MTH634 MEGA File

Mth634 Quiz#2 All in 1 file

Module (90to140)

Due Date (30july2021-5August2021)

MTH634:Quiz No. 2 Quiz Start T

Question # 1 of 4 (Start time: 02:39:20 PM, 14 August 2018)

If (X, τ) be a compact Hausdorff space then (X, τ) is not normal.

Select the correct option

●	True
○	False

Module 162

MTH634:Quiz No. 2 Quiz

Question # 2 of 4 (Start time: 02:41:37 PM, 14 August 2018)

Every discrete space is not a regular space.

Select the correct option

- True
- False ✓

Module 145, pg103

Question # 4 of 4 (Start time: 02:43:18 PM, 14 August 2018) Total Marks: 1

Which of the following statement is false?

Select the correct option

- ○ A set X with indiscrete topology is compact.
- ○ A set X with any topology containing finite number subsets of X is compact.
- ● A finite set X with any topology is compact.
- ○ An infinite set X with discrete topology is compact

Mod152, pg121

MTH634:Quiz No. 2 Quiz Start Time: 10:54

Question # 4 of 4 (Start time: 10:57:46 AM, 14 August 2018)

Which of the following statement is false?

Select the correct option

- Every metric space is second countable. ✓ Module 126, pg72
- Every metric space is Hausdorff space.
- Every metric space is normal space.
- none

MTH634:Quiz No. 2

Quiz Start Time: 09:29

Question # 1 of 4 (Start time: 09:29:56 AM, 14 August 2018)

Consider $\mathbb{R}$ with usual topology. Then $A = (0,1) \cup (3,5]$ is disconnected subset of $\mathbb{R}$.

Select the correct option

- [x] True

Module 166, pg147

- [ ] False

Question # 2 of 4 (Start time: 12:44:39 PM, 14 August 2018)

All the metric spaces are not normal spaces.

Select the correct option

- True
- False ✓ Module 148

MTH634:Quiz No. 2

Quiz Start Time: 10:54 AM, 14 August 2

Question # 2 of 4 (Start time: 10:54:27 AM, 14 August 2018)

Total Mar

Consider $\mathbb{R}$ with usual topology. There exits no homomorphism between an open interval of $\mathbb{R}$ and a closed interval of $\mathbb{R}$.

Select the correct option

Reload Math Equati

- True ✓ Module 160
- False

MTH634:Quiz No. 2 Quiz Start Time: 07:28 PM

Question # 4 of 6 (Start time: 07:29:33 PM, 17 February 2019)

A topological space X is called a connected space iff there exists a pair of subsets of X both nonempty and both open and closed.

Select the correct option

○	True
●	False ✓ Module 165

MTH634:Quiz No. 2

Question # 4 of 4 (Start time: 12:47:06 PM, 14 August 2018)

Which of the following statement is true?

Select the correct option

- ○ $\mathbb{R}$ with usual topology is compact.
- ● Module 152 — $\mathbb{R}$ with indiscrete topology is compact. ✓
- ○ $\mathbb{R}$ with discrete topology is compact.
- ○ $\mathbb{R}$ with cofinite topology is not compact.

Question # 3 of 6 (Start time: 07:28:57 PM, 17 February 2019) Total Marks: 1

Every compact subspace of a Hausdorff space is not closed.

Select the correct option

- ○ True
- ● False ✓ Module 157, pg133

MTH634:Quiz No. 2

Question # 6 of 6 (Start time: 07:30:56 PM, 17 February 2019)

$\mathbb{R}$ with usual topology is a $T_4 - space$.

Select the correct option

True

Module 147

False

Question # 10 of 10 (Start time: 03:25:35 PM, 09 February 2021) Total Marks: 1

Which of the following statement is true?

Select the correct option

- Restriction of a closed map may or may not be an open map.
- Restriction of an open map can never be an open map.
- Restriction of an open map is always an open map.
- Restriction of an open map may or may not be an open map. ✓ Pg8

MTH634:Quiz No. 2 Quiz Start Time: 07

Question # 1 of 6 (Start time: 07:28:26 PM, 17 February 2019)

Consider $\mathbb{R}$ with usual topology. Which of the following sets are separated?

Select the correct option

- A=(0,1), B=(4,9]
- A=(-1,3), B=[3,5)
- A=(0,2), B=[2,5)
- none

Pg142

Question # 3 of 6 (Start time: 03:28:22 PM, 17 February 2019)

A set X with cofinite topology is compact.

Select the correct option

- [x] True ✓ Module154, pg121
- [ ] False

MTH634:Quiz No. 2 Quiz Start Time: 03:27 PM, 17 February 2019

Question # 1 of 6 (Start time: 03:27:10 PM, 17 February 2019) Total Marks: 1

Image of a compact space under a continuous map is compact.

Select the correct option

- [x] True
- [ ] False

MTH634:Quiz No. 2 Quiz Start Time: 06:2

Question # 1 of 6 (Start time: 06:26:36 AM, 17 February 2019)

An open interval in

$\mathbb{R}$

with usual topology is not compact.

Select the correct option

●	True
○	False

Module 166, Pg147

✓

Question # 5 of 10 (Start time: 11:33:00 PM, 31 July 2021)

Total Marks: 1

Let X and Y be topological spaces and $f:X \to Y$ be a bijection then which of the following statement is true?

Select the correct option

- f is continuous and open.
- f is homeomorphism. R
- f is continuous and closed.
- All of them X R Wrong

Question # 4 of 10 (Start time: 10:40:19 PM, 31 July 2021) Total Marks: 1

Let X and Y be topological spaces and if a function $f:X \to Y$ is homeomorphism then which of the following statement is true?

Select the correct option

- f is continuous.
- f is bijection.
- All of them. R
- ${f^{-1}}$ is also continuous.

Question # 9 of 10 (Start time: 10:06:54 PM, 31 July 2021)

Which of the following statement is true?

Select the correct option

- All spaces are metrizable.
- All spaces are not metrizable. R

MTH634:Quiz No. 2

Question # 9 of 10 (Start time: 09:48:41 PM, 31 July 2021)

The usual metric d(x, y) =|x-y| defined on---------

Select the correct option

○	R^2
○	R^n
○	R
○	R^3

R

MTH634:Quiz No. 2

Question # 2 of 10 (Start time: 08:12:02 PM, 31 July 2021)

Let $X = \{1, 2, 3, 4\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$ be a topology on X, then which of the following is true ?

Select the correct option

- (X, τ) be a topological space.
- Every element of X has countable local base.
- (X, τ) be a first countable space.
- All of them. R

MTH634 Quiz No. 2

Question # 2 of 10 (Start time: 10:30:26 AM, 31 July 2021)

Let $X = \{1, 2, 3, 4, 5, 6\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$ be a topology on X. then the local base (B_x) of the point $x = 3, 4, 5$ is______

Select the correct option

- None of them.
- $\{\{1\}, \{2\}, X\}$
- $\{X\}$ ✓
- $\{\{2\}, \{1, 2\}, X\}$

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Question # 5 of 10 (Start time: 09:53:47 PM, 31 July 2021)

For a metric space (X, d) for all x, y belongs to X $d(x, y) = d(y, x)$ is called________property.

Select the correct option

- None of them
- Reflexive
- Non negativity
- Symmetric

Question # 1 of 10 (Start time: 05:18:36 PM, 31 July 2021) Total Marks: 1

If X has more than two points and $\left({X,\tau } \right)$ be an indiscrete topology then which of the following statement is true about $\left({X,\tau } \right)$?

Select the correct option

- It is not metrizable. ✓ Pg40
- It is metrizable.
- None of them.
- It is Haussdorff.

Question # 5 of 10 (Start time: 05:23:01 PM, 31 July 2021)

Total Marks: 1

A homoemorphic map between two topological spaces is always ....

Select the correct option

- surjective but not injective
- bijective ✓ Pg9
- discontinuous
- injective but not surjective

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Question # 2 of 10 (Start time: 05:27:16 PM, 31 July 2021)

Let X and Y be topological spaces. A map $f : X \to Y$ is called a Closed Map if______

Select the correct option

- for every open set $U \subseteq X$, the image $f(U) \subseteq Y$ is closed.
- for every open set $U \subseteq X$, the image $f(U) \subseteq Y$ is open.
- for every closed set $U \subseteq X$, the image $f(U) \subseteq Y$ is closed ✓ Module 87
- None of them

MTH634:Quiz No. 2

Question # 3 of 10 (Start time: 05:28:06 PM, 31 July 2021)

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ be a topology on X, then which of the following is NOT true ?

Select the correct option

○	All of them.
○	(X, τ) be a first countable space.
○	(X, τ) be a Topological space.
○	(X, τ) be a second countable space. ✔

MTH634:Quiz No. 2

Question # 9 of 10 (Start time: 05:35:20 PM, 31 July 2021)

Homomorphism of topological spaces is an equivalence relation.

Select the correct option

○	True Pg15
○	False

sec(s)

MTH634:Quiz No. 2

Quiz Start Time: 05:25 PM, 31 July 2021

Question # 8 of 10 (Start time: 05:34:21 PM, 31 July 2021)

Total Marks: 1

Let $X = \{1, 2, 3, 4, 5, 6\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$ be a topology on X, then the local base (B_x) of the point $x = 2$ is______

Select the correct option

Reload Math Equations

- $\{\{1\}, \{1, 2\}, \{2\}, X\}$.
- $\{\{1\}, \{2\}, X\}$.
- None of them.
- $\{\{2\}, \{1, 2\}, X\}$. ✔

MTH634:Quiz No. 2 Quiz St

Question # 10 of 10 (Start time: 05:36:51 PM, 31 July 2021)

Let $X = \{1, 2, 3, 4\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$ be a topology on X, then which of the following is NOT true ?

Select the correct option

- Every element of X has uncountable local base.
- (X, τ) be a first countable space.
- The local base of the element 4 is $\emptyset$. ✓
- (X, τ) be a topological space.

MTH634:Quiz No. 2

Question # 6 of 10 (Start time: 06:18:05 PM, 31 July 2021)

If d is a usual metric on R , then d(2,8)=

Select the correct option

- $1 - \int_{2}^{3} dx$
- $2 + \int_{0}^{2} 2x dx$ ✓
- $\int_{0}^{2} (2x+1) dx$
- $\int_{0}^{2} dx$

MTH634:Quiz No. 2

Quiz Start Time: 06:11 PM, 31 July 2021

Question # 9 of 10 (Start time: 06:20:31 PM, 31 July 2021)

Total Marks: 1

Let $X = \{1, 2, 3, 4\}$ and $\tau = \{\emptyset, \{1, 2\}, \{3, 4\}, X\}$ be a topology on X and $A = \{2, 3\}$ is a dense set, then which of the following is true?

Select the correct option

Reload Math Equations

- $\{1, 2\}$ is a closed set.
- None of them.
- (X, τ) must be a separable topology. ✓
- (X, τ) mar or may not be a separable topology.

Pg65

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sec(s)

MTH634:Quiz No. 2

Quiz Start Time: 06:11 PM, 31 July 2021

Question # 10 of 10 (Start time: 06:21:33 PM, 31 July 2021)

Total Marks: 1

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ be a topology on X, if $B = \{\emptyset, \{a\}, \{b\}, X\}$ be the base of τ, then which of the following is true ?

Select the correct option

Reload Math Equations

- B be the countable base.
- All of them
- (X, τ) be a second countable space.
- (X, τ) be a first countable space.

MTH634:Quiz No. 2 | Quiz Start Time: 06:33 PM, 31 July 2021

Question # 4 of 10 (Start time: 06:38:43 PM, 31 July 2021) | Total Marks: 1

Every Topological Space is a first countable space.

Select the correct option

- True
- False

Module 111, pg42

Question # 2 of 10 (Start time: 06:45:31 PM, 31 July 2021)

Let $X = Q$(Set of rational numbers) be a usual metric space and $Q \subseteq R$, then which of the following is true about Q ?

Select the correct option

○	All of them ✔
○	It has disjoint intersection with the set of its interior points.
○	None of its point is an interior point.
○	It is not an open set.

MTH634:Quiz No. 2

Question # 7 of 10 (Start time: 09:54:31 PM, 31 July 2021)

Metric topology induced by $d(x, y) = \begin{cases} 0 & if \quad x = y \\ 1 & if \quad x \neq y \end{cases}$ is called_____

Select the correct option

- discrete topology ✓
- indiscrete topology
- usual topology
- None of them

MTH634:Quiz No. 2

Question # 8 of 10 (Start time: 09:54:52 PM, 31 July 2021)

The property $d(x, y)0$ of a metric space is called------

Select the correct option

- Triangle property
- Reflexive property
- Symmetric property
- Non negativity

Question # 5 of 10 (Start time: 10:01:22 PM, 31 July 2021)

Let $X = R$(Set of real numbers) be a usual metric space and $N \subseteq R$, then which of the following is NOT true about N ?

Select the correct option

- It must be an open set.
- It is not a neighbourhood of any of its point.
- Neither of its point is an interior point.
- None of them.

MTH634:Quiz No. 2 Quiz

Question # 6 of 10 (Start time: 10:50:48 PM, 31 July 2021)

Let $X = R$(Set of real numbers) be a usual metric space and $N \subseteq R$, then which of the following is true about N ?

Select the correct option

- It has disjoint intersection with the set of its interior points.
- None of its point is an interior point.
- It is not an open set.
- All of them.

R

MTH634:Quiz No. 2 Quiz Start Time:

Question # 5 of 10 (Start time: 10:49:39 PM, 31 July 2021)

If (X, d) is a compact metric space, then_____

Select the correct option

- every Cauchy sequence converges.
- every sequence has a subsequence which is a Cauchy sequence.
- every sequence is a Cauchy sequence.
- every sequence which has a convergent subsequence is a Cauchy sequence.

sec(s)

MTH634:Quiz No. 2

Quiz Start Time: 11:21 PM, 31 July 2021

Question # 10 of 10 (Start time: 11:34:05 PM, 31 July 2021)

Total Marks: 1

Which of the following statement is false?

Select the correct option

Reload Math Equations

- Discrete topology on a countable set X is second countable.
- Any finite set with any topology is second countable.
- Discrete topology on a real line R is second countable. ✓
- The set $\mathbb{R}$ with usual topology is second countable.

MTH634:Quiz No. 2

Question # 1 of 10 (Start time: 10:54:48 PM, 31 July 2021)

Let X and Y be topological spaces and if the mapping $f : X \to Y$ is bicontinuous then____

Select the correct option

- it should not be bijective.
- it must be bijective.
- None of them.
- it may or may not be bijective.

Pg12

MTH634:Quiz No. 2

Quiz Start Time: 10:48 AM, 31 July 2021

Question # 3 of 10 (Start time: 10:50:35 AM, 31 July 2021)

Total Marks: 1

A map 'f' between two topological spaces is called …………… iff 'f' is bijective, continuous and f-1(f inverse) is continuous.

Select the correct option

- automorphism
- bijection
- isomorphism
- homeomorphism ✓ Pg9

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MTH634:Quiz No. 2

Question # 5 of 10 (Start time: 11:02:00 PM, 31 July 2021)

Let X and Y be topological spaces and $f : X \to Y$ be a bijection then f is said to be homeomorphism if____

Select the correct option

- f is continuous but its inverse not continuous.
- f and its inverse must be continuous. ✓ Pg9
- f and its inverse must be discontinuous.
- None of them.

MTH634:Quiz No. 2

Question # 4 of 10 (Start time: 11:24:48 PM, 31 July 2021)

Let (X, τ) be a metrizable then which of the following statement is true

Select the correct option

○	(X, τ) has the countable chain collection
○	(X, τ) is second countable.
○	(X, τ) is separable.
○	All of them ✓

Question # 6 of 10 (Start time: 11:26:57 PM, 31 July 2021) Total Marks: 1

Which of the following statement is correct?

Select the correct option

- None of them.
- Projection map is closed.
- Projection map is open.
- Projection map may or may not be closed. ✓

Pg3

MTH634:Quiz No. 2

Question # 8 of 10 (Start time: 11:29:30 PM, 31 July 2021)

Let X and Y be topological spaces. A map $f : X \to Y$ is called an Open Map if_______

Select the correct option

- for every open set $U \subseteq X$, the image $f(U) \subseteq Y$ is open. ✓

Pg9

- for every open set $U \subseteq X$, the image $f(U) \subseteq Y$ is closed.
- for every closed set $U \subseteq X$, the image $f(U) \subseteq Y$ is closed.
- None of them

MTH634:Quiz No. 2 Quiz Star

Question # 9 of 10 (Start time: 11:10:38 AM, 02 August 2021)

Let X and Y be topological spaces and $f : X \to Y$ be a bijection then which of the following statement is NOT true?

Select the correct option

- f is continuous and closed.
- f is homeomorphism.
- f is discontinuous and open. ✓
- None of them

MTH634:Quiz No. 2

Quiz Start Time: 11:01 AM, 02 August 2021

Question # 10 of 10 (Start time: 11:11:37 AM, 02 August 2021)

Total Marks: 1

Let $X = \{1, 2, 3, 4, 5, 6\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$ be a topology on X, then the local base (B_x) of the point $x = 1$ is_____

Select the correct option

Reload Math Equations

- $\{\{1\}, \{1, 2\}, X\}$.
- None of them
- $\{\{1\}, \{2\}, \{1, 2\}, X\}$.
- $\{\{1\}, \{2\}, X\}$.

MTH634:Quiz No. 2

Quiz Start Time: 03:56 PM, 05 August 2021

Question # 3 of 10 (Start time: 03:58:32 PM, 05 August 2021)

Total Marks: 1

Two spaces are called topologically equivalent if there exists a .... between two topological spaces.

Select the correct option

- homeomorphism ✓ Pg10
- an injective map
- surjective map
- bijection

MTH634:Quiz No. 2

Question # 9 of 10 (Start time: 04:04:15 PM, 05 August 2021)

Let (M, d) be a metric space then for all $a \in M$ and for all $r > 0$, $B(a, r)$ ____

Select the correct option

- must be a close ball.
- must be a close set.
- None of them.
- must be an open ball. ✓ Pg 72

MTH634:Quiz No. 2

Question # 10 of 10 (Start time: 04:05:20 PM, 05 August 2021)

If two sets A and B have same cardinality then which of the following is NOT true?

Select the correct option

- There is a one-to-one correspondence between the elements of A and B.
- $f : A \to B$ is an invertible function.
- $f : A \to B$ is not an invertible function ✓
- All of them.

MTH634 Quiz No. 2

Quiz Start Time: 08:59 PM, 05 August 2021

Question # 2 of 10 (Start time: 09:00:27 PM, 05 August 2021)

Total Marks: 1

Which of the following statement is Not correct?

Select the correct option

- Boundedness is a topological property. ✓ Pg17
- Length is not a topological property.
- Area is not a topological property.
- Cardinality of set X is a topological property.

MTH634:Quiz No. 2

Question # 1 of 10 (Start time: 08:59:01 PM, 05 August 2021)

Le X and Y be topological spaces. A function $f : X \to Y$ is continuous if______

Select the correct option

- the preimage of every closed set is open.
- the preimage of every open set is closed.
- the preimage of every open set is open. ✓
- None of them

MTH634:Quiz No. 2

Question # 6 of 10 (Start time: 09:02:59 PM, 05 August 2021)

Metric topology induced by $d(x, y) = |x - y|$ on $\mathbb{R}$ is called___\

Select the correct option

- indiscrete topology
- None of them
- discrete topology
- usual topology

Pg38

Question # 9 of 10 (Start time: 09:05:30 PM, 05 August 2021) Total Marks: 1

If $\left({X,\tau } \right)$ be a separable topology then it must have countable dense set.

Select the correct option

- True ✓ Pg 65
- False

Question # 2 of 10 (Start time: 03:56:49 PM, 05 August 2021)

Let $X = \{1, 2, 3, 4, 5, 6\}$ and $\tau = \{\emptyset, \{3\}, \{4\}, \{3, 4\}, X\}$ be a topology on

Select the correct option

- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open cover of the set $\{4\}$.
- All of them. ✓
- The set $\{\emptyset, \{3\}, \{4\}, X\}$ is an open cover of the set $\{4\}$.
- The set $\{\emptyset, \{4\}\}$ is an open cover of the set $\{4\}$.

MTH634:Quiz No. 2

Question # 7 of 10 (Start time: 09:54:24 PM, 05 August 2021)

Every metric space is first countable.

Select the correct option

- True

Pg45

- False

Question # 10 of 10 (Start time: 09:57:15 PM, 05 August 2021)

Let $X = \{1,2,3,4,5,6\}$ and $\tau = \{\emptyset, \{3\}, \{4\}, \{3,4\}, X\}$ be a topology on X, then which of the following is true?

Select the correct option

- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open cover of the set $\{4\}$. ✔
- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open cover of the set $\{2\}$.
- None of them.
- The set $\{\emptyset, \{3\}\}$ is an open cover of the set $\{4\}$.

Question # 8 of 10 (Start time: 09:57:46 PM, 05 August 2021) Total Marks: 1

A homoemorphic map between two topological spaces is always ....

Select the correct option

- piecewise continuous
- an identity map
- continuous ✓ Pg9
- discontinuous

Question # 9 of 10 (Start time: 09:14:01 PM, 05 August 2021)

Let $X = \{1,2,3,4,5,6\}$ and $\tau = \{\emptyset, \{3\}, \{4\}, \{3,4\}, X\}$ be a topology on X, then which of the following is true?

Select the correct option

- The set $\{\emptyset, \{3\}, \{4\}, X\}$ is an open cover of the set $\{4\}$.
- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open sub-cover of $\{\emptyset, \{3\}, \{4\}, X\}$.
- The set$\{\emptyset, \{3\}, \{4\}\}$ is an open cover of the set $\{4\}$.
- All of them ✓

and

$$A \subset B \subset \overline{A}.$$

Then B is connected.

In particular, Closure of a connected subset is also connected.

Examples: Compact Spaces

Example 4: Mca̋

An infinite set X with discrete topology $\mathcal{T}$ is not compact.

Reason:

$\mathcal{T}_{dis} = P(X)$

$G = \{\{x\} \mid x \in X\} \cup \{\{x_1, x_2\}\}$

fin. fin.

$G_i = \{x_i\}$

$X \neq \bigcup_{i \in I} G_i$

Subcover:

A subclass $\mathcal{S}$ of a cover $\mathcal{C}$ of X which is also a cover is called a subcover.

Open cover:

A cover $\mathcal{C}$ of X is said to be an open cover if and only if $\mathcal{C} \subset \mathcal{T}$.

Examples: Normal Spaces

Example 1: Consider $X = \{a, b, c, d\}$ with

Imp

$$\mathcal{T} = \{\emptyset, \{d\}, \{b, d\}, \{c, d\}, \{b, c, d\}, X\}.$$

Claim: $(X, \mathcal{T})$ is a normal space. But $(X, \mathcal{T})$ is not T_1

Explanation:

The closed collection is

$$\mathcal{C}_{\mathcal{T}} = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$$

Urysohn's Lemma

One important application of Urysohn's Lemma is the partial solution of metrization problem. Here we have this solution in the form of "Urysohn's Metrization Theorem"

Urysohn's Metrization Theorem:

Every normal-T_1 space with countable basis is metrizable.

Another version of Urysohn's Metrization Theorem is

Urysohn's Metrization Theorem:

Every regular-T_1 space with countable basis is metrizable.

VU

Examples: Normal Spaces

MCQ's

Example 5: $\mathbb{R}$ with usual topology is a T_4 space.

$(\mathbb{R}, \mathfrak{T}_u)$ is Normal space

F_1 F_2 $b < c$

$[a, b] \cap [c, d] = \phi$ b' d'

T_4 space.

Theorem: MCQ's

Every metric space is normal.

Definition:

Let (X, d) be a metric space and A be a nonempty subset of X. Then a continuous function

$$d_A : X \longrightarrow \mathbb{R}$$

defined as

$$d_A(x) = \inf\{d(x, a) | a \in A\}$$

provides the distance between x and A. We denote it as

MCQ's

$$\text{dist}(x, A) = d_A(x)$$

Urysohn's Lemma

One important application of Urysohn's Lemma is the partial solution of metrization problem. Here we have this solution in the form of "Urysohn's Metrization Theorem".

Urysohn's Metrization Theorem:
Every normal-T_1 space with countable basis is metrizable.

Another version of Urysohn's Metrization Theorem is

Urysohn's Metrization Theorem:
Every regular-T_1 space with countable basis is metrizable.

Recall the K-topology on ℝ ...
Topology $\mathcal{T}_K$ generated by

$$\mathcal{B}_K = \{(a,b) \mid a,b \in \mathbb{R}\} \cup \{(a,b) \setminus K \mid a, \ldots$$

where

$$K = \{1, 1/2, 1/3, \cdots\} \subset \mathbb{R}$$

MCQs

$(\mathbb{R}, \mathcal{T}_K)$ is not a regular space

In this topic ... discuss some examples of regular spaces.

Example 1:

Discrete space is a regular space.

Example 2:

Indiscrete space is a regular space.

collection of open subsets of the space.

imp MCQs

Consider $\mathbb{R}$, now

- $(\mathbb{R}, \mathcal{T}_u)$ is not compact.
- $(\mathbb{R}, \mathcal{T}_{indis})$ is compact.
- $(\mathbb{R}, \mathcal{T}_{dis})$ is not compact.
- $(\mathbb{R}, \mathcal{T}_{cof})$ is compact.

Closed Subspace...

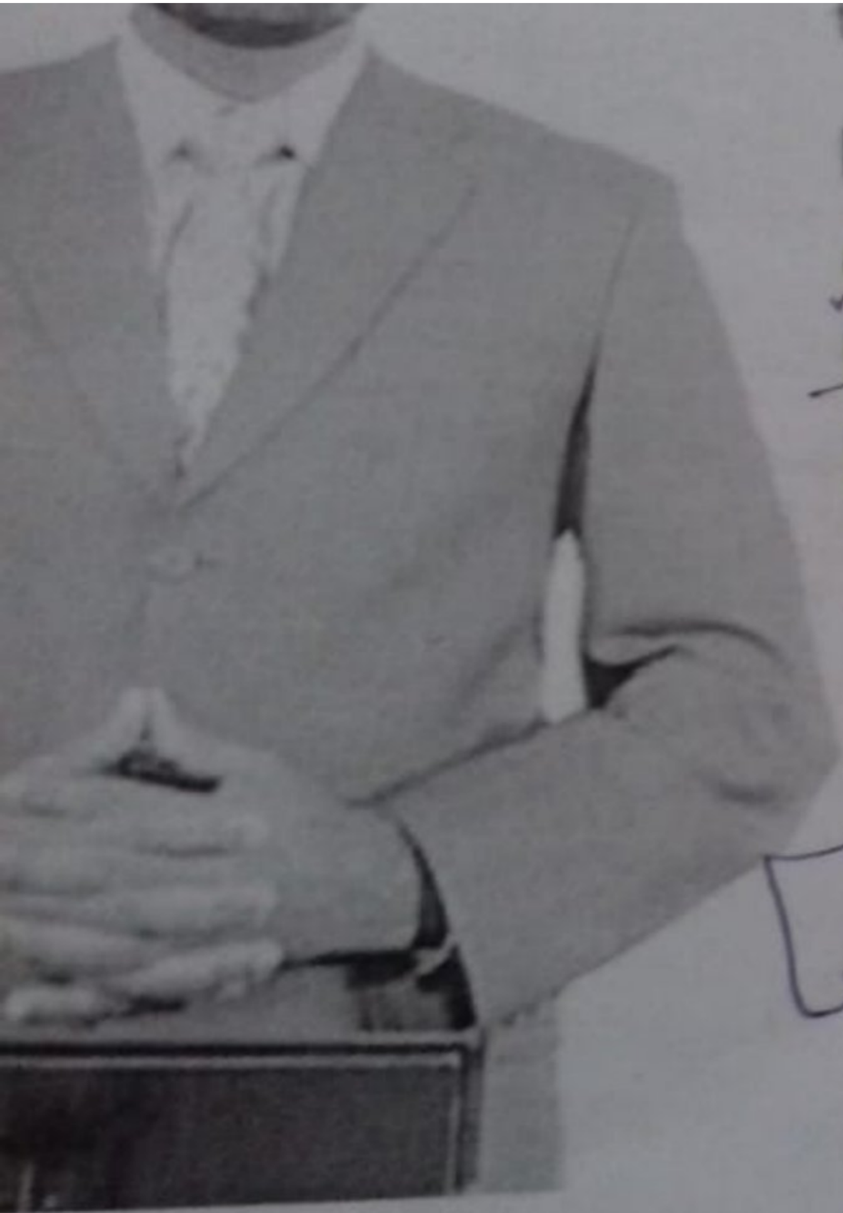
MCQs

Theorem:

Every closed subspace of a compact space is compact.

Proof:

Let (X, T) be a compact space and A be a closed subset of X.



the usual topology.

Fact:

An open interval in $\mathbb{R}$ with respect to usual topology is not compact.

An immediate consequence of this fact is

$(\mathbb{R}, \mathcal{T}_u)$ is not compact

with cofinite topology $\mathcal{T}_{cof}$.

Fact:

MCQs

A set X with cofinite topology $\mathcal{T}_{cof}$ is compact.

Examples: Separated Sets

Example 2:

Consider $(\mathbb{R}, \mathcal{T}_u)$. Then $A = (-1, 3]$ and $B = [3, 5)$ are not separated sets.

Reason:

By "Definition I" both sets should be disjoint but this is not the case here, i.e

$$(-1, 3] \cap [3, 5) = \{3\} \neq \emptyset$$

Examples: Separated Sets

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Example 1: MCQs

Consider $(\mathbb{R}, \mathcal{T}_u)$. Then $A = (0, 1)$ and $B = (4, 9]$ are separated sets

DEF

Reason: (By Definition I)

Limit points = Dri

$= A' \cap B$

$= A \cap B'$

Since

$$(0, 1) \cap (4, 9] = \emptyset$$

and

$[0,1]$ B

$$(0, 1)' \cap (4, 9] = \emptyset$$

Drive se

Corollary I:

Let $(X, \mathcal{T})$ be a compact space and $(Y, \mathcal{T}')$ be a Hausdorff space. Then a continuous map

$$f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$$

is a closed map.

V.V. imp

Compactn

space and

MCQ

$f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$

be a continuous map. Then $f(X)$ is compact.

i.e.

MCQ's

Image of compact space under a continuous map is compact.

MCQ

Theorem:

Let $(A, \mathcal{T}_A)$ be compact subspace of a Hausdorff space $(X, \mathcal{T})$.
Then A is closed in X.

i.e.

Every compact subspace of a Hausdorff space is closed.

MCQ's

Examples: Separated Sets

Example 3: MCQ's

Consider $(\mathbb{R}, \mathcal{T}_u)$. Then $A = (-1, 3)$ and $B = [3, 5)$ are not separated sets

Reason: (By Definition I)

Both sets are disjoint, i.e.

$$(-1, 3) \cap [3, 5) = \emptyset$$

By def. III $A \cap U_B$, $B \cap U_A$

$U_A = (-2, 4)$, $U_B = (2$

$A \cap U_B = (-1, 3) \cap (2, 6$

$B \cap U_A = [3, 5) \cap (-2$

But at the same time both sets should not contain each

Connected Spaces

Connected Set: MCQ's

A subset A of a topological space $(X, \mathcal{T})$ is said to be connected iff there exists no pair of nonempty open subsets U and V of X such that

$$A \cap U \quad \text{and} \quad A \cap V$$

Examples: Connected Spaces

MCQ's

Example 1: Consider $(\mathbb{R}, \mathcal{T}_u)$. Then $A = (0, 1) \cup (3, 5]$ is disconnected subset of $\mathbb{R}$.

Reason:

Since there exists a pair of nonempty open subsets

$$U = (0, 1) \text{ and } V = (3, 6)$$

of $\mathbb{R}$ such that (3,5]

$A \cap U$ and $A \cap V$ are nonempty disjoint sets and

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MTH634 Quiz#3 All in 1 File

Module (152-175)

Due Date (16-18August2021)

Question # 1 of 10 (Start time: 10:11:57 PM, 25 February 2021)

Which of the following statement is false?

Select the correct option

- Discrete topology on a countable set X is second countable.
- Any finite set with any topology is second countable.
- Discrete topology on a real line R is second countable. ✓
- The set $\mathbb{R}$ with usual topology is second countable.

MTH634:Quiz No 3 Quiz Start Time: 11:16 PM, 25 Febru

Question # 1 of 10 (Start time: 11:16:12 PM, 25 February 2021) Total

Let $X = \{1, 2, 3, 4\}$ and $\tau = \{\emptyset, \{1, 2\}, \{3, 4\}, X\}$ be a topology on X and $A = \{2, 3\}$ is a dense set, then which of the following is true?

Select the correct option

Reload Math Eq

- (X, τ) must be a separable topology.
- (X, τ) mar or may not be a separable topology.
- $\{1, 2\}$ is a closed set.
- None of them.

MTH634 Quiz No 3

Question # 6 of 10 (Start time: 09:56:32 PM, 25 February 2021)

Let $X = \{1,2,3,4,5,6\}$ and $\tau = \{\emptyset, \{3\}, \{4\}, \{3,4\}, X\}$ be a topology on X then which of the following is true?

Select the correct option

- The set $\{\emptyset, \{3\}\}$ is an open cover of the set $\{4\}$
- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open cover of the set $\{4\}$ R
- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open cover of the set $\{2\}$
- None of them

MTH634:Quiz No 3 Quiz Start Time:

Question # 9 of 10 (Start time: 10:19:50 PM, 25 February 2021)

Let $X = \{1, 2, 3, 4, 5, 6\}$ and $\tau = \{\emptyset, \{3\}, \{4\}, \{3, 4\}, X\}$ be a topology on X, then which of the following is true?

Select the correct option

- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open cover of the set $\{4\}$.
- The set $\{\emptyset, \{3\}, \{4\}, X\}$ is an open cover of the set $\{4\}$.
- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open sub-cover of $\{\emptyset, \{3\}, \{4\}, X\}$.
- All of them

MTH634:Quiz No 3

Quiz Start Time: 05:54 PM, 25 February 2021

Question # 9 of 10 (Start time: 06:03:29 PM, 25 February 2021)

Total Marks: 1

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ be a topology on X. if $B = \{\emptyset, \{a\}, \{b\}, X\}$ be the base of τ, then which of the following is true ?

Select the correct option

Reload Math Equations

- B be the countable base
- (X, τ) be a first countable space
- (X, τ) be a second countable space
- All of them

Question # 4 of 10 (Start time: 09:57:12 PM, 25 February 2021)

If $\left({X,\tau } \right)$ be a separable topology then it must have countable dense set.

Select the correct option

True

False

MTH634 Quiz No 3 Quiz Start Time: 10:34 PM, 25 February 2021

Question # 6 of 10 (Start time: 10:41:30 PM, 25 February 2021) Total Marks: 1

Every Topological Space is a first countable space.

Select the correct option

- True
- False

MTH634:Quiz No 3

Quiz Start Time: 10:56 PM, 2

Question # 7 of 10 (Start time: 11:01:54 PM, 25 February 2021)

Let $X = \{1, 2, 3, 4, 5, 6\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$ be a topology on X, then the local base (B_x) of the point $x = 3, 4, 5$ is_____

Select the correct option

Reload

- ○ $\{\{1\}, \{2\}, X\}$.
- ○ $\{\{2\}, \{1, 2\}, X\}$.
- ○ $\{X\}$. ✓
- ○ None of them.

Question # 9 of 10 (Start time: 10:45:45 PM, 25 February 2021)

Let $X = \{1, 2, 3, 4, 5, 6\}$ and $\tau = \{\emptyset, \{3\}, \{4\}, \{3, 4\}, X\}$ be a topology on X, then which of the following is true?

Select the correct option

- The set $\{\emptyset, \{4\}\}$ is an open cover of the set $\{4\}$.
- The set $\{\emptyset, \{3\}, \{4\}\}$ is an open cover of the set $\{4\}$.
- The set $\{\emptyset, \{3\}, \{4\}, X\}$ is an open cover of the set $\{4\}$
- All of them.

Question # 9 of 10 (Start time: 11:04:27 PM, 25 February 2021)

Let $X = \{1, 2, 3, 4\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$ be a topology on X, then which of the following is true ?

Select the correct option

- (X, τ) be a first countable space.
- (X, τ) be a topological space.
- Every element of X has countable local base.
- All of them. ✓

Question # 2 of 10 (Start time: 10:35:56 PM, 25 February 2021)

Let $X = \{1,2,3,4,5,6\}$ and $\tau = \{\emptyset, \{1\}, \{2\}, \{1,2\}, X\}$ be a topology on X, then the local base (B_x) of the point $x = 2$ is______

Select the correct option

- $\{\{1\}, \{2\}, X\}$
- $\{\{2\}, \{1,2\}, X\}$ ✓
- $\{\{1\}, \{1,2\}, \{2\}, X\}$
- None of them.

MTH634:Quiz No. 3 Quiz Start Time: 12

Question # 4 of 5 (Start time: 12:05:12 PM, 08 August 2021)

A topological space X is called a connected space iff there exists a pair of subsets of X both nonempty and both open and closed.

Select the correct option

- False

Module 165

- True

Question # 5 of 5 (Start time: 04:05:51 PM, 08 August 2021)

Consider $\mathbb{R}$ with usual topology. Which of the following sets are separated?

Select the correct option

- A=(-1,3), B=[3,5)
- A=(0,2), B=[2,5)
- A=(0,1), B=(4,9] Pg142 ✓
- none

MTH634:Quiz No. 3

Question # 5 of 5 (Start time: 07:31:16 PM, 08 August 2021)

$\mathbb{R}$ with usual topology is a $T_4 - space$

Select the correct option

○	*False*
○	*True* Module 147 ✓

Question # 1 of 5 (Start time: 04:35:58 PM, 08 August 2021)

Which of the following statement is true?

Select the correct option

Module 152

- $\mathbb{R}$ with indiscrete topology is compact. ✔
- $\mathbb{R}$ with usual topology is compact.
- $\mathbb{R}$ with cofinite topology is not compact.
- $\mathbb{R}$ with discrete topology is compact.

MTH634:Quiz No. 3 Quiz S

Question # 5 of 5 (Start time: 10:54:39 PM, 08 August 2021)

If (X, τ) be a compact Hausdorff space then (X, τ) is not normal.

Select the correct option

- True
- False

Module 162

MTH634:Quiz No. 3 Quiz Star

Question # 4 of 5 (Start time: 10:53:48 PM, 08 August 2021)

Consider R with usual topology. Then $A = (0, 1) \cup (3, 5]$ is disconnected subset of R.

Select the correct option

○	False
○	True

Module 166, pg147

Question # 1 of 5 (Start time: 07:27:11 PM, 08 August 2021) Total Marks: 1

Which of the following statement is false?

Select the correct option

- ○ A finite set X with any topology is compact.
- ○ An infinite set X with discrete topology is compact. ✓
- ○ A set X with indiscrete topology is compact.
- ○ A set X with any topology containing finite number subsets of X is compact.

Module 152, pg 121

sec(s)

MTH634:Quiz No. 3

Quiz Start Time: 05:12 PM, 08 August 2021

Question # 5 of 5 (Start time: 05:15:04 PM, 08 August 2021)

Total Marks: 1

Consider $\mathbb{R}$ with usual topology. There exits no homomorphism between an open interval of $\mathbb{R}$ and a closed interval of $\mathbb{R}$.

Select the correct option

Reload Math Equations

- ○ False
- ● True

Module 160 ✓

Question # 2 of 5 (Start time: 04:55:23 PM, 08 August 2021)

Total Marks: 1

A set X with cofinite topology is compact.

Select the correct option

- True

Module 154, pg121

- False

Question # 1 of 5 (Start time: 04:54:39 PM, 08 August 2021) Total Marks: 1

Which of the following statement is false?

Select the correct option

- Every metric space is normal space.
- Every metric space is Hausdorff space.
- none
- Every metric space is second countable. ✓

Module 126, pg72

MTH634:Quiz No. 3

Quiz Start Time: 04:35 PM, 08 August 20

Question # 3 of 5 (Start time: 04:37:57 PM, 08 August 2021)

Total Mark

Image of a compact space under a continuous map is compact.

Select the correct option

False

True

B

sec(s)

MTH634:Quiz No. 3

Quiz Start Time: 04:19 PM, 08 August 2021

Question # 3 of 5 (Start time: 04:21:41 PM, 08 August 2021)

Total Marks: 1

Every closed subspace of a compact space is compact.

Select the correct option

- False
- True

B

sec(s)

MTH634:Quiz No. 3

Quiz Start Time: 04:19 PM, 08 August 2021

Question # 3 of 5 (Start time: 04:21:24 PM, 08 August 2021)

Total Marks: 1

Every discrete space is not a regular space.

Select the correct option

- False — Module 145, pg103 ✓
- True

sec(s)

MTH634:Quiz No. 3

Quiz Start Time: 04:19 PM, 08 August 2021

Question # 1 of 5 (Start time: 04:19:48 PM, 08 August 2021)

Total Marks: 1

Every compact subspace of a Hausdorff space is not closed.

Select the correct option

- False

Module 157, pg133

- True

sec(s)

MTH634:Quiz No. 3

Quiz Start Time: 04:19 PM, 08 August 2021

Question # 1 of 5 (Start time: 04:19:29 PM, 08 August 2021)

Total Marks: 1

All the metric spaces are not normal spaces.

Select the correct option

True

False

Module 148

MTH634:Quiz No. 3

Question # 4 of 5 (Start time: 05:57:04 PM, 09 August 2021)

An open interval in

$\mathbb{R}$

with usual topology is not compact.

Select the correct option

- True

Module 153

- False

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MTH634 Topology

Question No : 10 of 41

Which of the following statement is true?

Answer (Please select your correct option)

- All spaces are metrizable
- all spaces are not metrizable.

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MTH634 Topology

Question No : 27 of 41

Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as $f(x) = x + 1$. Fixed point of f is:

Answer (Please select your correct option)

- 0
- 1
- 2
- no fixed point

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Question Summary (Attempted Question)

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MTH634 Topology MC

Question No : 24 of 41 **Marks: 1 (Budgeted Time Min)**

Consider $\mathbb{R}$ with usual topology. There exits no homomorphism between an open interval of $\mathbb{R}$ and a closed interval of $\mathbb{R}$.

Answer (Please select your correct option)

- True
- False

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MTH634 Topology

Question No : 17 of 41

A topological space is a T_1 − space if and only if its each finite subset is

Answer (Please select your correct option)

- a perfect set
- a closed set
- an open set
- none

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H634 Topology

estion No : 25 of 41

onsider $\mathbb{R}$ with usual topology. Then $A=(0,1)\cup(3,5]$ is disconnected subset of $\mathbb{R}$.

nswer (Please select your correct option)

True

False

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MTH634 Topology

Question No : 14 of 41

Every second countable space is not Lindelof space.

Answer (Please select your correct option)

- True
- False

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MTH634 Topology

Question No : 15 of 41

Which of the following statement is false.

Answer (Please select your correct option)

- Finite set X with any topology is separable.
- A set X with indiscrete topology is separable.
- □ with usual topology is separable.
- □ with discrete topology is separable. ⇦ R

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MTH634 Topology

Question No : 19 of 41

Every subspace of T_3 − space is a

Answer (Please select your correct option)

- $T_1 - space$
- $T_2 - space$
- $T_3 - space$
- none

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MTH634 Topology

Question No : 23 of 41

Every compact subspace of a hausdorff space is open.

Answer (Please select your correct option)

- True
- False

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MTH634 Topology

Question No : 8 of 41

Which of the following statement is true?

Answer (Please select your correct option)

- Projection map may or may not be closed.
- Projection map is open.
- Projection map is closed.

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MTH634 Topology

Question No : 21 of 41

□ with usual topology is a $T_4 - space$.

Answer (Please select your correct option)

True

False

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MTH634 Topology

Question No : 11 of 41

Every subspace of a second countable space is second countable.

Answer (Please select your correct option)

- true
- false

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MTH634 Topology

Question No : 18 of 41

Which of the following statement is false?

Answer (Please select your correct option)

- A nonempty set $X \neq \{x\}$ with indiscrete topology is Hausdorff.
- Every discrete space is Hausdorff.
- Every discrete space is T_0 space.
- Every discrete space is T_1 space

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MTH634 Topology

Question No : 9 of 41 **Marks: 1 (Budgeted**

A map $f : (X, \tau_X) \to (Y, \tau_Y)$ such that (i) f is bijective (ii) f is continuous (iii) f is an open map

Then the map f is called:

Answer (Please select your correct option)

- open map
- homomorphism
- closed map

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MTH634 Topology

Question No : 12 of 41 **Marks: 1**

For metric d on X, an open ball of radius r with center $x \in X$ is defined as:

Answer (Please select your correct option)

- $B(x,r) = (y \in X \mid d(x,y) \leq r)$
- $B(x,r) = (y \in X \mid d(x,y) > r)$
- $B(x,r) = (y \in X \mid d(x,y) < r)$ ⇦ R
- none

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MTH634 Topology

Question No : 16 of 41

Which of the following statement is true?

Answer (Please select your correct option)

- Every T_1 space is T_0.
- Every T_0 space is T_1.
- Every Hausdorff space is Normal.
- none

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MTH634 Topology

Question No : 7 of 41 Marks

A map $f:(X,\tau_X)\rightarrow(Y,\tau_Y)$ is an open map iff:

Answer (Please select your correct option)

- the image of each element of a basis β_X of τ_X is an open subset of Y
- the image of each element of a basis β_Y of τ_Y is an open subset of Y
- the image of each open subset of Y is open X
- none

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Favorites

MTH634 Topology

Question No : 29 of 41

A discrete space is not locally connected.

Answer (Please select your correct option)

- True
- False

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Favorites

MTH634 Topology

Question No : 22 of 41

A set X with cofinite topology is compact.

Answer (Please select your correct option)

- True
- False

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MTH634 Topology

Question No : 28 of 41

Let C be a connected component of a topological space X, then

Answer (Please select your correct option)

- C *is open*
- C *is closed* R
- C is infinite
- none X

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Question Summary : (Attempted Questions)

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MTH634 Topology

Question No : 20 of 41

A nonempty set $X \neq (x)$ with indiscrete topology is a

Answer (Please select your correct option)

- $T_0 - space$
- regular space
- Hausdorff space
- none

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MTH634 Topology

Question No : 13 of 41 **Marks: 1 (Budgeted Time**

Let $B_1(x,r_1)$ and $B_2(x,r_2)$ be two open balls centered at x with radii r_1 and r_2 respectively. Then the following statement is true.

Answer (Please select your correct option)

- $B_1(x,r_1) = B_2(x,r_2)$
- either $B_1(x,r_1) \subset B_2(x,r_2)$ or $B_1(x,r_1) \supset B_2(x,r_2)$
- none

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Done

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MTH634 Topology

Question No : 30 of 41

The image of connected topological space is path connected.

Answer (Please select your correct option)

- True
- False

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Favorites

MTH634 Topology

Question No : 26 of 41

Closure of a connected subset is also connected.

Answer (Please select your correct option)

- True
- False

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MTH634 Topology

Question No : 3 of 33

Which of the following statement is false?

Answer (Please select your correct option)

- The set $\mathbb{R}$ with usual topology is metrizable space.
- The set $\mathbb{R}^2$ with usual topology is metrizable space.
- Indiscrete topology on a set X containing more than one point is metrizable

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Question No : 8 of 33

Which of the following statement is true?

Answer (Please select your correct option)

- Every separable space is second countable.
- Every second countable space is separable
- $\mathbb{R}$ with lower limit topology is second countable

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Question No : 11 of 33

A finite set has ____________ in a T_1 space.

Answer (Please select your correct option)

- no isolated points
- no limit points
- infinte limit points
- none

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Question No : 2 of 33

Which of the following statement is true?

Answer (Please select your correct option)

- If a map $f : X \to Y$ is bicontinuos then it is also bijective.
- If a map $f : X \to Y$ is bicontinuos then it is not bijective.
- If a map $f : X \to Y$ is bicontinuos then it may or may not be bijective. ✓

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MTH634 Topology

Question No : 4 of 33

Which of the following statement is false?

Answer (Please select your correct option)

- Discrete topology on a countable set X is second countable.
- Any finite set with any topology is second countable.
- Discrete topology on a real line R is second countable. ✓
- The set $\mathbb{R}$ with usual topology is second countable.

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Question No : 12 of 33

A regular T_1 − space is called a

Answer (Please select your correct option)

- no isolated points
- T_2 − space
- T_3 − space
- none

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Question Summary : (Attempted Question)

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MTH634 Topology

Question No : 24 of 33

If (X,τ) be a path connected topological space, then (X,τ) is connected

Answer (Please select your correct option)

- True ✓
- False

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A real line ℝ with usual topology is path connected.

Answer (Please select your correct option)

True ✓

False

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Question Summary : (Attempted Question)

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MTH634 Topology

Question No : 40 of 41

Show that a set X with cofinite topology is compact

Q Show that a set X with cofinite topology is compact

Proof: If X is finite, Then

$\mathcal{T}_{cof} = P(X)$

$\Rightarrow (X, \mathcal{T}_{cof})$ is compact

But if X is not finite Then consider an open cover $\mathcal{D}$ of X.

Now let $\phi \neq D \in \mathcal{D}$ $\because D^c$ is finite

Now, for every $x \in D^c$ choose an element of $\mathcal{D}$, D_x containing x.

$\{D_x \mid x \in D^c\}$ is finite

$\because D^c$ is finite

Consider this subcover of $\mathcal{D}$

~~$X = X \{D\}$~~

$C = \{D\} \cup \{D_x \mid x \in D^c\}$

$\rightarrow C$ is a finite cover of $\mathcal{D}$

$\Rightarrow (X, \mathcal{T}_{cof})$ is compact

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MTH634 Topology

Question No : 39 of 41

Show that every second countable space is first countable.

Answer (Please click here to Add Answer)

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Second Countability implies...

Proof: Let X be a second countable space.

i.e $\exists$ $\mathcal{B}$ (a basis) s.t $\mathcal{B}$ is countable.

Now let $p \in X$ and consider

$\mathcal{B}_p \subset \mathcal{B}$ $\quad \mathcal{B}_p = \{B_p \mid p \in B_p\} \subset \mathcal{B}$

Now $\mathcal{B}_p$ is countable $\because \mathcal{B}$ is countable

Now $\mathcal{B}_p$ is local base at p.

Second Countability implies...

$\mathcal{B}_p \subset \mathcal{B}$ (basis) $\quad \forall\, U \in \mathcal{T}$ s.t $p \in U$

$\exists\, B_p \in \mathcal{B}_p$ s.t

$p \in B_p \subset U$

$\Rightarrow \mathcal{B}_p$ is a local base at p.

And this exists $\forall\, p \in X$.

$\Rightarrow X$ is first countable $\square$

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MTH634 Topology

Question No : 37 of 41

Show that a discrete space X is separable if and only if X is countable.

Answer (Please click here to Add Answer)

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Show that

Q5) A discrete space is seperable if and only if it is countable

Solu

$X, \mathcal{T}_{dis} = P(X)$

We know that the only dense subset in X is X itself.

So the only choice for A is X s.t

$\bar{A} = X$

$\Rightarrow$ X is seperable iff X is countable

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Done

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MTH634 Topology

Question No : 31 of 41

Let $X = \{a, b, c\}$ Write the basis for the discrete topology on X.

Answer (Please click here to Add Answer)

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Q11) Let $X = \{a, b, c\}$. Write the basis for the discrete topology on X.

Solu:- Let $X = \{a, b, c\}$

Consider the set of all singletons i-e

$B = \{\{a\}, \{b\}, \{c\}\}$

It is clear that this collection satisfies both the conditions of basis. Let us generate the topology from B.

Following is the topology

$\mathcal{T} = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$

It is discrete topology.

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Done

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MTH634 Topology

Question No : 36 of 41

Prove that every subspace of a second countable space is second countable.

Answe

Now B_{P_A} is local base at P for T_A and B_{P_A} is countable so A is countable

Every subspace of a second countable space is second countable

Proof:- is same as for first countable space

Let $A \subset X$ (X is second countable space)

So $\exists$ $B = \{B_A \mid n \in N\} \rightarrow$ Countable.

Now $B_A = \{A \cap B_n \mid n \in N\}$ is a basis for T_A and B_A is Countable

$\Rightarrow$ A is second countable.

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MTH634 Topology

Question No : 32 of 41

Give one example of second countable space.

Answer (Please click here to Add Answer)

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②

Second Countable example

Let X be a countable set with discrete topology. Then $(X, \mathcal{T}_{dis})$ is a second countable. Recall basis for discrete space is

$$B = \{\{x\} \mid x \in X\}$$

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MTH634 Topology

Question No : 32 of 41

Give one example of second countable space.

Answer (Please click here to Add Answer)

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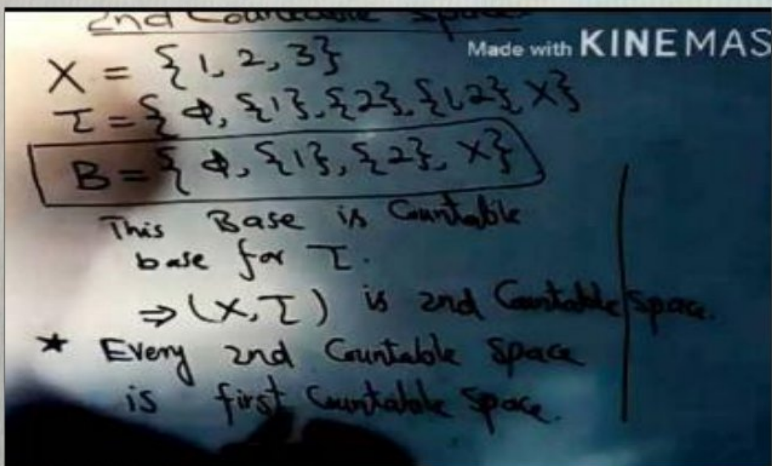

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MTH634 Topology

Question No : 34 of 41

Define separated sets.

Answer (Please click here to Add Answer)

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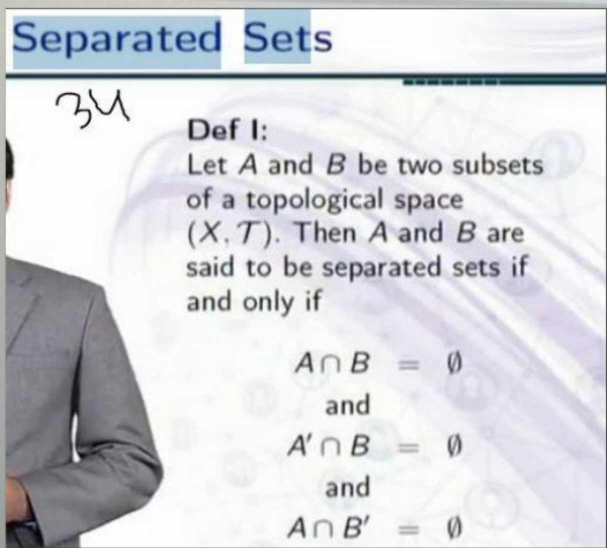

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MTH634 Topology

Question No : 33 of 41

Define T_1 – space.

Answer (Please click here to Add Answer)

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T_1 space

Def:
A topological space $(X, \mathcal{T})$ is said to be "T_1 Space" iff for each $x, y \in X$ such that $x \neq y$ there exist open subsets U_x, U_y of X containing x, y respectively such that $y \notin U_x$ and $x \notin U_y$.

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Question No : 27 of 33

Define Normal space.

Answer (Please click here to Add Answer)

Normal Spaces

Def:
A topological space $(X, \mathcal{T})$ is said to be "normal" iff for every pair of disjoint closed subsets, $F_1, F_2 \subset X$ there exist open subsets U_{F_1} and U_{F_2} containing F_1 and F_2 respectively such that

$$U_{F_1} \cap U_{F_2} = \emptyset.$$

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Question No : 26 of 33

Define metric topology.

Answer (Please click here to Add Answer)

Metric Topology:

Let X be a nonempty set with metric d. The topology $\mathcal{T}$ on X generated by the set of all open balls in X with respect to d is called the "Metric Topology."

We also say that a topology $\mathcal{T}$ on X induced by the metric d.

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Question Summary : (Attempted Question)

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MTH634 Topology

Question No : 30 of 33

Show that a set X with topology T containing finite number subsets of X is compact

Answer (Please click here to Add Answer)

Example 2:
A set X with topology $\mathcal{T}$ containing finite number subsets of X is compact.

Reason:
Since every open cover $\mathcal{C}$ of X is a subclass of $\mathcal{T}$ and $\mathcal{T}$ itself finite so $\mathcal{C}$ is finite too and any subcover $\mathcal{S}$ of $\mathcal{C}$ is also finite.

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Question No : 31 of 33

Define connected set

Answer (Please click here to Add Answer)

100%

Connected Set:
A subset A of a topological space $(X, \mathcal{T})$ is said to be connected iff there exists no pair of nonempty open subsets U and V of X such that

$$A \cap U \quad \text{and} \quad A \cap V$$

are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V)$$

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MTH634 Topology

Question No : 29 of 33

Consider the sets $X = \{1,2,3\}$ and $Y = \{a,b,c\}$ with topologies $\tau_X = \{\phi, \{1\}, \{1,2\}, X\}$ and $\tau_Y = \{\phi, \{a,b\}, \{b\}, Y\}$ repectively. Define a map $f : X \to Y$ as:

$f(1) = f(2) = a, f(3) = c,$

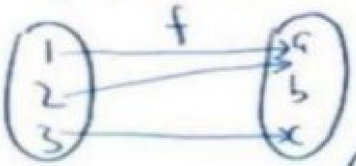
Show that f is a closed map.

Answer (Please click here to Add Answer)

Closed Mapping

Ex: $X = \{1,2,3\}$, $\tau_X = \{\phi, \{1\}, \{1,2\}, X\}$

$Y = \{a,b,c\}$, $\tau_Y = \{\phi, \{a,b\}, \{b\}, Y\}$



$\mathcal{C}_X = \{X, \{2,3\}, \{3\}, \phi\}$

$\mathcal{C}_Y = \{Y, \{c\}, \{a,c\}, \phi\}$

$f(\phi) = \phi$

$f(\{3\}) = \{c\}$ | $f(\{2,3\}) = \{a,c\}$ | $f(X) = \{a,c\}$

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Done

MTH634 Topology

Question No : 28 of 33

Define connected component in a topological space X.

Answer (Please click here to Add Answer)

100%

Def:
Consider a topological space $(X, \mathcal{T})$. Let $a \in X$ and C be a connected subset of X containing a.
Then

$$C_a = \bigcup_{a \in C} C$$

is called the connected component of X containing a.

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MTH634 Topology

Question No : 32 of 33

Show that any subspace (Y, τ_Y) of a first countable space (X, τ) is also first countable.

Answer (Please click here to Add Answer)

First Countable Spaces

Remark:
Every subspace of a first countable space is first countable.

$A \subset X$ (X is first countable)
$\Rightarrow$ A is also first countable.

Proof: Let X be a first countable space and $A \subset X$, let $p \in A$ ∵ X is first countable

So ∃ a countable local base at p.

$\mathcal{B}_p = \{ B_n \mid n \in \mathbb{N} \}$

Now $\mathcal{B}_{p_A} = \{ A \cap B_n \mid n \in \mathbb{N} \}$

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First Countable Spaces

Now $\mathcal{B}_{p_A}$ is a local base at p. for $\mathcal{T}_A$

And $\mathcal{B}_{p_A}$ is countable So A is first countable.

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Done

Question No : 33 of 33

Marks: 5 (Budgeted Time ...

Consider $X = \{a,b,c,d,e\}$ with topology $\tau = \{\phi,\{a\},\{a,b\},\{a,b,c\},\{a,d,e\},\{a,b,d,e\},X\}$. Show that the set $A = \{c,e\}$ is disconnected subset of X.

Answer (Please click here to Add Answer)

Examples: Connected Spaces

Example 2: Consider $X = \{a, b, c, d, e\}$ with $\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, \{a, d, e\}, \{a, b, d, e\}, X\}$. Then $A = \{c, e\}$ is disconnected subset of X.

Reason:

Since there exists a pair of nonempty open subsets $U = \{a, b, d, e\}$ and $V = \{a, b, c\}$ of X such that

$$A \cap U = \{e\} \quad \text{and} \quad A \cap V = \{c\}$$

$\{e\} \cap \{c\} = \phi$

are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V) = \{e\} \cup \{c\} = \{c, e\}$$

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Q9) Let $(X, \mathcal{J})$ be a path connected ~~topology~~ & topological space and $f : (X, \mathcal{J}) \longrightarrow (Y, \mathcal{J}')$ be a continuous map, Show that $f(X)$ is path connected.

Answer:-

$$f : (X, \mathcal{J}) \xrightarrow[\text{cont}]{} (Y, \mathcal{J}')$$

$f(X)$ is path connected

Let $a, b \in f(X)$

$\Rightarrow \exists \; x, y \in X$ s.t.

$$f(x) = a \quad , \; f(y) = b$$

Now $\because X$ is path connected

~~So~~, $\exists \; p : I \xrightarrow[\text{cont}]{} X \quad p(0) = x, \; p(1) = y$

$$q = f \circ p : I \longrightarrow f(X)$$

$$q(0) = f \circ p(0) = f(p(0)) = f(x) = a$$

$$q(1) = f \circ p(1) = f(p(1)) = f(y) = b$$

$\Rightarrow f(X)$ is path connected

MTH 634

Q: Consider $\mathbb{R}$ with usual $\mathcal{J}$. Show that there exist no homomorphism for (0,2) and (0,1] $\cup$ [3,4)

Q: Consider $\mathcal{J} = \{\phi, \{d\}, \{b,d\}, \{c,d\}, \{b,c,d\}, X\}$

$X = \{a,b,c,d\}$

show $(X, \mathcal{J})$ is normal but not T_4.

Q: Show homomorphism of Topological space is an equilance relation.

Q: $\mathbb{R}$ with usual $\mathcal{J}$, $B = (0,1)$ $C = [2,3)$ are seperable.

Q: Show a discrete space X is seperable iff X is countable.

Q: Show f Close map → Example

Q: Show $f: R_u \to R_u$ is continous iff $f(x) = [x]$

Q: Define seperable sets.

Q: T_0 space.

Q: Homomorphism Def / Q → A is open iff A = int(A)

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Mth634

Q1 Write one example of Second countable?

Q2 Let $X=\{1,2,3,4\}$ with discrete topology. Write Basis for it?

Q3 Let $X=\mathbb{R}$ and d is defined as $d(x,y)=|x-y|$ $\forall x,y \in \mathbb{R}$.
Show (X,d) metric space?

Q4 Let $(X, \mathcal{T}_{dis})$ be a discrete space $A \subset X$ then find closure and derived set?

Q5 Every cofinite space $\mathcal{T}_{cof}$ is compact.

Q6 Define Fixed point of a map.

Q7 Consider $X=\{a,b,c,d,e\}$ with
$\mathcal{T}=\{\phi,\{a,b\},\{d,e\},\{a,b,c\},\{a,b,d,e\},X\}$
Then X is a disconnected space.

Q8 Every subspace of T_1-space is T_1-space.

Q9 Let function $f: X \to X$. Show that there are no Homomorphism between $(0,2)$ and $(1,3] \cup [3,4)$.

$\forall\ a,b,c \in X$

① ~~Relative~~ Reflexive : $a \sim a$

② Symmetric : $a \sim b$ iff $b \sim a$

③ Transitive : if $a \sim b$ and $b \sim c$ then $a \sim c$

Proposition:

Homeomorphism of topological spaces is an equivalence relation.

Proof:

$\approx$

- $X \approx X$ $\quad I_d : X \to X$ (same $\mathcal{T}$)
  $x \mapsto x$ — biject + cont inverse

- Let $X \approx Y$. $f : X \to Y$ Homomorphic
  Bijective + Bicontinous.

  $f^{-1} : Y \to X$
  Bijective + continous + inverse . f is cn

$\Rightarrow X \approx Y \Leftrightarrow Y \approx X$

- $X \approx Y \quad , \quad Y \approx Z$

  $f : X \to Y$
  Bijective + Bicontinous | $g : Y \to Z$
  Bijective + Bicontinous

  $g \circ f : X \to Z$
  Bijective + Bicontinous.
  $g \circ f$ is continous

$\tau_x = \{\phi, \{1\}, \{1,2\}, X\}$ & $\tau_y = \{\phi, \{a,b\}, \ldots$

(6) Define a map $f : X \to Y$ as

$f(1) = f(2) = a$, $f(3) = c$.

Show that f is a closed map.

(7) Let X be a T_0 space & let B_p be a local base at $p \in X$. Show that if $q \in X$ is distinct from p, then some member of B_p does not contain q.

(8) Consider $X = \{a, b, c, d, e\}$ with topology $\tau = \{\phi, \{a,b\}, \{d,e\}, \{a,b,c\}, \{a,b,d,e\}, X\}$ Show that X is disconnected.

(9) Show that the plane R^2 with usual top. is a second countable space.

(10) Show that every T_1-space is a T_0 space

(11) Let (X, τ) be a compact space & (Y, τ') be a Housdorff space. Then show that a continous map $f : (X, \tau) \to (Y, \tau^*)$ is a closed map.

1) let $X = R$ with usual topology & $A = \emptyset$. Find the derived set of A.

2) Name two topological properties of a topological space (X, τ).

③ Define T_0 space?

④ Define locally connected spaces.

⑤ If A is a subset of a topological space X, then show that $Int(A) \cap boundary(A) = \phi$

⑥ Consider the sets $X = \{1,2,3\}$ & $Y = \{a,b,c\}$ with topologies $\tau_X = \{\phi, \{1\}, \{1,2\}, X\}$ & $\tau_Y = \{\phi, \{a,b\}, \{b\}, Y\}$ respectively

⑦ Define a map $f : X \to Y$ as
$f(1) = f(2) = a, f(3) = c.$

MTh 634

① Find limit points of $A = (0, 2)$ 2 mark
② open mapping (2 marks)
③ $X = \{a, b\}$ $A = \{a\}$ $B = \{b\}$
$\overline{A \cap B} \neq \bar{A} \cap \bar{B}$ (3 mark)
④ Prove $(X, T_X) \longrightarrow (Y, T_Y)$ continuous and restricted to f_A.
⑤ Prove $A' \subset A$ A is closed (5 Mark)

⑥ $(X, T_X) \longrightarrow (Y, T_Y)$ continuous $P \in X$ (5 Mark)

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① Let $X = \square$ with usual topology
then $B = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$,
limit point is
- 1
- 0 ✓ I picked.
- none

② In a discrete space X,
the derived set A' of any
subset A of X is

empty set ?

③ Closure of set X.

- $\bar{X} = X$
- $\bar{\emptyset} = \emptyset$
- $X = \emptyset$
- or i) + ii) ? ✓ I picked.

④ I discrete set, convergence to point a b

- $(a, a_2, \dots bbb)$?
- $bbb, \dots a a_1 a_2 a_3$?

① MTH634 27-12-18

① Give one example of comparable Topology.

② Prove that $\bar{A} = A$ ②

③ interior ki example ③

④ Continous show krna that sma

int(A) U (Boundary A)

⑤ Prove that The Given sma

Topology is interior.

MTH634.

1- Prove that
$\overline{A \cup B} \subseteq \bar{A} \cup \bar{B}$

2 Let $X = \{a, b, c, d, e\}$ $A = \{c, d\}$.
$J = \{\phi, \{b\}, \{a\}, \{a, c, d\}, \{a, b, c, d\}, X\}$
$int(A) = ?$ ~~int~~ $Ext(A) = ?$

3- Let $X = \{1, 2, 3, 4\}$. Derived set = ?
$A = \{2, 3\}$

4- Let $X = \{1, 2, 3, 4, 5\}$. $A = \{4, 5\}$ Find
Bais 'B' generated by A.

5- Let (X, J) be a topological space, then
prove that ~~derived set~~ $\bar{A}$ is in X
and also $\bar{A}$ is ~~in~~ dense in X

6- Let $X = \{a, b, c, d, e\}$, $\alpha = \{c, d, e\}$.
Find Bais "β" generated by α
and also find topology
J generated ~~α~~ β.

Example #1

Every discrete space is Hausdorff.

Example #2

A nonempty set $X \neq \{x\}$ with indiscrete topology is not Hausdorff.

Example #3

$\mathbb{R}$ with lower limit topology is Hausdorff.

$x, y \in \mathbb{R}$, $x < y$

$x < c < y < d$

$x \in \underbrace{[x, c)}_{U_x}, \underbrace{[y, d)}_{U_y} \ni y$

$U_x \cap U_y = \phi$

Example #4.

$\mathbb{R}$ with topology generated $\{(a, \infty) | a \in \mathbb{R}\}$ is not Hausdorff.

$3, 6 \in \mathbb{R}$

$a < 3$.

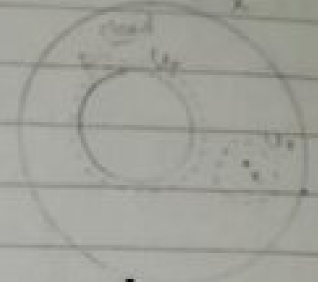
$6 \in (5, \infty)$. $3 \in (a, \infty) \ni 6$

Example #5.

An infinite set with cofinite topology is not Hausdorff space.

... is Regular iff for every closed subset F of X and for every $x \in F^c$ there exist open subsets U_F & U_x containing F and x Resp. Such that

$U_F \cap U_x = \phi$.



All used in different mcqs

T_3 Space:

A topological space (X, T) is said to be "T_3 Space" iff it is either

Regular + T_0 or

Regular + T_1 or

Regular + T_2.

Remarks:

1. Property of being regular is topological property.
2. Every subspace of a regular is a regular space.
3. T_3 property is a topological property.
4. Every subspace of a T_3 space is a T_3 space.

Examples: Regular Spaces.

Example #1

Discrete space is a Regular.

Example #2

Indiscrete space is a Regular space

$T_{ind} = \{\phi, X\}$

$C_{ind} = \{\phi, X\}$

$\phi^c = X \ni x$

$F = \phi$, $U_F = \phi$

$U_x = X$

$\boxed{\phi \cap X = \phi}$

$X \neq \{x\}$

MTH634

1) Define Neighborhood of P in Space (X, T). (2)

2) Define Metric Topology? (2)

3) Show that the set $\mathbb{R}$ with usual topology is separable?

4) Define connected component?

5) $X = \mathbb{R}$ with $A = (0,1]$, Find interior, Exterior boundary & partition of X.

6) $X = \{a, b, c\}$, $T, \{\phi, \{a\}, \{b, c\}, X\}$
Show that (X, T) is regular.

7) Discrete space is usually connected.

8) Show that T_1 is T_0 space. (5)

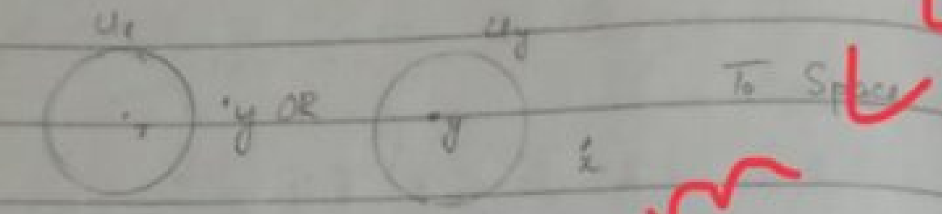
c, d open set is X. | so

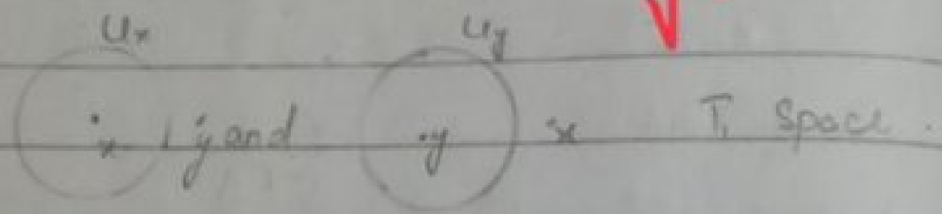
(a2) $T_0 \not\Rightarrow T_1$

T_1 property is much stronger property than T_0 property i.e.

Every T_1 space is T_0 but not every T_0 space is T_1

Let us prove first part of the claim and give an example for the second part.





$T_1 \Rightarrow T_0$

$T_0 \not\Rightarrow T_1$

Example:

A T_0 Space that is not T_1

Consider $\mathbb{R}$ with $\mathcal{T}$ generated by $\{(a,\infty) \mid a\in\mathbb{R}\}$.

$a,b\in\mathbb{R}$ $\quad a>b$

$a\in U_a=(c,\infty)\not\ni b$ $\quad b<c<a$

So T_0-Space.

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Define open function from topol space X to topo space Y.
Define metric topology.
Define normal space
Define fixed point of a map
Let $X = \{a, b\}$ with ind topo Let $A = \{a\}$, $B = \{b\}$
show $\overline{A \cap B} \neq \bar{A} \cap \bar{B}$
Let $X = \{a, b, c, d\}$ with $\mathcal{T}_1 = \{\phi, X\}$ and $Y = \{1, 2, 3, 4\}$ with $\mathcal{T}_2 = \{\phi, \{1\}, Y\}$
$f(a) = 1$ $f(b) = 1$ $f(c) = 1$ $f(d) = 1$ show open so cont
Consider $X = \{a, b, c\}$ with $\mathcal{T} = \{\phi, \{a\}, \{b, c\}, X\}$ show regular
$A = (0, 1) \cup (3, 5]$ is disconn?
Show any subspace of 1st count is 1st count
Show every T_1 space is T_0-space
Let $(X, \mathcal{T})$ compact $(Y, \mathcal{T}')$ Hand then show $f : (X, \mathcal{T}) \rightarrow (Y, \mathcal{T}')$ is closed map

MTH634

1) Define Neighborhood of P in Space (X, T). (2)

2) Define Metric Topology? (2)

3) Show that the set $\mathbb{R}$ with usual topology is separable?

4) Define connected component?

5) $X = \mathbb{R}$ with $A = (0,1]$, Find interior, Exterior boundary & partition of X.

6) $X = \{a, b, c\}$, $T, \{\phi, \{a\}, \{b, c\}, X\}$
Show that (X, T) is regular.

7) Discrete space is usually connected.

8) Show that T_1 is T_0 space. (5)

Q1 $X = \{1, 2, 3\}$ Show $B = \{\{1,2\}, \{2,3\}\}$ cannot be a base for any topology.

Q2 $X = \{a, b, c, d\}$ topology $T = \{\emptyset, \{a\}, \{a, b\}, \{a, c, d\}, X\}$ find open cover of X.

Q3 Show that X with discrete topology T is compact

Q4 Define locally connected space.

Q5 Show that a subset A of Topology space X is open if $\forall x \in X$ there exist U_x such that $U_x \subset A$

Q6 Prove that every subspace of second countable space is countable.

Q7 Show that every cofinite space is

Q8 let E be subset of real line R containing two points. Then show that if connected than E

Q9 Show that homeomorphism of topological space is equivalance relation

Q10 Show that every metric space is Hausdroff space

Q11 Consider $X = \{a, b, c, d, e\}$ $T = \{\emptyset, \{a\}, \{a, b\}, \{a, b, d, e\}, X\}$ show that $A = \{c, e\}$ disconne

MTH: 634

5:00 PM
24-02-2019

1. Define metric spaces.
2. Define connected components.
3. Example of regular space.
4. Fixed point map.
5. Normal space

} Short questions.

6. ~~Discuss~~ Discrete space is usually connected (5)
7. Show that T_1 is ~~a~~ T_0 space (5)

Define open function from topol space X to topo space Y.

Define metric topology.

Define normal space

Define fixed point of a map

Let $X = \{a, b\}$ with ind topo Let $A = \{a\}$, $B = \{b\}$

Show $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$

Let $X = \{a, b, c, d\}$ with $\mathcal{T}_1 = \{\phi, X\}$ and $Y = \{1, 2, 3, 4\}$ with $\mathcal{T}_2 = \{\phi, \{1\}, Y\}$

$f(a) = 1$ $f(b) = 1$ $f(c) = 1$ $f(d) = 1$ show open so cont

Consider $X = \{a, b, c\}$ with $\mathcal{T} = \{\phi, \{a\}, \{b, c\}, X\}$ show regular

$A = (0, 1) \cup [3, 5]$ is dissconn?

Show any subspace of 1st count is 1st count

Show every T_1 space is T_0-space

Let $(X, \mathcal{T})$ compact $(Y, \mathcal{T}')$ Hand then show $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is closed map

Define convergent sequence in $\mathcal{T}$
Define normal space, metric on X, regular space
Show int(A) is the union of all open subsets of A
Show that $(X, \mathcal{J})$ is regular.
Set is disconnected or not
Homeomorphism of topological space is equivalence relation
Every finite subset of T_1-space is closed

FALL/SPRING MID/FINAL TERM 20__ (Year)

Rough Sheet

Date: _/_/__

Superintendent Signature:

Student Signature:

Campus Code:

Course Code: MTH634

Student ID: MC170402187

$A = [0,2)$

(1) Let $X = \{a, b, c\}$. Write the basis for the discrete topology on X. (2 Marks)

(2) Give an example of second countable space. (2)

(3). Define T_0-space (2).

Define locally connected spaces (2).

(5). In a topological space X, show that $Ext(\phi) = Int(X)$. (3)

(6). Let $X = \{a, b, c, d\}$ with topology $\tau_x = \{\phi, X\}$ and $Y = \{1, 2, 3, 4\}$ with topology $\tau_y = \{\phi, \{1\}, Y\}$. Define a map $f : (X, \tau_x) \to (Y, \tau_y)$ such that $f(a) = 1$, $f(b) = 1$, $f(c) = 1$, $f(d) = 1$. Show that f is an open and continuous. (3)

(7) Show that every cofinite space is T_1.

(8) Show that every finite subset of T_1-space is closed. (5)

(9) Let (X, τ) be a path connected topological space and $f : (X, \tau) \to (X, \tau')$ be a continuous map. Show that $f(X)$ is path connected. (5)

(10) Find connected path of - - - - (3).

Note: Submit this sheet to Superintendent, before leaving the Examination Center.

Find close mapping?
Define lindelof space?
Find derive set ?
(B,r') subset of (B,r) jo open ball as open set wala tha.
Set with indiscrete topology is compact?
Subset connectede is interval wala jo quest tha?
Set ko disconnect prove krna tha?
Hausdroff space wala tha?
Open cover Kate jo subcover wala prove krna tha?
Every discrete space is To?

Q. Let T be the cofinite topology on any set X. Show that (X,T) is a separable i.e. contains a 2nd countable.

Q. Consider $\mathbb{R}$ with usual topology show that there exists no homeomorphism b/w $(0,2)$ and $(0,1)\cup(3,4]$.

Q. Let T be a topology on the real line $\mathbb{R}$ generated by the half-closed interval $[a,b)$. Show (X,T) is Hausdorff.

A topological [space] (X,T) is said to be closed subset F (2) ~~and for every x ∈ F~~ ~~& open ... containing ...~~

(1) Consider the topology $T = \{\phi, \{a\}, \{c\}, \{a,c\}, X\}$ on $X = \{a,b,c\}$. Determine the derived sets of $A = \{b\}$ (2)

(3) Define regular space (2)

~~Consider the ... = (0,1) and Y = (1,2)~~

~~of a set~~

~~A...~~

(2) 3. Define when two topological spaces X and Y are homeomorphic (2)

(4) 4. Consider $\mathbb{R}$ with usual topology. Show that the set A = (0,1) and B = (1,2) are separated sets (2)

(5) Let $X = \{a,b,c\}$ Show that $\beta = \{\{a,b\},\{b,c\}\}$ cannot be base for any topology on X (3)

(6) Give an example to show that first countable space need not be 2nd countable

(7) Consider with usual topology. Determine whether the set B = (0,1) and C = [2,3) are separated or not?

Campus Code: ________

Course Code: ________

MTh 634

① Define metric topology

② set with usual topology is separable

define locally connected space.

Prove every subspace of second counta

Countable

how every cofinite space is T_1

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Show homorphism of topology space equivalence relation.

> show evey metric space is hausdroff space.

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$A^\circ \cap (A^c)^\circ = \phi$

A° ... $A^\circ \subset A$

$A^c \subset A$

$A^\circ \subset A$ & $(A^c)^\circ \subset A^c$

$Int(A) = A^\circ$

$Ext(A) = (A^c)^\circ$

Since $A^\circ \subset A$ $(A^c)^\circ \subset A^c$

$\Rightarrow A^\circ \cap (A^c)^\circ \subset A \cap A^c$

$\Rightarrow A^\circ \cap (A^c)^\circ \subset \phi$

$\Rightarrow A^\circ \cap (A^c)^\circ = \phi$

$\Rightarrow Int(A) \cap Ext(A) = \phi$

Mth 634 Topology.

27-02-2019 8:00 am.

1- Prove Ext(A) ∩ Int(A) = ϕ

2- Define connected space.

3- Define Hausdroff space

3- Define metric topology.

4- Prove that every subspace of a second countable space is second countable

5- every colsed subspace of compact space is compact.

Regular Spaces (T_3 space)

✓	✗
Discrete Space	Indiscrete space
$(\mathbb{R}, \mathcal{T}_u)$	infinite set with $\mathcal{T}_{cof}$
	K-Topology on $\mathbb{R}$

Normal Spaces (T_4 spaces)

✓	✗
Discrete space	
metric space	
$(\mathbb{R}, \mathcal{T}_u)$	

$T_1 \not\Rightarrow T_2$

$T_2 \Rightarrow T_1$

T_2 space

✓	✗
Discrete space	$X \neq \{x\}$ with indiscrete topology
$\mathbb{R}$ with lower limit topo	$\mathbb{R}$ with topology $\{(a,\infty) \mid a \in \mathbb{R}\}$.
Metric Space.	An infinite set with cofinite topology.

The property that ensure the uniqueness of limit point is the Hausdorff property. (T_2 space)

T_0 space

$T_0 \not\Rightarrow T_1$
$T_1 \Rightarrow T_0$

✓ ✗

$(X, \mathcal{T}_u)$ Indiscrete space

Discrete space

T_1 Space

✓ ✗

Metric Space
Cofinite Space
Discrete Space.
Real line with usual topology

Properties of T_1

① Every finite subset of T_1 space is closed.
② Let X be a finite set. Then $(X, \mathcal{T})$ is a T_1 space iff $(X, \mathcal{T})$ is discrete space.
③ Let A be a finite subset of T_1 space X
Then A' is empty.

Campus Code: ________

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MTh 634

① Define metric topology

② set with usual topology is separable

define locally connected space.

prove every subspace of second countable

Countable

how every cofinite space is T_1

Metric Topology

Metric Topology:
Let X be a nonempty set with metric d. The topology $\mathcal{T}$ on X generated by the set of all open balls in X with respect to d is called the "Metric Topology."
We also say that a topology $\mathcal{T}$ on X induced by the metric d.

Metric Space:
Consider X with a metric d, this d induces a topology the "metric topology." (X, d) is called a metric space.

Examples: Separable Spaces

Example 3:

$\mathbb{R}$ with usual topology is separable.

Reason:

Since there exists $\mathbb{Q}$ in $\mathbb{R}$ that is countable and

$$\overline{\mathbb{Q}} = \mathbb{R}.$$

Example 4:

Locally Connected Spaces

Q3

Def:
Let $(X, \mathcal{T})$ be a topological space. X is said to be locally connected at a point $x \in X$ iff for every U_x, open subset of X containing x, there exists V_x, an open subset of X containing x, such that

- $V_x \subset U_x$
- V_x is connected.

Q4

Second Countable Spaces

Remark:
Every subspace of a second countable space is second countable.

Proof: Let $A \subset X$ (X is second countable space)

So $\exists$ $\mathcal{B} = \{B_n \mid n \in \mathbb{N}\}$. countable.

Now $\mathcal{B}_A = \{A \cap B_n \mid n \in \mathbb{N}\}$. is a basis for $\mathcal{T}_A$. And $\mathcal{B}_A$ is countable

$\Rightarrow$ A is second countable. $\square$

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Q5

Examples: T_1 Space

Example 3: Every cofinite space is T_1.

Sol) Let $(X, \mathfrak{T}_{cof})$

Let X is finite.

$\mathfrak{T}_{cof} = P(X)$ discrete

If X is not finite. Let $U \in \mathfrak{T}_{cof}$

iff $U = \phi$ or U^c = finite.

$x, y \in X$ $x \in U_x = X \setminus \{y\}$; $y \in U_y = X \setminus \{x\}$

$\not\ni y$ $\not\ni x$

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Differentiable Vector Valued ...

show homoophism of topology space equivalence relation.

show every metric space is hausdroff space.

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Homeomorphism...

Proposition:
Homeomorphism of topological spaces is an equivalence relation. ($\approx$)

Proof: • $X \approx X$ $Id : X \longrightarrow X$, $x \longmapsto x$

• Let $X \approx Y$ $f : X \xrightarrow{\text{Hom.}} Y$
Bijection + Bicont.

$f^{-1} : Y \to X$
Bijection + Cont + f is cont

$\Rightarrow X \approx Y \Leftrightarrow Y \approx X$.

VU

Q2

Metric Spaces are Hausdorff

Theorem:
Every metric space is Hausdorff.

Proof $(X, d) \longrightarrow (X, \mathfrak{T}_d)$

Let $x, y \in X$ $x \neq y$

$\because X$ is metric space. $d(x,y) \neq 0$

$d(x,y) = r$ say.

Now $U_x = B(x, r/2)$, $U_y = B(y, r/2)$

My claim is $U_x \cap U_y = \phi$

VU

Metric Spaces are Hausdorff

Let it is not true. So

$\exists$ $z \in U_x \cap U_y$

Now $d(x,y) \leq d(x,z) + d(z,y)$

$< r/2 + r/2 = r = d(x,y)$

X

So $\nexists$ $z \in U_x \cap U_y$

$\Rightarrow U_x \cap U_y = \phi \Rightarrow (X, \mathfrak{T}_d)$ is Hausdorff. □

VU

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mid

1- Prove Ext(A) ∩ Int(A) = ϕ

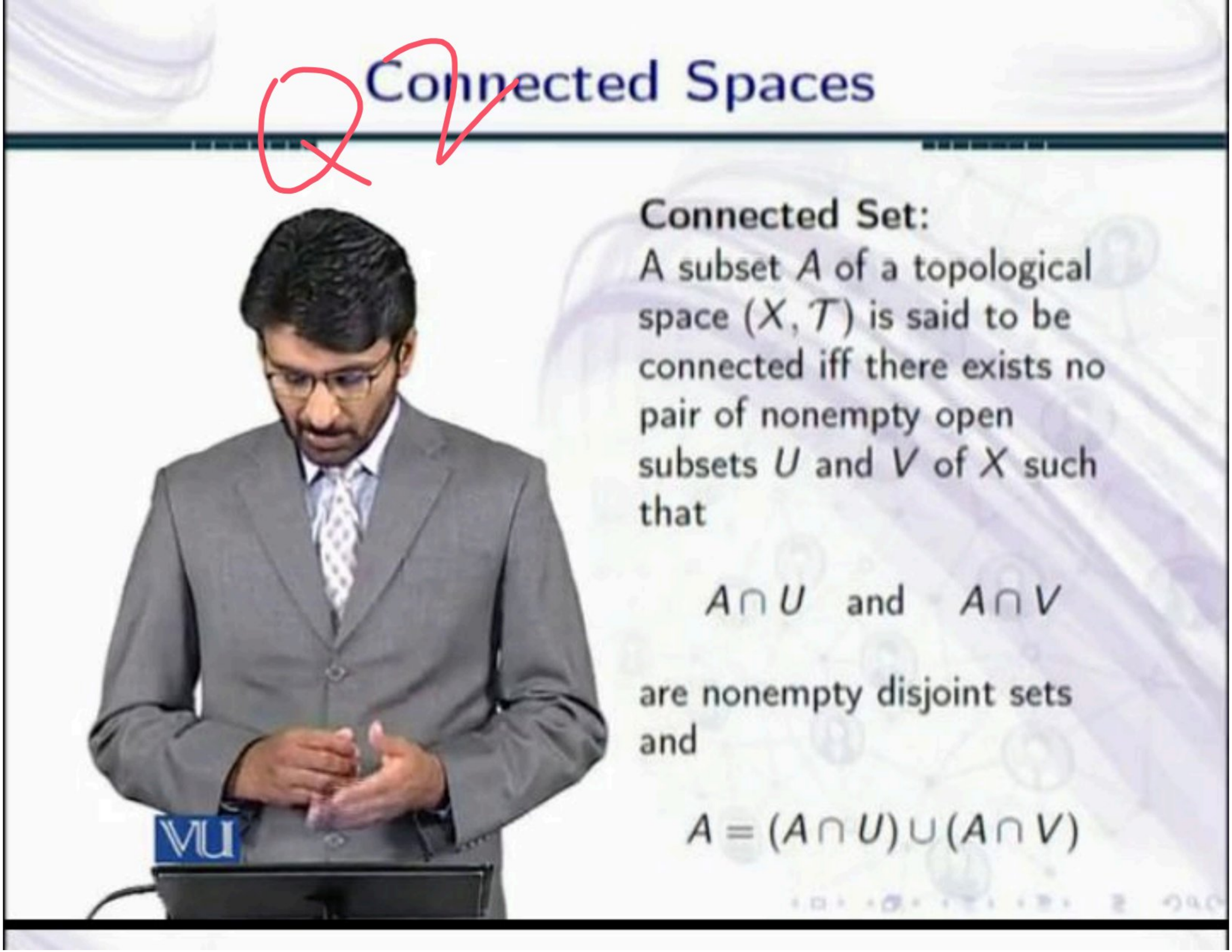
2- Define connected space.

3- Define Hausdroff space

4 3- Define metric topology.

5 4- Prove that every subspace of a second countable space is second countable

6- every colsed subspace of compact space is compact.



Connected Spaces
Connected Set:
A subset A of a topological space $(X, \mathcal{T})$ is said to be connected iff there exists no pair of nonempty open subsets U and V of X such that
$A \cap U$ and $A \cap V$
are nonempty disjoint sets and
$A = (A \cap U) \cup (A \cap V)$
VU

Connected Spaces

Connected Space:
A topological space $(X, \mathcal{T})$ is said to be connected if it can not be written as union of two nonempty open disjoint subsets U and V of X.

Connected Spaces

OR

Connected Space: (Def. II)
A topological space $(X, \mathcal{T})$ is said to be connected if

$$X = D_1 \cup D_2$$

such that D_1, D_2 both open and

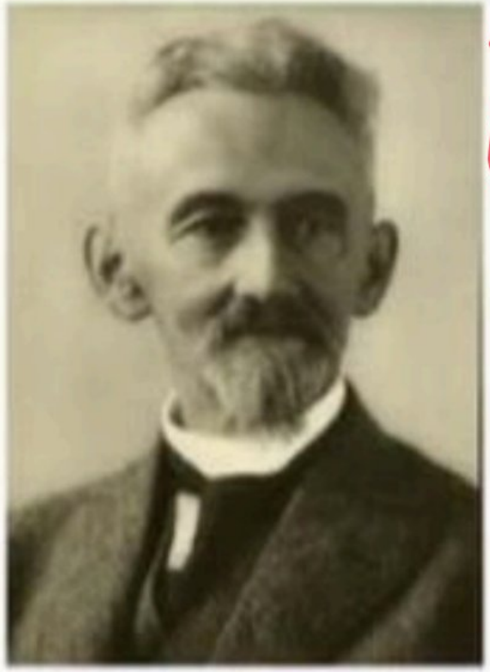
$$D_1 \cap D_2 = \emptyset$$

implies that either

$$D_1 = \emptyset \quad \text{or} \quad D_2 = \emptyset.$$

T_2/Hausdorff Spaces

Q3



Felix Hausdorff
(1868 - 1942)

Def:
A topological space $(X, \mathcal{T})$ is said to be "T_2 Space" iff for every distinct pair of elements $x, y \in X$ there exist open subsets U_x, U_y of X containing x, y respectively such that $U_x \cap U_y = \emptyset$.

T_2 spaces are also called Hausdorff Spaces.

Metric Topology

Metric Topology:
Let X be a nonempty set with metric d. The topology $\mathcal{T}$ on X generated by the set of all open balls in X with respect to d is called the "Metric Topology."
We also say that a topology $\mathcal{T}$ on X induced by the metric d.

Metric Space:
Consider X with a metric d, this d induces a topology the "metric topology." (X, d) is called a metric space.

Second Countable Spaces

Q5

Remark:
Every subspace of a second countable space is second countable.

Proof: Let $A \subset X$ (X is second countable space)

So $\exists$ $\mathcal{B} = \{ B_n \mid n \in \mathbb{N} \}$. countable.

Now $\mathcal{B}_A = \{ A \cap B_n \mid n \in \mathbb{N} \}$. is a basis for $\mathcal{T}_A$. And $\mathcal{B}_A$ is countable

$\Rightarrow$ A is second countable. $\square$

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Q5

Theorem: Closed Subspace...

Theorem:
Every closed subspace of a compact space is compact.

Proof:
Let $(X, \mathcal{T})$ be a compact space and A be a closed subset of X.
So $(A, \mathcal{T}_A)$ be closed subspace of X, where $\mathcal{T}_A$ is subspace topology on A.
Consider an open cover $\mathcal{D}$ of A, i.e.

Theorem: Closed Subspace...

$$A = \bigcup_{i\in I} D_i$$

where $D_i \in \mathcal{D} \subset \mathcal{T}_A$.
Now for each D_i there exists $U_i \in \mathcal{T}$ such that $D_i = A \cap U_i$.
So

$$A = \bigcup_{i\in I} D_i = \bigcup_{i\in I}(A \cap U_i)$$

But A is closed in X so A^c is an open subset of X. And we have an open covering of X.

$$X = A^c \cup (\bigcup_{i\in I} U_i)$$

Theorem: Closed Subspace...

$$A = \bigcup_{i\in I} D_i$$

where $D_i \in \mathcal{D} \subset \mathcal{T}_A$.
Now for each D_i there exists $U_i \in \mathcal{T}$ such that $D_i = A \cap U_i$.
So

$$A = \bigcup_{i\in I} D_i = \bigcup_{i\in I}(A \cap U_i) \Rightarrow A \subset \bigcup_{i\in I} U_i$$

But A is closed in X so A^c is an open subset of X. And we have an open covering of X.

$$X = A^c \cup (\bigcup_{i\in I} U_i)$$

Theorem: Closed Subspace...

But X is compact. So every open cover of X has a finite subcover, i.e. there exists $J \underset{finite}{\subset} I$ such that

$$X = A^c \cup (\underset{j \in J}{\cup} U_j)$$

$\Longrightarrow$

$$A \subset \underset{j \in J}{\cup} U_j$$

$\Longleftrightarrow$

$$A = A \cap (\underset{j \in J}{\cup} U_j) = \underset{j \in J}{\cup} D_j$$

$A \cap U_j = D_j$

Proved.

Theorem: Closed Subspace...

But X is compact. So every open cover of X has a finite subcover, i.e. there exists $J \underset{finite}{\subset} I$ such that

$$X = A^c \cup (\underset{j \in J}{\cup} U_j)$$

$\Longrightarrow$

$$A \subset \underset{j \in J}{\cup} U_j$$

$\Longleftrightarrow$

$$A = A \cap (\underset{j \in J}{\cup} U_j) = \underset{j \in J}{\cup} D_j$$

$A \cap U_j = D_j$

A is compact.

Proved.

T_0 space

$T_0 \not\Rightarrow T_1$
$T_1 \Rightarrow T_0$

✓ ✗

$(X, \mathcal{T}_u)$ Indiscrete space
Discrete space

T_1 Space

✓ ✗

Metric Space
Cofinite Space
Discrete Space.
Real line with usual topology

Question

Properties of T_1
① Every finite subset of T_1 space is closed.
② Let X be a finite set. Then $(X, \mathcal{T})$ is a T_1 space iff $(X, \mathcal{T})$ is discrete space.
③ Let A be a finite subset of T_1 space X. Then A' is empty.

Properties of T_1 Spaces: I

Q1

Let us discuss some properties of T_1 spaces. Some of these properties can also be used as a criterion of T_1 property.

Property I: Every finite subset of T_1 space is closed.

Properties of T_1 Spaces: I

Proof: Let $(X, \mathcal{J})$ be a T_1 space

$A \underset{\text{finite}}{\subset} X$ Let $A = \{a_1, \dots, a_n\}$

$$A = \bigcup_{i=1}^{n} \{a_i\} \qquad a_i \in A \ \forall i$$

$\because$ X is T_1 so every singleton subset of X is closed in X.

$\{a_i\}$ is closed $\forall\, i = 1, 2, \dots, n$.

$\Rightarrow$ A is a finite union of closed subset

$\Rightarrow$ A is closed in X.

Properties of T_1 Spaces: I

Property II:
Let X be a finite set. Then $(X, \mathcal{T})$ is a T_1 space if and only if $(X, \mathcal{T})$ is discrete space.

Proof: Let X is finite and (X, J) is T_1 space. So in X every finite subset is closed.
⇒ Every subset of X is also open in X
⇒ J = P(X) ⇒ X is discrete.

Now If (X, J) is discrete ⇒ (X, J) is T_1

Properties of T_1 Spaces: II

Property III:
Let A be a finite subset of a T_1 space X then A' is empty.
Proof:
Let A is finite subset of X, say A has n elements, i.e.

$$\{a_1, \dots, a_n\}$$

Since A is finite and X is a T_1 space and we know that every finite subset of a T_1 space is closed so

$$A' \subset A$$

Now let us check one by one the points of A as limits points of A.

Properties of T_1 Spaces: II

Let's start with a_1.
Now note that

$$U_{a_1} = \{a_2, \dots, a_n\}^c$$

is an open set containing a_1 and

$$A \cap (U_{a_1} \setminus \{a_1\}) = \emptyset$$

So a_1 is not a limit point of A.
Similarly, no other point of A is a limit point of A. So

$$A' = \emptyset$$

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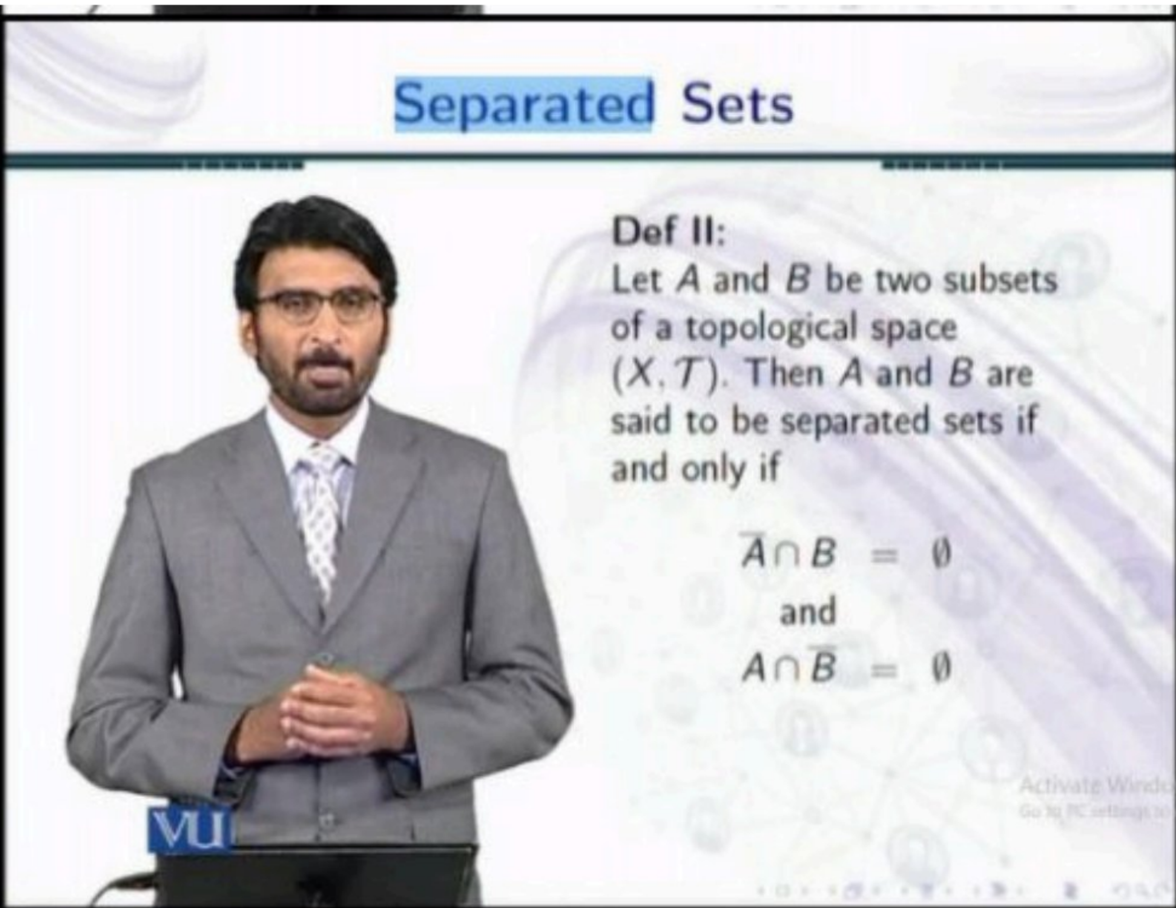
MTH634 Topology

Question No : 34 of 41

Define separated sets.


Separated Sets
Def II:
Let A and B be two subsets of a topological space $(X, \mathcal{T})$. Then A and B are said to be separated sets if and only if
$\overline{A} \cap B = \emptyset$
and
$A \cap \overline{B} = \emptyset$
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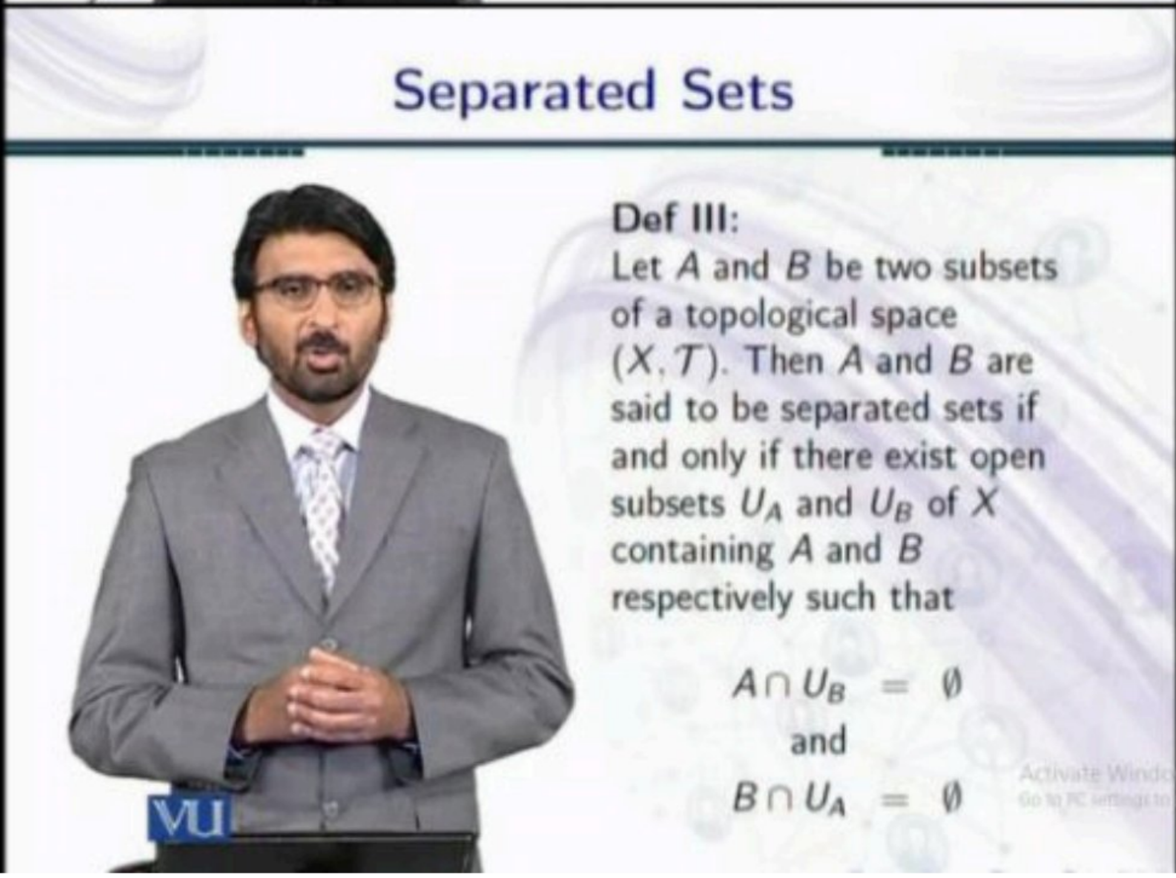
Separated Sets
Def III:
Let A and B be two subsets of a topological space $(X, \mathcal{T})$. Then A and B are said to be separated sets if and only if there exist open subsets U_A and U_B of X containing A and B respectively such that
$A \cap U_B = \emptyset$
and
$B \cap U_A = \emptyset$
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MTH634 Topology

Question No : 40 of 41

Show that a set X with cofinite topology is compact.

Q Show that a set X with cofinite topology is Compact? 68

Proof: If X is finite, Then

$$\mathcal{T}_{cof} = P(X)$$

$\Rightarrow (X, \mathcal{T}_{cof})$ is compact

But if X is not finite Then consider an open cover $\mathcal{D}$ of X.

Now let $\phi \in D \in \mathcal{D}$ $\because D^c$ is finite

Now, for every $x \in D^c$ choose an element of $\mathcal{D}$. D_x containing x.

$\{D_x \mid x \in D^c\}$ is finite

$\because D^c$ is finite

Consider this subcover of $\mathcal{D}$

~~$X = X \setminus \{D\}$~~

$$C = \{D\} \cup \{D_x \mid x \in D^c\}$$

$\rightarrow C$ is a finite cover of $\mathcal{D}$

$\Rightarrow (X, \mathcal{T}_{cof})$ is Compact

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MTH634 Topology

Question No : 37 of 41

Show that a discrete space X is separable if and only if X is countable.

Proposition 1:
A discrete space is separable if and only if it is countable.

Proof: X, $\mathfrak{T}_{dis} = P(X)$

We know that the only dense subset in X is X itself.

So the only choice for A is X s.t $\overline{A} = X$

$\Rightarrow$ X is separable iff X is countable.

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Activate Windows
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MTH634 Topology

Question No : 39 of 41

Show that every second countable space is first countable.

Q Every second countable space is first countable. 67

Proof:

Let X be a second countable space.

i-e $\exists$ B (a basis) s.t B is countable

Now let $p \in X$ and consider $B_p \subset B$

$$B_p = \{ B_p \mid p \in B_p \} \subset B$$

Now B_p is countable $\because B$ is countable

Now B_p is local base at p

$B_p \subset B$ (basis) $\forall$ $U \in \mathcal{J}$ s.t $p \in U$

$\exists$ $B_p \in B$ s.t.

$p \in B_p \subset U$

$\rightarrow B_p$ is a local base at p

and this exists $\forall$ $p \in X$

$\rightarrow X$ is first countable

Question No : 36 of 41

Prove that every subspace of a second countable space is second countable.

Answer (Please click here to Add Answer)

Second Countable Spaces

Remark:
Every subspace of a second countable space is second countable.

Proof: Let $A \subset X$ (X is second countable space)

So $\exists$ $\mathcal{B} = \{B_n \mid n \in \mathbb{N}\}$. countable.

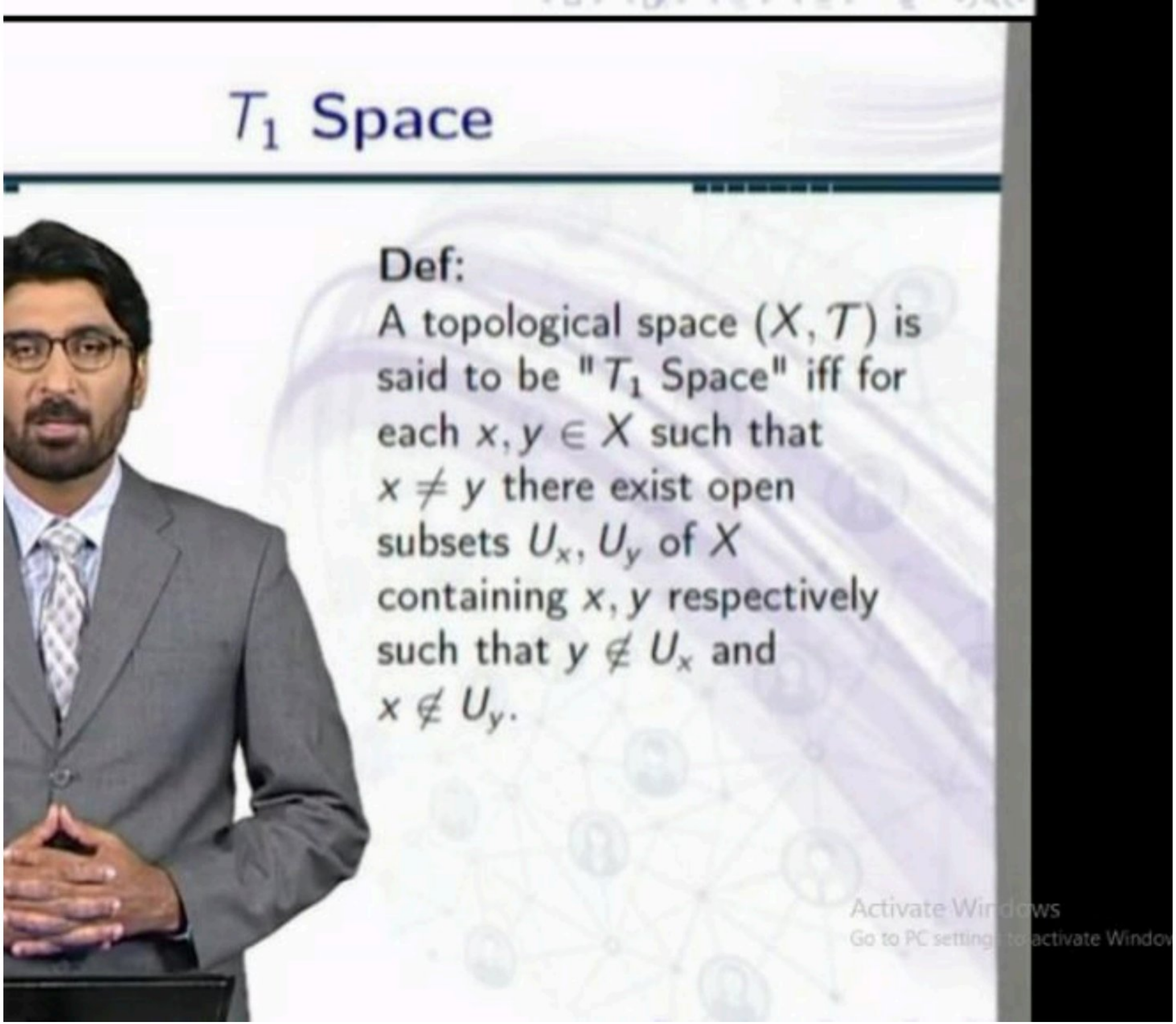
Now $\mathcal{B}_A = \{A \cap B_n \mid n \in \mathbb{N}\}$. is a basis for $\mathcal{T}_A$. And $\mathcal{B}_A$ is countable

$\Rightarrow$ A is second countable. □

MTH634 Topology

Question No : 33 of 41

Define T_1 - space.


T_1 Space
Def:
A topological space $(X, \mathcal{T})$ is said to be "T_1 Space" iff for each $x, y \in X$ such that $x \neq y$ there exist open subsets U_x, U_y of X containing x, y respectively such that $y \notin U_x$ and $x \notin U_y$.
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MTH634 Topology

Question No : 32 of 41

Give one example of second countable space.

Examples: Second Countable

Let us see discuss some examples of second countable spaces.

Example 1:

Let X be a countable set with discrete topology. Then $(X, \mathcal{T}_{Dis})$ is second countable. Recall basis for discrete space, i.e.

$$\mathcal{B} = \{\{x\} | x \in X\}$$

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Question No : 27 of 33

Define Normal space.

Normal Spaces

Def:

A topological space $(X, \mathcal{T})$ is said to be "normal" iff for every pair of disjoint closed subsets, $F_1, F_2 \subset X$ there exist open subsets U_{F_1} and U_{F_2} containing F_1 and F_2 respectively such that

$$U_{F_1} \cap U_{F_2} = \emptyset.$$

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Question No : 26 of 33

Define metric topology.

Metric Topology

Metric Topology:

Let X be a nonempty set with metric d. The topology $\mathcal{T}$ on X generated by the set of all open balls in X with respect to d is called the "Metric Topology."

We also say that a topology $\mathcal{T}$ on X induced by the metric d.

Metric Space:

Consider X with a metric d, this d induces a topology the "metric topology." (X, d) is called a metric space.

Question No : 31 of 33

Define connected set.

Answer (Please click here to Add Answer)

Connected Spaces

Connected Set:

A subset A of a topological space $(X, \mathcal{T})$ is said to be connected iff there exists no pair of nonempty open subsets U and V of X such that

$$A \cap U \quad \text{and} \quad A \cap V$$

are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V)$$

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MTH634 Topology

Question No : 30 of 33

Show that a set X with topology T containing finite number subsets of X is compact

xamples: Compact Spaces

Example 2:
A set X with topology $\mathcal{T}$ containing finite number subsets of X is compact.

Reason:
Since every open cover $\mathcal{C}$ of X is a subclass of $\mathcal{T}$ and $\mathcal{T}$ itself finite so $\mathcal{C}$ is finite too and any subcover $\mathcal{S}$ of $\mathcal{C}$ is also finite.

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MTH634 Topology

Question No : 28 of 33

Define connected component in a topological space X.

Connected Component

Def:
Consider a topological space $(X, \mathcal{T})$. Let $a \in X$ and C be a connected subset of X containing a.
Then

$$C_a = \bigcup_{a \in C} C$$

is called the connected component of X containing a.

Connected Component

(4) A space $(X, \mathcal{T})$ is connected iff it has only one connected component.

Remarks:

1. The connected component C_a is the largest connected subset of X containing a.
2. If $b \in C_a$ then $C_b = C_a$.
3. X can be written as disjoint union of its connected components. i.e.

$$X = \coprod_{C_i} C_i$$

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MTH634 Topology

Question No : 29 of 33

Consider the sets $X = \{1,2,3\}$ and $Y = \{a,b,c\}$ with topologies $\tau_X = \{\phi, \{1\}, \{1,2\}, X\}$ and $\tau_Y = \{\phi, \{a,b\}, \{b\}, Y\}$ repectively. Define a map $f : X \rightarrow Y$ *as* :

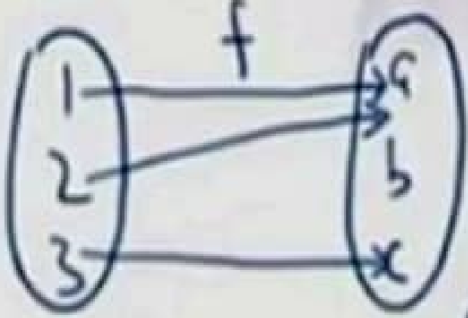
$f(1) = f(2) = a$, $f(3) = c$.

Show that f is a closed map.

Closed Mapping

Ex: $X = \{1, 2, 3\}$, $\mathfrak{T}_X = \{\phi, \{1\}, \{1, 2\}, X\}$

$Y = \{a, b, c\}$, $\mathfrak{T}_Y = \{\phi, \{a, b\}, \{b\}, Y\}$



$\mathcal{C}_X = \{X, \{2, 3\}, \{3\}, \phi\}$

$\mathcal{C}_Y = \{Y, \{c\}, \{a, c\}, \phi\}$.

$f(\phi) = \phi$ ✓

$f(\{3\}) = \{c\}$ ✓ | $f(\{2, 3\}) = \{a, c\}$ ✓ | $f(X) = \{a, c\}$ ✓

Question No : 33 of 33 **Marks: 5 (Budg**

Consider $X = \{a,b,c,d,e\}$ with topology $\tau = \{\phi,\{a\},\{a,b\},\{a,b,c\},\{a,d,e\},\{a,b,d,e\},X\}$. Show that the set $A = \{c,e\}$ is disconnected subset of X.

Answer (Please click here to Add Answer)

Examples: Connected Spaces

Example 2: Consider $X = \{a, b, c, d, e\}$ with
$\mathcal{T} = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}, \{a, d, e\}, \{a, b, d, e\}, X\}$.
Then $A = \{c, e\}$ is disconnected subset of X.

Reason:

Since there exists a pair of nonempty open subsets
$U = \{a, b, d, e\}$ and $V = \{a, b, c\}$ of X such that

$$A \cap U = \{e\} \quad \text{and} \quad A \cap V = \{c\}$$

$\{e\} \cap \{c\} = \emptyset$

are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V) = \{e\} \cup \{c\}. = \{c, e\}$$

2) Define metric Topology?

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Metric Topology: Let X be a non empty set with metric d. The topology T on X generated by the set of all open balls in X with respect to d is called metric topology.
A topology T on X induced by metric d.
Consider X with a metric d, this d induces a topology "metric topology". (x,d) is called **metric space.**
The topology generated by the set of all singletons is discrete topology.
Module : 108-110
Metrizable Space: A topological space (X,T) is called Metrizable space if there exists a metric d on X that induces topology T.

3) Show that the set R with usual Topology is separable.

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Examples: Separable Spaces

Example 3:
$\mathbb{R}$ with usual topology is separable.

Reason:
Since there exists $\mathbb{Q}$ in $\mathbb{R}$ that is countable and

$$\overline{\mathbb{Q}} = \mathbb{R}.$$

Example 4:
$\mathbb{R}$ with discrete topology is not separable.

Reason:
Since there exists no countable subset in $\mathbb{R}$ that is dense in $\mathbb{R}$.

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4) Defin Connected Components:

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A subset of A of $(\mathbb{R}, Tu)$ is connected iff A is an interval.
Fixed Point: A point $x \in X$ is said to be a fixed point of a map $f: X \longrightarrow X$ iff $f(x) = x$
Let A and B be two connected subsets of a topological spaces (X,T) such that $x \in A \cap B$ then $C = AUB$ is connected subset of X. **[ALSO SEE PROOF Page#179-180]**
Consider a topological space (X,T), Let $a \in X$ and C be a connected subset of X containing a then $C_a = U_{a \in C} C$ is called connected component of X containing a.
The connected component Ca is the largest connected subset of X containing a.
Prepare Page#182
Let C be connected component of X then $C \coloneqq \bar{C}$
A connected component is a closed subset of X.
In general a connected component C of X is not open.

(8) $X = \{a, b, c\}$, $\mathcal{I}\{\emptyset, \{a\}, \{b, c\}, X\}$

Show that $(X, \mathcal{I})$ is regular.

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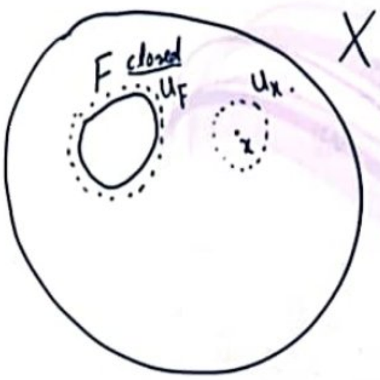
Regular Spaces

Def:
A topological space $(X, \mathcal{T})$ is said to be "regular" iff for every closed subset F of X and for every $x \in F^c$ there exist open subsets U_F and U_x containing F and x respectively such that

$$U_F \cap U_x = \emptyset.$$

Regular Spaces



Regular Spaces

T_3 Space:

A topological space $(X, \mathcal{T})$ is said to be "T_3 space" iff it is either

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1) Define Neighbourhood of P is Space (x, d)

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(5) $x = R$ with $A = (0,1]$ find interior, exterior boundary and Partionted of x

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(6) Discuss space is usually connected?

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(1) show that T_1 is T_0 space

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1) Define metric space.

Metric Space:
A set X with a metric d defined on it is called a metric space.

amples of Metric

Let us see some examples of metrics and verify all the three axioms for each metric.

Example 1:
Let $X = \mathbb{R}$ and d is defined as $d(x, y) = |x - y|$ where $x, y \in \mathbb{R}$.

2) Define connected components.

Def:
Consider a topological space $(X, \mathcal{T})$. Let $a \in X$ and C be a connected subset of X containing a.
Then

$$C_a = \bigcup_{a \in C} C$$

is called the connected

3) Example of regular space:

Examples: Regular Spaces

Example 3: Consider $X = \{a, b, c\}$ with

$$\mathcal{T} = \{\emptyset, \{a\}, \{b, c\}, X\}.$$

$(X, \mathcal{T})$ is a regular space.

4) Fixed point map.

Fixed Point:

A point $x \in X$ is said to be a fixed point of a map $f : X \to X$ iff $f(x) = x$.

Example:

1. $f : \mathbb{R} \to \mathbb{R}$, defined as $f(x) = x^2$. Here 0 is a fixed point of f. 1 is also a fixed point of f.
2. $g : \mathbb{R} \to \mathbb{R}$, defined as

$$g(x) = x + 1$$

g has no fixed point.

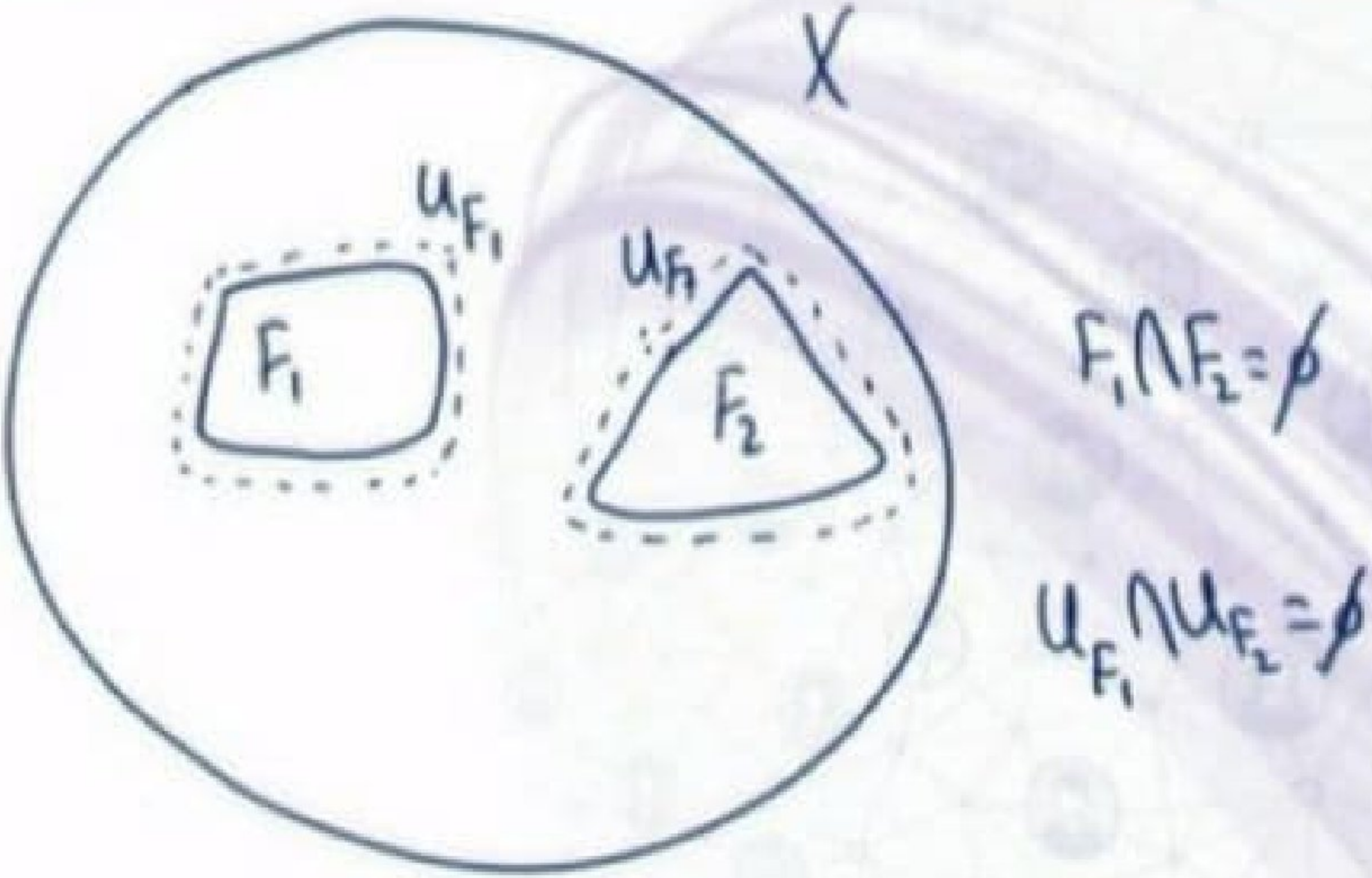
5) Normal Space.

Normal Spaces

Def:
A topological space $(X, \mathcal{T})$ is said to be "normal" iff for every pair of disjoint closed subsets, $F_1, F_2 \subset X$ there exist open subsets U_{F_1} and U_{F_2} containing F_1 and F_2 respectively such that

$$U_{F_1} \cap U_{F_2} = \emptyset.$$

Normal Spaces



6) Discrete space is locally connected.

Examples: Locally Connected...

Example 2: A discrete space is locally connected.

$X \qquad \mathfrak{S}_{dis} = \mathcal{P}(X)$

$x \in X \qquad U_x \in \mathcal{P}(X)$

$V_x = \{x\} \subset U_x.$

8) Every finite subset of T_0 space is closed.

Property I: Every finite subset of T_1 space is closed.

Properties of T_1 Spaces: I

Proof: Let $(X, \mathcal{T})$ be a T_1 space

$A \underset{\text{finite}}{\subset} X$ Let $A = \{a_1, \cdots, a_n\}$

$A = \bigcup_{i=1}^{n} \{a_i\}$ $a_i \in A \; \forall i$

$\because X$ is T_1 so every singleton subset of X is closed in X.

$\{a_i\}$ is closed $\forall\, i = 1, 2, \cdots, n$.

$\Rightarrow A$ is a finite union of closed subset

$\Rightarrow A$ is closed in X.

9) Homeomorphism of topological space is equivalance relation.

Proposition:
Homeomorphism of topological spaces is an equivalence relation. ($\approx$)

Proof:
- $X \approx X$ $Id : X \to X$, $x \mapsto x$
- Let $X \approx Y$ $f : X \to Y$ Hom.
  Bijection + Bicont.
  $f^{-1} : Y \to X$
  Bijection + Cont + f is cont
  $\Rightarrow X \approx Y \Leftrightarrow Y \approx X$.

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Homeomorphism...

- $X \approx Y$, $Y \approx Z$
  $f : X \to Y$ Bijection + Bicont. | $g : Y \to Z$ Bijective + Bicont.
  $g \circ f : X \to Z$
  Bijective + $g \circ f$ is bicont.
  → $g \circ f$ is cont
  → $f^{-1} \circ g^{-1}$ is cont
  $\Rightarrow X \approx Z$.

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Q: show that T_0 space that is not T_1.

$T_0 \not\Longrightarrow T_1$

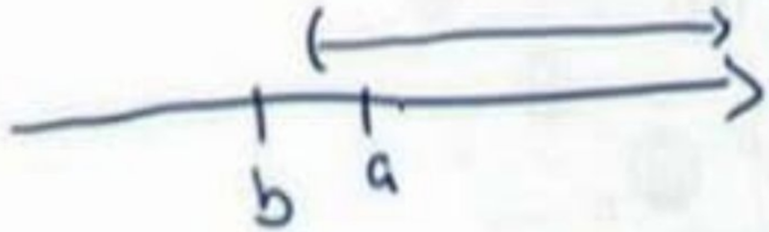
Example: A T_0 space that is not T_1.
Consider $\mathbb{R}$ with $\mathcal{T}$ generated by

$$\{(a, \infty) \mid a \in \mathbb{R}\}$$

$a, b \in \mathbb{R}$ $\quad a > b$

$b < c < a$

$a \in U_a = (c, \infty) \not\ni b$ $\quad$ So T_0-Space



Q: Define convergent Sequence.

Recall the definition of "Convergent Sequence".
Let $(X, \mathcal{T})$ be a topological space. A sequence of points $(x_i)_{i \in I}$ in X is said to be converges to a point $x \in X$ if for every open subset U_x, containing x, in X there exists $N \in \mathbb{N}$ such that for all $n \geq N$, $x_n \in U_x$.

discrete topology on X. (2 Marks)

② Give an example of second countable space. (2)

$\mathbb{R}$ with usual topology is second countable.
Consider the following basis for usual topology on $\mathbb{R}$.

$$\mathcal{B} = \{(a, b) \mid a, b \in \mathbb{Q}\}$$

mth 634

③. Define T_0 - space (2)

A topological space $(X, \mathcal{T})$ is said to be "T_0 Space" iff for each $x, y \in X$ such that $x \neq y$ there exists an open set $U_x \subset X$ containing x such that $y \notin U_x$

③. Define T_0 - space

Define locally connected spaces (2)

topological space X, show

Let $(X, \mathcal{T})$ be a topological space. X is said to be locally connected at a point $x \in X$ iff for every U_x, open subset of X containing x, there exists V_x, an open subset of X containing x, such that

- $V_x \subset U_x$
- V_x is connected.

Example 3: Every cofinite space is T_1.

Sol: Let $(X, \mathcal{T}_{cof})$

Let X is finite.

$\mathcal{T}_{cof} = P(X)$ discrete

If X is not finite. Let $U \in \mathcal{T}_{cof}$

iff $U = \phi$ or U^c = finite.

$x, y \in X$ $x \in U_x = X \setminus \{y\}$; $y \in U_y = X \setminus \{x\}$ ($\not\ni y$) ($\not\ni x$)

a map ... $f(a)=1$, $f(b)=3$, ... is an open and continuous. (3)

⑥ Show that every cofinite space is T_1.

⑦ Show that every finite subset of ... (5) ... topolog

Property I: Every finite subset of T_1 space is closed.

$f(a)=1$, ... is an open and ... show that every cofinite space is T_1 ...
(7) show that every finite subset of T_1-space
(8) show that every finite ... is closed. (5)
... be a path connected topological spa...

Proof: Let $(X, \mathcal{T})$ be a T_1 space

$A \subset_{\text{finite}} X$ Let $A = \{a_1, \dots, a_n\}$

$A = \bigcup_{i=1}^{n} \{a_i\}$ $a_i \in A \ \forall i$

$\because X$ is T_1 so every singleton subset of X is closed in X.

$\{a_i\}$ is closed $\forall\, i = 1, 2, \dots, n$.

$\Rightarrow$ A is a finite union of closed subset

$\Rightarrow$ A is closed in X.

(7) show that every [illegible]
(8) show that every finite ...
is closed. (5)
(9) let $(X, \mathcal{T})$ be a path connected topological spa...
and $f: (X, \mathcal{T}) \to (X, \mathcal{T}')$ be a continuous map. show that $f(X)$ is path connected. (5)
... connected path of ...... (3).

Note: Submit this sheet to Superintendent, before leaving the Examination Center.

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Let $(X, \mathcal{T})$ be a path connected topological space and

$$f : (X, \mathcal{T}) \longrightarrow (Y, \mathcal{T}')$$

be a continuous map. Then $f(X)$ is path connected.

Proof: $f: (X, \mathcal{T}) \xrightarrow[\text{Cont}]{} (Y, \mathcal{T}')$

X: Path connected.

$f(X)$ is path connected ??

Let $a, b \in f(X) \Rightarrow \exists\ x, y \in X$ s.t. $f(x) = a$, $f(y) = b$

Now $\because X$ is path connected.

So $\exists\ p: I \xrightarrow[\text{Cont}]{} X$ $p(0) = x$, $p(1) = y$.

$q = f \circ p : I \longrightarrow f(X)$

$q(0) = f \circ p(0) = f(p(0)) = f(x) = a$

$q(1) = f \circ p(1) = f(p(1)) = f(y) = b$

$\Rightarrow f(X)$ is path connected.

close mapping?
Define lindelof space?

Lindelöf Space:
A topological space $(X, \mathcal{T})$ is called a Lindelöf space if and only if every open cover of X has a countable subcover.

Set ko disconnect prove krna tha?

Example 1: Consider $(\mathbb{R}, \mathcal{T}_u)$. Then $A = (0,1) \cup (3,5]$ is disconnected subset of $\mathbb{R}$.
Reason:
Since there exists a pair of nonempty open subsets

$$U = (0,1) \text{ and } V = (3,6)$$

of $\mathbb{R}$ such that
$A \cap U$ and $A \cap V$ are nonempty disjoint sets and

$$A = (A \cap U) \cup (A \cap V).$$

Every discrete space is To?

Example 3: Consider $X = \{a, b, c, d\}$ with

$$\mathcal{T} = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$$

$(X, \mathcal{T})$ is T_0.

DATA for FINAL TERM ---- MTH634 ---- Module# 086 to 183
Module : 86-89
A map f: $(X,T_X) \longrightarrow (Y,T_Y)$ is called a closed map iff the image of each closed subset of X is closed in Y.
Projection map may or may not be closed.
Is a continuous map an open map. (May or may not be Open)
Is an open map a continuous map. (May or may not be Continuous)
Restriction of an open map may or may not be an open map.
Module : 90-100
Homeomorphism: A map f: X ⟶ Y is called a Homeomorphism between two topological space (X,Tx) and (Y,Ty) iff 1. " f " is bijective. 2. " f " is continuous 3. F^{-1} : Y ⟶ X or "f" is open map
Homeomorphic Space: Two topological space are called (X,Tx) and (Y,Ty) are called homeomorphic iff there exist a homeomorphism f: X ⟶ Y we also say such spaces are topologically equivalent spaces i.e . x ≅ y
Equivalence Relation: A binary relation ~ on a set X is said to be equivalence relation iff ~ satisfied the following three properties for all a,b,c ∈ X. (1) Reflexive: i.e $a \sim a$ (2) Symmetric: i.e $a \sim b \ iff \ b \sim a$ (3) Transitive: $if \ a \sim b \ and \ b \sim c \ then \ a \sim c$
Homomorphism of topological space is an equivalence relation.
A property "p" of a topological space (X,T) is called topological property if every space homeomorphic to X has property "p" i.e. A property that is unchanged under homeomorphism.
Topological property is also called topological invariant.
Some topological properties are: *_Cardinality of set X. *_Cardinality of T. *_Connectedness of space X. *_Being discrete.
Some non-topological properties are: *_ Length *_ Area
A metric is a distance function.
A function that defines a distance between each pair of elements of a set.
Metric: A metric on a non empty set X is a real valued function d defines on $X \times X$ i.e. $d: X \times X \longrightarrow R$
Metric Space: A set X with a metric "d" defined on it is called a metric space.
Usual metric on ℝ is $d(x,y) = \sqrt{(x-y)^2}$
Usual metric on $\mathbb{R}^2$: Consider $\mathbb{R}^2$ and "d" is defined as $d(x,y) = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}$ where $x = (x_1, x_2),\ y = (y_1, y_2) \in \mathbb{R}^2$
Usual metric on $\mathbb{R}^n$: Consider $\mathbb{R}^n$ and "d" is defined as $d(x,y) = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 \ldots\ldots (x_n-y_n)^2}$ where $x = (x_1, x_2, \ldots \ldots \ldots . . x_n),\ y = (y_1, y_{2 \ldots\ldots\ldots} y_n) \in \mathbb{R}^n$
Module : 101-107
Consider X with a metric d. For any $x \in X$ and any real number x>0, the set of all points $y \in X$ whose distance from x is less than r is called open ball of radius r and centered at x i.e. B(x,y)={ $y \in X \| d(x,y) < r$}
Prepare Page # 33-34-35

Metric Topology: Let X be a non empty set with metric d. The topology T on X generated by the set of all open balls in X with respect to d is called metric topology.

A topology T on X induced by metric d.

Consider X with a metric d, this d induces a topology "metric topology". (x,d) is called **metric space.**

The topology generated by the set of all singletons is discrete topology.

Module : 108-110

Metrizable Space: A topological space (X,T) is called Metrizable space if there exists a metric d on X that induces topology T.

Not all spaces are Metrizable.

Let X be an infinite set. Consider cofinite topology on X then it is not Metrizable space.

ℝ with lower limit topology is not Metrizable space.

Module : 111-112

A topological space (X,T) is called a **First Countable Space** if for each $p \in X$ there exists a countable local base at p.

Every subspace of first countable space is first countable.

Discrete space is first countable space.

Every finite set with any topology is first countable.

ℝ with usual topology is first countable i.e $(\mathbb{R}, Tu)$ is first countable.

Every metric space is first countable.

Module : 113-114

A topological space (X,T) is called **Second Countable Space** if there exist a countable a countable basis for T.

Every subspace of a second countable space is second countable.

Let X be a countable set with discrete topology then (X, T_{dis}) is second countable.

Any finite set with any topology is second countable.

ℝ with usual topology is second countable i.e $(\mathbb{R}, Tu)$ is second countable.

Module : 115-116

Second accountability implies first accountability.

Let a topological space (X,T) is second countable space then it is also first countable space.

First accountability does not imply second accountability.

Consider ℝ with usual topology then it is both first countable and second countable.

Consider ℝ with lower limit topology then it is first countable but not second countable.

A first countable space may or may not be second countable.

Module : 117-119

Cover: A cover of a set X is a collection of sets U={Ui | $i \in$ I} where I is index set such that $x \subset \cup_{i \in I} U_i$

Cover: Let (X,T) be topological space. A cover of a X is a collection of subsets of X i.e. $U = \{U_i \mid U_i \subset X \; i \in I\}$ where I is index set such that $X = \cup_{i \in I} U_i$

Open Cover: Let (X,T) be topological space. An open cover of a X is a collection of subsets of X i.e. $U = \{U_i \mid U_i \in T \; i \in I\}$ where I is index set such that $X = \cup_{i \in I} U_i$

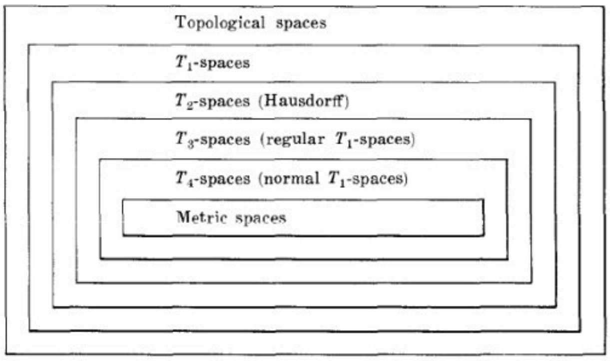
Module : 120-121

Lindelof Space: A topological space (X,T) is called a Lindelof space iff every open cover of X has a countable sub cover.
Let X be a finite set with any topology then it is Lindelof.
A set with indiscrete topology then it is Lindelof.
X= ℝ with usual topology is Lindelof.
A subspace of Lindelof space needs not to be Lindelof.
Every countable space is second Lindelof. (ALSO PREPARE ITS PROOF FROM P#69-70)
Module : 122-125
A topological space (X,T) is said to be separable if there exist a countable dense subset A of X.
Every second countable space is separable.
ℝ with lower limit topology is separable but not second countable.
Let X be a finite set then X with any topology is separable.
Consider a set X with indiscrete topology the X is separable.
ℝ with usual topology is separable.
ℝ with discrete topology is not separable.
A discrete space is separable iff it is countable. **[ALSO PROOF FROM PAGE # 74]**
X is separable iff X is countable.
(X,T_{cof}) is separable. **[ALSO PROOF FROM PAGE # 74-75]**
In general subspace of separable is not separable.
Module : 126
A metric space is not a second countable and not separable in general.
An infinite set X with trivial metric is not second countable and not separable.
A separable metric space is second countable. **[ALSO PROOF FROM PAGE # 78]**
Module : 127-143
There are 7 separation axioms. T_0-Spaces, T_1-Spaces, T_2-Spaces (Hausdorff), T_3-Spaces, Normal Spaces,T_4-Spaces,
T_0-Space: A topological space (X,T) is said to be **T_0-Space** iff for each $x, y \in X$ there exist an open set $U_x \subset X$ containing X such that $y \notin U_x$ OR There exist an open set $V_y \subset X$ containing y such that $x \notin V_y$.
T_0 Property is a topological property.
T_1-Space: A topological space (X,T) is said to be **T_1-Space** iff for each $x, y \in X$ such that $x \neq y$ there exist open sets U_x, U_y *of* X containing x,y respectively such that $y \notin U_x$ *and* $x \notin U_y$.
T_1-Propery is a topology property.
Every discrete space is T_1 space.
Every metric space is T_1 space.
Every cofinite space is T_1 space.
T_1 property is much stronger property than T_0 property.
A topological space (X,T) is T_1 iff every singleton subset $\{x\} \subset X$ is closed.**[ALSO PROOF FROM PAGE # 90]**

Every subspace of a T_1 space is also a T_1 space. **[ALSO PROOF FROM PAGE # 92]**
Every finite subset of T_1 space is closed.
Let A be a finite subset of a T_1 space X then $\acute{A}$ is empty.
<u>T_2-Space</u>: A topological space (X,T) is said to be **T_2-Space** iff for every distinct pair of element $x, y \in X$ there exist open sets U_x, U_y of X containing x,y respectively such that $U_x \cap U_y = \phi$.
T_2-space is also called Hausdorff discovered by Felix Hausdorff (1868-1942)
T_2 property is a topological property.
Every discrete space is Hausdorff.
A nonempty set $X \neq \{x\}$ with indiscrete topology is not Hausdorff.
$\mathbb{R}$ with lower limit topology is Hausdorff .
$\mathbb{R}$ with topology generated $\{(a, \infty) \mid a \in \mathbb{R}\}$ is not Hausdorff.
An infinite set with cofinite topology is not Hausdorff space.
The property of being a Hausdorff space is inherently.
Let (X,T) is a Hausdorff space and $A \subset X$ then (A, T_A) is Hausdorff. **[ALSO PROOF FROM PAGE # 102]**
An infinite set X with cofinite topology is not Metrizable.
Every metric space is Hausdorff. **[ALSO PROOF FROM PAGE # 104]**
T_2 property is a stronger property than T_1 property.
Every T_2 space is T_1 space but every T_1 space is not T_2 space.
An infinite set X with cofinite topology is T_1 space but not T_2 space.
Let X be a Hausdorff space then every convergent sequence of points has a unique limit point.
Module : 144
A topological space (X,T) is said to be regular iff for every closed subset F of X and for every $x \in F^c$ $\exists$ open subsets U_F and U_x containing F and x such that $U_F \cap U_x = \phi$
A topological space (X,T) is said to be "T_3 space" iff it is either Regular+T_0 or Regular+T_1 or Regular+T_2.
Every T_3 space will be regular but every regular space will not T_3 space.
Property of being regular is a topological property.
Every subspace of regular space is a regular space.
Every subspace of a T1 space is also a T1 space i.e. T1 space is heredity.
T_3 property is a topological property.
Every subspace of a T_3 space is a T_3 space.
Module : 145
Discrete space is a regular space.
Indiscrete space is a regular space.
A discrete topology space has a complete power set.
Every subset of discrete space is also closed as well as open so open collection is also close collection.
$\mathbb{R}$ with usual property is regular and this space is also T_3 space.
An infinite set with co finite topology is not regular so it is not T_3.

K-Topology on real line topology T_K.....($\mathbb{R}, Tk$) is not a regular space.
Module : 146
A topological space is said to be normal iff for every pair of disjoint closed subset $F_1,F_2 \subset X$, there exist open subset U_{F1} and U_{F2} containing F_1 and F_2 respectively such that $U_{F1} \cap U_{F2} = \phi$
A topological space (X,T) is said to be T_4 space iff it is either Normal +T_1 or Normal+T_2
Property of being normal is a topology property.
Subspace of normal space needs not to be normal space.
T_4 space is a topology property.
A topological space (X,T) is said to be complete normal space iff every subspace of X with subspace topology is normal.
Module : 147
Consider: x={a,b,c,d} T={ ϕ ,{d},{b,d},{c,d},{b,c,d},X} Claim: (X,T) is normal space but (X,T) is not T_4.
Consider: y={b,c,d} T={ ϕ ,{d},{b,d},{c,d},X} Claim: (y,T_4) is not a normal space.
Consider: x={a,b,c,d} T={ ϕ ,{d},{b,d},{c,d},{b,c,d},X} Claim: (X,T_4) is not completely normal space.
($\mathbb{R}$,T_4) is normal space.
$\mathbb{R}$ with usual topology is a T_4 space.
Module : 148
Metric spaces are T_4 space.
Every metric space is a Hausdorff space.
Every metric space is normal.
All the metric spaces are normal spaces.
Every separable metric space is second countable.
Module : 149
T_i properties: Topological space gives T0-spaces gives T1-spaces gives T2-spaces (Hausdorff) gives T3-spaces (Regular T1 spaces) gives T4-spaces (Normal T1-spaces) gives Metric Spaces.

diagram illustrates the relationship between the spaces discussed in this chapter.



Module : 150

Urysohn's Lemma is a classical and important result of topology.

An important application of Urysohn's Lemma is the partial solution of Metrization Theorem.

Urysohn's Metrization Theorem : Every **normal** T1 space with countable basis is metrizable.

Urysohn's Metrization Theorem : Every **regular** T1 space with countable basis is metrizable.

Module : 151

Cover: Let (X,T) be a topological space, Let C={Ci} be a class of subsets of X such that

$X=\cup_{i\epsilon 1} C_i$ then C is called a cover of X.

Subcover: A subclass S of a cover C of X which is also a cover is called a subcover.

Open Cover: A cover C of X is said to be Open Cover iff $C \subset T$

Compact Space: A topological space (X,T) is said to be compact if every open cover C of X contains a finite subcover S of X.

Module : 152

A set X with indiscrete topology T is compact.

A set X with topology T containing finite number subsets of X is compact.

A finite set X with topology T is compact.

A finite set X with discrete topology T is not compact.

Module : 153

An open interval in ℝ with respect to usual topology is not compact.

ℝ with usual topology/ $(\mathbb{R}, T_u)$ is not compact.

Module : 154

A set X with co finite topology T_{cof} is compact.

Proof:

If X is finite then $T_{cof}=P(X)$

$\Rightarrow (X,T_{cof})$ is compact

A set X with cofinite topology is compact.

Module : 155

ℝ with usual topology/ $(\mathbb{R},T_u)$ is not compact.

ℝ with usual topology/ $(\mathbb{R},T_{indis})$ is compact.

ℝ with usual topology/ $(\mathbb{R},T_{dis})$ is not compact.

ℝ with usual topology/ $(\mathbb{R}, T_{cof})$ is compact.
Module : 156
Every closed subspace of a compact space is compact.
If (X,T) is compact space and A be a closed subset of X then (A,T_A) will be closed subspace of X where T_A is subspace topology on A.
Module : 157
If (A,T_A) be compact subspace of a Hausdorff space (X,T) then A is closed in X.
Every compact subspace of a Hausdorff space is closed.
Module : 158
If (X,T) be a compact space and f: $(X,T)\longrightarrow(Y,T^{/})$ be continuous map then f(x) is compact.
Image of compact space under a continuous map is compact.
Module : 159
Let (X,T) be a compact space and $(Y,T^{/})$ be a Hausdorff space then a continuous map f: $(X,T)\longrightarrow(Y,T^{/})$ is a closed map.
Let (X,T) be a compact space and $(Y,T^{/})$ be a Hausdorff space then a bijective continuous map f: $(X,T)\longrightarrow(Y,T^{/})$ is a homeomorphism.
Module : 160
Consider ℝ with usual topology/ $(\mathbb{R}, T_u)$, there exist no homeomorphism between an open interval of ℝ and closed interval of ℝ.
Open interval in ℝ with usual topology/ $(\mathbb{R}, T_u)$ are not compact.
The only compact subset of ℝ are closed and bounded subsets [a,b] compact and (c,d) not compact.
$[0,1)\neq S^1$ since S^1 Compact and [0,1) not compact.
Module : 161
Let (X,T) be a topological space and $\{(X_i,T_i)\mid i\in I\}$ be a finite family of compact subspace of X then $\cup_{i\in I} X_i$ is compact.
Module : 162
Let (X,T) be a compact Hausdorff space then (X,T) is normal.
For every pair of disjoint closed subsets F_1, F_2 of a compact Hausdorff space X there exist a pair of of disjoint open subsets V_{F1}, V_{F2} of X containing F_1,F_2 respectively.
Module : 163
Let A and B be two subsets of a topological space (X,T) then A and B are said to be separated sets if and only if $A\cap B = \phi$ *and* $\acute{A}\cap B = \phi$ *and* $A\cap \acute{B} = \phi$
Let A and B be two subsets of a topological space (X,T) then A and B are said to be separated sets if and only if there exists open subsets U_A and U_B of X containing A and B respectively. $A\cap U_B = \phi$ *and* $B\cap U_A = \phi$
Let A and B be two subsets of a topological space (X,T) then A and B are said to be separated sets if and only if $\bar{A}\cap B = \phi$ *and* $A\cap \bar{B} = \phi$
Module : 164

Consider ℝ with usual topology/(ℝ, T_u) then A=(0,1) and B=(4,9] are separated sets.
Consider ℝ with usual topology/(ℝ, T_u) then A=(-1,3] and B=[3,5) are not separated sets.
Consider ℝ with usual topology/(ℝ, T_u) then A=(-1,3) and B=[3,5) are not separated sets.
Module : 165
Connected Set: A subset A of a topological space (X,T) is said to be connected if and only if there exist no pair of non empty open subset of U and V of X such that $A\cap U$ *and* $A\cap V$ are non empty disjoint sets and $A = (A\cap U)U(A\cap V)$
Disconnected Set: A subset A of a topological space (X,T) is said to be disconnected if it is not connected i.e If there exist a pair of non empty open subset of U and V of X such that $A\cap U$ *and* $A\cap V$ are non empty disjoint sets and $A = (A\cap U)U(A\cap V)$
Connected Space (Definition I): A topological space (X,T) is said to be connected space if it cannot be written as union of two non empty open disjoint subsets U and V of X.
Disconnected Space: A topological space (X,T) is said to be disconnected space if it can be written as union of two non empty open disjoint subsets U and V of X.
Connected Space (Definition II): A topological space (X,T) is said to be connected space if $X = D_1 U D_2$ such that D_1, D_2 both open and disjoint i.e. $D_1 \cap D_2 = \phi$ implies $D_1 = \phi$ *or* $D_2 = \phi$
Module : 166
Consider ℝ with usual topology/(ℝ, T_u) then A=(0,1)U(3,5] is disconnected subset of ℝ
Consider X={a,b,c,d,e} with $T = \{\phi, \{a\}, \{a,b\}, \{a,b,c\}, \{a,d,e\}, \{a,b,d,e\}, X\}$ then $A = \{c,e\}$ is disconnected subset of X.
Consider X={a,b,c,d,e} with $T = \{\phi, \{a,b\}, \{a,b,c\}, X\}$ then X is a connected space.
Consider X={a,b,c,d,e} with $T = \{\phi, \{a,b\}, \{d,e\}, \{a,b,c\}, \{a,b,d,e\}, X\}$ then X is a disconnected space.
Module : 167
Connected Space: A topological space (X,T) is said to be connected iff $X = D_1 U D_2$ such that D_1, D_2 both open and disjoint implies $D_1 = \phi$ *or* $D_2 = \phi$
Disconnected Space: A topological space (X,T) is said to be disconnected space iff $X = D_1 U D_2$ such that D_1, D_2 both open and disjoint implies that neither D_1 is empty nor D_2 is empty.
Module : 168-183
Let A be connected subset of a topological space (X,T) and $A \subset B \subseteq \bar{A}$ then B is connected.**[PROOF PAGE # 166]**
Image of a connected space under a continuous map is connected. **[ALSO SEE PROOF PAGE # 168]**

Prepare Page # 170 and 171
A subset of A of $(\mathbb{R}, Tu)$ is connected iff A is an interval.
Fixed Point: A point $x \in X$ is said to be a fixed point of a map $f: X \longrightarrow X$ iff $f(x) = x$
Let A and B be two connected subsets of a topological spaces (X,T) such that $x \in A \cap B$ then $C = AUB$ is connected subset of X. **[ALSO SEE PROOF Page#179-180]**
Consider a topological space (X,T), Let $a \in X$ and C be a connected subset of X containing a then $C_a = U_{a \in C} C$ is called connected component of X containing a.
The connected component Ca is the largest connected subset of X containing a.
Prepare Page#182
Let C be connected component of X then $C = \bar{C}$
A connected component is a closed subset of X.
In general a connected component C of X is not open.
A topological space (X,T) is locally connected iff it is locally connected at each of its points.
Prepare Page#186
Locally connectedness **does not imply** connectedness
Connectedness **does not imply** locally connectedness
A topological space (X,T) is said to be connected iff for every $x, y \in X$ there exist a path P from x to y in X.
Let (X,T) be a path connected topological space then (X,T) is connected.
Path connectedness **implies** connectedness.
Connectedness **does not imply** Path connectedness.
Let (X,T) be a path connected topology space and $f: (X, T) \longrightarrow (y, \acute{T})$ be a continuous map then f(x) is path connected.

QUESTION 1

Let ϕ: G → G' be a group morphism. Then, ϕ is injective if and only if Ker ϕ = {e}.

Proof :

If Ker(ϕ) = {e}, then for every a ∈ G, the elements mapped into ϕ(a) are precisely the elements of the left coset a { e} = {a}, which shows that ϕ is one to one.

Conversely, suppose ϕ is one to one. Now, we know that ϕ(e)=*e'*, the identity element of G'. Since ϕ is one to one, we see that *e* is the only element mapped into *e'* by ϕ, so Ker(ϕ)= {e}.

QUESTION 2

A homomorphism h: G→G' is injective if and only if Ker h={e}.

Proof:

Suppose h is injective, and let $x \in$ Ker h.

Then $h(x)=e'=h(e)$. Hence $x=e$.

Conversely, suppose Ker $h=\{e\}$.

Then $h(x)=h(y)$

$\Rightarrow h(xy^{-1})=h(x)h(y^{-1})$

$=h(x)h(y)^{-1}=e'$

$\Rightarrow xy^{-1} \in$ Ker h

$\Rightarrow xy^{-1}=e$

$\Rightarrow x=y$.

Hence, h is injective.

QUESTION 3

Classify the group $(\mathbb{Z}_4 \times \mathbb{Z}_2)/(\{0\} \times \mathbb{Z}_2)$ according to the fundamental theorem of finitely generated abelian groups.

Solution :

The projection map

$\pi_1 : \mathbb{Z}_4 \times \mathbb{Z}_2 \to \mathbb{Z}_4$ given by $\pi_1(x,y) = x$ is a homomorphism of $\mathbb{Z}_4 \times \mathbb{Z}_2$ onto $\mathbb{Z}_4$ with kernel $\{0\} \times \mathbb{Z}_2$. By fundamental theorem of homomorphism, we know that the given factor group is isomorphic to $\mathbb{Z}_4$.

The projection map

$\pi_1 : \mathbb{Z}_4 \times \mathbb{Z}_2 \to \mathbb{Z}_4$ given by

$\pi_1(x,y) = x.$

$K = \text{Ker } \pi_1 = \{0\} \times \mathbb{Z}_2$

$= \{(0,0),(0,1)\}.$

$(1,0)+K=\{(1,0),(1,1)\}$

$(2,0)+K=\{(2,0),(2,1)\}$

$(3,0)+K=\{(3,0),(3,1)\}$

QUESTION 4

The set of all inner automorphisms of G is a subgroup of Aut(G).

Proof:

(1) Let $i_a, i_b \in Inn(G)$.

Then $i_a(i_b(x)) = a(i_b(x))a^{-1} = abxb^{-1}a^{-1}$

$= abx(ab)^{-1} = i_{ab} \in Inn(G)$.

Hence the conjugation by b composed by conjugation by a is conjugation by ab.

(2) The inverse of i_a is conjugation by $a' = a^{-1}$.

$i_a((i_{a'})(x)) = i_a(a'x(a')^{-1}) = aa'xa'^{-1}a^{-1} = aa'x(aa')^{-1} = x$.

Thus Inn (G) is a subgroup.

QUESTION 6

Here the first factor $\mathbb{Z}_4$ of $\mathbb{Z}_4 \times \mathbb{Z}_6$ is left alone. The $\mathbb{Z}_6$ factor, on the other hand, is essentially collapsed by a subgroup of order 3, giving a factor group in the second factor of order 2 that must be isomorphic to $\mathbb{Z}_2$. Thus $(\mathbb{Z}_4 \times \mathbb{Z}_6)/\langle(0, 2)\rangle$ is isomorphic to $\mathbb{Z}_4 \times \mathbb{Z}_2$.The trivial subgroup N = {0} of $\mathbb{Z}$ is, of course, a normal subgroup. Compute $\mathbb{Z}$ /{0}.

Solution:

Since N={0} has only one element, every coset of N has only one element. That is, the cosets are of the form {m} for m $\in \mathbb{Z}$. There is no collapsing at all, and consequently, $\mathbb{Z}/\{0\} \cong \mathbb{Z}$. Each m $\in \mathbb{Z}$ is simply renamed {m} in $\mathbb{Z}$ /{0}.

QUESTION 5

A factor group of a cyclic group is cyclic.

Proof:

Let G be cyclic with generator a, and let N be a normal subgroup of G. We claim the coset aN generates G / N. We must compute all powers of aN. But this amounts to computing, in G, all powers of the representative a and all these powers give all elements in G. Hence the powers of aN certainly give all cosets of N and G / N is cyclic.

QUESTION 7

Let G = H x K be the direct product of groups H and K. Then $\overline{H}$={(h, e)| h ∈ H} is a normal subgroup of G. Also G/$\overline{H}$ is isomorphic to K in a natural way. Similarly, G / $\overline{K}$ ≃ H in a natural way.

Proof

Consider the map π_2: H x K → K given by

$\pi_2(h, k) = k$. The map π_2 is homomorphism since

$\pi_2(h_1h_2, k_1k_2) = k_1k_2 = \pi_2(h_1, k_1)\,\pi_2(h_2, k_2)$.

Because Ker(π_2) = $\overline{H}$, we see that $\overline{H}$ is a normal subgroup of H x K. Because π_2 is onto K, Fundamental Theorem of Homomorphism tells us that (H x K)/ $\overline{H}$ ≃ K.

QUESTION 8

The cyclic group G=$\mathbb{Z}/5\mathbb{Z}$ of congruence classes modulo 5 is simple.

Proof

If H is a subgroup of this group, its order must be a divisor of the order of G which is 5.

Since 5 is prime, its only divisors are 1 and 5, so either H is G, or H is the trivial group.

QUESTION 9

M is a maximal normal subgroup of G if and only if G / M is simple.

Proof :

Let M be a maximal normal subgroup of G. Consider the canonical homomorphism $\gamma: G \to G/M$. Now γ^{-1} of any nontrivial proper normal subgroup of G/M is a proper normal subgroup of G properly containing M. But M is maximal, so this can not happen. Thus G/M is simple.

Conversely, if N is a normal subgroup of G properly containing M, then $\gamma[N]$ is normal in G/M. If also $N \neq G$, then $\gamma[N] \neq G/M$ and $\gamma[N] \neq \{M\}$.

Thus, if G/M is simple so that no such $\gamma[N]$ can exist, no such N can exist, and M is maximal.

s

QUESTION 10

Show that Z(G) is a normal and an abelian subgroup of G.

Solution :

For each g ∈ G and z∈Z(G) we have $gzg^{-1}=zgg^{-1}=ze=z$, we see at once that Z(G) is a normal subgroup of G. It implies that gz=zg for g ∈ G and z∈Z(G). If G is abelian, then Z(G) = G; in this case, the center is not useful.

QUESTION 11

Let G be a group. The set of all commutators $aba^{-1}b^{-1}$ for a, b ∈ G generates a subgroup C of G.

Proof:

Let a, b ∈ G. Then,

$(aba^{-1}b^{-1})(aba^{-1}b^{-1})^{-1}$

$=aba^{-1}b^{-1}bab^{-1}a^{-1}$

$=e \in C$

since $e = eee^{-1}e^{-1}$ is a commutator.

QUESTION 12

The Klein 4-group V = {e, a, b, c} is generated.

Proof:

The Klein 4-group V = {e, a, b, c} is generated by {a,b} since ab=c.

It is also generated by {a,c}, {b,c }, and {a,b,c}.

If a group G is generated by a subset S, then every subset of G containing S generates G.

QUESTION 13

The group $\mathbb{Z}_6$ is generated.

Proof:

The group $\mathbb{Z}_6$ is generated by {1} and {5}.It is also generated by {2,3} since 2+3=5, so that any subgroup containing 2 and 3 must contain 5 and must therefore be $\mathbb{Z}_6$. It is also generated by {3,4}, {2,3,4}, {1,3}, and {3,5}. But it is not generated by {2, 4} since

<2> = {0, 2, 4} contains 2 and 4.

QUESTION 14

If N is a normal subgroup of G, then G/N is abelian if and only if $C \leq N$.

Proof :

If N is a normal subgroup of G and G/N is abelian, then

$(a^{-1}N)(b^{-1}N)=(b^{-1}N)(a^{-1}N)$; that is, $aba^{-1}b^{-1}N=N$,

so $aba^{-1}b^{-1} \in N$, and $C \leq N$.

Finally, if $C \leq N$, then

$(aN)(bN)=abN$

$=ab(b^{-1}a^{-1}ba)N$

$= (abb^{-1}a^{-1})baN$

$= baN$

$= (bN)(aN)$.

QUESTION 15

The set Aut(G) of all automorphisms of a group G is a group under composition of mappings, and Inn(G) ◁ Aut(G). Moreover, G/Z(G)≃Inn(G).

Proof:

Clearly, Aut(G) is nonempty. Let $\sigma, \tau \in$ Aut(G). Then for all x, y ∈ G, $\sigma\tau(xy)=\sigma((\tau(x)\tau(y)) = (\sigma\tau(x))(\sigma\tau(y))$.

Hence, $\sigma\tau \in$ Aut(G). Again,

$\sigma(\sigma^{-1}(x)\sigma^{-1}(y))=$
$\sigma\sigma^{-1}(x)\sigma\sigma^{-1}(y)$

=xy.

Hence $\sigma^{-1}(x)\sigma^{-1}(y)=$
$\sigma^{-1}(xy)$. Therefore, $\sigma^{-1} \in$
Aut(G). This proves that Aut(
G) is a subgroup of the
symmetric group S_G and,
hence, is itself a group.