension of time to form a four-dimensional spacetime.

Minkowski space is often contrasted with Euclidean space. While a Euclidean space has only spacelike dimensions, a Minkowski space also has one timelike dimension. 66

Therefore the symmetry group of a Euclidean space is the Euclidean group and for a Minkowski space it is the Poincare group. 67

Locally flat spacetime:

Locally flat spacetime:

The use of the Minkowski space to describe physical systems over finite distances applies only in the Newtonian limit of systems without significant gravitation.

In the case of significant gravitation, spacetime becomes curved and one must abandon special relativity in favor of the full theory of general relativity. 68 69

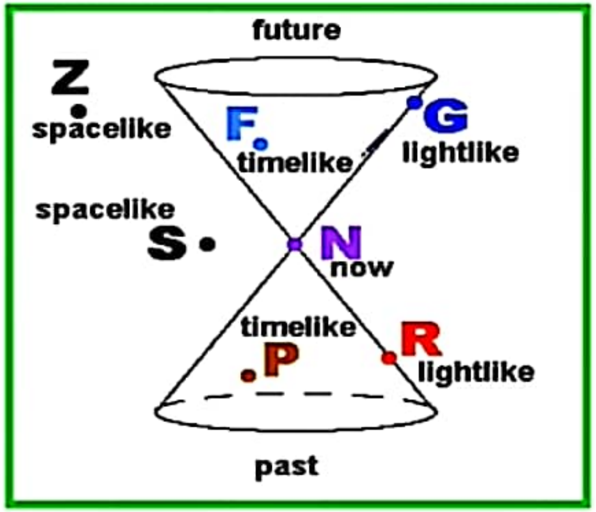
In the presence of gravity, spacetime is described by a curved 4-dimensional manifold for which the tangent space to any point is a 4-dimensional Minkowski space. Thus, the structure of Minkowski space is still essential in the description of general relativity.

Minkowski Spacetime is:

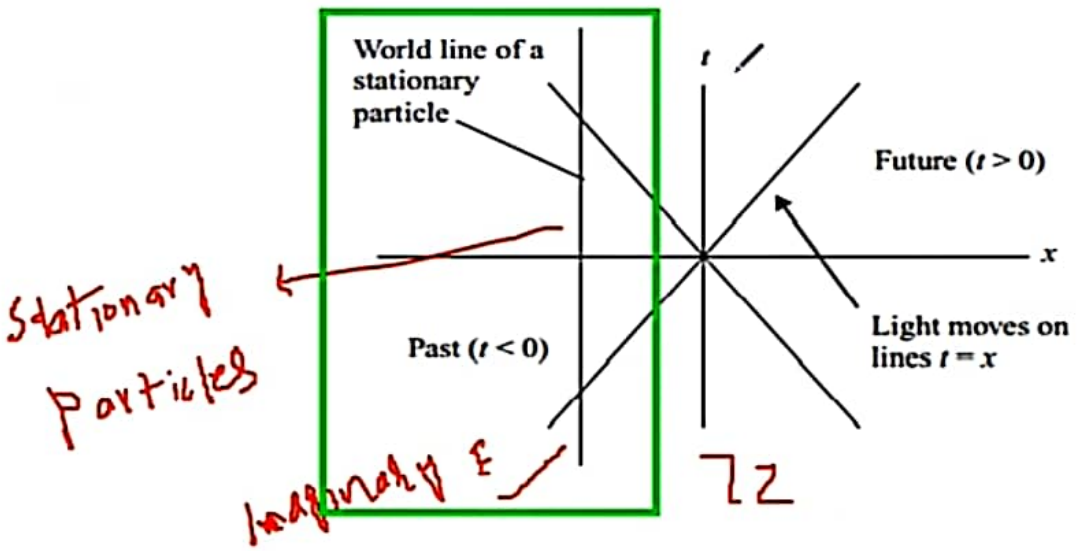
$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = g^{\mu\nu}$$

This metric is denoted by $\eta_{\mu\nu}$ and its inverse $\eta^{\mu\nu}$ is also same.

light emitted at the origin. Such a flash of light is described by a spherical wavefront. However, our minds cannot visualize four dimensions and it's not possible to draw it on paper. So we do the next best thing and suppress one or more of the spatial dimensions.



71



The simplest of all particle motion is a particle just sitting somewhere. To indicate the worldline of a stationary particle on a somewhere. To indicate the worldline of a stationary particle on a spacetime diagram, we simply draw a straight vertical line that passes through the x-axis at the spatial location of the particle. This makes sense because the particle is located at some position x that does not change, but time keeps marching forward.

Special relativity tells us that a particle cannot move faster than the speed of light. On a spacetime diagram, this is indicated by the fact that particle motion is restricted to occur only inside the light cone. The region inside the light cone is called timelike. Regions outside the light cone, which are casually unrelated to the event E, are called spacelike.

This is due to postulate of special relativity, i.e.,

$$\boxed{ds^2 = d\tilde{s}^2.}$$

The relativistic distance b/w two events is also called interval.

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Design form fields

$$\Rightarrow g_{ij}\, \delta^i_r\, \delta^j_s = g_{ij}\, a^i_r\, a^j_s$$

$$\Rightarrow g_{rs} = g_{ij}\, a^i_r\, a^j_s$$

or In matrix form:

$$A^t G A = G$$

75

or: $(a^0_0)^2 - (a^1_0)^2 - (a^2_0)^2 - (a^3_0)^2 = 1$

$$(a^0_j)^2 - (a^1_j)^2 - (a^2_j)^2 - (a^3_j)^2 = -1 \quad (j = 1, 2, 3)$$

182 / 441

100%

Simple Lorentz Transformations:

The simple Lorentz transformations are

$$\boxed{\tilde{x}^0 = \frac{x^0 - \beta x^1}{\sqrt{1-\beta^2}}, \quad \tilde{x}^1 = \frac{-\beta x^0 + x^1}{\sqrt{1-\beta^2}}, \quad \tilde{x}^2 = x^2, \quad \tilde{x}^3 = x^3}$$

OR in matrix form:

$$A = \begin{bmatrix} 1/\sqrt{1-\beta^2} & -\beta/\sqrt{1-\beta^2} & 0 & 0 \\ -\beta/\sqrt{1-\beta^2} & 1/\sqrt{1-\beta^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \dots \end{bmatrix}$$

Any 4x4 matrix of the form:

$$\boxed{A = \begin{bmatrix} a & b & 0 & 0 \\ b & a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{where} \quad a^2 - b^2 = 1}$$

is called Simple Lorentz and the relative velocity is recovered as:

$$\beta = -b/a$$

The inverse of a simple Lorentz matrix is:

$$\boxed{A^{-1} = \begin{bmatrix} a & -b & 0 & 0 \\ -b & a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

78

which is itself a simple Lorentz matrix, corresponding to $-\beta$.

i.e: If the velocity of $\bar{O}$ (frame of reference) w.r.t O is v, then the velocity of

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$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

which is itself a simple Lorentz matrix, corresponding to $-\beta$.

i.e: If the velocity of $\bar{O}$ (frame of reference) w.r.t O is v, then the velocity of O with respect to $\bar{O}$ is $-v$.

Composition of Simple Lorentz Transformation:

For any fixed x^0, simple Lorentz transformation gives:

$$\boxed{\Delta\tilde{x}^1 = \frac{1}{\sqrt{1-\beta^2}}\Delta x^1 \text{ or } \Delta x^1 = \sqrt{1-\beta^2}\,\Delta\tilde{x}^1 < \Delta\tilde{x}^1}$$

So

If the frame $\tilde{O}$ is moving at a uniform velocity v relative to O, distance in $\tilde{O}$ appear to observer O to run slow by the factor $\sqrt{1-\beta^2}$.

Time Dilation:

For any fixed $\tilde{x}^1$,

$$\boxed{\Delta x^0 = \frac{1}{\sqrt{1-\beta^2}}\,\Delta\tilde{x}^0 \quad \text{or} \quad \Delta t = \frac{1}{\sqrt{1-\beta^2}}\,\Delta\tilde{t} > \Delta\tilde{t}}$$

If the frame $\tilde{O}$ is moving at a uniform velocity v relative to O, the clock of observer $\tilde{O}$ appears to observer O to run slow by the factor $\sqrt{1-\beta^2}$.

Four vectors

80

In special relativity, we work with a unified entity called spacetime rather than viewing space as an arena with time flowing in the background.

As a result, a vector is going to have a time component in addition to the spatial components we are used to. This is called a four vector.

In general, (V^i) is a four vector if it transforms according to the law $\boxed{\tilde{V}^i = a^i_j V^j}$, where (a^i_j) is the Lorentz matrix.

$$\bar{V}^i = V^j \frac{\partial \bar{x}^i}{\partial x^j}$$

82

$$(x^i) = (ct, r(t)) = (ct, x(t), y(t), z(t))$$

$$ct \to x^0,\quad x(t) \to x^1,\quad y(t) \to x^2,\quad z(t) \to x^3$$

Then we have the formulas:

8) $$\boxed{v_i = \frac{dx^i}{dt} = (c, \vec{v})}$$

$$V_i = (c, \vec{v})$$

where $\vec{v} = \dfrac{d\vec{r}}{dt}$ and $\|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$$(a_i) = \left(\frac{d^2 x^i}{dt^2}\right) = (0, \vec{a})$$

$$c = \gamma\,(c - \beta V_x)\,\frac{dt}{d\tilde{t}} \quad \text{or}$$

$d\tilde{t}$ dt $d\tilde{t}$

$$\frac{dt}{d\tilde{t}} = \frac{c}{\gamma(c - \beta V_x)} = \frac{c}{\gamma c\,(1 - \frac{\beta}{c} V_x)}$$

Also $\boxed{\beta = v/c}$

$$\Rightarrow \boxed{\frac{dt}{d\tilde{t}} = \frac{1}{\gamma\left(1 - \frac{v V_x}{c^2}\right)}}$$

8h

Let us reparameterize the curve, choosing the quantity: $\sqrt{C^2 - V_x^2 - V_y^2 - V_z^2} = c\sqrt{1 - V^2/c^2}$

Arc length

$$\boxed{\tau = \frac{s}{c}} = \frac{1}{c}\int_{t_0}^{t}\sqrt{g_{ij}\frac{dx^i}{du}\frac{dx^j}{du}}\, du \qquad (31)$$

86

or $$\boxed{\frac{d\tau}{dt} = \sqrt{1 - \hat{V}^2/c^2}} \qquad 87$$

where $\boxed{\hat{V} < c}$. The new parameter τ is known as the proper time for the particle.

No connection

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the corresponding velocity and acceleration become proper velocity and acceleration. That is we have:

$$\boxed{u^i = \frac{dx^i}{d\tau}, \quad b^i = \frac{du^i}{d\tau} = \frac{d^2x^i}{d\tau^2}}$$

Vel

Acc

88

★ The particle's motion (with respect to frame O) is uniformly accelerated if its spatial acceleration relative to an instantaneous rest frame $\tilde{O}$ does not vary, i.e.,

$$\hat{a} = \sqrt{a_x^2 + a_y^2 + a_z^2} = \alpha$$

89

Relativistic Mass and Force:

1) Rest Mass and Relativistic Mass:

The relativistic mass of a particle with spatial velocity $\vec{v}$ is:

$$\boxed{\hat{m} = \frac{m}{\sqrt{1 - \hat{v}^2/c^2}}}$$

$$v/c = \beta$$

where m is the rest mass of the particle and $\hat{m}$ is relativistic mass.

219 (219 / 441)

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Relativistic Momentum and Force:

The 4-momentum of Special Relativity is defined by:

$p = \hat{m}v$

$$(P^i) = (P^0, \vec{P}) = (\hat{m} V_i)$$

and the Lorentz force $(F_0, \vec{F})$ is defined as the time derivative of the 4-momentum:

$$\boxed{F_0 = \frac{dP^0}{dt}} = \frac{d}{dt}(\hat{m} V_0) = \frac{d}{dt}\left(\frac{mc}{\sqrt{1-\hat{v}^2/c^2}}\right)$$

219 / 441 100%

$$(P^i) = (P^0, \vec{P}) = (\hat{m} V_i)$$

and the Lorentz force $(F_0, \vec{F})$ is defined as the time derivative of the 4-momentum:

$$\boxed{F_0 = \frac{dP^0}{dt}} = \frac{d}{dt}(\hat{m} V_0) = \frac{d}{dt}\left(\frac{mc}{\sqrt{1-\hat{v}^2/c^2}}\right)$$

$$\vec{F} = \frac{d}{dt}\left(\frac{m\vec{v}}{\sqrt{1-\hat{v}^2/c^2}}\right)$$

In terms of proper time τ. The 4-force also called Minkowski force is defined as:

$$(K^i) = \left(\frac{dp^i}{d\tau}\right)$$

$$\Rightarrow \boxed{K^i = \frac{dp^i}{dt} \cdot \frac{dt}{d\tau} = F_i \frac{dt}{d\tau} = \frac{F_i}{\sqrt{1 - \hat{v}^2/c^2}}}$$

Force

derivative with respect to x^j and then with respect to x^k produces the third order tensor

$$\boxed{((V_i)_{;j})_{;k} = V_{i;jk}}$$

$$V_{i;j;j} = V_{i;j} - \Gamma^t_{ij} T_t$$

$$V^i_{;j} = V^i_{j} + \Gamma^i_{jk} T^b$$

The Question is:

Does the order of differentiation matter?

OR does $V_{i;jk} = V_{i;kj}$ hold in general?

By definition of the covariant derivative:

$V_{i;jk}$

$$V_{i;jk} - V_{i;kj} = R^s_{\ ijk} V_s \longrightarrow (*)$$

where
$$\boxed{R^s_{\ ijk} = \frac{\partial \Gamma^s_{ik}}{\partial x^j} - \frac{\partial \Gamma^s_{ij}}{\partial x^k} + \Gamma^l_{ik}\Gamma^s_{lj} - \Gamma^l_{ij}\Gamma^s_{lk}}$$

The Quotient theorem immediately implies:

" The n^4 components defined by Equation (X) are those of a forth-order tensor, contravariant of order one and covariant of order three."

R^i_{jkl} is called the Riemann or Riemann-Christoffel tensor of the contravariant of order one and covariant of order three."

R^i_{jkl} is called the Riemann or Riemann-Christoffel tensor of the second kind. Lowering the contravariant index gives

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Second kind. Lowering the contravariant index gives

$$\boxed{R_{ijkl} = g_{ir} R^{r}_{jkl}}$$ 1st kind

is the Riemann tensor of first kind.

Note: We may now say that covariant differentiation is order-dependent is the Riemann tensor of first

Note: We may now say that covariant differentiation is order-dependent unless the metric is such as to make Riemann tensor vanish.

Question: Show that at any point where the Christoffel symbols vanish,

$$\boxed{R^i_{jkl} + R^i_{klj} + R^i_{ljk} = 0}$$

Solution: Given that the christoffel symbols vanish, the expression for Riemman tensor becomes:

$$R^i_{\cdot} = \partial\Gamma^i_{jl} \quad \partial\Gamma^i_{jk}$$

Below are the symmetry properties of Riemann tensor:

1) $R^{i}_{jkl} = - R^{i}_{jlk}$

2) $R_{ijkl} = - R_{jikl}$ (first skew symmetry)

3) $R_{ijkl} = - R_{ijlk}$ (second skew symmetry)

4) $R_{ijkl} = R_{klij}$

5) $R_{ijkl} + R_{iklj} + R_{iljk} = 0$ (Bianchi's Identity

(A) Type R_{abab}, $a<b$:

$\to$ 1212, 1313, 1414

$$n_A = {}^nC_2 = \frac{n(n-1)}{2!}$$

(B) Type R_{abac}, $b<c$:

$\to R_{1213}$, 1314

$$n_B = 3\,{}^nC_3 = \frac{n(n-1)(n-2)}{2}$$

all different indices

(C) Type R_{abcd} or R_{acbd}, $a<b<c<d$

$$n_B = 3\,{}^nC_3 = \frac{n(n-1)(n-2)}{2}$$

(C) Type R_{abcd} or R_{acbd}, $a<b<c<d$ all different indices

$$n_C = 2\,{}^nC_4 = \frac{n(n-1)(n-2)(n-3)}{12}$$

Theorem: There are a total of $\frac{n^2(n^2-1)}{12}$

components of the Riemann tensor R_{ijkl} that are

The Ricci Tensor and Ricci Scalar: (86)

The Riemann tensor can be used to derive two more quantities that are used to define the Einstein tensor. The first of these is the Ricci tensor, which is calculated from the Riemann tensor by contraction on the first and third

$$R_{ab} = R^{c}{}_{acb} \qquad 2$$

The Ricci tensor is symmetric, so

$$R_{ab} = R_{ba}$$

Using contraction on the Ricci tensor, we obtain the Ricci scalar:

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The Ricci tensor is Symmetric, So

$$R_{ab} = R_{ba}$$

Using Contraction on the Ricci tensor, We obtain the Ricci Scalar:

$$R = g^{ab} R_{ab} = R^{a}{}_{a}$$

Question: Show that $\boxed{R=2 \text{ for}}$ ^{3} the unit 2-Sphere. Given

$$R^{1}_{212} = \sin^2\theta, \quad R_{1212} = \sin^2\theta$$

Solution:

The Symmetry conditions of the Riemann tensor implies that:

$$R_{1212} = R_{2121} = -R_{1221} = -R_{2112}$$

Solution: Using the formula:

$$\boxed{R_{ij} = R^{k}_{ikj} = R^{1}_{i1j} + R^{2}_{i2j} + R^{3}_{i3j}}$$

$$= g^{11} R_{1i1j} + g^{22} R_{2i2j} + g^{33} R_{3i3j}$$

$$\boxed{R^{\cdot}_{ikj} = g^{kk} R_{kikj}}$$

Implies:

$$R_{11} = g^{11} \overset{\to 0}{R_{1111}} + g^{22} R_{2121} + g^{33} \overset{\to 0}{R_{3131}}$$

$$= g^{22} R_{2121}$$

Physical Interpretation:

Curvature describes how a geometric object such as a curve deviates from a straight line or a surface from a flat plane.

Curvature can be expressed simply as a scalar that represents the magnitude of this deviation. Curvature can also be described as a vector that takes into account the direction of the curve along with the magnitude.

For more complex objects such as surfaces or n-dimensional spaces, a more complex object is needed to describe the curvature. **One such object, is the Riemann curvature tensor.**

Intrinsic objects are those which can be obtained by measuring distances and computing the derivatives of those distances in some space.

The **metric tensor** is an example of an intrinsic object. The metric tensor describes how to compute distances and lengths of curves in a given space.

Riemann generalized this idea and extended it to spaces of n-dimensions. Because the metric tensor is an intrinsic object, subsequent objects that can be described in terms of the metric tensor and its derivatives are also intrinsic.

One object that can be derived from the metric tensor is the Christoffel symbol computing the derivatives of those distances in some space.

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Riemann generalized this idea and extended it to spaces of n-dimensions. Because the metric tensor is an intrinsic object, subsequent objects that can be described in terms of the metric tensor and its derivatives are also intrinsic.

One object that can be derived from the metric tensor is the Christoffel symbol. The Christoffel symbol describes the variation in basis vectors from one point to another in curvilinear coordinate systems. The Christoffel symbols measure the rate of change of the covariant basis with respect to the coordinate variables.

The Riemann tensor, Ricci tensor, and Ricci scalar are all derived from the metric tensor and are therefore intrinsic measures of curvature.

A **manifold** is a space that can be curved but is locally flat. That is, near each point the space resembles Euclidean space. One example of a curved manifold is the surface of a sphere.

The **Riemann ten**sor is the most common tool used to describe the curvature of a Riemannian manifold.

The Riemann curvature tensor is a tool used to describe the curvature of n-dimensional spaces such as Riemannian manifolds in the field of differential geometry. The Riemann

Contraction, of the Riemann tensor produces the **Ricci tensor**. The Ricci tensor provides a way measure the degree to which a space differs from Euclidean space.

Contraction of the Ricci tensor produces the scalar curvature or **Ricci scalar**. The Ricci scalar is the simplest curvature invariant of a manifold.

$$\boxed{K = K(x^i; U^i, V^i) = \frac{R_{ijkl}\, U^i V^j U^k V^l}{G_{pqrs}\, U^p V^q U^r V^s}}$$

R curvat..

where, $G_{pqrs} = g_{pr}\, g_{qs} - g_{ps}\, g_{qr}$

$$G_{pqrs}\, U^p V^q U^r V^s$$ 13

where, $G_{pqrs} = g_{pr}\, g_{qs} - g_{ps}\, g_{qr}$

This curvature can be associated to **Gaussian curvature** of the surface.

This sort of curvature depends not only on position, but also on a pair of directions selected at each point.

15

By contrast, the curvature k of a curve depends only on the points along the curve.

In the calculations of K it is helpful

$R_{ijkl} v^i v^j v^k v^l \longrightarrow$ is an invariant

$G_{pqrs} v^p v^q v^r v^s \longrightarrow$ is also an invariant

It follows that $\boxed{K \text{ is an invariant}}$ and it serves to generalize the Gaussian curvature of a surface to higher dimensions.

Question: Using the fact that in a two-dimensional Space the only non-zero components of Riemann tensor are

$$\boxed{R_{1212} = R_{2121} = -R_{1221} = -R_{2112}} \quad \text{In } 2D$$

simplify the expression of Riemannian Curvature.

Leo

Remember me in prayer

Riemannian Space: A Riemannian space is the $\boxed{\text{Space } R^n}$ coordinatized by (x^i), together with a fundamental form or Riemannian metric, $g_{ij}\, dx^i dx^j$, where $g = (g_{ij})$ obeys conditions for a metric.

Coordinate transformation, $(g_{ij}) = \delta_{ij}$.

Now a coordinate system $(\tilde{x}^i)$ in which $\boxed{(\tilde{g}_{ij}) = \delta_{ij}}$ is a rectangular coordinate system. Then $\tilde{K} = 0$. Since all Christoffel

symbols vanish in $(\tilde{x}^i)$.

But Riemannian curvature is an ... in the original

Flat Riemannianian Spaces: 88

$\boxed{\text{A Riemannian Space is flat if and only if } K=0 \text{ at all points.}}$

Theorem: If $K=0$, then $R=0$.

Proof: If $K=0$, then the space is flat and hence the $\tilde{g}_{ij}$ are constant for some

Question: Verify that

$$\boxed{ds^2 = (dx^1)^2 + (x^1)^2 (dx^2)^2}$$

represent the Euclidean metric.

Solution: This is actually polar metric:

$$ds^2 = dr^2 + r^2 d\theta^2 \qquad (x^1, x^2 \to r, \theta)$$

with christoffel symbols:

$\Gamma^1_{22} = -x^1, \quad \Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{x^1}$ (remaining all zero)

Accordingly, for each P in $\mathcal{N}$, define the coordinates of P as:

$$\boxed{y^i = s\,p^i}$$

where s is the distance along the geodesic from O to P. The numbers (y^i) are called the normal coordinates or the geodesic coordinates

$$\boxed{y^i = s\,p^i}$$

where s is the distance along the geodesic from 0 to P. The numbers (y^i) are called the normal coordinates
or the geodesic coordinates
or the Riemannian coordinates

of point P.

of point P.

Theorem: At the origin of a Riemannian coordinate system (y^i), all $\frac{\partial g_{ij}}{\partial y^k}$, $\frac{\partial g^{ij}}{\partial y^k}$, Γ_{ijk} and Γ^i_{jk} are zero. 11

22

Proof: If (y^i) are normal coordinates, ... through O and any

The Einstein Tensor: 42

The Einstein tensor is defined in terms of the Ricci tensor R_{ij} and the curvature invariant R as:

$$\boxed{G^i_{\ j} = R^i_{\ j} - \frac{1}{2}\delta^i_j R}$$

This is a mixed tensor of

The Einstein tensor is defined in terms of the Ricci tensor R_{ij} and the curvature invariant R as:

$$\boxed{G^i_{\ j} = R^i_{\ j} - \frac{1}{2}\delta^i_j R}$$

24

$$G^i_{\cdot j} = R^i_{\cdot j} - \delta^i_j R$$

This is a mixed tensor of order 2.

Theorem: For any Riemannian metric, the divergence of the Einstein tensor is zero at all points.

Proof: We have to show that

$$\boxed{G^{r}_{i;r} = 0 \qquad A^{i}_{;i} = 0}$$

consider,

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Energy Momentum Tensor:

93

- The quantity T^{ij}; $i, j = 0,1,2,3$ is the so-called energy-momentum-stress tensor or in a shorter version the energy-momentum tensor or the stress-energy tensor.
- It is a rank-2 symmetric tensor encoding all the information about energy density, momentum density, stress, pressure etc.
- In general relativity, the stress-energy or energy-momentum tensor T^{ij} acts as the source of the gravitational field.

$$T^{\alpha\beta} = \left(\begin{array}{c|c} \overset{T^{00}}{\text{energy density}} & \overset{T^{0i}}{\text{energy flux}} \\ \hline \overset{T^{i0}}{\text{mom. density}} & \overset{T^{ij} \to i,j=1,2,3}{\text{stress tensor}} \end{array}\right).$$

the rate of transfer of momentum across a unit area.

- T^{ij} is the 3-momentum flux or stress tensor, i.e the rate of flow of the ith momentum component per unit area in the plane orthogonal to the jth -direction. The component T^{ii} encodes the isotropic pressure in the ith direction while the components T^{ij} with $i \neq j$ refer to the viscous stresses of the fluid.

$$T^{ij} = \begin{bmatrix} T^{11} & T^{12} & T^{13} \\ T^{21} & T^{22} & T^{23} \\ T^{31} & T^{32} & T^{33} \end{bmatrix} \rightarrow \text{viscous stresses.}$$

isotropic stresses $\rightarrow$ pressure

Covariant Derivative of Energy Momentum Tensor:

The covariant derivative of energy momentum tensor is given by:

$$\boxed{\nabla_j T^{ij} = 0}$$

The covariant derivative of energy momentum tensor is actually the generalization of the conservation of energy and momentum in flat space.

This relation is called the local conservation of energy-momentum because it reduces to the conservation law

$\boxed{\frac{\partial T^{ij}}{\partial x^j} = 0 \text{ (in flat spacetime) in a local inertial frame.}}$ However, it is not a conservation law like in flat space, as the energy of matter

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$\boxed{\frac{\partial T^{ij}}{\partial x^j} = 0 \text{ (in flat spacetime) in a local inertial frame.}}$ However, it is not a conservation law like in flat space, as the energy of matter is not conserved in the presence of dynamic spacetime curvature but changes in response to it.

In this case, there are only two quantities that can be used to describe the matter field in the problem, the energy density and how fast (and in what direction) the dust is moving.

The simplest way to obtain the first quantity, the energy density, is to jump over to the co-moving frame. If you're in the comoving frame, then you're moving along with the dust particles. In that case there is a number of dust particles per unit volume n, and each dust particle has energy m. So the energy density is given by $\boxed{\rho = mn.}$

The second item of interest is none other than the velocity four vector u. This of course will give us the momentum carried by the dust. Generally speaking, to get the stress-energy tensor for dust, we put this together with the energy density. So for dust, the stress-energy tensor is given by

$$\boxed{T^{ij}=\rho u^i u^j}$$

Where u^i is the proper velocity.

The Components of Energy Momentum Tensor of Dust in Minkowski spacetime (Flat space)

Here we discuss two cases: when frame is comoving and non comoving.

Case I:

A frame moving with the flow of the dust fluid is called comoving frame. For a co-moving observer, the four velocity reduces to

$$u^i = (c, 0, 0, 0)$$

In this case, the stress-energy tensor takes on the remarkably simple form

$$T^{ij} = \rho u^i u^j \rightarrow T^{00} = \rho u^0 u^0 = \rho c^2$$

$$T^{ij} = \begin{bmatrix} c^2\rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$T^{ij} = \begin{bmatrix} c^2\rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Case II:

Now consider the case of a stationary observer seeing the dust particles go by with four velocity u. In that case, we have

$$u = (\gamma c, \gamma v^x, \gamma v^y, \gamma v^z)$$

where u^i are the ordinary components of three velocity and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. In this case the stress-energy tensor is

$$T^{ij} = \rho\gamma^2 \begin{bmatrix} c & cv^x & cv^y & cv^z \\ cv^x & (v^x)^2 & v^x v^y & v^x v^z \\ cv^y & u^y u^x & (v^y)^2 & v^y v^z \\ cv^z & v^z v^x & v^z v^y & (v^z)^2 \end{bmatrix}$$

the equation of continuity of a fluid. (in flat space)

Solution: In a flat space the conservation equation is

$$\boxed{\frac{\partial T^{ij}}{\partial x^j} = 0}$$

3 6

Question: Find trace of energy momentum tensor defined by $\quad x^0 = ct$

$$\boxed{T = T^i_i = T^{ij} g_{ij}}$$

when $g_{ij} = \eta_{ij} = (c\,dt)^2 - dx^2 - dy^2 - dz^2$

and $T^{ij} = \rho u^i u^j$ in a comoving frame.

Solution: here only

frame.

Solution:

$$\boxed{T = T^{ij}\eta_{ij}} \qquad 38$$

$$= T^{00}\eta_{00} + T^{11}\eta_{11} + T^{22}\eta_{22} + T^{33}\eta_{33}$$

$$= \rho\gamma^2\left[c^2 + (v^x)^2(-1) + (v^y)^2(-1) + (v^z)^2(-1)\right]$$

$$\Rightarrow T = \rho\gamma^2\left[c^2 - (v^x)^2 - (v^y)^2 - (v^z)^2\right]$$

Energy-Momentum Tensor of Perfect Fluid

96

A perfect fluid is a fluid that has no heat conduction or viscosity. Such fluid is characterized by its mass density ρ and the pressure P. The stress-energy tensor that describes a perfect fluid is

$$\boxed{T^{ij} = \left(\rho + \frac{P}{c^2}\right) u^i u^j - P g^{ij}}$$

THE EINSTEIN FIELD EQUATIONS

91

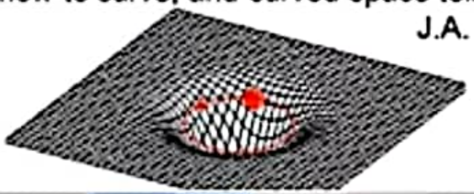
- To take into account spacetime curvature, tensor calculus was necessary in Einstein's General Relativity.

THE EINSTEIN FIELD EQUATION

$$\boxed{G_{\mu\nu} = 8\pi T_{\mu\nu}}$$

"matter tells spacetime how to curve, and curved space tells matter how to move"

J.A. Wheeler



Einstein's Formulation of the Field Equation:

Einstein did not derive the field equation, i.e., the law of gravitation, but formulated it from a number of considerations which we will discuss here.

$\boxed{\text{4 Points}}$ 43

1) At first, the field equation must be a tensor equation.

2) Secondly, under appropriate conditions the new law of gravitation must yield an equation of the form

$$\boxed{\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=4\pi G\rho \;\Big\}\text{Poisson law}}$$

ψ of Newton's theory.

3) Thirdly, the right hand side of Poisson law should be related to $T_{\mu\nu}$.
Obeying the conservation law

$$\boxed{T^{\alpha\beta}{}_{;\beta} = 0 \quad \text{or} \quad \frac{\partial T^{\alpha\beta}}{\partial x^{\beta}} = 0 \ (\text{in flat space})}$$

4) Finally,

4) Finally,

$$\left(R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R\right)_{;\beta} = 0$$

Inspired by these consideration, Einstein write down the general relativistic analogue of the Poisson equation as:

equation as:

$$\boxed{R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = -\kappa T_{\alpha\beta} \quad \text{where } \kappa = 8\pi G/c^4}$$

Question: Write down the field equations for the cosmological model $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$, in case

It is the branch of Physics which deals with rapidly changing electric and magnetic fields.

Here we will discuss the applications of tensors or use of tensors in the well known Maxwell field equations of electrodynamics.

electric field

charge density

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

permittivity of Vacuum

$$\nabla \cdot \mathbf{B} = 0$$

Magnetic flux

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

speed of light

electric ...t. permeability

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electric field

charge density

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

permittivity of Vacuum

$$\nabla \cdot \mathbf{B} = 0$$

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electric ...t.   pemeability of vacuum

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

speed of light

Leo

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$$\nabla \cdot \mathbf{B} = 0$$

Magnetic flux

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$

electric current

permeability of vacuum

speed of light

Activate Windows

Go to Settings to activate Windows.

Maxwell Field Equations 48.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$ → Gauss' law of electricity

$$\nabla \cdot \mathbf{B} = 0$$ → Gauss' law of Magnetism

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ → Faraday's law of Induction

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$ → Ampere's law

frame S. Now if charge density is defined as

$$\rho_0 = \boxed{\rho_0 = \frac{Ne}{l_0^3}}$$

in the frame S'.

{ N → No of electrons
e → charge
l_0 → length of one side of cube

Then in frame S the charge density appears to be

$$\boxed{\rho = \frac{Ne}{l_0^3\sqrt{1 - v^2/c^2}}}$$

→ this factor appears due to length contraction.

However, for frame S we have:

$$\boxed{\vec{j} = \rho \vec{V}}$$

$$\Rightarrow \vec{j} = \frac{\rho_0 \vec{V}}{\sqrt{1 - V^2/c^2}}$$

or in component form:

$$j_x = \frac{\rho_0 v_x}{}, \quad j_y = \frac{\rho_0 V_y}{}, \quad j_z = \frac{\rho_0 v_z}{}$$

$$\boxed{j = \rho V}$$

$$\Rightarrow \vec{j} = \frac{\rho_0 \vec{V}}{\sqrt{1 - v^2/c^2}}$$

or in component form:

$$j_x = \frac{\rho_0 v_x}{\sqrt{1 - v^2/c^2}}, \quad j_y = \frac{\rho_0 v_y}{\sqrt{1 - v^2/c^2}}, \quad j_z = \frac{\rho_0 v_z}{\sqrt{1 - v^2/c^2}}$$

density ρ_0 according to frame S', then mass according to S is:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \frac{\rho}{m} = \frac{\rho_0}{m_0} \Rightarrow \rho = \left(\frac{\rho_0}{m_0}\right) m$$

and $\vec{j} = \rho \vec{u}$

$$\boxed{\rho = \left(\frac{\rho_0}{m_0}\right) m}$$

$$\Rightarrow \vec{j} = \left(\frac{\rho_0}{m_0}\right) m \vec{u} \Rightarrow \boxed{\vec{j} = \left(\frac{\rho_0}{m_0}\right) \vec{p}}$$

Four current:

Mathematically, $J_\mu = g_{\mu\nu} J^\nu$

$= g_{00} J^0 + g_{11} J^1 + g_{22} J^2 + g_{33} J^3$

$= \rho c - J^1 - J^2 - J^3$

$= (\rho c, -\vec{j})$

$$\boxed{J^\mu = (\rho c, \vec{j})}$$

where $\vec{j} = j_x \hat{i} + j_y \hat{j} + j_z \hat{k}$

$\rho \rightarrow$ charge density

or
$$\boxed{J_\mu = (\rho c, -\vec{j}) \; [\text{in Minkowski space}]}$$

Conservation equation for four current density is:

$$\boxed{\nabla_\alpha J^\alpha = 0 \text{ or } J^\alpha{}_{;\alpha} = 0}$$

In Special relativity:

$J^\alpha \quad J^0 \quad J^1 \quad J^2 + J^3 = 0$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$$

Electromagnetic four potential:

The four potential has time and space components defined as

$$\boxed{A^{\mu} = \left(\frac{\phi}{c}, \vec{A}\right)}$$

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and space components defined as

$$\boxed{A^\mu = \left(\frac{\phi}{c}, \vec{A}\right)}$$

$\vec{A} \rightarrow$ vector potential

$\phi \rightarrow$ scalar potential

The scalar and vector potential are related to the potential energy U and potential momentum $\vec{Q}$ of a particle having charge energy $\underline{U}$ and potential momentum $\vec{Q}$ of a particle having charge q by:

$$\boxed{\begin{aligned} U &= q\phi \\ \vec{Q} &= q\vec{A} \end{aligned}}$$

$$\boxed{Q = qA}$$

The magnetic field $\vec{B}$ is defined in terms of vector potential as:

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

and electric field as:

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

and electric field as:

$$\boxed{\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}}$$

When charged particles are in motion then magnetic field is also produced such that both fields are $\boxed{\text{orthogonal}}$ to each

61

other as well as to the direction

Maxwell field equations describe the evolution of electromagnetic field.

Electromagnetic field tensor: 101

The electromagnetic field tensor also called Faraday's tensor is defined as

$$\boxed{F^{\alpha\beta} = \nabla^{\alpha} A^{\beta} - \nabla^{\beta} A^{\alpha}}$$

Also, $A^{\alpha} = \left(\phi/c, \vec{A}\right)$

In flat space:

$$\boxed{F^{\alpha\beta}} = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

$$\begin{vmatrix} E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{vmatrix}$$

$$\boxed{F^{\alpha\beta} \text{ is an antisymmetric tensor.}}$$

and

$$F_{\alpha\beta} = g_{\alpha\gamma} F^{\gamma\delta} g_{\delta\beta}$$

103

In Short, we can write

$$\boxed{F_{ij} = \varepsilon_{ijk} B^{k}} \; ; \; i, j, k = 1, 2, 3$$

and $F_{i0} = \dfrac{-E_i}{c}$; $i = 1, 2, 3$

$$F^{i0} = \frac{E_i}{c}$$

Maxwell field equations in Tensor form:

These equations in tensor form are given below:

$$\boxed{F_{\alpha\beta;\gamma} + F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} = 0}$$

Electromagnetic Energy Momentum Tensor:

It is defined by:

$$\boxed{T^{\alpha}{}_{\beta} = \frac{1}{4}\,\delta^{\alpha}_{\beta}\left(F_{\mu\gamma}F^{\mu\gamma}\right) - F_{\mu\beta}F^{\mu\alpha}}$$

or

$$T^{\alpha\beta} = \underline{1}\,g^{\alpha\beta}\left(F_{\mu\nu}F^{\mu\nu}\right) - F^{\alpha\mu}F^{\beta}{}_{\mu}$$

Properties of $F^{\mu\nu}$:

1) Antisymmetry $F^{\mu\nu} = -F^{\nu\mu}$

2) It has six independent components.

3) $F_{\mu\nu} F^{\mu\nu} = 2\left(B^2 - E^2/c^2\right)$

1) Antisymmetry

2) It has Six independent components.

3) $F_{\mu\nu} F^{\mu\nu} = 2\left(B^2 - E^2/c^2\right)$

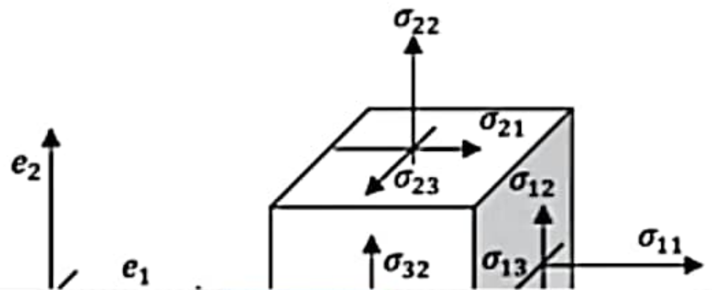
4) Trace:

$$F = F^{\mu}{}_{\mu} = 0$$

Normal and Shear Stresses:

- σ_{ii} are called the normal stresses acting on plane i.

- Whereas σ_{ij}; $i \neq j$ are called the tangential or shear stresses, acting on the plane perpendicular to the i-axis in the direction of j-axis.



Cauchy's Equations of Motion:

Cauchy's first law of motion

According to the principle of conservation of linear momentum, if the body is in static equilibrium it can be demonstrated that the components of the Cauchy stress tensor in every material point in the body satisfy the equilibrium equations.

$$\boxed{\sigma_{ji,j} + F_i = 0}$$

here F_i represents body force component.

★A body force is a force that acts throughout the volume of the body for example:

According to the principle of conservation of angular momentum, equilibrium requires that the summation of moments with respect to an arbitrary point is zero, which leads to the conclusion that the stress tensor is symmetric, thus having only six independent stress components, instead of the original nine in three dimensions.

$$\boxed{\sigma_{ij} = \sigma_{ji}}$$

★ Moment: The moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis.

★ Angular momentum is the rotational analog

Principle Stresses:

- The Eigen values of σ_{ij} tensor; represented as σ_1 , σ_2 , σ_3 are referred to as the principle stresses. The Eigen vectors are the principle stress directions.

$$\boxed{\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}} \rightarrow \text{principal stress matrix}$$

- The stress tensor is a symmetric 2nd order tensor so its eigenvalues are real numbers.

For the eigenvalue λ and its corresponding eigenvector v:

$$\boxed{\sigma_{ij} \cdot v = \lambda v}$$

$$\Rightarrow \left(\sigma_{ij} - \lambda I\right) \cdot v = 0$$

1) Mean stress:

Given the cauchy stress tensor and its principal stresses, mean stress is defined as

$$\boxed{\sigma_m = \frac{1}{3} Tr(\sigma_{ij})} = \frac{1}{3}\sigma_{ii} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

2) Mean Pressure:

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In a hydrostatic state of stress, the stress tensor is isotropic and, thus, its components are the same in any Cartesian coordinate system. As a consequence, any direction is a principal direction where $\boxed{\varepsilon_{ij} = e_{ij} - \tilde{\omega}_{ij}}$

is defined as the strain

is defined as rigid body rotation.

and

$$\varepsilon_{ij} = \frac{1}{2}\left(e_{ij} + e_{ji}\right) \rightarrow \varepsilon_{ji} = \frac{1}{2}\left(e_{ji} + e_{ij}\right) = \varepsilon_{ij}$$

$$\begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} = \begin{pmatrix} e_{11} & \frac{1}{2}(e_{12}+e_{21}) \\ \frac{1}{2}(e_{21}+e_{12}) & e_{22} \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2}(e_{12}-e_{21}) \\ \frac{1}{2}(e_{21}-e_{12}) & 0 \end{pmatrix} \rightarrow \tilde{\omega}_{ij}$$

* ε_{ij} is a symmetric tensor and $\tilde{\omega}_{ij}$ is an antisymmetric tensor; the leading diagonal of $\tilde{\omega}_{ij}$ is always zero.
* The tensor ε_{ij} has Eigen values which are called the principal strains $\varepsilon_1, \varepsilon_2, \varepsilon_3$.

Relation of Elasticity with stress and Strain:

If the amount of stress (σ) is infinitesimally small then the amount of strain (ε), which is also infinitesimal, is linearly proportional to the strain and may be written as:

$$\boxed{\begin{aligned} \varepsilon &= s\sigma \\ \text{or } \sigma &= C\varepsilon \end{aligned}}$$

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Where s is the elastic compliance and c is the elastic stiffness. In order to relate $\underbrace{\text{two second rank tensors}}_{\text{stress and strain}}$, a fourth rank tensor is necessary.

Elasticity Tensor:

so,

$$\boxed{\begin{aligned} \varepsilon_{ij} &= S_{ijkl}\,\sigma_{kl} \\ \text{or}\quad \sigma_{ij} &= C_{ijkl}\,\varepsilon_{kl} \end{aligned}}$$

* S_{ijkl} → elasticity tensor → related to change

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* S_{ijkl} → elasticity tensor → related to change in volume when subjected to an applied force. It is reciprocal of stiffness tensor.

* $\boxed{C_{ijkl} \rightarrow \text{Stiffness tensor}}$